Anti-Exceptionalism and the Justification of Basic Logical Principles*

Matthew Carlson

May 6, 2022

To appear in *Synthese*. Please cite the published version.

Abstract

Anti-exceptionalism about logic is the thesis that logic is not special. In this paper, I consider, and reject, a challenge to this thesis. According to this challenge, there are basic logical principles, and part of what makes such principles basic is that they are epistemically exceptional. Thus, according to this challenge, the existence of basic logical principles provides reason to reject anti-exceptionalism about logic. I argue that this challenge fails, and that the exceptionalist positions motivated by it are thus unfounded. I make this case by disambiguating two senses of 'basic' and showing that, once this disambiguation is taken into account, the best reason we have for thinking that there are basic principles actually implies that those principles do not require a special epistemology. Consequently, the existence of basic logical principles provides reason to accept, rather than reject, anti-exceptionalism concerning the epistemology of logic. I conclude by explaining how an abductivist, anti-exceptionalist approach to the epistemology of logic can accommodate the notion of basic logical principles.

1 Introduction

Anti-exceptionalism about logic is the thesis that logic is not special. While logic might be said to be unexceptional in various ways, including methodologically [Martin and Hjortland(2021)] and normatively [Russell(2020)], in this paper I am solely concerned with logic's epistemology. Anti-exceptionalism about the epistemology of logic holds that logic does not require a special epistemology. More to the point, logical principles are to be justified, on this view, in the same way as other scientific principles. Following in the wake of [Hjortland(2017)],

*This paper is dedicated to the memory of David McCarty, who taught me a great deal about logic and its philosophy.

there has been a spate of recent work on this topic. In this paper, I aim to take on a challenge to anti-exceptionalism about logic, which has not yet, to my mind, been adequately considered in the literature. According to this challenge, there are *basic* logical principles, and part of what makes such principles basic is that they are epistemically exceptional. This is in conflict with anti-exceptionalism because it holds hold that basic logical principles demand a special epistemology to account for their justification.Thus, according to this challenge, the fact that there are basic logical principles provides reason to reject anti-exceptionalism about logic.

In this paper, I argue that the challenge from basic principles fails, and that the exceptionalist positions motivated by it are thus unfounded. My argument proceeds as follows. In §2, I argue that we must disambiguate two senses of 'basic' at play. I call them *systematically basic* and *epistemically basic*, respectively.[1] Employing this disambiguation in §3, I argue that the best reason we have for thinking that there are basic principles actually implies that those principles do not require a special epistemology. Consequently, the existence of basic logical principles provides reason to accept, rather than reject, anti-exceptionalism about logic. In §4 I explain how an abductivist, anti-exceptionalist approach to the epistemology of logic can accommodate the notion of basic logical principles.

2 Two Senses of 'Basic'

By 'logical principle', I mean an *argument-text schema* in the sense explained by [Corcoran and Hamid(2016)]. That is, a logical principle is a schema each of whose instances is an *argument-text*, a two-part system consisting of a set of sentences (the premises) and a single sentence (the conclusion). I use the symbol '$\Rightarrow$' to mark the division between these two parts. Some logical principles have special names, for instance,

$$A \to B, A \Rightarrow B \,(\textit{modus ponens}) \tag{1}$$

and

$$A \to B, B \Rightarrow A \,(\textit{affirming the consequent}). \tag{2}$$

Note that, as these examples illustrate, some logical principles are valid, and some are not. Most logical principles, in the sense in which I use the expression, lack special names, for instance,

$$(A \vee B) \vee \neg C \Rightarrow \neg A \to (\neg B \to \neg C) \tag{3}$$

and

$$\forall x(\exists y Rxy \to \forall y Ryx), \exists x \exists y Rxy \;\Rightarrow\; \forall x \forall y Rxy.^2 \tag{4}$$

[1] I take the term 'systematically basic' from [Carlson(2015), §1.3], and I use it in largely the same sense here. The only significant difference is that here I am concerned with the basic status of individual logical principles, as opposed to that of logical theory as a whole.

