Theoretical necessity of the Accelerating Universe

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Abstract

We obtain that Universe space has to be flat, but also, at the same time, it has to be finite, homogeneous (on global scale) and isotropic. So rays of light can be observed moving along parallel trajectories in an expanding finite hypersphere. We show this implies an accelerated expansion of the Universe. Considering the energy conservation problem, also we argue about the necessity of matter-antimatter asymmetry.

Introduction

The accelerated expansion of the Universe was discovered in 1998 [11] [12] [13] [14]. The mysterious Dark energy is still today considered the cause of acceleration. But apart from some hypotheses about its nature, such as energy vacuum or modified gravity [3] [8], nothing is known. This work has the objective to explain this phenomenon, starting with application of gravitational equations to a global, isotropic, energy-matter homogeneous and finite space. Then precisely, an hyper-spherical space. But this application is disregarded, contrary to what has always been thought, and also we show a violation of the general covariance principle. These results imply the existence of an ”Euclidean” space (or also hyperbolic?), but in a finite space. Two rays of light moving in parallel trajectories in an expanding hypersphere, or

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more simply in an its bidimensional subspace, give an accelerated expansion. It is very suggestive that to accomplish to the energy conservation we have to assign a negative energy to the vacuum space, getting the matter-antimatter asymmetry, although in a qualitative manner.

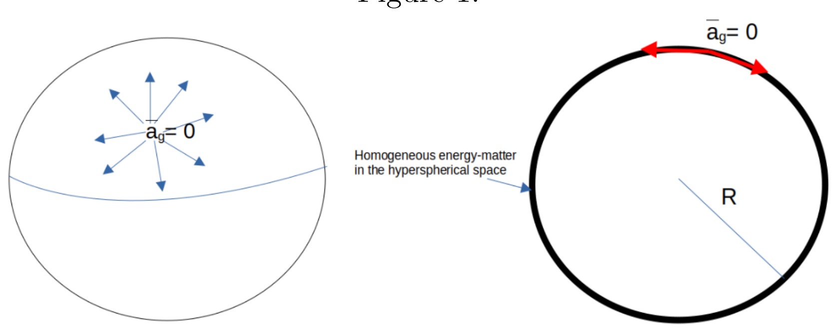
1 No gravitational contribution to the Universe space global curvature

Einstein, De Sitter, Lemaitre and many others after them applied gravitational field equations $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \chi T_{\mu\nu}$ to the global curvature of the homogeneous Universe space [5] [7] [9], in a similar manner as they are applied to every local gravitational field inside a not homogeneous portion of space. An intrinsic, auto-consistent curvature of the entire Universe space, due to the gravitational field of all energy-matter, is a very suggestive and persuading assumption.

But no reason seems to justify this arbitrary assumption in a finite (closed), isotropic and homogeneous, then hyperspherical Universe.

Moreover, a local curvature in a space point, due to a localized energy-mass, is always related with a not null field in that point. But it is not considered that in a hyperspherical space, in which energy and matter are uniformly distributed, the total gravitational field, globally permeating this intrinsic space and acting in each point, is zero, **Fig.1**;

Figure 1:



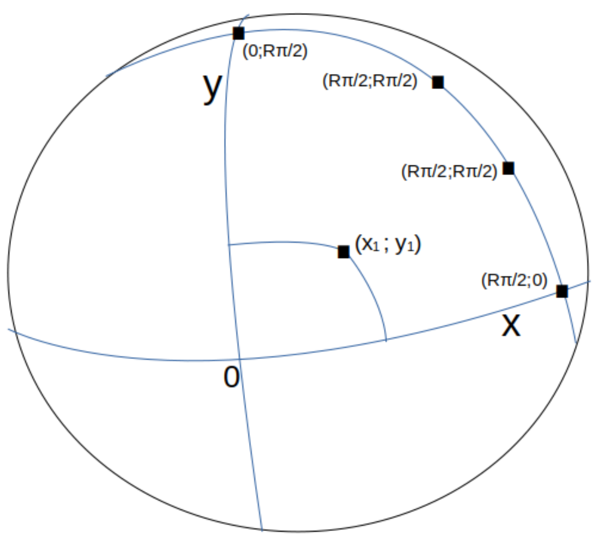
in each point, the total field (including each type of field component), generated in each point of the global space with a spherical geometry, then with no boundary, is null (perhaps can gravitational field have only a not-null resultant along time dimension with effects in space, exactly as an external spatial dimension? However it appears difficult to understand).

On the other hand, inversely, it is clear that the presence of a curvature, also global in time-space, has to imply a not-null gravitational field; we will see in section 3 how to solve this problem.

