

Beyond Witches, Angels and Unicorns. The Possibility of Expanding Russell’s Existential Analysis

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Abstrakt: This paper attempts to be a contribution to the epistemological project of explaining complex conceptual structures departing from more basic ones. The central thesis of the paper is that there are what I call “functionally structured concepts”, these are non-harmonic concepts in Dummett’s sense that might be legitimized if there is a function that justifies the tie between the inferential connection the concept allows us to trace. Proving this requires enhancing the russellian existential analysis of definite descriptions to apply to functions and using this in proving the legitimacy of such concepts. The utility of the proposal is shown for the case of thick ethical terms and an attempt is made to use it in explaining the development of natural numbers. This last move could allow us to go one step lower in explaining the genesis of natural numbers while maintaining the notion of abstract numbers as higher order entities.

Keywords: Russell, functionally structured concepts, definite descriptions.

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1 The Analysis of Definite Descriptions

The suggestion that we tend to be confused by surface similarities in the use of language into wrong ontological assumptions found in Russell's (1905) treatment of definite descriptions together proof, explanation and treatment. Including the refinement of the logical tools available to make the problem explicit and allow further implementation. It seems to me, though, that the way concepts might fail Russell's analysis, the way they are deceiving, should also be object of attention and might give us further clues in how to expand its use in contemporary philosophy.

Beyond those clear cases of existentially empty descriptions, as illustrated by the case of fictional characters such as Unicorns, Fairies, etc. the more worrisome ones are those concepts which do seem to have a referent, are used as having one and are often treated as true on its behalf. Cases of what can be called "non-harmonic concepts." It is in these last cases that I want to focus.

2 Non-Harmonic Concepts

Concepts might fail *a priori* the russellian analysis by being descriptions of logically impossible objects such as meignonian ones, some might be considered even *a priori* existentially empty in the actual world if their existence, while logically possible, goes against the natural laws. Nevertheless, we might want to playfully speculate about entities that defy the known, imaginable entities that respond to unknown natural laws or question our knowledge of existing ones to some extent. Problematic is this just when we talk about entities or properties as if they would exist, while no one has ever being able to detect them, without making a difference; when no difference is made between the mental and the ontological postulate. This discussion forms the background of Russell's analysis, whose usefulness is not so much in detecting clear cases of non-existents (those never perceived) but unclear ones.

Witches, Sorcerers, Saints and Angels are typical cases of this. They do not belong to the fictional world nor do they have a fictional referent. For the experts there were always recognizable signs or pieces of evidence that allowed us to recognize an exemplar of the sort "witch" or "sorcerer". These were very seriously considered in witchcraft trails to accuse a suspect. There are also identifying signs considered to determine sanctity, and angels might themselves leave traces in the world for us to know when they have made an appearance. We could say in a more inferentialist terminology that the introduction rules for the use of any such a concept, the grounds on whose basis they were rightly applied fall much too short of the consequences extracted on their behalf by way of the elimination rules. The assimilation of right conditions of application and truth, however, misleads us in thinking that we are dealing with true attributions and, what is worse, with corresponding facts. With truth applied on a narrow ground but ascribed to the whole surplus of content that the concept expresses and with talk of facts finishing the job of ontologically blessing the asserted, we do not simply invent fictional entities, as is the case of unicorns and superheroes, but we confirm their existence on an application basis.

Take "Witch", for instance, since it offers an easy example, and let's consider first what was accepted evidence to pick up witches.

wrinkled face, furred brow, hairy overlip, gobber tooth, squint eye, squeaking voice, scolding tang²; floats when thrown tied into a river; uses unusual potions and ointments; has moles, birthmarks, scars, or extra nipples; talk to themselves; cannot say the lords prayers (although they might deceptively do); has a number of pets

Call this *Witch Proofs* (WP for a shortage). Consider then further non-provable aspects attributed to witches.

Has pacts with the devil and has supernatural powers (can harm or hail others, can fly, can make potions ointments and dictums with magical capacities, can transform themselves into other creatures or transform others into such)

Call this *Witch Excess* (WE for a shortage).

