A categorical model of the Elementary Process Theory incorporating Special Relativity

Marcoen J.T.F. Cabbolet*

Department of Philosophy, Free University of Brussels

Abstract — The purpose of this paper is to show that the Elementary Process Theory (EPT) agrees with the knowledge of the physical world obtained from the successful predictions of Special Relativity (SR). For that matter, a recently developed method is applied: a categorical model of the EPT that incorporates SR is fully specified. Ultimate constituents of the universe of the EPT are modeled as point-particles, γ-rays, or time-like strings, all represented by integrable hyperreal functions on Minkowski space. This proves that the EPT agrees with SR.

1 Introduction

The Elementary Process Theory (EPT) is, in a sentence, a collection of seven mathematically abstract formulas that can be interpreted as process-physical principles describing the individual processes at supersmall scale by which interactions have to take place for the gravitational interaction between matter and antimatter to be repulsive [1]. One of its two main issues is that there is no proof that the EPT is consistent with existing knowledge of the fundamental interactions—that is, there is no proof that the interactions as we know them can take place in the individual processes as described by the EPT. Recently a method has been developed for proving that the EPT agrees with a modern interaction theory T: a categorical model $\mathscr{C}$ of the EPT has to be specified such that $\mathscr{C}$ reduces empirically to T [2].

The purpose of this paper is to demonstrate the method by fully specifying a categorical model $\mathscr{C}_{SR}$ of the EPT that reduces empirically to Special Relativity (SR), first published in [3]. This categorical model $\mathscr{C}_{SR}$ is thus a *category* in the sense of category theory as introduced in [4]; it consists of

(i) a collection of objects, each of which is a set-theoretic model of the EPT in the reference frame of an inertial observer;

(ii) a collection of arrows, each of which corresponds to a Lorentz transformation that transforms one set-theoretic model into another.

The specification of the category $\mathscr{C}_{SR}$ is straightforward but some elaboration is in place on how the components of the universe of the EPT have been modeled. It has to be taken that the EPT is a mathematically abstract theory that states elementary principles in terms of ultimate components but without reference to any coordinate system of an observer, while each model M_p in $\{M_i\}_{i\in F_1}$ is a mathematically concrete interpretation of these principles in the reference frame of an inertial observer. Recall that the universe described by the EPT consists of *world and antiworld*: a component of this universe is designated by a 2×1 matrix $\left[\begin{array}{c}\phi\\ \overline{\phi}\end{array}\right]$, where the abstract set ϕ designates a constituent of the world and the abstract set $\overline{\phi}$ a constituent of the antiworld—observers who live in "our" forward time-direction thus only observe a manifestation (i.e., a state) of the constituent ϕ of the world, while a (hypothetical) observer in opposite time-direction would observe a manifestation of the constituent $\overline{\phi}$ of the antiworld. In this study, however, only inertial observers are considered who live in "our" forward time-direction: all models M_p in $\{M_i\}_{i\in F_1}$ are thus models of the *world*, not of the *antiworld*.

The outline of this paper is as follows. The next section describes the purely pragmatic approach taken towards specifying a categorical model of the EPT. The section thereafter introduces the main result of this study: the categorical model $\mathscr{C}_{SR}$ of the EPT. The final section elaborates on the corresponding world view in terms of particles and events, and states the conclusions.

---

*e-mail: Marcoen.Cabbolet@vub.ac.be

2 Pragmatic approach: pointillism

The EPT describes *without reference to any coordinate system* how new ultimate building blocks are formed from existing ones by discrete transitions that take place in individual processes. The idea of a set-theoretic model of the EPT is then that the symbols of the EPT that refer to ultimate building blocks are interpreted in a concrete set-theoretical domain D, such that an interpretation $I(\phi)$ of a symbol ϕ mathematically models the state of that building block *in the reference frame of an observer.* In the model, the discrete transitions of the EPT then become state transitions in the reference frame of an observer: from there, quantitative predictions can be derived.

Recall from the introduction that the aim here is to prove that the EPT agrees with SR *and nothing more than that.* This calls for a purely pragmatic approach: it is enough to specify the simplest categorical model of the EPT that reproduces SR. First of all, the definition of a reference frame of an observer can be taken from SR:

Definition 2.1 (IRF) *The* ***reference frame of an inertial observer*** *is Minkowski space* $\mathcal{M} = \mathbb{R}^4$ *with signature* $(-,+,+,+)$. *Such an inertial reference frame will henceforth be referred to by the acronym 'IRF'. For a point* $X = (x^0, x^1, x^2, x^3) \in \mathcal{M}$, *the real number* x^0 *is the time coordinate, the three real numbers* x^1, x^2, x^3 *are the spatial coordinates.* ***Planck units*** *are used: both Planck length and Planck time are scaled to 1.* □

Def. 2.1 thus implies that the present categorical model of the EPT only applies for inertial observers: it is, thus, a presupposition that all observers are inertial observers. Furthermore, for the sake of simplicity we will use rectangular coordinates so that we can use the components $\eta_{\alpha\beta}$ of the metric tensor $\eta = \text{diag}(-1, 1, 1, 1)$.

Secondly, to show agreement with SR it suffices that the set-theoretic models of the EPT in the category $\mathscr{C}_{SR}$ are *pointillistic.* Originally referring to a technique in painting, the term 'pointillism' in physics is defined as

> *the doctrine that a physical theory's fundamental quantities are defined at points of space or of spacetime, and represent intrinsic properties of such points or point-sized objects located there; so that properties of spatial or spatiotemporal regions and their material contents are determined by the point-by-point facts* [5].

Thus speaking, a pointillistic model of the EPT is one in which the state of a phase quantum—an ultimate constituent of the universe of the EPT—in the IRF of an observer at every moment of its existence is modeled by a point-particle. Butterfield made a case against pointillism in [5], but it is once more emphasized that we take a purely pragmatic approach in this study: the pointillistic model of the EPT is an *idealization* that is purely intended to prove agreement with SR—the model needs to be refined to have a wider area of application.

Applying hyperreal Dirac delta functions, introduced in [6], to represent the state of a system made up of point-particles, we come to the following *pointillistic state postulate*:

Postulate 2.2 *In the categorical model* $\mathscr{C}_{SR}$ *of the EPT, the state of a phase quantum in the IRF of an observer* $\mathcal{O}$ *is represented by a function* $f : \mathcal{M} \to {}^*_+\mathbb{R}$ *for which*

$$f : (t, x, y, z) \mapsto E \cdot \chi(t)\delta^3_{(r^1,r^2,r^3)}(x, y, z) \tag{1}$$

where E *is the energy of the state and* $\chi : \mathcal{M} \to {}^*_+\mathbb{R}$ *is a characteristic function having the value 0 at times* t *when the state doesn't exist, and the value 1 at times* t *when the state exists. That is, at every time* t *that the state exists, the energy* E *of the state is then (i.e. at the time* t*) distributed over the one point* $(t, r^1(t), r^2(t), r^3(t), n) \in \mathcal{M}$. □

Recall that the EPT is not a quantum theory, so in the present categorical model of the EPT the above state postulate is to be viewed as an equivalent of e.g. the state postulate of standard quantum mechanics, which states that a quantum state is represented by an element ψ of a Hilbert space $\mathscr{H}$ with norm $\|\psi\| = 1$—this goes back to Schrödinger's early works, e.g. [7]. Similarly, here we have that the state of a phase quantum is represented by an element f of the function space ${}^*_+\mathbb{R}^{\mathcal{M}}$ for which

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(t, x, y, z)dxdydz = E \tag{2}$$

at any time t with $\chi(t) = 1$. In the next section, set-theoretic models of the EPT are specified in accordance with this pointillistic state postulate.

3 Result: the categorical model $\mathscr{C}_{SR}$

3.1 Overview

In Sect. 3.2 a *generic set-theoretic model* $M_{\mathbb{Z},\omega,\mathcal{O}}$ of the EPT is specified in a number of *interpretations* that apply the state postulate 2.2. In this generic model, the set of all integer-valued degrees of evolution is modeled by $\mathbb{Z}$, and the number of individual processes from any integer-valued degree of evolution n to the next is ω: this is a generic constant which does not depend on n.[1] Correspondingly, the set S_ω is the section of positive integers up to and including ω:

$$S_\omega := \{1, 2, \ldots, \omega\} \tag{3}$$

For the constant k in the k^{th} process from the n^{th} to the $(n+1)^{\text{th}}$ degree of evolution we thus have $k \in S_\omega$. There is, thus, a class of concrete set-theoretic models for each value of ω. That said, the objects of the category $\mathscr{C}_{SR}$ can then be defined as follows:

Definition 3.1 An **object of the category** $\mathscr{C}_{SR}$ is a concrete set-theoretic model $M_{\mathbb{Z},\omega,\mathcal{O}}$ of the EPT, that is, a structure $\langle |M_{\mathbb{Z},\omega,\mathcal{O}}|, \mathbb{E}, I_{\mathbb{Z},\omega,\mathcal{O}}(R)\rangle$ consisting of:

(i) the set of individuals $|M_{\mathbb{Z},\omega,\mathcal{O}}|$, the *universe* of $M_{\mathbb{Z},\omega,\mathcal{O}}$, which is the union of the following sets:

  - the set $A_{\omega,\mathcal{O}}$ specified by Def. 3.4;
  - the set $\langle G_{\mathbb{Z},\omega,\mathcal{O}}\rangle$ specified by Ints. 3.7, 3.9, 3.11, 3.12, 3.13, Def. 3.19, and Rem. 3.26;
  - the set $\Theta_{\mathbb{Z},\omega,\mathcal{O}} = \{\Theta^n_k \mid n \in Z, k \in S_\omega\}$ made up of the sets Θ^n_k specified by Int. 3.25;
  - the set $\phi_{\mathbb{Z},\omega,\mathcal{O}}$ made up of the choice functions specified by Int. 3.25.

(ii) the unary existence relation $\mathbb{E}$ specified by Int. 3.24, which can be identified with a subset of $\langle G_{\mathbb{Z},\omega,\mathcal{O}}\rangle$;

(iii) the ternary relation $I_{\mathbb{Z},\omega,\mathcal{O}}(R)$ specified by Ints. 3.14, 3.15, 3.16, 3.17, and Rems. 3.23 and 3.26, which can be identified with a subset of $\langle G_{\mathbb{Z},\omega,\mathcal{O}}\rangle \times \langle G_{\mathbb{Z},\omega,\mathcal{O}}\rangle \times \langle G_{\mathbb{Z},\omega,\mathcal{O}}\rangle$.

In this structure, the axioms of the EPT and the universality of the speed of light are valid. □

The collection of objects of $\mathscr{C}_{SR}$ is (uncountably) infinite; which model applies to the physical world depends, then, on the system to be modeled. E.g. for a system consisting of a single electron, for any $r \in \mathbb{R}$ there are models in which the electron at $t = 0$ is co-moving with the observer at spatial distance r, but there are also models in which the electron at $t = 0$ is moving relative to the observer; moreover, there are models in which the 4-momentum of the electron remains constant in the time interval $(0, 2)$, and there are models in which the 4-momentum of the electron remains changes in the time interval $(0, 2)$.

In Sect. 3.3 the arrows of the category $\mathscr{C}_{SR}$ are specified in a number of definitions. If for an inertial observer $\mathcal{O}$ a concrete set-theoretic model $M_{\mathbb{Z},\omega,\mathcal{O}}$ of the EPT applies to a given physical system, then for a *different* inertial observer $\mathcal{O}'$ a *different* model $M_{\mathbb{Z},\omega,\mathcal{O}'}$ applies *to the same physical system.* The point is, then, that these models are related by an arrow T in the collection of arrows of $\mathscr{C}_{SR}$. That being said, we can define precisely what such an arrow is.

Definition 3.2 Let the objects of the category $\mathscr{C}_{SR}$ be structures as in Def. 3.1. Then an **arrow of the category** $\mathscr{C}_{SR}$ is an isomorphism T of a structure $M_{\mathbb{Z},\omega,\mathcal{O}} = \langle |M_{\mathbb{Z},\omega,\mathcal{O}}|, \mathbb{E}, I_{\mathbb{Z},\omega,\mathcal{O}}(R)\rangle$ and a structure $M_{\mathbb{Z},\omega,\mathcal{O}'} = \langle |M_{\mathbb{Z},\omega,\mathcal{O}'}, \mathbb{E}, I_{\mathbb{Z},\omega,\mathcal{O}'}(R)\rangle$, which maps $|M_{\mathbb{Z},\omega,\mathcal{O}}|$ bijectively to $|M_{\mathbb{Z},\omega,\mathcal{O}'}|$ such that

$$T(f_1) + T(f_2) = T(f_1 + f_2) \quad \text{for any } f_1, f_2 \in \langle G_{\mathbb{Z},\omega,\mathcal{O}}\rangle \tag{4}$$

$$\mathbb{E}T(f) \Leftrightarrow \mathbb{E}f \quad \text{for any } f \in \langle G_{\mathbb{Z},\omega,\mathcal{O}}\rangle \tag{5}$$

$$\langle T(f_1), T(f_2), T(f_3)\rangle \in I_{\mathbb{Z},\omega,\mathcal{O}'}(R) \Leftrightarrow \langle f_1, f_2, f_3\rangle \in I_{\mathbb{Z},\omega,\mathcal{O}}(R) \quad \text{for any } f_1, f_2, f_3 \in \langle G_{\mathbb{Z},\omega,\mathcal{O}}\rangle \tag{6}$$

Every arrow of the category $\mathscr{C}_{SR}$ corresponds with a Lorentz transformation. □

So, once we have a concrete set-theoretic model $M_{\mathbb{Z},\omega,\mathcal{O}}$ that applies to a given system for inertial observer $\mathcal{O}$, then the arrows of $\mathscr{C}_{SR}$ transform this to models $M_{\mathbb{Z},\omega,\mathcal{O}'}, M_{\mathbb{Z},\omega,\mathcal{O}''}, \ldots$ that will apply to *the same physical system* for other inertial observers $\mathcal{O}', \mathcal{O}'', \ldots$ That is, the arrows relate the predictions of observer $\mathcal{O}$ to those of observers $\mathcal{O}', \mathcal{O}'', \ldots$ This way, the categorical model $\mathscr{C}_{SR}$ of the EPT reproduces relativity of length and time as in standard SR.

