Theoretical mass value of electron, neutrino and other particles by means of QED together with Gravitational theory

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Abstract

From time involved in pair production process, in the photon-photon collision, we consider energy and then mass uncertainty. Taking the latter as the mass of a particle, we repeat pair production process obtaining another mass uncertainty, then another particle mass, and so on. Starting with the heaviest charged elementary particle that is possible using Planck length for electromagnetic mass, we obtain a mass value close to electron mass. It is supposed exactly electron mass, considering the presence of uncertainties in the process. Then plausible neutrino mass and other particles masses are obtained.

1 Introduction

Photon-photon pair production is defined by Breit-Wheleer cross-section equation and consequent Γ rate process equation. Then it is necessary to obtain photon density. It is achieved hypothesizing and calculating a minimum volume around a photon. Considering Γ and then time process $T \equiv \Delta t$, ΔE, Δm and Δm_0 are obtained. Δm_0 is assumed equal to an existing elementary particle mass, even if not known. Well, at first, the photon production of the heaviest charged particle (obtained considering Planck length for classical radius in electromagnetic mass) and related Δm_0 is taken. Then $\Delta m_0 \equiv$ particle mass is also considered for particles produced in another photon-photon pair production process, and so on. At one point a mass value close to electron mass is obtained. For the presence of theoretic uncertainties, the process is supposed really considering electron mass. With the precise electron mass value, an accurate formula for providing several particles masses, for each photon-photon pair production process, is obtained. So unknown charged particles mass and at last a neutral one are provided by formula together other physical data.

2 Equivalence between energy-mass uncertainties and particles mass in photon-photon pair production

We assume that a quantum uncertainty ΔA of a size A has to correspond to a real observable object. As 1 mm in a ruler corresponds to a real object with the same extension, then this is for all uncertainties of all sizes. So Δm corresponds to a real isolated portion of matter $M = \Delta m$; if M didn't exist, why should $\Delta m = M$ exist? In particular we can consider the uncertainty on rest mass, so that Δm_0 could amount to a real particle with rest mass M_0. We observe that Δm_0, because of uncertainty definition itself, is not divisible, in analogy with an elementary particle.

We focus attention on pair production process $\gamma\gamma \longrightarrow e^+e^-$ defined by Breit-Wheeler equation and more general $\gamma\gamma \longrightarrow p^+p^-$ with p a generic particle. We take the time of process (inverse rate) as indetermination on time in $\Delta t \cdot \Delta E \geq \frac{\hbar}{2}$ and then in $\Delta t \cdot \Delta m \geq \frac{\hbar}{2 \cdot c^2}$. Breit-Wheeler cross section for unpolarized photons in the mass center system [1] [5] [4]:

$$\sigma = \frac{\pi}{2}\left(\frac{\alpha\hbar}{m_0 c}\right)^2 (1-\beta^2)\left[(3-\beta^4)\ln\frac{1+\beta}{1-\beta} - 2\beta(2-\beta^2)\right] \qquad \beta = \frac{v}{c} \quad (1)$$

Photons velocity is the same but opposite ($|\overrightarrow{c}_1| = |\overleftarrow{c}_2|$), and also particles velocity is the same but opposite ($|\overleftarrow{v}_1| = |\overrightarrow{v}_2|$).

3 Obtaining energy-mass uncertainties

Rate process is:

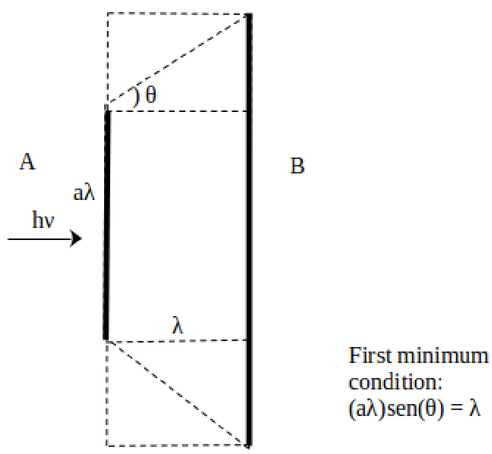
$$\Gamma = \sigma \rho c \quad (2)$$

$\rho = \frac{1}{V}$ = photon density , c = light speed. $T = \frac{1}{\Gamma} = \Delta t$.