Then, the introduction and elimination rules for such a concept would be something like

WITCH INTRODUCTION

x satisfies WP

x is a “Witch”³

WITCH ELIMINATION

x is a “Witch”

x has WP and WE

The result is that if WP can be ratified to a good extent (some characteristics of WP can be evidentially proven) then WE is assumed and truth is attributed to an assertion that covers both WP+WE and says that x is “a witch”.

It is surely true that it is a very extended practice to rely on just partial proofs of what justifies us in applying a concept, assuming that the rest will apply too, but that is a quite different problem: One thing is to say that you don’t bother yourself to make an exhaustive proof of whether those conditions of application that justify us in applying the concept are satisfied, those conditions competent speakers or at least experts, in some cases, would accept as such, although they could be proven, (the same ones that will be assumed to be the case when the concept is applied in harmonic cases); another is that no one ever considers the possibility of proving them, or we couldn’t, even if we tried, while we are all just assuming them to be the

² During an English witch frenzy of the 1640s, the Rev John Gaule insisted that “every old woman with a wrinkled face, a furred brow, a hairy lip, a gobber tooth, a squint eye, a squeaking voice, or a scolding tongue is not only suspected, but pronounced for a witch.” In “how to spot witches” <http://www.mirror.co.uk> 21.10.2014.

³ Thanks to an anonymous referee of *E-Logos* for a very helpful commentary on my reconstruction of this.

case when the others are present and furthermore assign truth alone on the prior basis. The cases of non-harmonic concepts considered are of this second sort.

3 Types of Non-Harmonic Concepts and their Treatment

Actually the standard problematic cases of non-harmonic concepts, both in recent discussion and in the original literature, have been pejoratives. Dummett (1973), who introduced the very idea of *conceptual harmony* appointed “Boche”⁴ as a standard example of its failure. I do not want to abound much more in the analysis of pejoratives, since it is not the object of my present study. But I just want to remark that, although pejoratives are of course paradigms of non-harmonic concepts, I do agree with Boghossian (2003) and Williamson (2003) in saying that they are nevertheless perfectly working and legitimate concepts. My reasons are though slightly different from theirs, since I see pejoratives ‘working successes’ as derived from a structure of *pretense* or *make-believe*. Pejoratives illegitimately but purposely tie a descriptive aspect to an exceeding content, the pejorative evaluation. Non-harmonic is this because they are introduced on the basis of some, for example, ‘racial’ physical characteristics and make non-derivable derogative associations on their behalf (boche’s case). It is offensive, which is the role for which they were brought into existence, precisely because they attempt to give to understand that the association is in good standing. Being a black person or a German already makes you that despicable. The association is a cognitive one and not merely the expression of an illocutionary force⁵, since there are parallel concepts that could have already been used with such a force. There was no need, that is, to invent a new concept. What is the difference between screaming: “you are a negro!” and saying or writing “You are a niger”? In a sense none, but in the first case you need the shouting to do the work, in the second you already have it in the cognitive content, so even if you just write it or read it, it is there. Of course, for the receiver (and a good number of others) it is known to be a falsehood – with a damaging goal – like fictions are known to be fictions and, nevertheless, use the simulation of reality for a given purpose. An equivalent might be a caricature, such as those issued in Nazi times painting the Jews as dark wicked figures counting money. Often these work through an hyperbole, they present a real fact, that some of them worked in banks, in the form of a vice due to insane and wicked ambition. This might be done too by way of adding a feature that gives to understand that some aspects (possibly not known to others) are to be associated with them: for example, painting the target person with a big pig face to suggest the existence of some dirty dealings or costumes.