---

[1] Here ω is a finite integer, not to be confused with the hyperreal number with the same symbol; in the remainder of this text it is assumed that it will be clear from the context to which number the symbol ω refers.

3.2 The objects of the category $\mathscr{C}_{SR}$

Agreement 3.3 Greek indices α, β, etc. for the components of vectors and tensors can take all values from 0 to 3, but Roman indices i, j, k, etc. can only take a value from 1 to 3. So x^α can be any of the components of the 4-tuple $(x^0, ..., x^4)$, while x^j refers only to x^1, x^2, or x^3. Furthermore, $(\vec{x})^\alpha$ denotes the α^{th} component of the 4-vector $\vec{x}$. □

To specify the generic set-theoretic model $M_{\mathbb{Z},\omega,\mathcal{O}}$ of the EPT, we must begin by defining the set of *monads*. A 'monad' in the EPT is an abstraction of an indivisible massive particle: in this model, a monadic state is an indivisible building block of the world as seen by observer $\mathcal{O}$—its properties then relate to the properties of the monad defined below.

Definition 3.4 (Monads) Let $M_{\mathbb{Z},\omega,\mathcal{O}}$ be a set-theoretic model of the EPT. The **set of all monads** in $M_{\mathbb{Z},\omega,\mathcal{O}}$ is then the set

$$A_{\omega,\mathcal{O}} = \{\langle k, \sigma_k, \chi_k\rangle \mid k \in S_\omega\} \tag{7}$$

For any $k \in S_\omega$, the three-tuple $\langle k, \sigma_k, \chi_k\rangle \in S$ is the $\mathbf{k}^{\text{th}}$ **monad**; the constant σ_k is the **rest mass spectrum** of the k^{th} monad; the constant $\chi_k \in \{-1, 1\}$ is the **characteristic number of normality** of the k^{th} monad. In this model, the rest mass spectrum is a constant function

$$\sigma_k : \mathbb{Z} \to \mathbb{R}, \sigma_k : n \to m_k \tag{8}$$

that adds the number $m_k > 0$, the rest mass of the k^{th} monad, to a degree of evolution n. □

The interpretations of constants and axioms of the EPT make use of the following notation and definition:

Notation 3.5 Let $M_{\mathbb{Z},\omega,\mathcal{O}}$ be a set-theoretic model of the EPT; let $I_{\mathbb{Z},\omega,\mathcal{O}}$ be the interpretation function that maps any constant ϕ of the EPT to its interpretation $I_{\mathbb{Z},\omega,\mathcal{O}}(\phi)$ in the language of $M_{\mathbb{Z},\omega,\mathcal{O}}$. For a constant ϕ of the EPT referring to a phase quantum, the expression

$$\phi \xrightarrow{\mathcal{O}} f \tag{9}$$

is then a notation for $I_{\mathbb{Z},\omega,\mathcal{O}}(\phi) = f$, and has to be read as: 'the state of the phase quantum, designated by ϕ, in the coordinate system of the observer $\mathcal{O}$ is represented by f'. (This notation is loosely based on a notation used in [8].) □

Definition 3.6 Let $\{t\} \subset \mathbb{R}$ be any singleton, and let $(t, u) \subset \mathbb{R}$ be any open real interval. Then the **characteristic functions** $\chi_{\{t\}} : \mathbb{R} \to \mathbb{R}$ and $\chi_{(t,u)} : \mathbb{R} \to \mathbb{R}$ are given by

$$\chi_{\{t\}} : x \mapsto \begin{cases} 1 \text{ iff } x = t \\ 0 \text{ iff } x \neq t \end{cases} \tag{10}$$

$$\chi_{(t,u)} : x \mapsto \begin{cases} 1 \text{ iff } x \in (t, u) \\ 0 \text{ iff } x \notin (t, u) \end{cases} \tag{11}$$

The latter equation is to include the cases that $(t, u) = (t, \infty)$. □

Interpretation 3.7 For integers $n \in \mathbb{Z}$ and $k \in S_\omega$, the constant ${}^{EP}\mu^n_k$ of the EPT designates the *extended particlelike matter quantum at the* n^{th} *degree of evolution associated to the* k^{th} *monad.* In the model $M_{\mathbb{Z},\omega,\mathcal{O}}$ we then have

$${}^{EP}\mu^n_k \xrightarrow{\mathcal{O}} s^n_k \tag{12}$$

$$s^n_k : (t, x, y, z) \mapsto E^{EP}_{n,k} \cdot \chi_{\{t_{n,k}\}}(t)\delta^3_{(x_{n,k},y_{n,k},z_{n,k})}(x, y, z) \tag{13}$$

So, the state of the particlelike matter quantum, designated by the symbol ${}^{EP}\mu^n_k$ in the EPT, in the IRF of the observer $\mathcal{O}$ is modeled as a **point-particle** with energy $E^{EP}_{n,k} > 0$ at the spatiotemporal position $X_{n,k} = (t_{n,k}, x_{n,k}, y_{n,k}, z_{n,k})$, represented by the function $s^n_k : \mathcal{M} \to {}^*_+\mathbb{R}$. □

Note that the support of the function s^n_k is a singleton: supp $s^n_k = \{(t_{n,k}, x_{n,k}, y_{n,k}, z_{n,k})\} = \{X_{n,k}\}$.[2] That means that in the IRF of $\mathcal{O}$, the point-particle only exists at the spatiotemporal position $X_{n,k}$.

---

[2] Recall that, for any nonempty set X and any a vector space V, the **support** of a function $f : X \to V$ is the set denoted by 'supp f' for which supp $f = \{x \in X \mid f(x) \neq 0\}$; see e.g. [9].

Agreement 3.8 We will henceforth refer to the state represented by the function s^n_k as the 'particle state of the k^{th} monad at the n^{th} degree of evolution in the IRF of the observer $\mathcal{O}$'. □

Now that we have the monadic particle states, we are going to let these evolve according to the principles of the EPT, which are formulated in terms of *phase quanta*: the idea for this model is that the particlelike state of the k^{th} monad at the n^{th} degree of evolution is the initial state at the start of the k^{th} process from the n^{th} to the $(n+1)^{\text{th}}$ degree of evolution. So we first interpret the constants of the EPT referring to phase quanta, and then we interpret the principles of the EPT.

Interpretation 3.9 For integers $n \in \mathbb{Z}$ and $k \in S_\omega$, the constant ${}^{EP}\Phi^n_k$ of the EPT designates the *extended particlelike phase quantum occurring in the k^{th} process from the n^{th} to the $(n+1)^{\text{th}}$ degree of evolution.* In the model $M_{\mathbb{Z},\omega,\mathcal{O}}$ we then have

$$ {}^{EP}\Phi^n_k \xrightarrow{\mathcal{O}} {}^{EP}f^n_k \tag{14}$$

$$ {}^{EP}f^n_k = s^n_k \tag{15}$$

□

Thus speaking, in $M_{\mathbb{Z},\omega,\mathcal{O}}$ the state of the phase quantum, designated by the symbol ${}^{EP}\Phi^n_k$ in the EPT, in the IRF of the observer $\mathcal{O}$ is the particle state of the k^{th} monad at the n^{th} degree of evolution in the IRF of the observer $\mathcal{O}$. Thus speaking, in the IRF of the observer $\mathcal{O}$, the k^{th} process from the n^{th} to the $(n+1)^{\text{th}}$ degree of evolution starts with a point-particle with energy $E^{EP}_{n,k}$ at spatiotemporal position $X_{n,k}$. Moreover, Int. 3.9 associates the k^{th} process from the n^{th} to the $(n+1)^{\text{th}}$ degree of evolution with the k^{th} monad: the properties of the monad defined in Def. 3.4 will thus occur in the said process.

Remark 3.10 To emphasize it: in a more elaborate model of the EPT the phase quantum ${}^{EP}\Phi^n_k$ will be modeled as an *aggregation* of monadic particle states, and these do not have to be point-particles. Thus speaking, Int. 3.9 forces us to treat, for example, a deuterium nucleus as a monadic state—although we already know that it is composed of a neutron and a proton. The crux here is that we are only interested in showing that the EPT agrees with SR: therefore, we keep the internal states of massive particles as simple as possible—that is, all massive particles are modeled as elementary point-particles. □

Interpretation 3.11 For integers $n \in \mathbb{Z}$ and $k \in S_\omega$, the constant ${}^{NW}\Phi^n_k$ of the EPT designates the *non-local wavelike phase quantum occurring in the k^{th} process from the n^{th} to the $(n+1)^{\text{th}}$ degree of evolution.* In the model $M_{\mathbb{Z},\omega,\mathcal{O}}$ we then have

$$ {}^{NW}\Phi^n_k \xrightarrow{\mathcal{O}} {}^{NW}f^n_k \tag{16}$$

$$ {}^{NW}f^n_k : \begin{cases} (t,x,y,z) \mapsto E^{NW}_{n,k} \cdot \omega^3 & \text{if} \quad (t,x,y,z) \in \overline{\Delta X}_{n,k} \\ (t,x,y,z) \mapsto 0 & \text{if} \quad (t,x,y,z) \notin \overline{\Delta X}_{n,k} \end{cases} \tag{17}$$

for a line segment $\overline{\Delta X}_{n,k}$ in the IRF of the observer $\mathcal{O}$ determined by the spatiotemporal position $X_{n,k}$ of Int. 3.7 and a displacement vector $\Delta\vec{x}_{n,k} = (\Delta t_{n,k}, \Delta x_{n,k}, \Delta y_{n,k}, \Delta z_{n,k})$ in spacetime with $\Delta t_{n,k} > 0$ such that

$$ \overline{\Delta X}_{n,k} = \{X_{n,k} + \lambda \cdot \Delta\vec{x}_{n,k} \in \mathcal{M} \mid \lambda \in (0,1)\} \tag{18}$$

$$ \eta(\Delta\vec{x}_{n,k}, \Delta\vec{x}_{n,k}) = -1 \tag{19}$$

Thus speaking, the state of the phase quantum, designated by the symbol ${}^{NW}\Phi^n_k$ in the EPT, in the IRF of the observer $\mathcal{O}$ is a **time-like string** with energy $E = E^{NW}_{n,k} > 0$ and spatiotemporal extension $\overline{\Delta X}_{n,k}$, represented by the above function ${}^{NW}f^n_k \in {}^*_+\mathbb{R}^{\mathcal{M}}$. At every point $X = X(\lambda)$ of its spatiotemporal extension (with the above parametrization), the time-like string is associated with a **4-momentum** $\vec{p}^{NW}_{n,k}$ for which

$$ \vec{p}^{NW}_{n,k} = m_k \cdot (\frac{dx^0}{d\lambda}, \frac{dx^1}{d\lambda}, \frac{dx^2}{d\lambda}, \frac{dx^3}{d\lambda}) = (E^{NW}_{n,k}, p^1_{n,k}, p^2_{n,k}, p^3_{n,k}) \tag{20}$$

$$ \eta(\vec{p}^{NW}_{n,k}, \vec{p}^{NW}_{n,k}) = -(E^{NW}_{n,k})^2 + (p^1_{n,k})^2 + (p^2_{n,k})^2 + (p^3_{n,k})^2 = -(m_k)^2 \tag{21}$$

where m_k in Eq. (20) is the rest mass of the k^{th} monad as given by Def. 3.4. □

Note that the components $p^\alpha_{n,k}$ of $\vec{p}^{NW}_{n,k}$ in Eq. (20) are constants that do not depend on λ, so $\frac{d^2x^\alpha}{d\lambda^2} = 0$. We can view the time-like string ${}^{NW}f^n_k$ therefore as a *wave traveling in a straight line*, associated with energy $E^{NW}_{n,k}$ and **constant** spatial momenta $p^j_{n,k}$.

