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The Classical Aristotelian hexagon versus the Modern Duality hexagon

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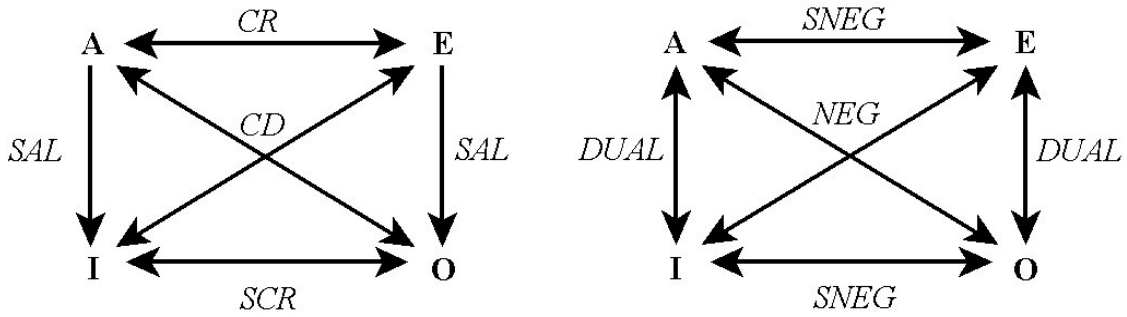
1 Aristotelian versus Duality Square

(1)



Piaget (1949)	inversion	réciprocation	corrélation
Gottschalk (1953)	complement	contradual	dual
Löbner (1990)	negation (NEG)	subnegation (SNEG)	dual (DUAL)
Westerstahl (2010)	outer negation	inner negation	dual

(2)

NEG (Q1, Q2)	⇔ Q1 (A,B) = ¬ Q2 (A,B) ⇔ Q2 (A,B) = ¬ Q1 (A,B)	NEG (**All**, **Not all**)	NEG (**Some**, **No**)
SNEG (Q1, Q2)	⇔ Q1 (A,B) = Q2 (A,¬B) ⇔ Q2 (A,B) = Q1 (A,¬B)	SNEG (**All**, **No**)	SNEG (**Some**, **Not all**)
DUAL (Q1, Q2)	⇔ Q1 (A,B) = ¬ Q2 (A,¬B) ⇔ Q2 (A,B) = ¬ Q1 (A,¬B)	DUAL (**All**, **Some**)	DUAL (**No**, **Not all**)

Quantifier (domain/restrictor,	predicate/nuclear scope)	2-place second-order predicates
All (children ,	be asleep)	<<et>,<et>,t>

(3)

All children are asleep	= **No** children are awake/are **not** asleep.
No children are asleep	= **All** children are awake/are **not** asleep.
Some children are asleep	= **Not all** children are awake/are **not** asleep.
Not all children are asleep	= **Some** children are awake/are **not** asleep.

(4)

All children are asleep	= It is **not** the case that **some** children are awake/are **not** asleep.
Some children are asleep	= It is **not** the case that **all** children are awake/are **not** asleep.
Not all children are asleep	= It is **not** the case that **no** children are awake/are **not** asleep.
No children are asleep	= It is **not** the case that **not all** children are awake/are **not** asleep.

(5)

Aristotelian relations	**Duality relations**
4 types: diagonal CD horizontal CR + SCR vertical SAL	3 types: diagonal NEG horizontal SNEG vertical DUAL
3 symmetric CD/CR/SCR vs 1 asymmetric SAL	3 symmetric NEG/SNEG/DUAL
non-recursive	recursive (Quaternality)

2 From Square to Hexagon

2.1 The standard quantifiers

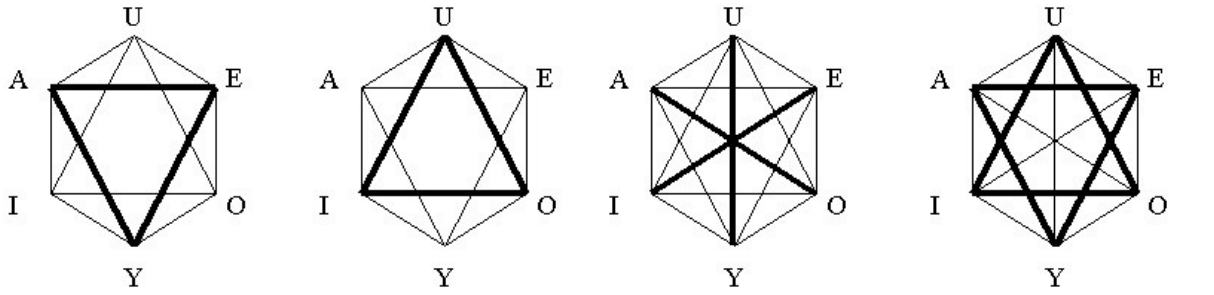
2.1.1 Generalizing the square

(6) U (= A ∨ E, all or no) Y (= I ∧ O, some but not all)