These are standard ways of literally facilitating a negative perception of the victim from potential others. The pejorative too simply incorporates a negative light to the perception of a given group or person by possible spectators or hearers, the public humiliation adding thereby an intensifying factor. The derogative element is part of the content just as much as it is in the caricature. The goal is a similar one, to give to understand that there are mighty reasons for such negative associations even if others should not know them well. But the public aspect actually is not strictly necessary. The offense exercises its power also when no third parties are there to hear. We might put in words the variant of the above caricature, saying things such as: “You are a swine”. Both parties, the insulting and the insulted one, of course know this is literally false and just metaphoric. The connection to what someone is as a person is not as tight as in thick cases such as *niger*, *boche* and similar ones, but there is a connection with the person’s

⁴ Pp.454-455

⁵ Pace Williamson’s (2003) *gricean analysis*

behavior. The simulating aspect is in the first case totally obvious and lays open the family resemblance between pejoratives, fictions and other types of make-believe. It is enough for the offense to go through that the other party gives to understand that you are to be seen that way, that the way you are or behave justifies making that association. The public humiliation and the promotion of negative considerations about the person in question simply strengthens the point. I do not pretend this to be a complete analysis, but I think it is a plausible line of interpretation that does not seem to have gotten much attention in the literature.

My interest, however, is to focus on those cases whose non-harmonic structure is not always kept in mind and might give raise to confusion from an ontological perspective; or which are overseen and considered true on an unreliable basis; cases too whose exact connection with the application basis remains obscure or unproved and could benefit from a more careful elucidation.

Besides those already seen of what I might call “fictional-existents” there are other interesting non-harmonic cases: a good example is delivered by theoretical scientific terms. Whose existential instability we are not unaware of but whose status has being subject to continuous controversy and explanatory demands. That they are non-harmonic is self-understood. Take Boghossian’s (2003) ‘neutrino’ example. The concept is applied on the basis of some experience, some track in a cloud chamber, what is ratified, though, in correct conceptual application goes beyond the strict acknowledgment of merely “a track in a cloud chamber” and asserts the existence of an extra entity, *the neutrino*, that is responsible for causing it. So we take the effect as basis for the introduction rule to presume the existence of the cause in the elimination rule. The existential analysis that gets rids of the designing term in favour of a quantified definite description shows that there is no such entity unless there exists the individual that satisfies the presumptions of the theory. That is exactly what the *Carnap cum Ramsey solution* brings.

T (Neutrino)

(S) *Ex (Tx)*

(M) *Ex ((Tx) → T(Neutrino))*

(Boghossian 2003, p. 245)

Carnap’s (1966) analysis expresses that the use of the term by speakers is conditional upon the truth of a corresponding existential sentence, where the suppositions of the neutrino theory are ontologically satisfied. That is, they are conditional upon the truth of the sentence (S). This can be seen as a way to express the hypothetical character of theoretical concepts. Instead of demanding the existence of a special kind of entities, that is *theoretical entities*, these concepts would be hypothetically presuming the existence of imaginably (per extension, refinement etc., of our usual procedures) experientially capturable *entities*, whether we do capture them or not. At least the procedure makes clear what it is that we would have to register in order to say that the theoretical postulates that define what the concept expresses are fully satisfied (or made further probable). Whatever it is, either the requirement of harmony between such an hypothetical experiential basis and what the theory specifies as the content of the concept must be satisfied, or, at least, it must be explained how we come from the one to the other, how it might be proved.

The similarities with the case of *fictional-existents* considered before can be seen by drawing a parallel ‘Angel Theory’ (*T Angel*), which based on some signs in the world presume Angels as their causes. Of course, the huge difference lies in the available explanation of why such a

causal link is assumed to be there. But from the point of view of using some given symptoms as application conditions not just to assert the existence of the symptoms themselves but to assert the existence of something extra (referred to by the concept) whose ontological backing is not given, the cases are similar enough. The utility of the *Carnap cum Ramsey* analysis of theoretical terms was precisely specifying their dependency relations with experience.