Furthermore, note that the function prescription (17)—in which the symbol ω refers, of course, to the hyperreal number—can be rewritten in the form of Eq. (1) of Post. 2.2. We have

$$ {}^{NW}f^n_k : (t,x,y,z) \mapsto E^{NW}_{n,k}\, \chi_{(t_{n,k},\ t_{n,k}+\Delta t_{n,k})}(t)\delta^3_{\langle x^1(t), x^2(t), x^3(t)\rangle}(x,y,z) \tag{22} $$

with $\chi_{(t_{n,k},\ t_{n,k}+\Delta t_{n,k})}$ the characteristic function of the interval $(t_{n,k}, t_{n,k}+\Delta t_{n,k})$ as in Def. 3.6, and with

$$ \begin{pmatrix} x^1(t) \\ x^2(t) \\ x^3(t) \end{pmatrix} = \begin{pmatrix} x_{n,k} \\ y_{n,k} \\ z_{n,k} \end{pmatrix} + \frac{t - t_{n,k}}{\Delta t_{n,k}} \cdot \begin{pmatrix} \Delta x_{n,k} \\ \Delta y_{n,k} \\ \Delta z_{n,k} \end{pmatrix} \tag{23} $$

This gives precisely the same function values of ${}^{NW}f^n_k$.

Interpretation 3.12 For integers $n \in \mathbb{Z}$ and $k \in S_\omega$, the constant ${}^{NP}\Phi^{n+1}_k$ of the EPT designates the *non-extended particlelike phase quantum occurring in the* k^{th} *process from the* n^{th} *to the* $(n+1)^{\text{th}}$ *degree of evolution.* In the model $M_{\mathbb{Z},\omega,\mathcal{O}}$ we then have

$$ {}^{NP}\Phi^{n+1}_k \xrightarrow{\mathcal{O}} {}^{NP}f^{n+1}_k \tag{24} $$

$$ \text{supp } {}^{NP}f^{n+1}_k = \{(t_{n+1,k}, x_{n+1,k}, y_{n+1,k}, z_{n+1,k}, 0)\} = \{X_{n+1,k}\} \quad , \quad t_{n+1,k} = t_{n,k} + \Delta t_{n,k} \tag{25} $$

$$ {}^{NP}f^{n+1}_k : (t,x,y,z,u) \mapsto E^{NP}_{n+1,k}\chi_{\{t_{n+1,k}\}}(t)\delta^3_{\langle x_{n+1,k},\ y_{n+1,k},\ z_{n+1,k}\rangle}(x,y,z) \tag{26} $$

Thus speaking, in $M_{\mathbb{Z},\omega,\mathcal{O}}$ the state of the phase quantum, designated by the symbol ${}^{NP}\Phi^{n+1}_k$ in the EPT, in the IRF of the observer $\mathcal{O}$ is modeled by a **point-particle** with energy $E = E^{NP}_{n+1,k} > 0$ represented by the above function ${}^{NP}f^{n+1}_k \in {}^*_+\mathbb{R}^{\mathcal{M}}$. Note that the point-particle only exists at the one spatiotemporal position $X_{n+1,k}$ in the IRF of $\mathcal{O}$, so $\chi_{\{t_{n+1,k}\}}(t) = 1$ if $t) = t_{n+1,k}$ and $\chi_{\{t_{n+1,k}\}}(t) = 0$ else. □

Interpretation 3.13 For integers $n \in \mathbb{Z}$ and $k \in S_\omega$, the constant ${}^{LW}\Phi^{n+1}_k$ of the EPT designates the *local wavelike phase quantum occurring in the* k^{th} *process from the* n^{th} *to the* $(n+1)^{\text{th}}$ *degree of evolution.* In the model $M_{\mathbb{Z},\omega,\mathcal{O}}$ we then have

$$ {}^{LW}\Phi^{n+1}_k \xrightarrow{\mathcal{O}} \gamma^{n+1}_k \tag{27} $$

$$ \gamma^{n+1}_k : \begin{cases} (t,x,y,z) \mapsto \Delta E_{n+1,k}\cdot\omega^3 & \text{if} \quad (t,x,y,z) \in \ell^\gamma_{n+1,k} \\ (t,x,y,z) \mapsto 0 & \text{if} \quad (t,x,y,z) \notin \ell^\gamma_{n+1,k} \end{cases} \tag{28} $$

for a line segment $\ell^\gamma_{n+1,k} \subset \mathcal{M}$ in the IRF of the observer $\mathcal{O}$ determined by the spatiotemporal position $X_{n+1,k}$ of Int. 3.12 and a null vector $(1, v^1, v^2, v^3) \in \mathcal{M}$:

$$ \ell^\gamma_{n+1,k} : \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix} = \begin{pmatrix} t_{n+1,k} \\ x_{n+1,k} \\ y_{n+1,k} \\ z_{n+1,k} \end{pmatrix} + \mu \cdot \begin{pmatrix} 1 \\ v^1 \\ v^2 \\ v^3 \end{pmatrix} \quad , \quad \mu \in (0, t_{\text{end}}) \tag{29} $$

$$ \eta((1,v^1,v^2,v^3),(1,v^1,v^2,v^3)) = -1 + (v^1)^2 + (v^2)^2 + (v^3)^2 = 0 \tag{30} $$

Thus speaking, in $M_{\mathbb{Z},\omega,\mathcal{O}}$ the state of the phase quantum, designated by the symbol ${}^{LW}\Phi^{n+1}_k$ in the EPT, in the IRF of the observer $\mathcal{O}$ is modeled by a γ**-ray** with spatiotemporal extension $\ell^\gamma_{n+1,k}$ and with energy $E = \Delta E^{NP}_{n+1,k} > 0$, represented by the above function $\gamma^{n+1}_k \in {}^*_+\mathbb{R}^{\mathcal{M}}$. If the γ-ray gets absorbed at a time $t > t_{n+1,k}$, then t_{end} in Eq. (29) has the finite value $t - t_{n+1,k}$; if no absorption takes place, then we have $(0, t_{\text{end}}) = (0, \infty)$. At every point $X(\mu)$ of its path (with the above parametrization), the γ-ray is associated with a **4-momentum** $\vec{p}^{LW}_{n+1,k}$ for which

$$ \vec{p}^{LW}_{n+1,k} = \Delta E_{n+1,k} \cdot \left(\frac{dx^0}{d\mu}, \frac{dx^1}{d\mu}, \frac{dx^2}{d\mu}, \frac{dx^3}{d\mu}\right) = (\Delta E_{n+1,k}, \Delta p^1_{n+1,k}, \Delta p^2_{n+1,k}, \Delta p^3_{n+1,k}) \tag{31} $$

$$ \eta(\vec{p}^{LW}_{n+1,k}, \vec{p}^{LW}_{n+1,k}) = -(\Delta E_{n+1,k})^2 + (\Delta p^1_{n+1,k})^2 + (\Delta p^2_{n+1,k})^2 + (\Delta p^3_{n+1,k})^2 = 0 \tag{32} $$

□

Given Eq. (33) we here also have $\frac{d^2x^\alpha}{d\mu^2} = 0$, so we associate the γ-ray with **constant** spatial momenta $\Delta p^j_{n+1,k}$. The idea of the γ-ray implements a ray theory of light in this model, with the front of the ray being a photon. We thus conveniently ignore that phenomena like interference and diffraction require wave theory. But recall that the aim is to show that the EPT agrees with SR: in the framework of SR, photons are point-particles too!

Furthermore, similar to the case of the time-like strings, the function prescription (28) can be rewritten in the form of Eq. (1) of Post. 2.2. We get

$$\gamma_k^{n+1} : (t,x,y,z) \mapsto \Delta E_{n+1,k} \cdot \chi_{(t_{n+1,k},\ t_{n+1,k}+t_{\text{end}})}(t)\delta^3_{\langle x^1(t),x^2(t),x^3(t)\rangle}(x,y,z) \tag{33}$$

with $\chi_{(t_{n+1,k},\ t_{n+1,k}+t_{\text{end}})}$ the characteristic function of the interval $(t_{n+1,k},\ t_{n+1,k}+t_{\text{end}})$ as in Def. 3.6, and with

$$\begin{pmatrix} x^1(t) \\ x^2(t) \\ x^3(t) \end{pmatrix} = \begin{pmatrix} x_{n+1,k} \\ y_{n+1,k} \\ z_{n+1,k} \end{pmatrix} + (t - t_{n+1,k}) \cdot \begin{pmatrix} v^1 \\ v^2 \\ v^3 \end{pmatrix} \tag{34}$$

where the v^j's are the spatial components of the null vector from Eq. (29). This gives precisely the same function values of γ_k^{n+1}.

Having modeled the *objects* in the universe of the EPT in terms of point-particles, time-like strings and gamma-rays, we are now ready to model the *elementary principles* of the EPT.

Interpretation 3.14 For integers $n \in \mathbb{Z}$ and $k \in S_\omega$, in the model $M_{\mathbb{Z},\omega,\mathcal{O}}$ the expression

$$\models\ 0 :\ {}^{EP}f_k^n \to\ {}^{NW}f_k^n \tag{35}$$

models the Elementary Principle of Nonlocal Equilibrium, the first of seven axioms of the EPT; here the symbol '0' refers to the function $0 : \mathcal{M} \to {}^*_+\mathbb{R}$, $0 : X \mapsto (0,\ldots,0)$. Since ${}^{EP}f_k^n = s_k^n$, cf. Int. 3.9, this expression means that in the IRF of the observer $\mathcal{O}$, the particle state of the k^{th} monad at the n^{th} degree of evolution, located at the spatiotemporal position $X_{n,k}$, transforms spontaneously into the time-like string ${}^{NW}f_k^n$, which over time occupies the open line segment $\overline{\Delta X}_{n,k}$. □

Interpretation 3.15 For integers $n \in \mathbb{Z}$ and $k \in S_\omega$, in the model $M_{\mathbb{Z},\omega,\mathcal{O}}$ the expression

$$\models\ {}^{NW}f_k^n :\ {}^{EP}f_k^n \to\ {}^{NP}f_k^{n+1} \tag{36}$$

models the Elementary Principle of Nonlocal Mediation, the second of seven axioms of the EPT. Since we have ${}^{EP}f_k^n = s_k^n$, cf. Int. 3.9, this expression means that in the IRF the observer $\mathcal{O}$, the time-like string ${}^{NW}f_k^n$ effects a transition from the particle state of the k^{th} monad at the n^{th} degree of evolution, located at the spatiotemporal position $X_{n,k}$ in the IRF of the observer $\mathcal{O}$, to the point-particle ${}^{NP}f_k^{n+1}$ located at the spatiotemporal position $X_{n+1,k}$ in the IRF of $\mathcal{O}$. This has to be taken that at $t = t_{n+1,k}$, the time-like string "collapses" into, i.e. transforms into, the point-particle ${}^{NP}f_k^{n+1}$. □

Interpretation 3.16 For integers $n \in \mathbb{Z}$ and $k \in S_\omega$, in the model $M_{\mathbb{Z},\omega,\mathcal{O}}$ the expression

$$\models\ 0 :\ {}^{NP}f_k^{n+1} \to \gamma_k^{n+1} \tag{37}$$

models the Elementary Principle of Local Equilibrium, the third of seven axioms of the EPT; here '0' has the same meaning as in Int. 3.14. This expression means that in IRF of the observer $\mathcal{O}$, the point-particle ${}^{NP}f_k^{n+1}$ *spontaneously* emits a γ-ray γ_k^{n+1}. □

Interpretation 3.17 For integers $n \in \mathbb{Z}$ and $k \in S_\omega$, in the model $M_{\mathbb{Z},\omega,\mathcal{O}}$ the expression

$$\models\ \gamma_k^{n+1} :\ {}^{NP}f_k^{n+1} \to\ s_k^{n+1} \tag{38}$$

models the Elementary Principle of Local Mediation, the fourth of seven axioms of the EPT. This expression means that in the IRF of the observer $\mathcal{O}$, the emitted γ-ray γ_k^{n+1} causes the transition of the point-particle ${}^{NP}f_k^{n+1}$ to the particle state of the k^{th} monad at the $(n+1)^{\text{th}}$ degree of evolution. Note that supp ${}^{NP}f_k^{n+1}$ = supp ${}^{EP}f_k^{n+1} = \{X_{n+1,k}\}$, cf. Ints. 3.7 and 3.12, so the discrete transition ${}^{NP}f_k^{n+1} \to\ {}^{EP}f_k^{n+1}$ involves no spatiotemporal displacement. The particle state of the k^{th} monad at the $(n+1)^{\text{th}}$ degree of evolution is then the starting point of the k^{th} process from the $(n+1)^{\text{th}}$ to the $(n+2)^{\text{th}}$ degree of evolution. □

At the level of abstractness of the EPT, the phase quanta in terms of which the elementary principles are stated are abstracted from their properties. In the present model, however, we have endowed the phase quanta with properties, in particular (spatiotemporal) position, energy and spatial momentum. To exclude inapplicability to the physical world the formulation of conservation laws is required; this has the status of an additional postulate.