(7)

a.	CD(all, not all)	CD(some,no)	**CD(some but not all, all or no)**
b.	CR(all, no)	**CR(all, some but not all)**	**CR(no, some but not all)**
c.	SCR(some, not all)	**SCR(some, all or no)**	**SCR(not all, all or no)**
d.	SAL(all, some)	**SAL(all, all or no)**	**SAL(some but not all, some)**
	SAL(no, not all)	**SAL(no, all or no)**	**SAL(some but not all, not all)**

(8)



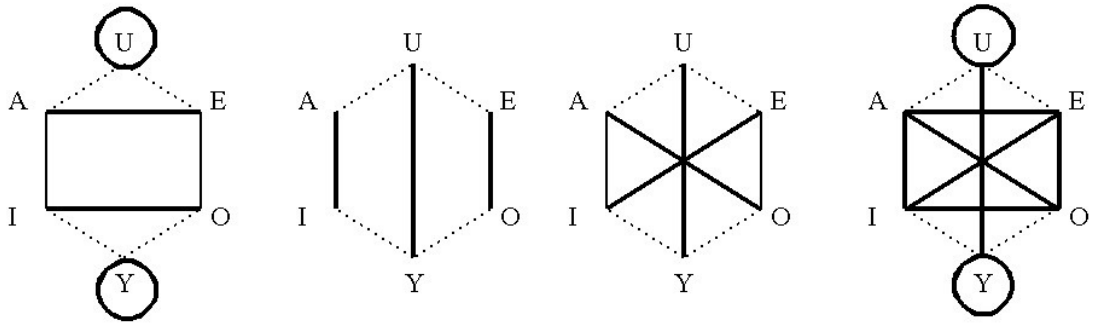
contrary + *subcontrary* + *contradiction* = *Aristotelian Hexagon*

(9) NEG (**Some but not all**, **No or all**)
It is **not** the case that **some but not all** children are asleep = **No or all** children are asleep
It is **not** the case that **no or all** children are asleep = **Some but not all** children are asleep

(10) SNEG (**Some but not all**, **Some but not all**) SNEG (**No or all**, **No or all**)
Some but not all children are asleep = **Some but not all** children are awake/are **not** asleep
No or all children are asleep = **No or all** children are awake/are **not** asleep.

(11) DUAL (**Some but not all**, **No or all**)
Some but not all children are asleep = It is **not** the case that **no or all** children are **not** asleep.
= It is **not** the case that **no or all** children are awake.
No or all children are asleep = It is **not** the case that **some but not all** children are **not** asleep.
= It is **not** the case that **some but not all** children are awake.

(12)



subnegation + *dual* + *negation* = *Duality Hexagon*

(13)

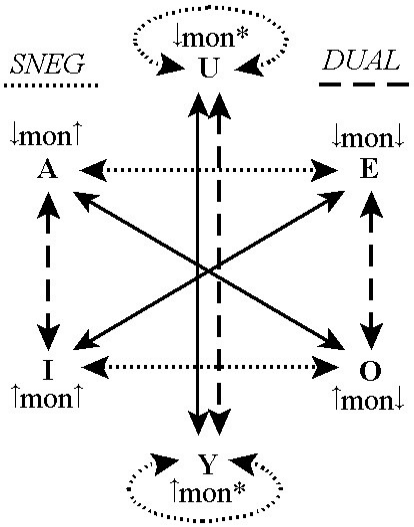
Aristotelian hexagon	Duality hexagon
2 triangles + 3 diagonals = "***star***"	square + pair = "***shield and spear***"
two extra nodes integrate	two extra nodes remain autonomous

2.1.2 Monotonicity properties

(14) a. D is left-monotone increasing(↑mon) ≡ [D(A,B) ∧ A ⊆ A'] → D(A', B)
b. D is left-monotone decreasing (↓mon) ≡ [D(A,B) ∧ A' ⊆ A] → D(A', B)
c. D is right-monotone increasing (mon↑) ≡ [D(A,B) ∧ B ⊆ B'] → D(A, B')
d. D is right-monotone decreasing (mon ↓) ≡ [D(A,B) ∧ B' ⊆ B] → D(A, B')
(Partee, ter Meulen and Wall, 1990: 381)