The problem is to obtain a relation for volume V containing a photon.

Definable minimum length in the motion direction of the photon is its wave length λ. Minimum length perpendicular to the motion direction could be defined considering the minimum diffraction figure area for a given slit. A slit is equivalent to a space uncertainty. In particular we consider the minimum average between a circular slit (A) and its figure diffraction maximum area (B) corresponding to the first minimum diffraction condition (Fig. 1 and Eq. (3)-(4)).

Figure 1:



$$\overline{V} = \frac{V_A + V_B}{2} = \frac{\lambda \cdot \pi (\frac{a\lambda}{2})^2 + \lambda \cdot \pi \left[\frac{a\lambda + 2\lambda \frac{\lambda}{a\lambda} \frac{1}{\sqrt{1 - \frac{\lambda^2}{(a\lambda)^2}}}}{2} \right]^2}{2} \tag{3}$$

Then the minimum average of related cylindrical volumes is given equaling the derivative of this formula to zero.

$$\frac{d\overline{V}}{da} = 0 \tag{4}$$

We have a minimum (in Eq. (3)) for a = 1,63420, then $\overline{V} = 5,02378 \cdot \lambda^3$. This is the smallest (cylindrical) volume where we can observe an unpolarized photon.

Eq. (1) has a maximum for $\beta = 0,701316$, then $\sigma = \frac{\pi}{2}\left(\frac{\alpha\hbar}{m_0 c}\right)^2 \cdot 1,36341$ is the maximum for cross section.

Eq. (2) becomes: $\Gamma = \frac{\pi}{2}\left(\frac{\alpha\hbar}{m_0 c}\right)^2 \cdot 1,36341 \cdot \frac{1}{5,02378\cdot\lambda^3} \cdot c$.

$\lambda = \frac{h}{mc} = \frac{h}{\frac{m_0}{\sqrt{1-\beta^2}}c} = \frac{h}{m_0 c} \cdot 0,712850 = \lambda_0 \cdot 0,712850$.

So Eq. (2) becomes: $\Gamma = \frac{\alpha^2}{8\pi}\lambda_0^2 \cdot 1,36341 \cdot \frac{1}{5,02378\cdot 0,362238\cdot\lambda_0^3} \cdot c = \frac{\alpha^2}{8\pi} \cdot \frac{1,36341}{5,02378\cdot 0,362238} \cdot \frac{m_0 c^2}{h} = \frac{\alpha^2 m_0 c^2}{8\pi h} \cdot 0,749207$. Then $\Delta t = \frac{8\pi h}{\alpha^2 m_0 c^2 \cdot 0,749207}$, and $\Delta m = \frac{\hbar}{c^2}\frac{\alpha^2 m_0 c^2}{8\pi h} \cdot 0,749207 = \frac{\alpha^2 m_0}{16\pi^2} \cdot 0,749207$.

Because $m_0 \pm (\Delta m)_0 = \sqrt{1-\beta^2}\,(m \pm \Delta m)$ we have: $(\Delta m)_0 = \sqrt{1-\beta^2}\Delta m = \sqrt{1-\beta^2}\frac{\alpha^2 m_0}{16\pi^2} \cdot 0,749207 = 0,712850 \cdot \frac{\alpha^2 m_0}{16\pi^2} \cdot 0,749207 = \frac{\alpha^2 m_0 \cdot 0,534072}{16\pi^2}$.