Postulate 3.18 (Conservation of 4-momentum) Upon the collapse of the time-like string ${}^{NW}f^n_k$ with 4-momentum $\vec{p}^{NW}_{n,k}$ to the point-particle ${}^{NP}f^{n+1}_k$ the momenta are conserved, so we associate ${}^{NP}f^{n+1}_k$ with a 4-momentum

$$\vec{p}^{NP}_{n+1,k} := \vec{p}^{NW}_{n,k} = (E^{NW}_{n,k}, p^1_{n,k}, p^2_{n,k}, p^3_{n,k}) \tag{39}$$

The γ-ray γ^{n+1}_k with associated 4-momentum $\vec{p}^{LW}_{n+1,k}$ emitted by the point-particle ${}^{NP}f^{n+1}_k$ then causes the latter to transform to the point-particle ${}^{EP}f^{n+1}_k$, so we associate ${}^{EP}f^{n+1}_k$ with a 4-momentum $\vec{p}^{EP}_{n+1,k}$ for which

$$\vec{p}^{EP}_{n+1,k} := \vec{p}^{NP}_{n+1,k} - \vec{p}^{LW}_{n+1,k} \tag{40}$$

$$\eta(\vec{p}^{EP}_{n+1,k}, \vec{p}^{EP}_{n+1,k}) = -m_{n+1} \tag{41}$$

By a discrete state transition, the point-particle ${}^{EP}f^{n+1}_k$ subsequently transforms into the time-like string ${}^{NW}f^{n+1}_k$ with 4-momentum $\vec{p}^{NW}_{n+1,k}$. If a γ-ray γ^p_m with associated 4-momentum $\vec{p}^{LW}_{p,m}$ is absorbed, that is, if a γ-ray γ^p_m has a path $\{X(t) \mid t \in (0, t_{\text{end}}\} \subset \mathcal{M}$ such that

$$\lim_{t \to t_{\text{end}}} X(t) = X_{n+1,k} \tag{42}$$

then 4-momentum is conserved according to

$$\vec{p}^{NW}_{n+1,k} = \vec{p}^{EP}_{n+1,k} + \vec{p}^{LW}_{p,m} \tag{43}$$

If no γ-ray is absorbed, then Eq. (43) holds with $\vec{p}^{LW}_{p,m} = 0$. ☐

Definition 3.19 Let $G_{\mathbb{Z},\omega,\mathcal{O}} = \{{}^{EP}f^n_k,\ {}^{NW}f^n_k,\ {}^{NP}f^{n+1}_k, \gamma^{n+1}_k \mid n \in Z,\ k \in S_\omega\}$; then $\langle G_{\mathbb{Z},\omega,\mathcal{O}} \rangle$ is the **commutative monoid generated by the set $G_{\mathbb{Z},\omega,\mathcal{O}}$ under function addition**, for which

$$f + g : X \mapsto f(X) + g(X) \tag{44}$$

Note that $s^n_k \in \langle G_{\mathbb{Z},\omega,\mathcal{O}} \rangle$ since $s^n_k = {}^{EP}f^n_k$. ☐

Interpretation 3.20 For integers $n \in \mathbb{Z}$ and $k \in S_\omega$, the constant ψ^n_k of the EPT designates the *state of the k^{th} monad from the n^{th} to the $(n+1)^{\text{th}}$ degree of evolution.* In the model $M_{\mathbb{Z},\omega,\mathcal{O}}$ we then have

$$\psi^n_k \xrightarrow{\mathcal{O}} t^n_k \tag{45}$$

$$t^n_k : \mathcal{M} \to {}^*_+\mathbb{R} \tag{46}$$

such that the expression

$$\models\ t^n_k = {}^{EP}f^n_k + {}^{NW}f^n_k \tag{47}$$

models the Elementary Principle of Binad Composition, the fifth of seven axioms of the EPT. Recall that in the EPT the constant $\beta^n_k \equiv {}^{EP}\Phi^n_k + {}^{NW}\Phi^n_k$ designates the *binad occurring in the k^{th} process from the n^{th} to the $(n+1)^{\text{th}}$ degree of evolution*; the expression (47), thus, means that the state of the binad β^n_k in the IRF of the observer $\mathcal{O}$ is modeled by the monadic state t^n_k which is made up of the point-particle ${}^{EP}f^n_k$ and the time-like string ${}^{NW}f^n_k$. ☐

In a more advanced model of the EPT the state of the binad $\beta^n_k = {}^{EP}\Phi^n_k + {}^{NW}\Phi^n_k$ may be identified with an aggregation of monadic states. The next two examples will formalize electrons and positrons in the present framework, but it works the same way for neutrons, antineutrons, protons, antiprotons, and all other massive particles and their antimatter counterparts.

Example 3.21 Suppose that the k^{th} monad, introduced in Def. 3.4, is an *electronic* monad: then the rest mass spectrum σ_k maps any degree of evolution n to the rest mass $\sigma_k(n) = m_k = m_e$ of an electron; the characteristic number of normality χ_k has then the value $+1$. The particle state s_k^n of the k^{th} monad at the n^{th} degree of evolution in the IRF of the observer $\mathcal{O}$, introduced in Int. 3.7, is then *a particle state of an electron*: the lowest possible value of its energy $E_{n,k}^{EP}$ is the rest mass of an electron m_e, which is thus predetermined by the rest mass spectrum σ_k, and it is a *normal* particle state as indicated by the value $+1$ of the characteristic number of normality χ_k. The time-like string ${}^{NW}f_k^n$, created from the particle state of the electron on account of the principle stated in Int. 3.14, can be viewed as *a wave state of an electron.* Together, the particle state of the electron and the wave state of the electron form the state t_k^n, which is the (temporally extended) state of the electron from the n^{th} to the $(n+1)^{\text{th}}$ degree of evolution—see Int. 3.20. □

Example 3.22 Suppose that the j^{th} monad is a *positronic* monad, then the rest mass spectrum σ_j is the same as that of an electronic monad: σ_j maps any degree of evolution n to the rest mass of an electron, so $\sigma_j(n) = m_j = m_e = \sigma_k(n)$. However, the characteristic number of normality χ_j has now the value -1. The particle state s_j^n of the j^{th} monad at the n^{th} degree of evolution in the IRF of the observer $\mathcal{O}$ is then a positron in a particle state: the lowest possible value of its energy $E_{n,j}^{EP}$ is the rest mass of an electron m_e, which is thus predetermined by the rest mass spectrum σ_j, and it is an *abnormal* particle state as indicated by the value -1 of the characteristic number of normality χ_j. The state t_k^n is then the (temporally extended) state of the positron from the n^{th} to the $(n+1)^{\text{th}}$ degree of evolution. In this as well as in the previous example, the characteristic number of normality has the same value as the lepton quantum number in quantum theory. □

Remark 3.23 Formulas (35), (36), (37), and (38) describe *all* individual processes in the IRF of the observer $\mathcal{O}$: there are no other processes (but see Rem. 3.26). In the EPT, the corresponding four elementary principles all use expressions of the form $\begin{bmatrix} a \\ \bar{a} \end{bmatrix} : \begin{bmatrix} x \\ \bar{x} \end{bmatrix} \begin{matrix} \rightarrow \\ \leftarrow \end{matrix} \begin{bmatrix} y \\ \bar{y} \end{bmatrix}$, which are notations for

$$\langle \begin{bmatrix} a \\ \bar{a} \end{bmatrix}, \begin{bmatrix} x \\ \bar{x} \end{bmatrix}, \begin{bmatrix} y \\ \bar{y} \end{bmatrix} \rangle \in R \tag{48}$$

where R is a ternary relation on a finitely generated commutative monoid $(\langle g_1, g_2, g_3, \ldots, g_\Omega \rangle, +)$; an individual $\begin{bmatrix} a \\ \bar{a} \end{bmatrix}$, $\begin{bmatrix} x \\ \bar{x} \end{bmatrix}$, or $\begin{bmatrix} y \\ \bar{y} \end{bmatrix}$ in an expression (48) can, thus, be a sum of generators g_j. In the present model $M_{\mathbb{Z},\omega,\mathcal{O}}$, however, by these formulas (35), (36), (37), and (38) this relation R is interpreted as a ternary relation $I_{\mathbb{Z},\omega,\mathcal{O}}(R)$ on the set $\langle G_{\mathbb{Z},\omega,\mathcal{O}} \rangle$. □

Having described the elementary processes in this model, we can now interpret the unary existence relation M_E of the EPT, which is straightforward.

Interpretation 3.24 For any generator $f \in G_{\mathbb{Z},\omega,\mathcal{O}}$ and for any finite sum $f_1 + \ldots + f_n \in \langle G_{\mathbb{Z},\omega,\mathcal{O}} \rangle$ of such generators, the expressions

$$\models \ \mathbb{E}f \Leftrightarrow f \neq 0 \tag{49}$$

$$\models \ \mathbb{E}f_1 + \ldots + f_n \Leftrightarrow \mathbb{E}f_1 + \ldots + f_{n-1} \wedge ((\mathbb{E}f_n \wedge f_1 \neq f_n \wedge f_2 \neq f_n \wedge \ldots \wedge f_{n-1} \neq f_n) \vee f_n = 0) \tag{50}$$

model the existence relation for the objects in the IRF of the observer $\mathcal{O}$, where '$\mathbb{E}f$' denotes $f \in \mathbb{E}$ with $\mathbb{E} = I_{\mathbb{Z},\omega,\mathcal{O}}(M_E)$. □

So, in the model $M_{\mathbb{Z},\omega,\mathcal{O}}$ we have $\mathbb{E}\ {}^{EP}f_k^n$ for any $n \in Z,\ k \in S_\omega$, but we do not necessarily have $\mathbb{E}\gamma_k^{n+1}$ for any $n \in Z,\ k \in S_\omega$. The point is that there may be elementary processes in which no γ-ray is emitted: in that case $\gamma_k^{n+1} = 0$, and thus $\neg\mathbb{E}\gamma_k^{n+1}$; formula (37) is then trivially true.

It remains to be established that the present model is a *deterministic* model of the EPT, which contains an elementary principle of choice. In the IRF of the observer $\mathcal{O}$, a choice takes place at every event that a time-like string ${}^{NW}f_k^n$ with spatiotemporal extension $\overline{\Delta X}_{n,k}$ transforms into a point-particle ${}^{NP}f_k^{n+1}$ at spatiotemporal position $X_{n+1,k}$. The time-like string corresponds to a displacement vector $\Delta\vec{x}_{n,k} = (\Delta t_{n,k}, \Delta x_{n,k}, \Delta y_{n,k}, \Delta z_{n,k})$ in $\mathcal{M}$, but although we have from Eq. (25) for the time coordinate that $t_{n+1,k} = t_{n,k} + \Delta t_{n,k}$ it **does not** follow from the foregoing that $X_{n+1,k} = X_{n,k} + \Delta\vec{x}_{n,k}$. It is, thus, the principle of choice that guarantees continuity. That is to say: the point-particle ${}^{NP}f_k^{n+1}$ is *chosen* from a set of possibilities Θ_k^{n+1}.

Interpretation 3.25 Let Θ_k^{n+1} be the set of all functions ${}^{NP}h_k^{n+1} : \mathcal{M} \to {}^*_+\mathbb{R}$ for which

$$ {}^{NP}h_k^{n+1} : (t,x,y,z) \mapsto E^{NW}_{n.k} \cdot \chi_{\{t_{n+1,k}\}} \cdot \delta^3_{\langle,x^1,x^2,x^3\rangle}(x,y,z) \tag{51}$$

so that ${}^{NP}h_k^{n+1}(t_{n+1,k}, x^1, x^2, x^3) = {}^{NP}f_k^{n+1}(X_{n+1,k})$: the support is a singleton $\{X\}$ whose element $X = (t_{n+1,k}, x^1, x^2, x^3)$ differs only with respect to the spatial coordinates x^j from $X_{n+1,k}$. Let, for $Y = (y^0, y^1, y^2, y^3)$ with $y^0 = t_{n+1,k}$, the choice function $\phi_Y : \{\Theta_k^{n+1}\} \to \Theta_k^{n+1}$ be given by

$$\phi_Y(\Theta_k^{n+1}) = {}^{NP}h_k^{n+1} \Leftrightarrow \text{supp } {}^{NP}h_k^{n+1} = \{Y\} \tag{52}$$

Let $n \in \mathbb{Z}$, $k \in S_\omega$, and $X(t) \in \overline{\Delta X}_{n,k}$ with $x^0 = t$; then in the model $M_{\mathbb{Z},\omega,\mathcal{O}}$ the expression

$$\models {}^{NP}f_k^{n+1} = \phi_Y(\Theta_k^{n+1}) \wedge Y = \lim_{t \to t_{n+1,k}} X(t) = X_{n+1,k} \tag{53}$$

models the Elementary Principle of Choice, the sixth of seven axioms of the EPT. This expression means that in the IRF of the observer $\mathcal{O}$, the point-particle ${}^{NP}f_k^{n+1}$ is a choice from a set of possibilities Θ_k^{n+1} strictly determined by the spatiotemporal extension $\overline{\Delta X}_{n,k}$ of the time-like string ${}^{NW}f_k^n$. See Fig. 1 for an illustration in a spacetime diagram. □