(15) a. **Some** young women are cycling fast. → Some *women* are cycling fast.
↑**mon**↑ → Some young women are *cycling*.
b. There are **no** women cycling. → There are no *young women* cycling.
↓**mon**↓ → There are no women *cycling fast*.
c. **Not all** young women are cycling. → Not all *women* are cycling.
↑**mon**↓ → Not all young women are *cycling fast*.
d. **All** women are cycling fast. → All *young women* are cycling fast.
↓**mon**↑ → All women are *cycling*.

(16) a. DUAL = reverse left-monotonicity
b. SNEG = reverse right-monotonicity
c. NEG = reverse left- and right-monotonicity



(17) a. **Some but not all** young women are cycling.
→ Some but not all *women* are cycling.
b. **Some but not all** women are cycling fast.
*→ Some but not all *women* are cycling.
c. **Some but not all** women are cycling.
*→ Some but not all *women* are cycling fast.
↑**mon***

d. **No or all** women are cycling.
→ No or all *young women* are cycling.
e. **No or all** women are cycling fast.
*→ No or all *women* are cycling.
f. **No or all** women are cycling.
*→ No or all *women* are cycling fast.
↓**mon***

2.2 Other [+Aristotelian, +Duality] hexagons

2.2.1 One-place second-order predicates: alethic modalities

(18) Quantifier (proposition) Be possible (he is asleep) 1-place second order predicates <t, t>

(19) a. CD(possible, impossible) CD(necessary, not necessary) CD(contingent, not contingent)
b. CR(impossible, contingent, necessary)
c. SCR(possible, not contingent, not necessary)

(20) a. SNEG(possible, not necessary)
It is possible that he is asleep It is not necessary that he is awake
b. SNEG(impossible, necessary)
It is impossible that he is asleep It is necessary that he is awake
c. SNEG(contingent, contingent)
It is contingent that he is asleep It is contingent that he is awake
He may but needn't be asleep He may but needn't be awake
d. SNEG(not contingent, not contingent)
It is not contingent that he is asleep It is not contingent that he is awake
He must be or can't be asleep He must be or can't be awake

(21) a. DUAL(possible, necessary) It is possible that he is asleep
= It is **not** the case that he is necessarily awake/**not** asleep
b. DUAL(impossible, not necessary) It is impossible that he is asleep
= It is **not** the case that he is not necessarily awake/**not** asleep
c. DUAL(contingent, not contingent)
It is contingent that he is asleep = He may but needn't be asleep
= It is **not** the case that he must be or can't be awake/**not** asleep

2.2.2 Two-place second-order predicates: deontic modalities

(22) Quantifier (entity, predicate) Be allowed (he, to stay) 2-place 2nd-order pred. <<e>,<et>,t>

(23) a. CD(allowed, forbidden) CD(obliged, not obliged)
CD(allowed but not obliged, forbidden or obliged)
b. CR(forbidden, allowed but not obliged, obliged)
c. SCR(allowed, forbidden or obliged, not obliged)

(24) a. SNEG(allowed but not obliged, allowed but not obliged)
He is allowed but not obliged to stay He is allowed but not obliged to leave
b. SNEG(forbidden or obliged, forbidden or obliged)
He is forbidden or obliged to stay He is forbidden or obliged to leave

(25) DUAL(allowed but not obliged, forbidden or obliged)
He is allowed but not obliged to stay
= It is **not** the case that he is forbidden or obliged to leave/**not** to stay

2.2.3 Two-place second-order predicates: proportional quantifiers

(26) Quantifier (domain/restrictor, predicate/nuclear scope) 2-place second-order predicates
Less than 20% (of the children, be asleep) <<et>,<et>,t>

(27) a. CD(less than 20%, at least 20%) CD(more than 80%, at most 80%)
CD(between 20% and 80%, less than 20% or more than 80%)
b. CR(less than 20%, between 20% and 80%, more than 80%)
c. SCR(at least 20%, less than 20% or more than 80%, at most 80%)