The analysis, though, doesn't go beyond being an application of the russellian method of existential quantification for definite descriptions. The question is whether we need to go further than this, in refining our explanations of the relations of some concepts to experience

4 The russellian analysis in metaethics

I believe a more clarifying treatment of thick ethical concepts could be given through the suggested russellian path. Thick concepts also exhibit a non-harmonic structure, but their treatment requires an enhancement of the possibilities opened by the russellian analysis. Unlike theoretical scientific terms whose peculiarity is widely agreed upon, thick ethical concepts have been seen by many to dispute special treatment as ontologically queer in any form. John McDowell (1981) and others have argued that thick-ethical concepts refer to thick ethical facts and there is no going beyond this in understanding. Reality is not to be thought of as distinct from our conceptual apprehensions, and it is this conceptually understood whole intertwined of factual and evaluative aspects that we are capturing in applying a thick ethical term. From this realist perspective, of course, thick ethical terms are harmonic. It is on the basis of the purported presence of this entangled mixture of descriptive and evaluative aspects that the concept is introduced so the elimination rule will be harmonic in allowing us to derive the presence of both from conceptual use too.

However it should be noticed that the defence of disentanglement does not require, contrary to what McDowell (1981) argues, defending the existence of a non-evaluative feature of the world, at a non-conceptual naturalistic level, we would be each time appealing (or reacting) to in evaluating and using the concept. Just if you accept a requirement to the contrary McDowell's argument against Non-Cognitivism and the possibility of a rule following explanation of thick-ethical terms on a disentangled account is successful. Instead of this, it is possible to defend that we would be appealing to a conceptually graspable but non-evaluative set of application conditions (sorting out a specific kind of behavior, for instance), which deliver the required common basis for conceptual use. The disentangled account would be this way made plausible since the moral evaluation of the described behavior would not already belong to the application conditions but would be added on their behalf. Actually, that some concepts should lack the kind of common features McDowell demands at such a naturalistic non-conceptual level is far from rare and not exclusive of evaluative concepts. Being "the Chef" or "a game", "the Senate", a "marriage" or a "Dean", in none of these cases we will easily find them and this need not imply that there aren't any conceptually expressible ones. I am going to defend therefore that a disentangled account on the lines described is not just possible but more adequate in order to understand our evaluative behavior. Starting with its capacity to liberate us from ontologically sanctifying attributions of values to behaviors, which with the course of time might not seem as appealing as they once were; or having to accept the thick truth and factuality of similar claims in other cultures we do not identify with, simply because they have correct application conditions for their thick terms.

From the disentangled perspective adopted the non-harmonic structure of these terms becomes soon apparent. Since the suggestion is that we learn to prove whether some conditions are

fulfilled and on their basis allow ourselves to extract (via the elimination rule) the corresponding evaluation.

SCHEME I. Non-harmonic disentangled reading of thick concept

INTRODUCTION RULE

x has application cond(n)

x is “thick ethical cpt(n)”

ELIMINATION RULE

x is “thick ethical cpt(n)”

x has application cond(n) and x is (morally) wrong/rightAs opposed to this, consider a realist harmonic reading

SCHEME 2

INTRODUCTION RULE

x has application cond(n), (which includes moral evaluation r/w)

x is “thick ethical cpt(n)”

ELIMINATION RULE

x is “thick ethical cpt(n)”

x has application cond(n) (which includes moral evaluation r/w)

Whereas in this last case (following the realist reading) we have to (make an effort to) figure out that the evaluation is somehow indistinguishable from the rest of the application conditions.

The problem in the non-harmonic case is that, like in the Witch case, what we ratify through correct application goes beyond what we had as an application basis. Therefore identifying application conditions and truth we directly obtain the truth and factuality of the whole, as we saw before. But, is the non-harmonic reading presented above convincing? The problem with non-harmonic concepts is precisely that they seem to allow inferences for which we lack any further justification. We can give no naturalistic explanation in terms of reactions to some common features of the world at the non-conceptual level since we already rejected that model to avoid McDowell’s critique. But we might not give us satisfied either with a stipulation that simply prescribes our transition from the one to the other without giving an adequate justification for it. Why at all I am to accept as a matter of course that such and such described behavior is to be morally condemned? Of course, there might be cases where such behavior might be a special case of a general kind of behavior that the apprentice for whichever reasons, justified or not, already condemns (or might be associated with some infraction of a religion he professes and hence condemnable too) and there the transition is something like a deduction. But this need not always be the case. Furthermore, the more distant from ours the community is, historically or simply in worldview, the more likely it will be that we do not see such transitions as self-understood.