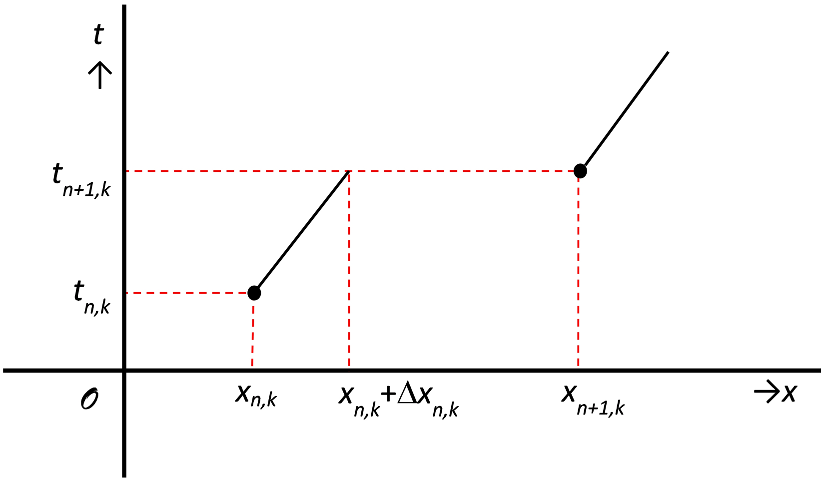

Figure 1: Spacetime diagram illustrating the elementary principle of choice, Eq. (53). The two black dots represent the positions $X_{n,k}$ and $X_{n+1,k}$ as indicated: these are the positions of the particle states ${}^{EP}f_k^n$ and ${}^{NP}f_k^{n+1}$, respectively. The two diagonal line segments represent the line segments $\overline{\Delta X}_{n,k}$ and $\overline{\Delta X}_{n+1,k}$ as indicated: these are the spatiotemporal extensions of the time-like strings ${}^{NW}f_k^n$ and ${}^{NW}f_k^{n+1}$, respectively (cf. Int. 3.11). The spacetime diagram shows a discontinuity: **without** the principle of choice there is no guarantee that $X_{n+1,k} = X_{n,k} + \Delta\vec{x}_{n,k}$, so the transition from the time-like string ${}^{NW}f_k^n$ to the point-particle ${}^{NP}f_k^{n+1}$ at the position $X_{n+1,k}$ could then involve a discontinuity as shown in the diagram. But the principle of choice, as given by Int. 3.25, guarantees that we have $X_{n+1,k} = X_{n,k} + \Delta\vec{x}_{n,k}$ and thus that **no such discontinuity occurs**. So in the IRF of the observer $\mathcal{O}$, the particle state s_k^{n+1} is located where the spatiotemporal extension of the time-like string ${}^{NW}f_k^n$ ends. (In $M_{\mathbb{Z},\omega,\mathcal{O}}$, the higher black dot thus continues the lower line segment).

Remark 3.26 We leave constants ${}^{S}\Phi_k^{n+2}$, which designate the *spatial phase quanta* that occur in the universe of the EPT, **uninterpreted**; the same then goes for the Elementary Principle of Formation of Space, the last of seven axioms of the EPT. The reason for this omission is that these interpretations are not needed for showing that the EPT agrees with SR—after all, in SR spacetime is not a substance. For those who find this omission unacceptable, we can interpret an individual constant ${}^{S}\Phi_k^{n+2}$ as a function ${}^{S}f_k^{n+2} : \mathcal{M} \to {}^*_+\mathbb{R}$ for which ${}^{S}f_k^{n+2}(X) = \gamma_k^{n+1}(X - E_1)$ where $E_1 = (1,0,0,0) \in \mathcal{M}$. The Elementary Principle of Formation of Space, which involves a continuous process, then becomes the expression

$$\models \mathbb{E}\gamma_k^{n+1} \Rightarrow \mathbb{E}\,{}^{S}f_k^{n+2} \tag{54}$$

(with '$\mathbb{E}$' as in Int. 3.24, and with the assumption that the set $G_{\mathbb{Z},\omega,\mathcal{O}}$ now also contains the functions ${}^{S}f_k^{n+2}$) meaning that in the IRF of the observer $\mathcal{O}$, an existing γ-ray leaves a (vanishing) trace of substantial space. To emphasize it: this is just to trivially complete the model. □

3.3 The arrows of the category $\mathscr{C}_{SR}$

There are, then, three kinds of special arrows ('ur-arrows') in the collection of arrows of $\mathscr{C}_{SR}$:

- **permutation arrows** that correspond to a permutation of counting numbers;
- **translation arrows** that correspond to a translation in spacetime;
- **Lorentz arrows** that correspond to a Lorentz transformation.

Below these ur-arrows will be defined precisely; all other arrows are then compositions of these ur-arrows. To define such an ur-arrow, it suffices to define how the individuals in the set $A_{\omega,\mathcal{O}}$ and the individuals in the set $G_{\mathbb{Z},\omega,\mathcal{O}}$ of generators of $\langle G_{\mathbb{Z},\omega,\mathcal{O}}\rangle$ transform: that determines everything else. To see that, let T be an arrow $T : M_{\mathbb{Z},\omega,\mathcal{O}} \to M_{\mathbb{Z},\omega,\mathcal{O}'}$; if $T({}^{NW}f^n_k)$ and $T({}^{NP}f^{n+1}_k)$ are known for all $n \in \mathbb{Z}, k \in S_\omega$, then $\Theta_{\mathbb{Z},\omega,\mathcal{O}'}$ and $\phi_{\mathbb{Z},\omega,\mathcal{O}'}$ are determined by Int. 3.25.

Definition 3.27 Let $M_{\mathbb{Z},\omega,\mathcal{O}}$ be a concrete set-theoretic model of the EPT, and let Σ_ω be the set of all permutations on the section of positive integers S_ω. Then for every $\pi \in \Sigma_\omega$ there is a **permutation arrow** $T_{\mathbb{Z},\omega,\mathcal{O},\pi}$ and a concrete set-theoretic model $M_{\mathbb{Z},\omega,\mathcal{O}'}$ of the EPT given by

$$T_{\mathbb{Z},\omega,\mathcal{O},\pi} : M_{\mathbb{Z},\omega,\mathcal{O}} \to M_{\mathbb{Z},\omega,\mathcal{O}'} \tag{55}$$

$$T_{\mathbb{Z},\omega,\mathcal{O},\pi} : \langle k, \sigma_k, \chi_k\rangle \mapsto \langle \pi(k), \sigma_{\pi(k)}, \chi_{\pi(k)}\rangle \wedge \sigma_{\pi(k)} = \sigma_k \wedge \chi_{\pi(k)} = \chi_k \tag{56}$$

$$T_{\mathbb{Z},\omega,\mathcal{O},\pi} : \ {}^{\alpha}f^n_k \mapsto {}^{\alpha}f'^{\,n}_{\pi(k)} \wedge {}^{\alpha}f^n_k = {}^{\alpha}f'^{\,n}_{\pi(k)} \tag{57}$$

(here α denotes EP, NP, NW, LW, S). □

Loosely speaking, for every inertial observer $\mathcal{O}$ there is an equivalent inertial observer $\mathcal{O}'$ such that the k^{th} process from the n^{th} to the $(n+1)^{\text{th}}$ degree of evolution in the IRF of $\mathcal{O}$ is the $\pi(k)^{\text{th}}$ process from the n^{th} to the $(n+1)^{\text{th}}$ degree of evolution in the IRF of $\mathcal{O}'$. The point is that the numerical value that an observer gives to the label k is trivial: it is only important that the same value is maintained for its successor and its predecessor, and for the events (i.e. the state transitions) in that process.

Definition 3.28 Let $M_{\mathbb{Z},\omega,\mathcal{O}}$ be a concrete set-theoretic model of the EPT. Then for every function τ for which $\tau : S_\omega \times \mathbb{Z} \to \mathbb{Z}$, $\tau : (k, n) \mapsto n + j(k)$, there is a **permutation arrow** $T_{\mathbb{Z},\omega,\mathcal{O},\tau}$ and a concrete set-theoretic model $M_{\mathbb{Z},\omega,\mathcal{O}'}$ of the EPT given by

$$T_{\mathbb{Z},\omega,\mathcal{O},\tau} : M_{\mathbb{Z},\omega,\mathcal{O}} \to M_{\mathbb{Z},\omega,\mathcal{O}'} \tag{58}$$

$$T_{\mathbb{Z},\omega,\mathcal{O},\tau} : \langle k, \sigma_k, \chi_k\rangle \mapsto \langle k, \sigma_k, \chi_k\rangle \tag{59}$$

$$T_{\mathbb{Z},\omega,\mathcal{O},\tau} : \ {}^{\alpha}f^n_k \mapsto {}^{\alpha}f'^{\,\tau(n,k)}_{k} \wedge {}^{\alpha}f^n_k = {}^{\alpha}f'^{\,n+j(k)}_{k} \tag{60}$$

(here α denotes EP, NP, NW, LW, S). □

Loosely speaking, for every inertial observer $\mathcal{O}$ there is an equivalent inertial observer $\mathcal{O}'$ such that the k^{th} process from the n^{th} to the $(n+1)^{\text{th}}$ degree of evolution in the IRF of $\mathcal{O}$ is the k^{th} process from the $(n+j(k))^{\text{th}}$ to the $(n+j(k)+1)^{\text{th}}$ degree of evolution in the IRF of $\mathcal{O}'$. The point is that the numerical value that an observer gives to the degree of evolution n is trivial *in this categorical model*: only the displacement in degrees of evolution matters (vide infra).

Definition 3.29 Let $M_{\mathbb{Z},\omega,\mathcal{O}}$ be a concrete set-theoretic model of the EPT. Then for every $\Delta X \in \mathcal{M}$ with $(\Delta X)^4 = 0$ there is a **translation arrow** $T_{\mathbb{Z},\omega,\mathcal{O},\Delta X}$ and a concrete set-theoretic model $M_{\mathbb{Z},\omega,\mathcal{O}''}$ of the EPT given by

$$T_{\mathbb{Z},\omega,\mathcal{O},\kappa} : M_{\mathbb{Z},\omega,\mathcal{O}} \to M_{\mathbb{Z},\omega,\mathcal{O}''} \tag{61}$$

$$T_{\mathbb{Z},\omega,\mathcal{O},\kappa} : \langle k, \sigma_k, \chi_k\rangle \mapsto \langle k, \sigma_k, \chi_k\rangle \tag{62}$$

$$T_{\mathbb{Z},\omega,\mathcal{O},\kappa} : \ {}^{\alpha}f^n_k \wedge \ \mapsto {}^{\alpha}f''^{\,n}_{k} \wedge {}^{\alpha}f''^{\,n}_{k}(X) = {}^{\alpha}f^n_k(X + \Delta X) \tag{63}$$

(here α denotes EP, NP, NW, LW, S). □

Loosely speaking, for every inertial observer $\mathcal{O}$ there is an equivalent inertial observer $\mathcal{O}''$ who does not move relative to $\mathcal{O}$, such that the constituents of the IRF of $\mathcal{O}''$ are the constituents of the IRF of $\mathcal{O}$ shifted by ΔX. The set of monads $A_{\omega,\mathcal{O}}$ is thus invariant under translation.

Definition 3.30 Let $M_{\mathbb{Z},\omega,\mathcal{O}}$ be a concrete set-theoretic model of the EPT. Then for every Lorentz transformation Λ there is a **Lorentz arrow** $T_{\mathbb{Z},\omega,\mathcal{O},\Lambda}$ and a concrete set-theoretic model $M_{\mathbb{Z},\omega,\mathcal{O}'''}$ of the EPT given by

$$T_{\mathbb{Z},\omega,\mathcal{O},\Lambda} : M_{\mathbb{Z},\omega,\mathcal{O}} \to M_{\mathbb{Z},\omega,\mathcal{O}'''} \tag{64}$$

$$T_{\mathbb{Z},\omega,\mathcal{O},\Lambda} : \langle k, \sigma_k, \chi_k \rangle \mapsto \langle k, \sigma_k, \chi_k \rangle \tag{65}$$

$$T_{\mathbb{Z},\omega,\mathcal{O},\Lambda} : \ {}^{\alpha}f_k^n \mapsto \ {}^{\alpha}{f'''_k}^{n} \wedge \text{supp } {}^{\alpha}{f'''_k}^{n} = \Lambda[\text{supp } {}^{\alpha}f_k^n] \tag{66}$$

$$T_{\mathbb{Z},\omega,\mathcal{O},\Lambda} : \vec{p}(X) \mapsto \Lambda(\vec{p}(X)) \tag{67}$$

where α denotes EP, NP, NW, LW, S, and $\vec{p}(X)$ is any 4-momentum of any object at the point X in the IRF of the observer $\mathcal{O}$. □

Loosely speaking, for every inertial observer $\mathcal{O}$ there is an equivalent inertial observer $\mathcal{O}'''$ who moves relative to $\mathcal{O}$ with constant speed, such that the origins of the IRFs of $\mathcal{O}$ and $\mathcal{O}'''$ coincide, and such that the support of the individuals in $G_{\mathbb{Z},\omega,\mathcal{O}}$ and $G_{\mathbb{Z},\omega,\mathcal{O}'''}$, as well as the 4-momenta at any point in the support, are related by a Lorentz transformation Λ. In other words, an object that has 4-momentum $\vec{p}$ at position X in the IRF of $\mathcal{O}$ has 4-momentum $\Lambda(\vec{p})$ at position $\Lambda(X)$ in the IRF of $\mathcal{O}'''$.