(28) a. SNEG(between 20% and 80%, between 20% and 80%)
Between 20% and 80% of the boys are asleep
Between 20% and 80% of the boys are awake/**not** asleep
b. SNEG(less than 20% or more than 80%, less than 20% or more than 80%)
Less than 20% or more than 80% of the boys are asleep
Less than 20% or more than 80% of the boys are awake/**not** asleep

(29) DUAL(between 20% and 80%, less than 20% or more than 80%)
Between 20% and 80% of the boys are asleep
= It is **not** the case that less than 20% or more than 80% of the boys are awake/**not** asleep

3 [+Aristotelian, -Duality] hexagons

3.1 One-place first-order predicates

(30) Quantifier (entity) Male (John) 1-place first-order predicates <<e>,t>

(31) a. CD(male, not male) CD(female, not female) CD(sexual, asexual)
b. CR(male, asexual, female)
c. SCR(not male, sexual, not female)

(32) a. CD(black, not black) CD(white, not white) CD(coloured, not coloured)
b. CR(black, coloured, white)
c. SCR(not black, not coloured, not white)

3.2 Two-place first-order predicates

(33) Quantifier (entity, entity) 2-place first-order predicates <<e>,<e>,t>
=> ***Blanché's fundamental "ordering" hexagon <, >, =, ≤, ≥, ≠***

3.2.1 Linear ordering predicates

(34) a. CD(A precedes B, A does not precede B)
CD(A coincides with B, A does not coincide with B)
CD(A follows B, A does not follow B)
b. CR(A precedes B, A coincides with B, A follows B)
c. SCR(A does not precede B, A does not coincide with B, A does not follow B)

3.2.2 Temporal ordering predicates

(35) a. CD(A before B, A not before B = A from B onwards)
CD(A at B, A not at B)
CD(A after B, A not after B = A until B)
b. CR(A before B, A at B, A after B)
c. SCR(A until B, A not at B, A from B onwards)

3.2.3 Comparative quantity predicates

(36) a. CD(A has less money than B, A has at least as much money than B)
CD(A has exactly as much money as B, A does not have exactly as much money as B)
CD(A has more money than B, A has at most as much money than B)
b. CR(A has less money than B, A has exactly as much money as B,
A has more money than B)
c. SCR(A has at least as much money than B, A does not have as much money as B,
A has at most as much money than B)

3.2.4 Comparative size predicates

(37) a. CD(A is smaller than B, A is at least as big as B)
CD(A is exactly equal to B, A is not exactly equal to B)
CD(A is bigger than B, A is at most as big as B)
b. CR(A is smaller than B, A is exactly equal to B, A is bigger than B)
c. SCR(A is at least as big as B, A is not exactly equal to B, A is at most as big as B)

3.3 Two-place second-order predicates

3.3.1 Numerical quantifiers

(38) a. CD(More than 5 women are cycling, At most 5 women are cycling)
CD(Fewer than 5 women are cycling, At least 5 women are cycling)
CD(Exactly 5 women are cycling, Not exactly 5 women are cycling)
b. CR(More than 5, Exactly 5, Fewer than 5)
c. SCR(At least 5, Not exactly 5, At most 5)

3.3.2 Standard and numerical quantifiers

(39) a. CD(some, no)
CD(at most 5, more than five)
CD(some but at most five, no or more than five)
b. CR(no, some but at most five, more than five)
c. SCR(some, no or more than five, at most five)

4 Conclusion

(40)	***Aristotelian Relations***	
	+ Duality Relations	*- Duality Relations*
first-order predicates		one-place predicates two-place predicates (linear and temporal ordering) (comparative quantity and size)
second-order predicates	standard quantifiers alethic and deontic modalities proportional quantifiers (propositional connectives)	numerical quantifiers standard and numerical quantifiers