Since non-harmonic concepts are in this way owing, someone might try to do better with a non-realist harmonic explanation. Instead of a non-explained non-cognitivist transition, according

to which given some description we are allowed to derive a predetermined evaluation, he could offer something like this.

INTRODUCTION RULE

x has applic. cond(n) & x is (morally) w/r

x is “thick ethical cpt(n)”

ELIMINATION RULE

x is “thick ethical cpt (n)”

x has applic. cond(n) & x is (morally) w/r

This way for the concept to apply both the application conditions and the evaluation are to be proved independently. We would have to prove, that is, whether we have to do with a specific kind of behavior, for example, (determined by the application conditions) and then whether it is morally right or wrong. But, there are several shortcomings to this proposal. First of all, we would have to determine separately what is to prove that some behavior or circumstance is *morally right or wrong*. But even if we should manage to do that, what is the point of having a concept that requires us to do so? Am I to take the time to consider on each occasion whether both conditions are fulfilled? As if it could be the case that the one is given without the other. What is the purpose of having a thick concept then if it not to economize some work on our side? Actually I think the contrary is the case. What we are trying do is precisely to equip ourselves with some readymade evaluations to spar ourselves all that trouble. Since we are not evaluating tokens of behavior anew, but making a connection between a given type and the evaluation, so that it can be taken for granted without any more thinking that if the one is given the other will be given too. The non-harmonic reading records this idea much better. But still this has not yet solved the question posed before of justifying the transition, we should take it for granted. Why are we to do so?

Now, I was saying before that the solution given to non-harmonic concepts in the case of theoretical scientific terms such as “neutrino” is not helpful here. Why not? Because the strategy to give account of them, as we saw, is to see whether there is an individual such that it satisfies the requirements of the theory and causes the track in the cloud chamber. If we try to propose a parallel reconstruction of a thick term understood along the non-harmonic lines proposed for neutrino above we would come to something more or less like this:

T (Thick-n)

(S) *Ex Tx*

(M) *Ex (Tx → T (Thick – n))*

Thick theory = T (Thick-n), T =Theory, (M) Conditionalized Sentence

(S) Existential Sentence

We could read the above scheme as saying that according to (M) the use of thick concepts is conditional upon the existence of (at least) an individual that satisfies the requirements of a correspondingly specified thick theory (as would be specified by sentence (S)). But the problem is here not whether there is such a token satisfying our disentangled approach. That is, whether we find an instance satisfying the described behavior and satisfying the corresponding moral

evaluation. That might allow us to turn the conceptual inference into a harmonic one: we find an individual that satisfies the description of the application conditions and satisfies the evaluation too. Good, but that is not what we need. Not just because having to prove both aspects separately would be uneconomic and absolutely beyond the point of having a thick concept at all, as argued before when talking about a disentangled harmonic proposal. But because what we need to prove, as the concept allows us to infer, is that for whichever instance *to come*, if it satisfies the application condition it *necessarily must* be suitable for the (positive or negative) evaluation too. Simply to claim that whenever the one is given the other will be given too won't do. An apprentice might learn that way and children (and some adults too) possibly use many such concepts so. But that is not to give an explanation as to *why* the value is to be applied to the sorted out conditions of a given behavioral type. Or worry is whether there is a proof that justifies the transition. So, if our question was what is the difference between this and the neutrino case, the answer is that the legitimacy of the concept cannot be proven in the ethical case upon ontological satisfaction. Actually no instance of such a behavioral type needs to have been ever given for it to be valid for us that should anyone behave in that specific manner it would be morally wrong. The legitimacy of the concept here depends rather on the validity of the *connection* presumed by it. So what we are looking for is some kind of *function* that ties the one with the other. Correspondingly if we were to make a russellian analysis we would have to say that the use speakers make of thick concepts is justified just if there is some function allowing the inferential connection the concept permits.