The collection of arrows of the categorical model is then generated by the ur-arrows defined above under arrow composition; for any arrows $T : \text{dom } T \to \text{cod } T$ and $T' : \text{dom } T' \to \text{cod } T'$ with $\text{cod } T' = \text{dom } T$ there is thus an arrow $T \circ T' : \text{dom } T' \to \text{cod } T$. See Fig. 2 for a diagrammatic illustration.

4 Discussion and conclusions

4.1 Worldview

In this section we want to establish a firm contact with the world view of standard SR by formalizing notions of 'events', 'massive particles', and 'massless particles' in the language of $\mathscr{C}_{SR}$.

Definition 4.1 (Events) In the IRF of an inertial observer $\mathcal{O}$, an **event** $\mathcal{E}$ is the manifestation of a discrete transition $g_1 \to g_2$ at a spatiotemporal position X in the IRF of $\mathcal{O}$; we formalize an event $\mathcal{E}$ as a three-tuple $\langle \alpha^1, \alpha^2, \alpha^3 \rangle$ for which

$$\mathcal{E} = \langle X, I_{\mathbb{Z},\omega,\mathcal{O}}(g_1), I_{\mathbb{Z},\omega,\mathcal{O}}(g_2) \rangle \tag{68}$$

An event $\mathcal{E}$ in the IRF of an inertial observer $\mathcal{O}$ and an event $\mathcal{E}'$ in the IRF of an equivalent inertial observer $\mathcal{O}'$ are **equivalent**, notation: $\mathcal{E} \sim \mathcal{E}'$, if and only if $\mathcal{E}$ and $\mathcal{E}'$ are manifestations of *the same* discrete transition in the IRFs of $\mathcal{O}$ adn $\mathcal{O}'$, respectively. □

Notation 4.2 An expression '$\mathcal{E} \xrightarrow{\mathcal{O}} X$', meaning: 'for the observer $\mathcal{O}$ the event $\mathcal{E}$ takes place at spatiotemporal position X', is a notation for '$M_{\mathbb{Z},\omega,\mathcal{O}} \models (\mathcal{E})^1 = X$', that is, the first component of the three-tuple $\mathcal{E}$ is X. (This notation is based on a notation used in [8].) □

Thus speaking, for any $n \in \mathbb{Z}$ and for any $k \in S_\omega$, the following events take place in the k^{th} process from the n^{th} to the $(n+1)^{\text{th}}$ degree of evolution in the IRF of an observer $\mathcal{O}$:

- the **initial event** $\mathcal{E}^I_{n,k}$: this is the discrete transition ${}^{EP}f_k^n \to \ {}^{NW}f_k^n$ at the spatiotemporal position $X_{n,k}$, so that $\mathcal{E}^I_{n,k} \xrightarrow{\mathcal{O}} X_{n,k}$;
- the **collapse event** $\mathcal{E}^C_{n,k}$: this is the discrete transition ${}^{NW}f_k^n \to \ {}^{NP}f_k^{n+1}$ at the spatiotemporal position $X_{n+1,k}$, so that $\mathcal{E}^C_{n,k} \xrightarrow{\mathcal{O}} X_{n+1,k}$;
- the **emission event** $\mathcal{E}^E_{n,k}$: this is the discrete transition ${}^{NP}f_k^{n+1} \to \gamma_k^{n+1}$ at the spatiotemporal position $X_{n+1,k}$, so that $\mathcal{E}^E_{n,k} \xrightarrow{\mathcal{O}} X_{n+1,k}$;
- the **final event** $\mathcal{E}^F_{n,k}$: this is the discrete transitions ${}^{NP}f_k^{n+1} \to \ {}^{EP}f_k^{n+1}$ at the spatiotemporal position $X_{n+1,k}$, so that $\mathcal{E}^F_{n,k} \xrightarrow{\mathcal{O}} X_{n+1,k}$.

The point here is that in particular the absorption and emission of a γ-ray is an event: if γ-rays are absorbed, it is at these events $\mathcal{E}^I_{n,k}$; if γ-rays are emitted, it is at these events $\mathcal{E}^E_{n,k}$.

$$\begin{tikzcd}
 & M_{\mathbb{Z},\omega,\mathcal{O}'} \arrow[loop above, "T_{\mathbb{Z},\omega,\mathcal{O}',I}"] \arrow[d, shift left, "T_{\mathbb{Z},\omega,\mathcal{O}',\pi^{-1}}"] & \\
 & M_{\mathbb{Z},\omega,\mathcal{O}} \arrow[u, shift left, "T_{\mathbb{Z},\omega,\mathcal{O},\pi}"] \arrow[dl, shift left, "T_{\mathbb{Z},\omega,\mathcal{O},\Lambda}"] \arrow[dr, shift left, "T_{\mathbb{Z},\omega,\mathcal{O},\Delta X}"] & \\
M_{\mathbb{Z},\omega,\mathcal{O}'''} \arrow[loop left, "T_{\mathbb{Z},\omega,\mathcal{O}''',I}"] \arrow[ur, shift left, "T_{\mathbb{Z},\omega,\mathcal{O}''',\Lambda^{-1}}"] & & M_{\mathbb{Z},\omega,\mathcal{O}''} \arrow[loop right, "T_{\mathbb{Z},\omega,\mathcal{O}'',0}"] \arrow[ul, shift left, "T_{\mathbb{Z},\omega,\mathcal{O}'',-\Delta X}"] \arrow[ll, bend left, "T_{\mathbb{Z},\omega,\mathcal{O},\Lambda} \circ T_{\mathbb{Z},\omega,\mathcal{O}'',-\Delta X}"]
\end{tikzcd}$$

Figure 2: Diagram illustrating various ur-arrows, identity arrows and composite arrows in the categorical model. The four dots represent the models $M_{\mathbb{Z},\omega,\mathcal{O}}$, $M_{\mathbb{Z},\omega,\mathcal{O}'}$, $M_{\mathbb{Z},\omega,\mathcal{O}''}$, $M_{\mathbb{Z},\omega,\mathcal{O}'''}$ in the collection of objects as indicated. The vertical arrows between $M_{\mathbb{Z},\omega,\mathcal{O}}$ and $M_{\mathbb{Z},\omega,\mathcal{O}'}$ represent two permutation arrows $T_{\mathbb{Z},\omega,\mathcal{O},\pi} : M_{\mathbb{Z},\omega,\mathcal{O}} \to M_{\mathbb{Z},\omega,\mathcal{O}'}$ and $T_{\mathbb{Z},\omega,\mathcal{O}',\pi^{-1}} : M_{\mathbb{Z},\omega,\mathcal{O}'} \to M_{\mathbb{Z},\omega,\mathcal{O}}$ as defined by Def. 3.27. The circular arrow at the top middle is the identity arrow $T_{\mathbb{Z},\omega,\mathcal{O}',I} : M_{\mathbb{Z},\omega,\mathcal{O}'} \to M_{\mathbb{Z},\omega,\mathcal{O}'}$ corresponding with the identity permutation $I : S_\omega \to S_\omega$, $I : k \mapsto k$. Permutation arrows as defined by Def. 3.28 are not shown. The diagonal arrows between $M_{\mathbb{Z},\omega,\mathcal{O}}$ and $M_{\mathbb{Z},\omega,\mathcal{O}''}$ represent two translation arrows $T_{\mathbb{Z},\omega,\mathcal{O},\Delta X} : M_{\mathbb{Z},\omega,\mathcal{O}} \to M_{\mathbb{Z},\omega,\mathcal{O}''}$ and $T_{\mathbb{Z},\omega,\mathcal{O}'',-\Delta X} : M_{\mathbb{Z},\omega,\mathcal{O}''} \to M_{\mathbb{Z},\omega,\mathcal{O}}$ as defined by Def. 3.29. The circular arrow at the lower right is the identity arrow $T_{\mathbb{Z},\omega,\mathcal{O}'',0} : M_{\mathbb{Z},\omega,\mathcal{O}''} \to M_{\mathbb{Z},\omega,\mathcal{O}''}$ corresponding with the zero displacement in $\mathcal{M}$. The diagonal arrows between $M_{\mathbb{Z},\omega,\mathcal{O}}$ and $M_{\mathbb{Z},\omega,\mathcal{O}'''}$ represent two Lorentz arrows $T_{\mathbb{Z},\omega,\mathcal{O},\Lambda} : M_{\mathbb{Z},\omega,\mathcal{O}} \to M_{\mathbb{Z},\omega,\mathcal{O}'''}$ and $T_{\mathbb{Z},\omega,\mathcal{O}''',\Lambda^{-1}} : M_{\mathbb{Z},\omega,\mathcal{O}'''} \to M_{\mathbb{Z},\omega,\mathcal{O}}$ as defined by Def. 3.30. The circular arrow at the lower left is the identity arrow $T_{\mathbb{Z},\omega,\mathcal{O}''',I} : M_{\mathbb{Z},\omega,\mathcal{O}'''} \to M_{\mathbb{Z},\omega,\mathcal{O}'''}$ corresponding with the identity transformation $I : \mathcal{M} \to \mathcal{M}$, $I : X \mapsto X$. The bent arrow at the bottom represents the composite arrow $T_{\mathbb{Z},\omega,\mathcal{O},\Lambda} \circ T_{\mathbb{Z},\omega,\mathcal{O}'',-\Delta X} : M_{\mathbb{Z},\omega,\mathcal{O}''} \to M_{\mathbb{Z},\omega,\mathcal{O}'''}$; for the sake of clarity other (composite) arrows are omitted in the diagram. The diagram commutes.

Definition 4.3 (Massive particles) Let $M_{\mathbb{Z},\omega,\mathcal{O}}$ be a set-theoretic model of the EPT that is an object of $\mathscr{C}_{SR}$. Then for any $k \in S_\omega$, the function t_k, for which

$$t_k : \mathcal{M} \to^*_+ \mathbb{R} \ , \ t_k = \sum_{n=-\infty}^{\infty} t^n_k \tag{69}$$

represents the k^{th} **massive particle**—i.e. an ultimate constituent of matter having rest mass—in the IRF of $\mathcal{O}$, moving on a **world line** ℓ_k for which

$$\ell_k = \text{supp } t_k = \bigcup \{\{X_{n,k}\}, \overline{\Delta X}_{n,k} \mid n \in \mathbb{Z}\} \qquad \text{(for some } k \in S_\omega) \tag{70}$$

(Recall that $t^n_k = {}^{EP}f^n_k + {}^{NW}f^n_k$.) At any $X \in \ell_k$ where ℓ_k is differentiable, the **4-velocity** $\vec{u}(X)$ is given by

$$\vec{u}(X) = \frac{1}{m_k} \cdot \vec{p}(X) = (u^0, u^1, u^2, u^3) \tag{71}$$

where $\vec{p}(X)$ is the 4-momentum at X and m_k the rest mass as given by Def. 3.4. ☐

Definition 4.4 (Massless particles) Let $M_{\mathbb{Z},\omega,\mathcal{O}}$ be a set-theoretic model of the EPT that is an object of $\mathscr{C}_{SR}$. Then any function $\gamma_k^{n+1} \in G_{\mathbb{Z},\omega,\mathcal{O}}$ for which $\mathbb{E}\gamma_k^{n+1}$ represents a **massless particle**—i.e. an ultimate constituent of matter having no rest mass—in the IRF of the inertial observer $\mathcal{O}$, moving on a **world line** $\ell_{k,n+1}^{\gamma}$ for which

$$\ell_{k,n+1}^{\gamma} = \text{supp } \gamma_k^{n+1} \tag{72}$$

The notion of a 4-velocity, as given by Eq. (71), does not apply to massless particles. □

We are now finally in a position to reap the fruits of all the definitions and interpretations by establishing contact between the language of this model of the EPT and existing physical language. For that matter, a description will be given of the k^{th} process from the n^{th} to the $(n+1)^{\text{th}}$ degree of evolution:

(i) the **initial state** of the process is the particle state ${}^{EP}f_k^n$ of the k^{th} massive particle, having position $X_{n,k}$ and 4-momentum $\vec{p}_{n,k}^{\,EP}$—its rest mass m_k is **predetermined** by the rest mass spectrum σ_k;

(ii) the **initial event** of the process is the event $\mathcal{E}_{n,k}^{I}$—by the state transition ${}^{EP}f_k^n \to {}^{NW}f_k^n$ the k^{th} massive particle gets in the wave state ${}^{NW}f_k^n$ with 4-momentum $\vec{p}_{n,k}^{\,NW}$;

(iii) the **law of conservation of 4-momentum** applies: the 4-momentum of the wave state is identical to the 4-momentum of the initial particle state plus 4-momentum of a possibly observed γ-ray;

(iv) the **collapse event** of the process is the next event $\mathcal{E}_{n,k}^{C}$—by the state transition ${}^{NW}f_k^n \to {}^{NP}f_k^{n+1}$ an intermediate particle state ${}^{NP}f_k^{n+1}$ with momentum $\vec{p}_{n+1,k}^{\,NP}$ is produced at the position $X_{n+1,k}$ from the wave state of the k^{th} massive particle;

(v) the **law of conservation of 4-momentum** applies: the 4-momentum of the intermediate particle state is identical to the 4-momentum of the preceding wave state;

(vi) upon the collapse event, we thus have $\mathbb{E}\ {}^{NP}f_k^{n+1}$;

(vii) the **emission event** of the process is the next event $\mathcal{E}_{n,k}^{E}$—by the state transition ${}^{NP}f_k^{n+1} \to \gamma_k^{n+1}$ the γ-ray γ_k^{n+1} is emitted from the spatiotemporal position $X_{n+1,k}$;

(viii) the **final event** of the process is the event $\mathcal{E}_{n,k}^{F}$—upon the emission of the the γ-ray γ_k^{n+1}, by the state transition ${}^{NP}f_k^{n+1} \to {}^{EP}f_k^{n+1}$ the intermediate particle state turns into the next particle state ${}^{EP}f_k^{n+1}$ of the k^{th} massive particle, having position $X_{n+1,k}$ and 4-momentum $\vec{p}_{n+1,k}^{\,EP}$

(ix) the **law of conservation of 4-momentum** applies: the 4-momentum of the new particle state of the k^{th} massive particle is identical to the 4-momentum of the intermediate particle state minus the 4-momentum of the emitted γ-ray;

(x) the **spatiotemporal separation**, i.e. the invariant interval, between the spatiotemporal positions $X_{n,k}$ and $X_{n+1,k}$ of initial and final event is always unity: $\Delta s = 1$.