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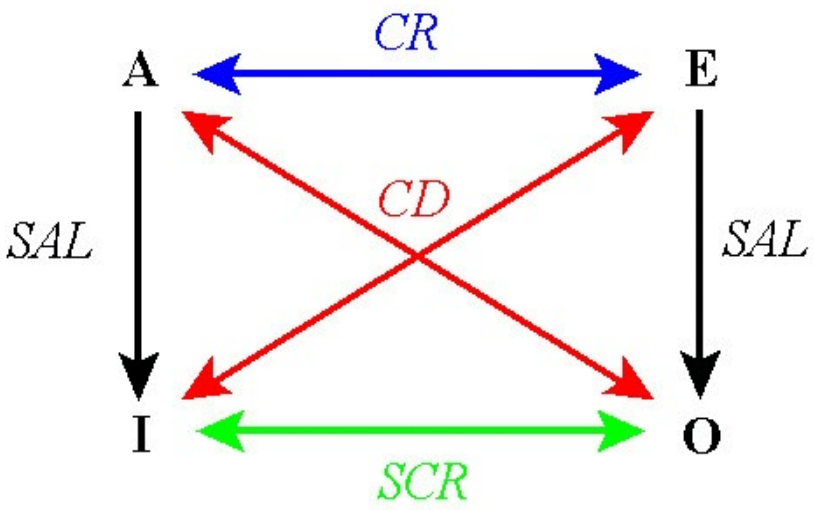
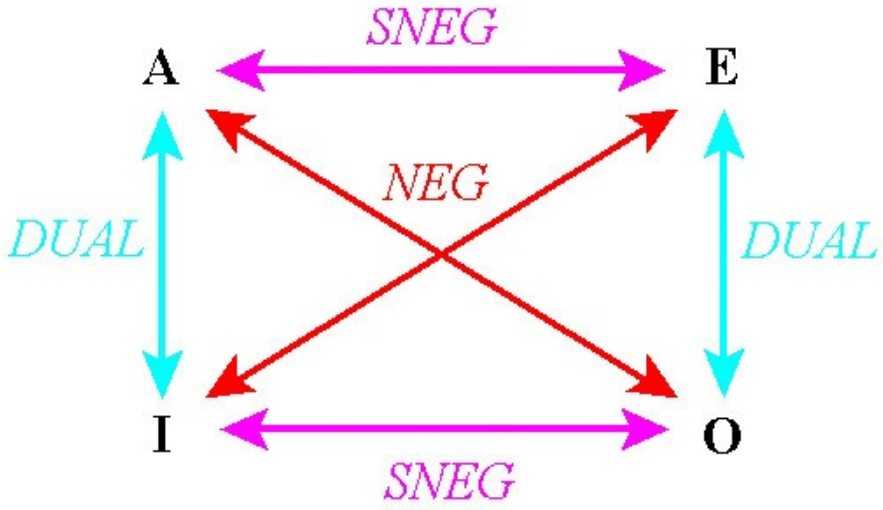
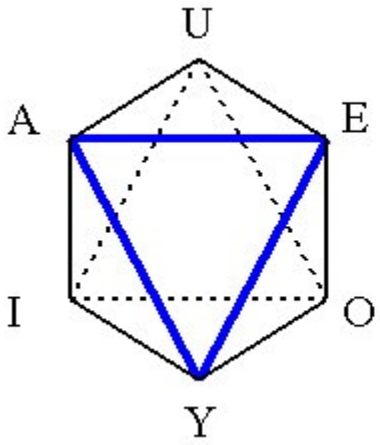
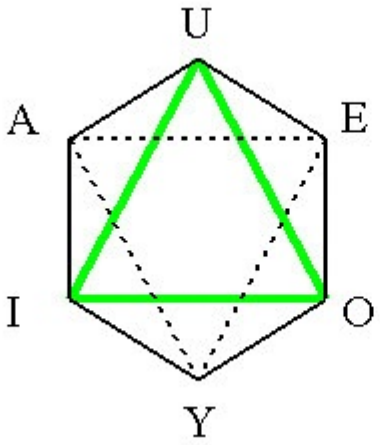
The Aristotelian Square versus the Duality Square