2 General covariance principle and not equivalence between Riemannian spherical and Euclidean space

In a Riemannian closed space, an hiperspherical space, a Cartesian reference system is not equivalent to one with polar coordinates, at the finite. In fact, referring to a bi-dimensional space for simplicity (a subspace of a tridimensional one), the point $(\frac{\pi}{2}R, \frac{\pi}{2}R)$ in Cartesian coordinates (R is the radius of the sphere defined in an fictitious space) corresponds to infinite points, along the geodetic that connects the two points with coordinates $(\frac{\pi}{2}R, 0)$ and $(0, \frac{\pi}{2}R)$, **Fig.2**.

Figure 2:



So, points on this line cannot be described in Cartesian coordinates.

On the other hand, the same points can be described in polar coordinates. Then general covariance principle ([9]) is not respected; the two reference systems are not equivalent.

In an Euclidean space, at the finite, Cartesian and polar reference systems are both able to describe all points in the space.

Also this is true in an Hyperbolic space, but there are Cartesian coordinates (x, y) with $x = y$ not corresponding to points of the space, as it is demonstrable by hyperbolic trigonometry. This doesn't happen with polar coordinates. Then in some sense, Cartesian and polar coordinates aren't equivalent in an hyperbolic space too.

Anyway, in an Euclidean space, at the finite, Cartesian and polar reference systems are equivalent to describe points in the space and phenomena in them. So general covariance principle is not violated in this space.

3 Flat space and accelerating finite Universe

For that previously said, in section 1 and 2, we assume:

1) a flat space of the Universe.

At the same time we assume:

2) an isotropic and homogeneous, finite (closed) and then a Universe with no boundary (hyperspherical); these being empirical, intuitive and concrete prescriptions (assuming a finite quantity of energy-matter implies a finite density and a finite space), but also see [1] for a rational denial of actual infinity.

But how to accommodate these two statements?

First of all, for physical Euclidean geometry we intend a space in which two rays of light can be observed moving along two parallel trajectories (parallel condition). This is a necessary condition for an Euclidean geometry.

We have to see if this condition can be satisfied in an expanding hyperspherical space, and, in case, the behavior in time.

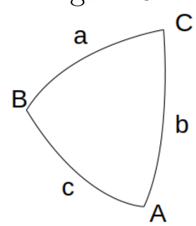
We can consider a spherical surface, then a subspace of an hyperspherical surface, defined by a radius R, in a fictitious space, only depending on time t.

Using spherical trigonometry, "parallel condition" is as follows.

Fundamental formula for a triangle is:

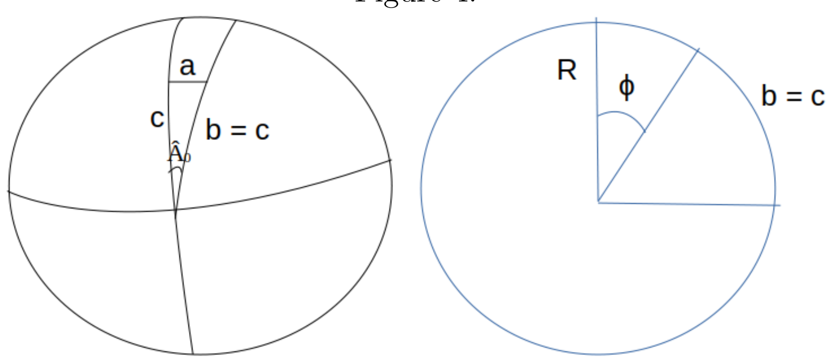
$\cos(\frac{a}{R}) = \cos(\frac{b}{R})\cos(\frac{c}{R}) + \sin(\frac{b}{R})\sin(\frac{c}{R})\cos\hat{A}$, **Fig.3**.

Figure 3:



In our case we have:
$\frac{b}{R} = \frac{c}{R} = \frac{\pi}{2} - \phi$, and "a" the distance between the two rays of light, **Fig.4**.

Figure 4:



So: $\cos(\frac{a}{R}) = \sin^2\phi + \cos\hat{A}_0 \cos^2\phi$,
then: $\cos(\frac{a}{R}) = 1 - \cos^2\phi + \cos\hat{A}_0 \cos^2\phi$, and:

$$\phi = \arccos\sqrt{\frac{1 - \cos(\frac{a}{R})}{1 - \cos\hat{A}_0}} \tag{1}$$

Parallel condition simply corresponds to consider $a = constant$ **during the expansion** (then increasing R), **Fig.5**.