But, since behaviors can be evaluated and called 'good' in many difference senses, they can be favorable to my interests, contribute to very different kind of expectations of a group or a community, they might be good in fulfilling some religious dogma etc. we need a function that specifies when is it that they are to be considered good (or evil) *in the moral sense* the concept expresses. Therefore we are looking for a *moral* function. Different authors, might depict this function differently, but the point is that if we are to be able to explain and prove the legitimacy of thick concepts the classic russellian existential analysis is not enough, we need a functional one. This amounts to expanding Russell's analysis to functions. Conceptual use can be proven as legitimate not just when we can show that there is an entity that satisfies the concept. But also when we can give some form of proof that justifies the connections the concept expresses.

Accordingly we should distinguish two possibilities of analysis

- a) Quantifying over entities: to prove if there is something there making true the theoretical assumptions that have been made about some presumed existent entities.
- b) Quantifying over functions: to prove the rightness of the connection between different entities or properties, or, for example, between some specific behavior and its evaluation that some concepts allow us to take for granted.

One might wonder, though, whether and when we should need proof for the inferential connections our concepts allow us to make. It seems that in many cases a direct transition need not be problematic and no one would come to the idea to ask for a proof. So when should we ask for one? Well, we could say so much: a proof seems required when the transition the concept allows is not self understood, doesn't go of itself, at least not for those who have not previously been educated to consider it so.

5 Functionally Structured Concepts

The existence of such concepts, which apparently are perfectly standard denoting ones, but whose non-harmonic inferential structure requires extra justification through some mediating

function, allows us to speak of *functionally structured concepts*. Proving the validity of such concepts, thus, will be a matter of backward testing whether there is such an acceptable function through a quantified functional sentence and whether it really does justify the transition the concept expresses. Actually, to the extent that the conclusions arrived at regarding the validity of the connection might require occasional revisions, it is a healthy attitude to remind ourselves that talking about rightness (or truth) in the case of such concepts, just like with neutrino, will always be dependent on the justification (or truth) of the corresponding (S) sentence. Of course an independent issue will be our acceptance of the requirements the function puts forward. Why we should have the goal on whose behalf behaviors are proved. That is from my perspective the real issue of justifying the need of morality. But precisely by disclosing which is the moral function being used to justify the evaluations it becomes also possible to reconsider that. We could first see if there is a justificatory tie and second whether it is an acceptable one.

A few aspects to keep in mind about what I have been calling ‘functionally structured concepts’

- 1) They are non-harmonic
- 2) The inferences they allow need not be self understood for any non trained subjects
- 3) Because of 1) and 2) a justification of their connection is needed.
- 4) Their truth depends upon the truth of a corresponding functional Sentence
- 5) Therefore the identification of correct application and truth is to be seen with care in some cases.
- 6) Even more important (for all the points above) we should not be too quick in talking about facts resulting from their assertion in realist terms.

Now, is the proposed analysis applicable in other fields?

6 Natural numbers as functionally structured concepts

There are certainly many concepts that describe functional results. Mathematics is full of such cases. Take ‘prime numbers’, their selection requires to subsume any candidate number to the proof of whether they are merely divisible by one and themselves. Prime numbers are considered such because they just allow these operations. However, the concept “prime” clearly includes as part of its cognitive content the fulfilling of the requirements mentioned. What is sorted out doesn't present itself as any kind of extra entity, but as numbers (whatever they are is simply taken for granted) that satisfy certain operational possibilities. That is, the introduction and elimination rules for the concept “prime” are fully harmonic (*Introduction-Prime*: x is a n^0 and x can be divided by one and itself / x is a prime n^0 ; *Elimination-Prime*: x is a prime n^0 / x is a n^0 and x can be divided by one and itself). These concepts might be considered functional concepts but are not ‘functionally structured concepts’. There are no further conclusions we are legitimized to infer as part of their cognitive content that go beyond those that allowed us to apply the concept in the first instance. The concept of natural numbers, on the contrary, does give rise to misunderstandings in that sense. Like with theoretical scientific terms they make us think that there are some especial entities, abstract entities, we are referring to. But, actually, besides what we might consider concrete differentiated unities, natural numbers can be defined as operational results. According to Peano’s Axioms (1889) *two*, for example, is a natural number since it is the successor of a natural number. So *two* could define itself as the successor