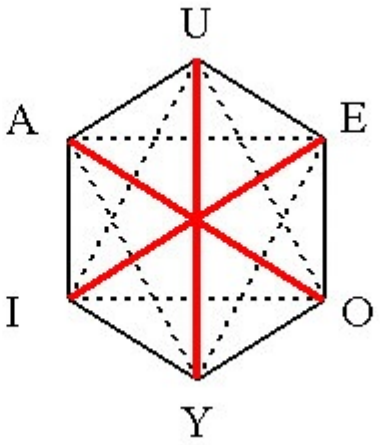


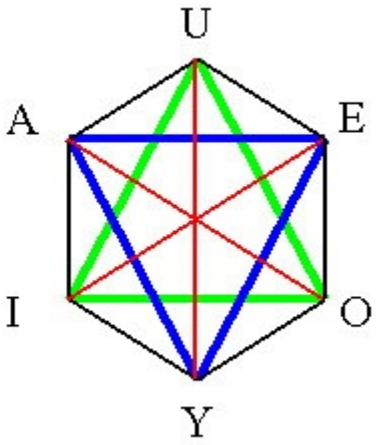


The Aristotelian Hexagon



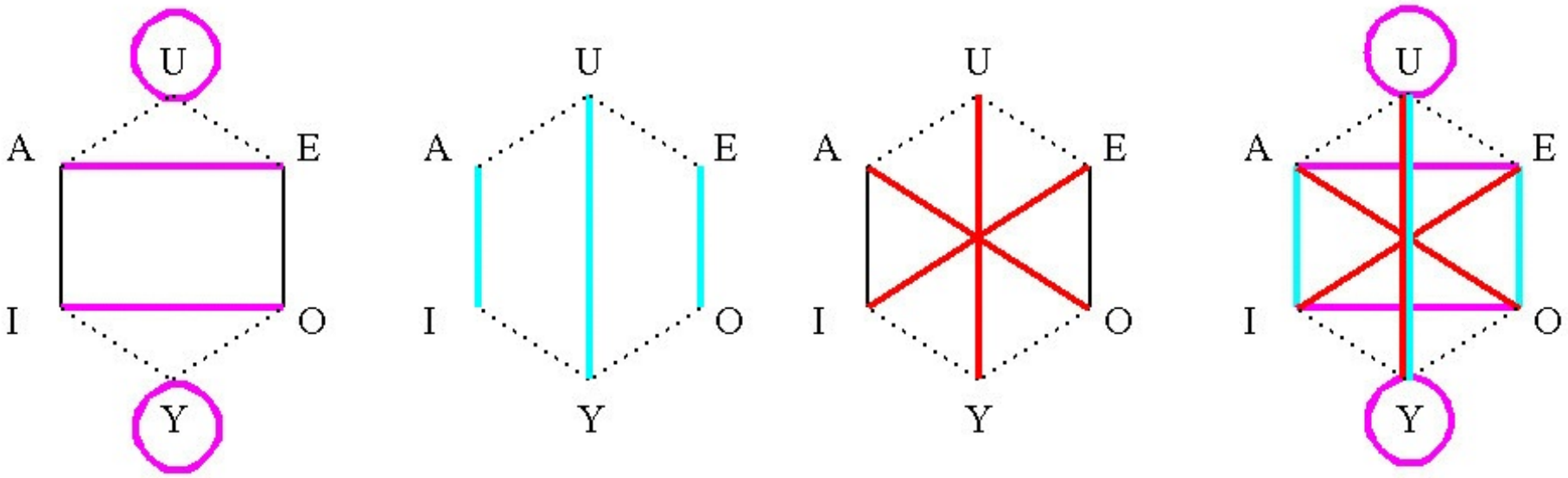






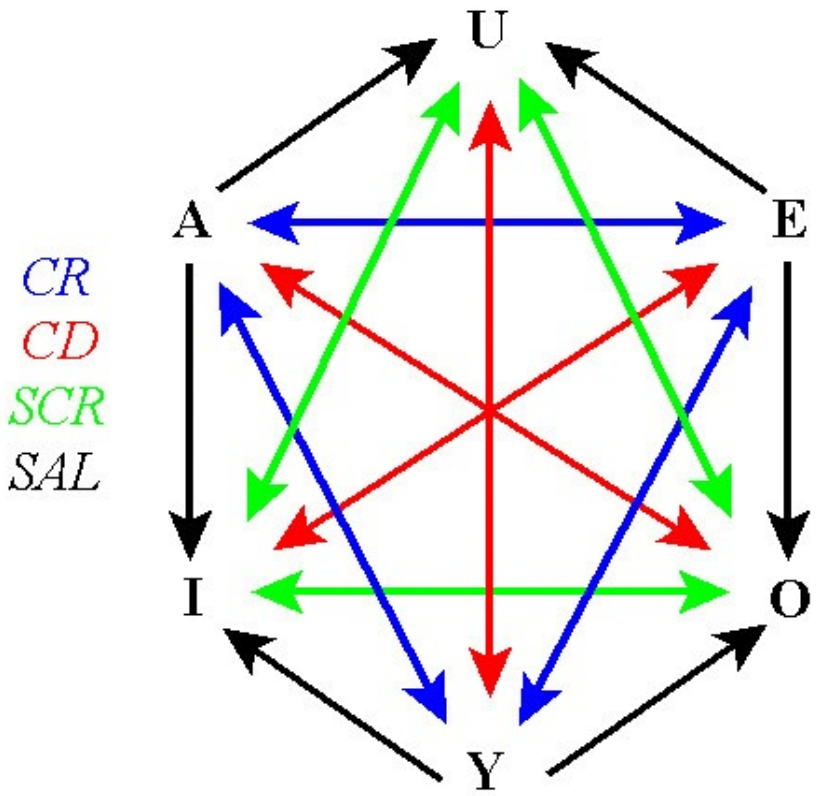
contrary + subcontrary + contradiction = Aristotelian Hexagon