This holds for any $n \in \mathbb{Z}, k \in S_\omega$ and in the IRF of any observer $\mathcal{O}$: for any observer, all individual processes are essentially the same. By these processes, massive particles alternate between a particle state and a wave state. It doesn't matter whether the massive particle concerns an ultimate constituent of matter or an ultimate constituent of antimatter: the course of events is the same, regardless of the value of the particle's characteristic number of normality. That is a feature that will remain the same also in a more elaborate model of the EPT that includes interactions, but of course then the displacement that takes place will become a function of the particle's properties.

That said, below some lemma's are stated without proof, as well as some remarks: these contribute to an understanding of the categorical model $\mathscr{C}_{SR}$ in terms of particles and events.

Lemma 4.5 For any inertial observer $\mathcal{O}$, any massive particle moves on a continuous, piecewise differentiable world line (i.e., path) in the IRF of $\mathcal{O}$, so that we have

$$\eta(\vec{u}(X), \vec{u}(X)) = 1 \tag{73}$$

for the 4-velocity $\vec{u}(X)$ at any spatiotemporal position X on the particle's world line ℓ in the IRF of $\mathcal{O}$ (provided ℓ is differentiable at X).(See [10] for a definition of a continuous piecewise differentiable function.) □

Lemma 4.6 For any inertial observer $\mathcal{O}$, any massive particle moves piecewise unaccelerated; that is, at any point X of any massive particle's world line ℓ we have for the 4-acceleration

$$\vec{a}(X) = \frac{d}{d\tau}\vec{u}(X) = (0,0,0,0,0) \tag{74}$$

provided ℓ is differentiable at X; here τ is the proper time. □

Remark 4.7 One should realize, however, that the fact that the motion of massive particles is piecewise unaccelerated as defined in Lemma 4.6 **does not imply** that there is no accelerated motion. It is merely the case that if we want to speak about a '4-acceleration' in the present context, then this has to be understood in terms of a change in the 4-velocity of a particle on *subsequent pieces* of its world line. A formal definition is omitted here, since it is not important for the aim of this paper. □

Lemma 4.8 (Universality of light speed) For any inertial observer $\mathcal{O}$, any massless particle moves with the speed of light $c = 1$ through space. That is, at any point X on its world line ℓ we have

$$\left(\frac{dx^1}{dt}\right)^2 + \left(\frac{dx^2}{dt}\right)^2 + \left(\frac{dx^3}{dt}\right)^2 = 1 \tag{75}$$

□

Remark 4.9 (Degrees of evolution vs. invariant interval) The numerical 'degrees of evolution', which occur in the EPT, are a numbering of states in the direction of evolution: every individual process by which a massive particle alternates once between a particle state and a wave state then corresponds to a 'jump' in degrees of evolution of precisely one. In this categorical model $\mathscr{C}_{SR}$ of the EPT, every such jump thus effects a displacement in spacetime with unit Minkowski measure: the difference in degrees of evolution between consecutive particle states of *any* massive particle is identical to the spatiotemporal separation between their (spatiotemporal) positions in the IRF of any inertial observer $\mathcal{O}$. Likewise, photons do not evolve: they remain at the same degree of evolution, and correspondingly the Minkowski measure of any displacement of any photon is zero. □

Remark 4.10 (Reality of Planck time) The unit spatiotemporal displacement between initial and final events of the elementary processes, to which massive particles are subjected, means that there a minimum time quantum: for any inertial observer $\mathcal{O}$, this is the time difference between initial and final events of the elementary processes by which a co-moving massive particle evolves. In this model, this minimum time quantum has been identified with the Planck time: it is, thus, implicitly postulated that the individual processes take place at Planck scale. This identification of the minimum 'process-physical time unit' (pptu) with Planck time is somewhat arbitrary: this has the status of a conjecture—it may very well be that the pptu is orders of magnitude larger than Planck time. But nevertheless, a minimum time quantum is real in this model—its identification with Planck time gives reality to the Planck scale, and leads to verifiable predictions. □

4.2 Kinematics of some physical processes

The objective of this section is to describe three kinds of processes—inertial motion of massive particles, Bremsstrahlung, and laser cooling—in the language of $\mathscr{C}_{SR}$. The statements are purely descriptive: there is no 'why' to the inertial motion or to the Bremsstrahlung.

4.2.1 Inertial motion of massive particles

Definition 4.11 (Inertial motion in $\mathscr{C}_{SR}$) For integers $n \in \mathbb{Z}$ and $k \in S_\omega$, in the model $M_{\mathbb{Z},\omega,\mathcal{O}}$ the k^{th} process from the n^{th} to the $(n+1)^{\text{th}}$ degree of evolution is a process of **inertial motion** if and only if

(i) no γ-ray is absorbed at the initial event $\mathcal{E}^I_{n,k}$, that is, at the discrete transition ${}^{EP}f^n_k \to {}^{NW}f^n_k$ at the position $X_{n,k}$: we thus have $\vec{p}^{\,NW}_{n,k} = \vec{p}^{\,EP}_{n,k}$ as in Eq. (43) with $\vec{p}^{\,LW}_{p,m} = 0$;

(ii) no γ-ray is emitted at the emission event $\mathcal{E}^E_{n,k}$ upon the discrete transition ${}^{NW}f^n_k \to {}^{NP}f^{n+1}_k$ at the position $X_{n+1,k}$: we thus have $\neg\mathbb{E}\gamma^{n+1}_k$ and, from Eqs. (39) and (41), $\vec{p}^{\,NP}_{n+1,k} = \vec{p}^{\,EP}_{n+1,k}$.

□

Translated into terms of particles and events, this means for an inertial observer $\mathcal{O}$ that if a particle exhibits inertial motion between the events $\mathcal{E}_1 \xrightarrow{\mathcal{O}} (t_1, x_1, y_1, z_1, n_1)$ and $\mathcal{E}_2 \xrightarrow{\mathcal{O}} (t_2, x_2, y_2, z_2, n_2)$, $t_2 > t_1$ on its world line ℓ, then the 4-momentum of the particle is a constant, and there is no event $\mathcal{E}_3 \xrightarrow{\mathcal{O}} (t_3, x_3, y_3, z_3, n_3)$ on ℓ with $t_2 > t_3 > t_1$ where a massless particle is emitted or absorbed. See Fig. 3 for an illustration with a spacetime diagram.

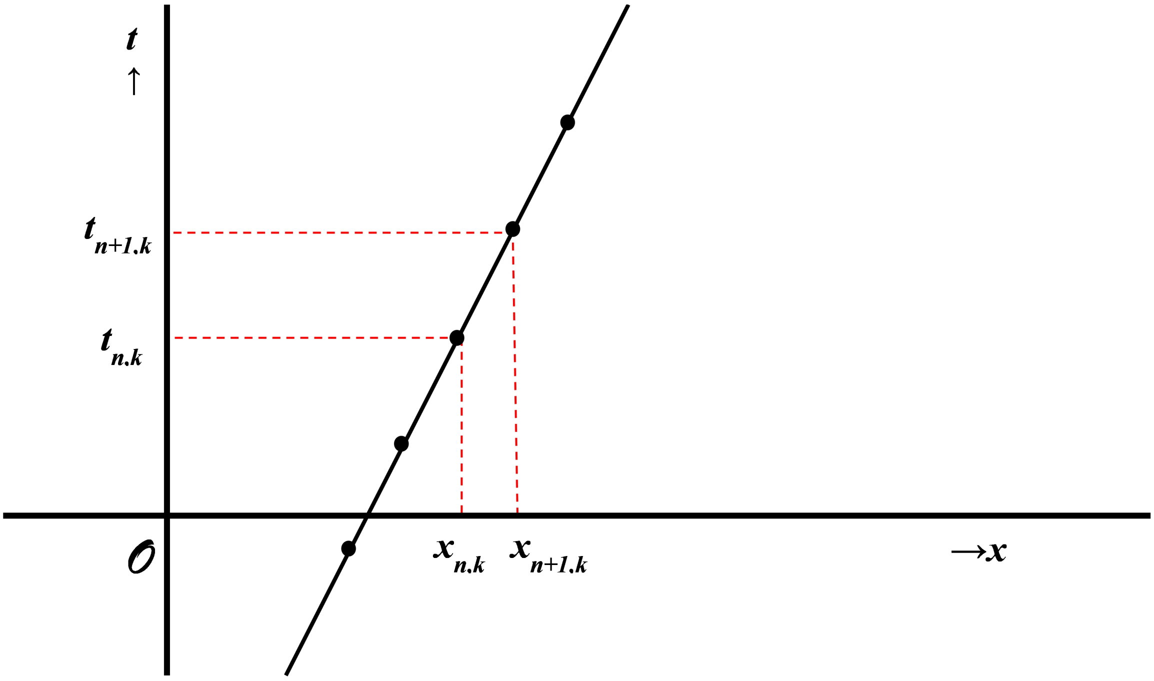

Figure 3: Spacetime diagram of a sequence of processes of inertial motion. Horizontally the spatial coordinates x of the IRF of an inertial observer $\mathcal{O}$, vertically the time coordinates t. The five dots represent subsequent point-particle $s^n_k = {}^{EP}f^n_k$, the line segments connected by the dots represent subsequent time-like strings ${}^{NW}f^n_k$. Together this represents the k^{th} massive particle on its world line ℓ_k; the constant slope of ℓ_k reflects the constant 4-momentum.

4.2.2 Bremsstrahlung

Definition 4.12 (Bremsstrahlung in $\mathscr{C}_{SR}$) For integers $n \in \mathbb{Z}$ and $k \in S_\omega$, in the model $M_{\mathbb{Z},\omega,\mathcal{O}}$ the k^{th} process from the n^{th} to the $(n+1)^{\text{th}}$ degree of evolution is a process with **Bremsstrahlung** if and only if

(i) no γ-ray is absorbed at the initial event $\mathcal{E}^I_{n,k}$, that is, at the discrete transition ${}^{EP}f^n_k \to {}^{NW}f^n_k$ at the position $X_{n,k}$: we thus have $\vec{p}^{\,NW}_{n,k} = \vec{p}^{\,EP}_{n,k}$ as in Eq. (43) with $\vec{p}^{\,LW}_{p,m} = 0$;

(ii) a γ-ray is emitted at the emission event $\mathcal{E}^E_{n,k}$, that is, at the discrete transition ${}^{NP}f^{n+1}_k \to \gamma^{n+1}_k$ at the position $X_{n+1,k}$: we thus have $\mathbb{E}\gamma^{n+1}_k$ and, from Eqs. (39) and (41), $\vec{p}^{\,EP}_{n+1,k} := \vec{p}^{\,NP}_{n+1,k} - \vec{p}^{\,LW}_{n+1,k}$.

□

So as a simple example, consider that the point-particle ${}^{EP}f^n_k$ has 4-momentum $(E, p_x, 0, 0)$, such that $p_x > 0$ and $-E^2 + (p_x)^2 = -m^2$. At its transition to the time-like string ${}^{NW}f^n_k$, this 4-momentum is conserved, so at any point on the line segment occupied by the time-like string ${}^{NW}f^n_k$, the 4-momentum is also $(E, p_x, 0, 0)$. Upon the transition of the time-like string ${}^{NW}f^n_k$ to the intermediate point-particle ${}^{NP}f^{n+1}_k$, the latter emits a γ-ray with 4-momentum $(\Delta E, \Delta p_x, 0, 0)$ with $p_x > \Delta p_x > 0$ and $\Delta E = \Delta p_x$. Upon emission, the point-particle ${}^{NP}f^{n+1}_k$ then transforms into the new point-particle ${}^{EP}f^{n+1}_k$: its 4-momentum is then $(E', p_x - \Delta p_x, 0, 0)$ for which $-(E')^2 + (p_x - \Delta p_x)^2 = -m^2$ so $E' < E$.