$(\Delta m)_0 = m_0^I$, then $m_0^{II} = \frac{\alpha^2 m_0^I \cdot 0,534072}{16\pi^2} = \left(\frac{\alpha^2 \cdot 0,534072}{16\pi^2}\right)^2 m_0$. In general we have:

$$m_0^{(n)} = \left(\frac{\alpha^2 \cdot 0,534072}{16\pi^2}\right)^n m_0 = \left(1,80098 \cdot 10^{-7}\right)^n m_0 \qquad (5)$$

4 Heaviest charged particle and other particles

Considering $M_0 c^2 = \frac{q^2}{4\pi\varepsilon_0 c^2 r_0}$, for $r_0 = l_P = \sqrt{\frac{\hbar G}{c^3}}$ and $\alpha = \frac{q^2}{4\pi\varepsilon_0 c\hbar}$, we have $M_0 = \alpha\sqrt{\frac{\hbar c}{G}} = \alpha M_P$ (l_P *and* M_P are Planck length and Planck mass). $M_0 = 1,58822 \cdot 10^{-10}$ Kg is the heaviest charged elementary particle mass that is possible. We put $m_0 = M_0$.

Here we don't consider possible gravitational corrections to M_0 and eventually to Eq. (1). It could be of the order of $\frac{GM_0^2}{r_0 c^2} = \frac{G\alpha^2 M_P^2}{r_0 c^2} = \frac{\alpha^2\hbar c}{r_0 c^2} = \alpha\frac{q^2}{4\pi\varepsilon_0 r_0 c^2} = \alpha M_0$, then about $\frac{1}{137}M_0$.

From (5), for $m_0 = M_0$ and n = 3 we obtain $m_0^{III} = 9,27764 \cdot 10^{-31}$ Kg. This value is close to electron rest mass ($9,10938215 \cdot 10^{-31}$ Kg). We have to consider that Eq. (1) is approximated at the first order in QED. A superior order make a negative contribution to the cross section [2], then we have to expect a value even closer to electron rest mass (at the moment we don't have more precise calculations available). So we assume m_0^{III} = electron rest mass. On this way we can obtain a more precise coefficient in (5):

$$k = \sqrt[3]{\frac{m_0^{III}}{m_0}} = 1,79003 \cdot 10^{-7} (m_0 = M_0). \tag{6}$$

The difference between this value and that one in (5) is approximately $0,6\%$, about the possible order of magnitude of second order approximation ($\propto \alpha \cdot \sigma$ referring to cross section σ [2]).

Equation (5) becomes:

$$m_0^{(n)} = \left(1,79003 \cdot 10^{-7}\right)^n m_0 \tag{7}$$

With this we obtain the following table.

	rest mass (Kg)	charge
$m_0(M_0)$	$1,58822 \cdot 10^{-10}$	± 1
m_0^I	$2,84296 \cdot 10^{-17}$	± 1
m_0^{II}	$5,08896 \cdot 10^{-24}$	± 1
$m_0^{III}(electron)$	$9,10938 \cdot 10^{-31}$	± 1
m_0^{IV}	$1,63060 \cdot 10^{-37}$	0

The last particle m_0^{IV} has a null charge; in fact charged particles lighter than electron would be easily observable if they existed. We suppose m_0^{IV} is a neutrino mass eigenvalue about the three possible (or, for a specific neutrino flavor, the mixing of the three eigenvalues ?). It is in agreement with neutrino experimental rest mass upper limit [3] . This "generation" of particles stops with m_0^{IV} because the process, described by Eq. (1) and following, is applicable only to charged particles (± 1).

Muon and Tauon mass values aren't included in this scheme, but they could be found by another way. However $m_{0\mu}^I = \left(1,79003 \cdot 10^{-7}\right) m_{0\mu}$ and $m_{0\tau}^I = \left(1,79003 \cdot 10^{-7}\right) m_{0\tau}$ could be neutrino mass eigenvalues.

5 Conclusion

Electron, neutrino and other unknown particles mass values have been obtained starting with the heaviest charged particle that is possible, using Planck length. So gravitational theory (General Relativity theory) and Quantum electrodynamics theory (QED) are involved in obtaining these particles mass values. It can be considered a result in demonstrating a deep connection between QED, GR and particles in nature. Obviously experimental results in particles physics will confirm, reject or approach these mass values.

References

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