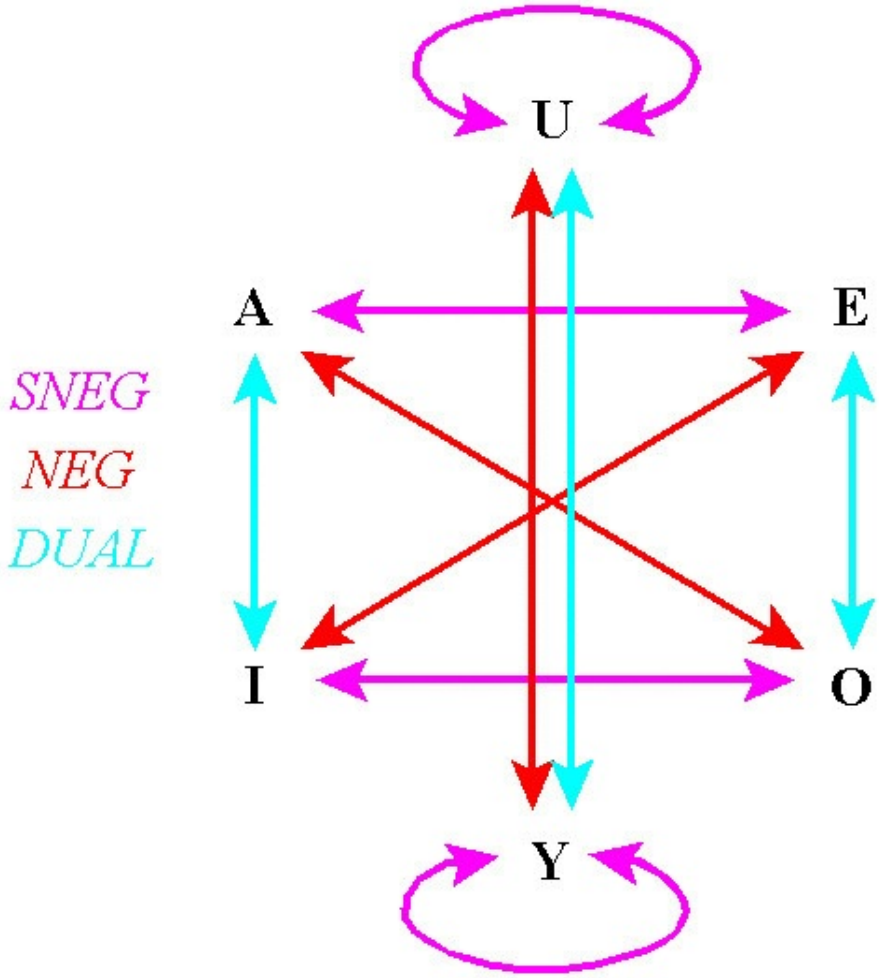
The Duality Hexagon



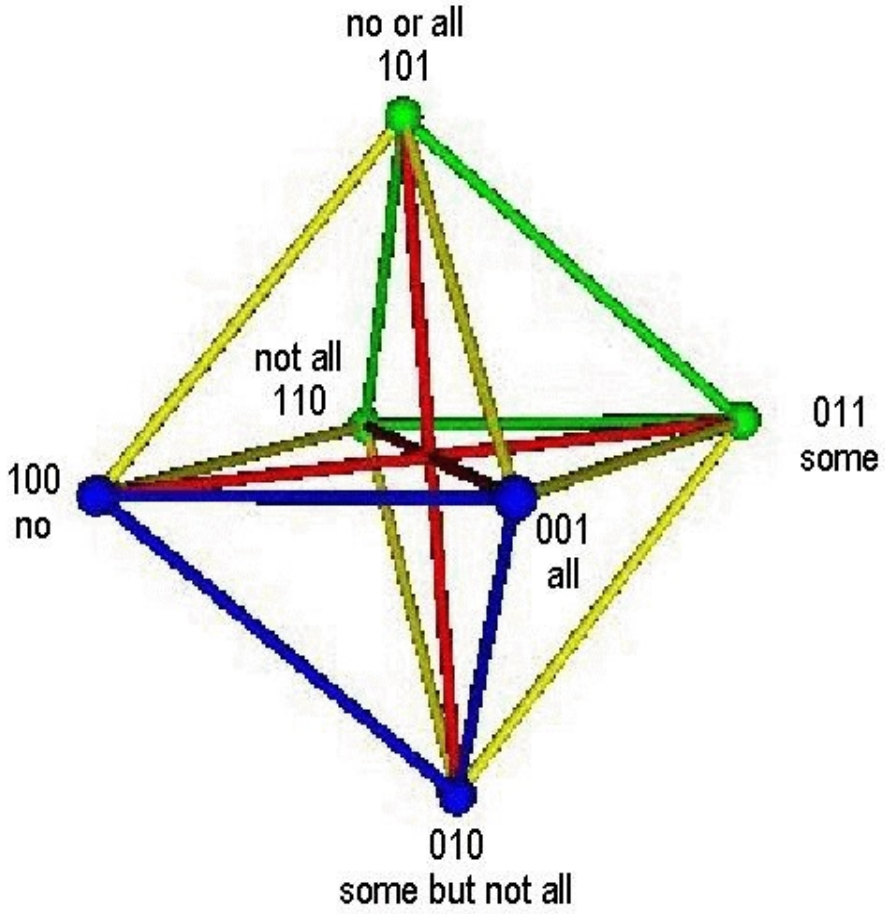
subnegation + *dual* + *negation* = *Duality Hexagon*