Translated into terms of particles and events, this means for an inertial observer $\mathcal{O}$ that if a particle emits Bremsstrahlung between the events $\mathcal{E}_1 \xrightarrow{\mathcal{O}} (t_1, x_1, y_1, z_1, n_1)$ and $\mathcal{E}_2 \xrightarrow{\mathcal{O}} (t_2, x_2, y_2, z_2, n_2)$ on its world line ℓ, $t_2 > t_1$, then the energy and spatial momentum of the particle decrease stepwise through the emission of massless particles (photons). See Fig. 4 for an illustration with a spacetime diagram.

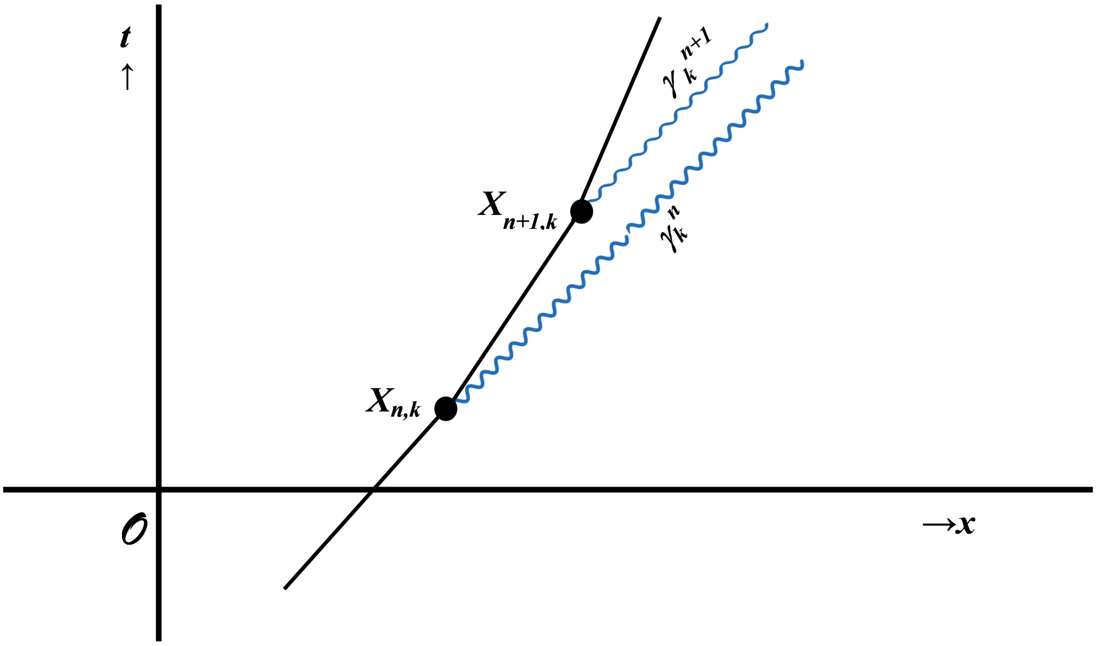

Figure 4: Spacetime diagram of subsequent processes with Bremsstrahlung. Horizontally the spatial coordinates x of the IRF of an inertial observer $\mathcal{O}$, vertically the time coordinates t. The two dots represent subsequent point-particles $s^n_k = {}^{EP}\!f^n_k$ and $s^{n+1}_k = {}^{EP}\!f^{n+1}_k$, the line segments connected by the dots represent subsequent time-like strings ${}^{NW}\!f^{n-1}_k$, ${}^{NW}\!f^n_k$, and ${}^{NW}\!f^{n+1}_k$. The wavy blue lines represent emitted γ-rays γ^n_k and γ^{n+1}_k. Together this represents the k^{th} massive particle on its world line ℓ_k, plus two emitted photons; the increasing slope of ℓ_k reflects the stepwise deceleration.

4.2.3 Laser cooling

Definition 4.13 (Laser cooling in $\mathscr{C}_{SR}$) For integers $n \in \mathbb{Z}$ and $k \in S_\omega$, in the model $M_{\mathbb{Z},\omega,\mathcal{O}}$ the k^{th} process from the n^{th} to the $(n+1)^{\text{th}}$ degree of evolution is a process with **laser cooling** if and only if

(i) a γ-ray γ^p_m from a laser source is absorbed at the initial event $\mathcal{E}^I_{n,k}$, that is, at the discrete transition ${}^{EP}\!f^n_k \to {}^{NW}\!f^n_k$ at the position $X_{n,k}$: for some $p \in \mathbb{Z}$ and $m \in S_\omega$ we thus have $\vec{p}^{\,NW}_{n,k} = \vec{p}^{\,EP}_{n,k} + \vec{p}^{\,LW}_{p,m}$ as in Eq. (43), but in particular with $E^{NW}_{n,k} < E^{EP}_{n,k}$ (decreasing energy);

(ii) no γ-ray is emitted at the emission event $\mathcal{E}^E_{n,k}$ upon the discrete transition ${}^{NW}\!f^n_k \to {}^{NP}\!f^{n+1}_k$ at the position $X_{n+1,k}$: we thus have $\neg\mathbb{E}\gamma^{n+1}_k$ and, from Eqs. (39) and (41), $\vec{p}^{\,NP}_{n+1,k} = \vec{p}^{\,EP}_{n+1,k}$.

□

So as a simple example, consider that the point-particle ${}^{EP}\!f^n_k$ has 4-momentum $(E^{EP}_{n,k}, p_x, 0, 0)$, such that $p_x > 0$ and $-(E^{EP}_{n,k})^2 + (p_x)^2 = -m^2$. At its transition to the time-like string ${}^{NW}\!f^n_k$, a γ-ray is absorbed with 4-momentum $(\Delta E, -\Delta p_x, 0, 0)$ with $-\Delta p_x < 0$ and $\Delta E = \Delta p_x$. Then at any point on the line segment occupied by the time-like string ${}^{NW}\!f^n_k$, the 4-momentum is $(E^{NW}_{n,k}, p_x - \Delta p_x, 0, 0, m)$ for which $-(E^{NW}_{n,k})^2 + (p_x - \Delta p_x)^2 = -m^2$ so that $E^{NW}_{n,k} < E^{EP}_{n,k}$.

Translated into terms of particles and events, this means for an inertial observer $\mathcal{O}$ that if a particle is laser cooled between the events $\mathcal{E}_1 \xrightarrow{\mathcal{O}} (t_1, x_1, y_1, z_1, n_1)$ and $\mathcal{E}_2 \xrightarrow{\mathcal{O}} (t_2, x_2, y_2, z_2, n_2)$ on its world line ℓ, $t_2 > t_1$, then the energy and spatial momentum of the particle decrease stepwise through the absorption of massless particles (photons) emitted by a laser tube. See Fig. 5 for an illustration with a spacetime diagram.

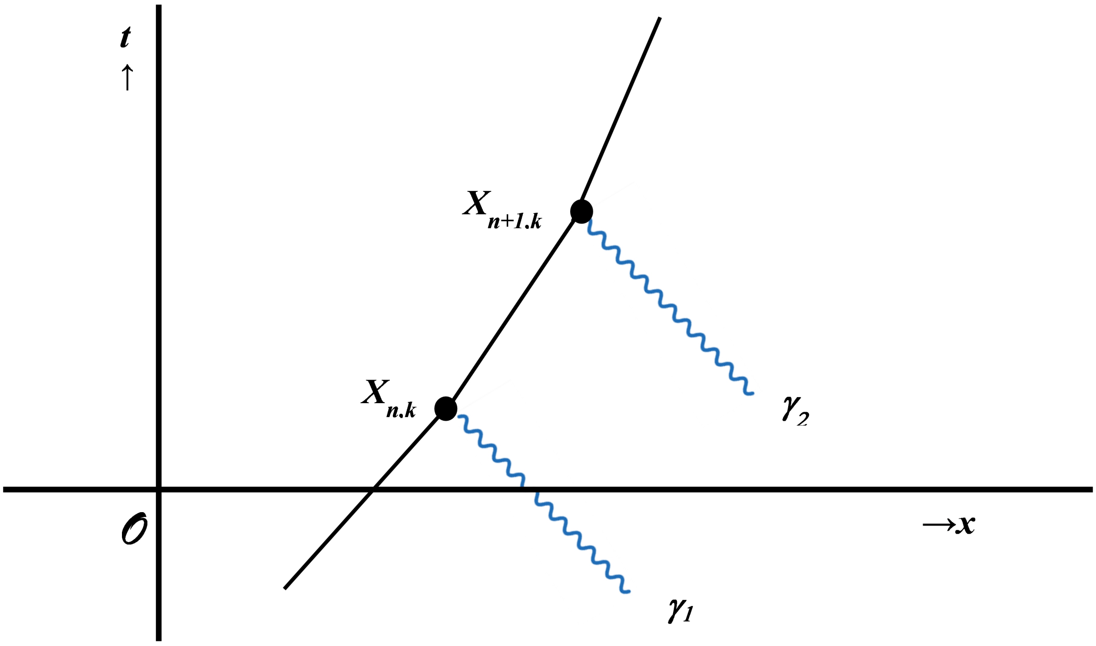

Figure 5: Spacetime diagram of subsequent processes with laser cooling. Horizontally the spatial coordinates x of the IRF of an inertial observer $\mathcal{O}$, vertically the time coordinates t. The two dots represent subsequent point-particles $s_k^n = {}^{EP}f_k^n$ and $s_k^{n+1} = {}^{EP}f_k^{n+1}$, the line segments connected by the dots represent subsequent time-like strings ${}^{NW}f_k^{n-1}$, ${}^{NW}f_k^n$, and ${}^{NW}f_k^{n+1}$. The wavy blue lines represent γ-rays γ_1 and γ_2 from a laser source that are absorbed at the points $X_{n,k}$ and $X_{n+1,k}$, respectively. Together this represents the k^{th} massive particle on its world line ℓ_k, plus two absorbed photons; the increasing slope of ℓ_k reflects the stepwise deceleration by laser cooling.

4.3 Conclusions

In this paper a categorical model of the EPT incorporating SR has been specified: the main conclusion of this tedious exercise is that this proves that the EPT agrees with SR. This result renders the EPT consistent with the outcome of real-world experiments and observations that can be described as predictions of SR—examples are the null result of the Michelson-Morley experiment [11], and the observed prolonged lifetime of fast muons [12]. In addition, it has been shown that laser cooling and Bremsstrahlung can be described in the language of the categorical model $\mathscr{C}_{SR}$.

A main outcome is that an individual process effects a unit jump in space-time: for any individual process and for any observer, the spatiotemporal separation Δs between the spatiotemporal positions of the initial particle state and final particle state of the process is always unit—that is, always satisfies $\Delta s = \sqrt{\Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2} = 1$. This directly relates the process-physical principles of the EPT to observable motion of massive (anti)particles.

The present study doesn't purport to yield an advancement in relativity theory. In addition, a limitation of this study is that it has been focused purely at demonstrating the agreement of the EPT with SR, and with SR alone. Further research is therefore required to establish whether or not the EPT agrees with the knowledge of the physical world obtained from the experimentally confirmed predictions of modern, relativistic interaction theories.

Acknowledgements This study has been facilitated by the Foundation Liberalitas.

References

[1] M.J.T.F. Cabbolet, *Ann. Phys.* **522**, 699-738 (2010); **523**, 990-994 (2011); **528**, 626-627 (2016)

[2] M.J.T.F. Cabbolet, *A methodolocal note on proving agreement between the Elementary Process Theory and modern interaction theories*, Preprint: philpapers.org/rec/MARAGN-2 (2018)

[3] A. Einstein, *Ann. Phys. (Leipzig)* **17**, 891-921 (1905)

[4] S. Eilenberg, S. MacLane, *Trans. Amer. Math. Soc.* **58**, 231-294 (1945)

[5] J. Butterfield, *Br. J. Philos. Sci.* **57**(4), 709-753 (2006)

[6] M.J.T.F. Cabbolet, *Hyperreal delta functions as a new general tool for modeling physical states with infinitely high densities* (Unpublished, 2018)

[7] E. Schrödinger, *Phys. Rev.* **28**(6), 1049-1070 (1926)

[8] B.F. Schutz, *A First Course in General Relativity*, Cambridge: Cambridge University Press (1990)

[9] L. Debnath, P. Mikunski, *Introduction to Hilbert Spaces with Applications*, Academic Press, San Diego, p. 39 (1990)

[10] N.E. Sofronidis, *Real Anal. Exchange* **31**(1), 13-22 (2005)

[11] A.A. Michelson, E.W. Morley, *Am. J. Sci.* **34**, 333-345 (1887)

[12] B. Rossi, D.B. Hall, *Phys. Rev.* **59**(3), 223-228 (1941)