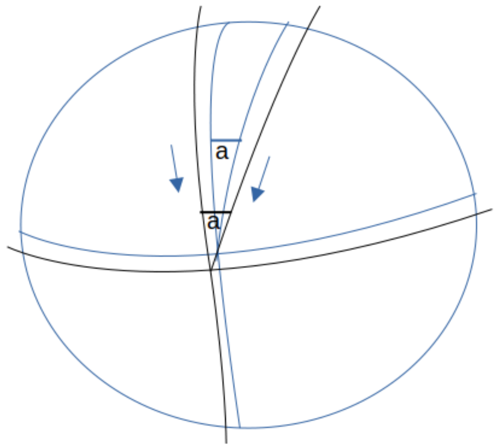
R is time-dependent ($R \equiv R(t)$) and $\hat{A}_0 = constant$ during the expansion; geodetics remain along the same direction, and proportions are preserved.

To find time-dependency of R we have to consider:

$$R\frac{d\phi}{dt} = c \tag{2}$$

that is the local light speed in the local space. We have to solve $\frac{d\phi}{dt}$ with (1). So:

Figure 5:



$$\frac{d\phi}{dt} = \frac{aR^{'}\sin(\frac{a}{R})}{2R^2\sqrt{1-\cos\hat{A}_0}\sqrt{1-\cos(\frac{a}{R})}\sqrt{1-\frac{1-\cos(\frac{a}{R})}{1-\cos\hat{A}_0}}} \tag{3}$$

with "a" and $\hat{A}_0$ constant over time.

For $\frac{a}{R} \to 0$ ($R \to \infty$), $\hat{A}_0 \simeq 0$ and $\sqrt{1-\cos x} \simeq \frac{\sqrt{2}}{2}x - \frac{\sqrt{2}}{48}x^3$ with $x \simeq 0$ we have:

$$\frac{d\phi}{dt} \simeq \frac{aR^{'}}{\hat{A}_0 R^2} \qquad \frac{a}{\hat{A}_0} = r_0 \tag{4}$$

with $r_0 = k_0 R_0$ an initial undetermined condition (R_0 is R for t = 0, $0 < k_0 \leq 1$ depends on ϕ_0). It is independent of the "a" value ($\hat{A}_0$ is proportional to "a").

Substituting (4) in (2):

$$R^{'} \simeq \frac{c}{r_0}R \tag{5}$$

And:

$$R^{''} \simeq \frac{c}{r_0}R^{'} \simeq \frac{c^2}{r_0^2}R \tag{6}$$

$R^{'}$ defines the space expansion speed and $R^{''}$ the space expansion acceleration.

So, to accomplish the parallel condition of two rays of light (physical Euclidean geometry) and in any case for avoiding a physical Riemannian-spherical geometry, acceleration of the Universe expansion is necessary.

Considering (5) and (6), the solution for R is:

$$R \simeq r_1\, e^{\frac{c}{r_0}t} \tag{7}$$

With r_1 another constant. Equation (7) isn't automatically valid for $t \simeq 0$ (being $\frac{a}{R_0} \not\simeq 0$).

Exponential trend is compatible with models including cosmological constant [16].

We note it can be $R' \gg c$, but we are considering a space expansion, not a motion in space.

4 Energy conservation principle and matter-antimatter asymmetry

We note the difficulty to accomplish to energy conservation, a finite energy-matter being not able to support an undefined over time matter acceleration. This problem requires explanations, then theories that are currently unknown or that are under construction [3] [8].

We hypothesize, only in a qualitative manner, that expansion and then matter acceleration is generated by vacuum space production (the ordinary motion of an object is the motion inside space). If we attribute to vacuum space a negative energy [4] [10] [15], then also positive energy has to be produced to accomplish energy conservation principle (total energy variation = 0). This positive energy would be the kinetic energy by acceleration.

Keeping in mind that one just said above, also we note an interesting fact: a Universe, only consisting of photons, violates energy conservation. In fact positive energy, kinetic energy, cannot be produced with the expansion (moreover photons lose energy, for frequency decrease during the expansion [6]). Existence of matter would be necessary. Then an equal presence in the early Universe of matter and anti-matter wouldn't be possible; in fact, for an homogeneous distribution, statistically this equality would involve only photons by the annihilation (should we admit a very improbable fluctuation to avoid annihilation? Moreover not observed? [2]). So matter-antimatter asymmetry appear necessary.

Conclusion

The accelerated expansion of the Universe has been obtained with general and basic considerations. Future observations could confirm the trend of equations (6) and (7). Also initial conditions could be defined.

The energy conservation has to be treated with more accuracy in a quantitative manner, especially with regards to the vacuum negative energy value; for this, a theoretical and experimental development in this area is necessary.

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