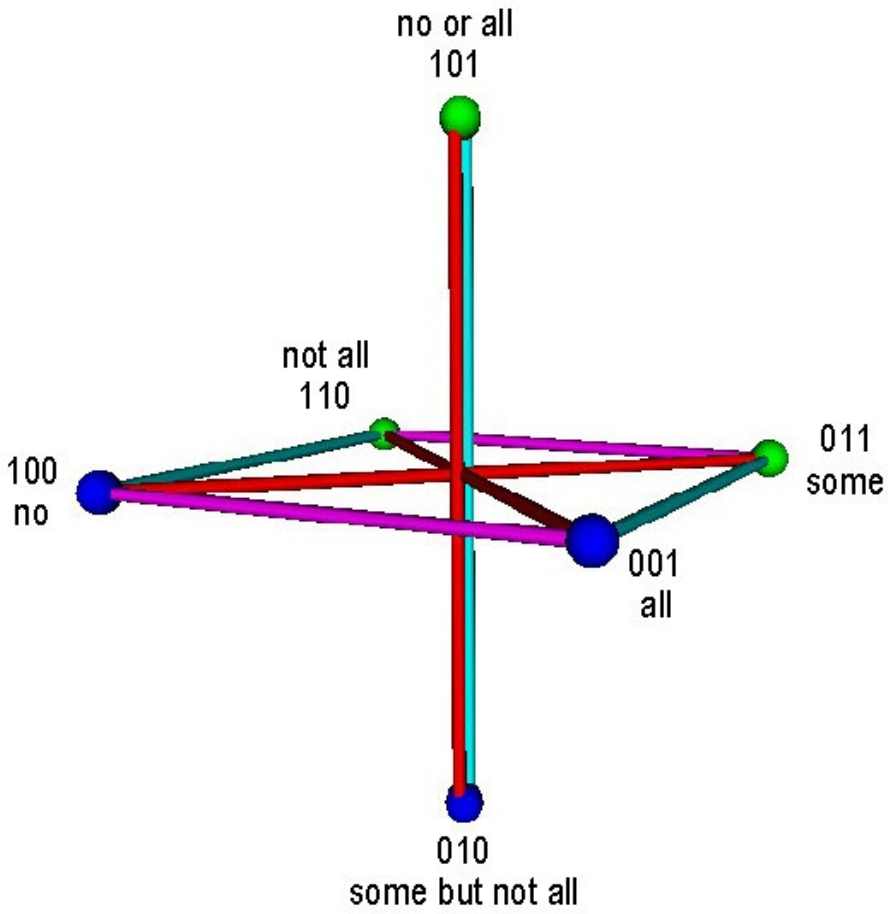
The Aristotelian Hexagon vs The Duality Hexagon





Aristotelian Octahedron vs Duality Octahedron





Monotonicity Properties in the Duality Hexagon