of *one*, etc.⁶ Natural numbers, unlike prime numbers, aren't operations done upon some 'entities' that are already there and taken for granted (that is natural numbers). Rather they do not exist previous to the introduction of an operation upon single unities (not numbers). Actually, if we want to keep the unquestionable utility of talking of numbers as 'abstract entities' or abstract representations, it could be said that they refer rather to what we might consider *second order representative 'entities'*. The first of which, the unity and the zero might be considered proper representations of something too, since they do represent the absence and presence of concrete unities⁷, while all the others would be determined through their ordered adding to the succession of unities and so to be seen as derived operational results. But, once they are there, nothing speaks against representing these operational results synthetically in the form of abstract entities. We can, thus, say that something is a natural number if it results out of the application of such a function to a set of concrete unities (whether real unities or concrete figurations) and, if it does so result, it can *represent* what all sets of concrete unities that satisfy the same functional requirements have in common. I take this to be what Fregeans, or maybe I should say neo-Fregeans, such as Wright (1983) would be actually saying.

So, to be sure, the concept of natural numbers refers to this new set of entities which, strictly taken, did not exist as such before the introduction of the operational process that gave them birth. In this sense they are misleading. The application conditions would say that if there is a succession of unities to which this function can be applied, their amount of repetitions allows us to refer to something else: *number 5* for example. If we put it this way the concept shows itself as non-harmonic.

“Number Five” Introduction, Rules

Set (x) = (a, b, c, d, e)

Set (x) is “number Five”

Elimination rule for “Number Five”

Set (x) is “number Five”

Set (x) is (a, b, c, d, e) and is 5

What is in this reconstruction missing, as in the case of thick concepts, is the function that justifies the transition from the set of unities to the abstract entity and it is this function that explains how we come from the one to the other. So, if we take a Russellian approach to the issue, we could reconstruct the apparently referring expression “number Five” as a quantified sentence that makes the transition dependent upon the existence of such a function. Something that we might tentatively represent along these lines

$$E(F) \forall Set S ((F(S) \wedge S = (a,b,c,d,e)) \rightarrow S = 5)$$

⁶ I did not use *one* to avoid confusion with the concrete unities mentioned before I want to distinguish it from but, of course, *one* considered as a number can be defined operationally too.

⁷ *Number one* might have a double character representing the concrete unity while being derivable operationally too. What I mean by this is that a group of unities is a group of unities and they become numbers when considered as an order succession, while the number *one* can be considered to refer to the unity even if there is no succession considered, it just needs the contrast with nothing or zero.

If there is a function that allows us to transform all sets consistent of such a repetition of unities into an abstract number 5, then our use of the term “Number Five” to allude to such an abstract number is considered justified by it. This reconstruction enables us to make the use of abstract terms and entities dependent upon our operations with concrete reality. The difference with a russellian or ‘carnapian’ reconstruction of theoretical scientific entities is that theoretical scientific entities are made dependent upon their possible *real* existence. Here, on the contrary, we are introducing *a new* abstract entity justified operationally, that is, dependent on the operation that gives them birth. So talk of abstract entities in this second ordered sense should not lead us to forget their derived status in an ontological setting. This reconstruction would seem to support classifying the concept of natural numbers as a functionally structured concept. Most probably the formulation given allows improvement, but I just wanted to put on the table the possibility of making an analysis along these lines.

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