[2] I should also note that the explicatory emphasis here is on 'principle', not 'logical'. There are, of course, difficult theoretical issues concerning the "bounds" of logic; concerning whether,

My aim in the remainder of this section is to explain how reflections on the epistemology of logic motivate the thought that there are *basic* logical principles, and to clarify what it means to call a logical principle 'basic'. I do this by disambiguating two senses in which logical principles are said to be 'basic', which I call *systematically* and *epistemically* basic, respectively.

2.1 The Reduction Picture of the Justification of Logical Principles

We can make inroads on our main agenda by considering how logical principles, in the sense characterized above, can be justified. For example, consider the following logical principle of distribution.

$$(A \wedge B) \vee C \Rightarrow (A \vee C) \wedge (B \vee C) \tag{5}$$

How can this principle be justified? Here is a common, and compelling, answer to this question. This principle can be justified by *proof*; i.e. by a cogent series of inferential steps which show that, if the premise is true, the conclusion must also be true.[3] For the sake of clarity, let's say that an inference (or more generally, a series of inferential steps) is *cogent* just in case that inference confers justification on its conclusion.[4] Considering the case at hand, you might reason as follows.

Suppose that $(A \wedge B) \vee C$. There are two possibilities: either $A \wedge B$ or C. Take the first case. If we have $A \wedge B$, then we have A. But then it follows that $A \vee C$. By parallel reasoning, we have B, and hence $B \vee C$. Thus, since we have $A \vee C$ and $B \vee C$, it follows that $(A \vee C) \wedge (B \vee C)$. Now consider the second case. Assuming C, then we have $A \vee C$ and $B \vee C$. Thus, as before, it follows that $(A \vee C) \wedge (B \vee C)$. Hence, either way, it must be the case that $(A \vee C) \wedge (B \vee C)$. Articulating this reasoning would seem to suffice to justify your belief that the principle in question is valid. Justification in logic and mathematics amounts, at least primarily, to proof, and here we have produced a proof of the validity of this logical principle. But how exactly does producing such a proof suffice to justify one's belief that the principle is valid?

Let's consider an answer to this question that is explicitly endorsed, or at least presupposed, in much of the literature on the epistemology of logic. Here is Michael Dummett, for instance: "There can, of course, be such a thing as a demonstration of the validity of some particular form of argument, namely the kind of demonstration which constitutes a non-trivial proof of syntactic validity: by use of rules of inference taken as primitive, the conclusion of a given form of argument may be shown to be derivable from its premisses" [Dummett(1978),

e.g., second order logic is really logic properly so-called. I'll skirt this issue by focusing on principles that are, I think, clearly logical.

[3] If you don't find this so compelling, consider instead as an example an extremely complicated valid principle; a principle whose validity you cannot simply determine "on sight."

[4] This is roughly the sense in which the term is used in [Wright(2002)]. While it may be a proof, a one-line proof consisting solely of a statement of an axiom of the sort described in [Williamson(forthcoming)] is not a cogent inference in this sense.

291]. By this procedure, Dummett writes, we have "reduced" the principle in question "to a series of applications" of simpler principles.[5] On Dummett's account, inferential justification is a sort of *reduction.* That is, on this picture, by proving that a principle is valid, I have in effect reduced the question of the principle's validity to the question of the validity of the resources that I employ in the proof. In the above example, I showed that principle (5) is valid by the use of a variety of other principles, e.g. the following principle of *conjunction elimination.*

$$A \wedge B \Rightarrow A \tag{6}$$

By doing this, I reduced the question "Is this distribution principle valid?" to the question "Are the principles employed in the proof valid?"[6] Let's fix this general idea for the moment. In logic, proof is the primary vehicle of justification, and we can understand the relation of justification to proof by thinking of a proof as a reduction of the question of the justification of the item proved to the question of the justification of the resources employed in the proof. This is the *reduction picture* of the relationship of proof to justification.

It appears that the reduction picture provides a straightforward account of the justification of many logical principles. Logical principles are justified because they can be proven to be valid, i.e., it is possible to articulate cogent series of inferences that establish that they are valid. But in constructing these proofs, we must rely on other logical principles. In our example above, for instance, we relied on principles like *conjunction elimination.* Is *this* principle justified? It seems that it had better be. How could our above reasoning confer justification on its conclusion if it relied on logical principles that are themselves unjustified? Indeed, recall that on the reduction picture, by means of proof one "reduces" the question of the justification of a principle to the question of the justification of the principles employed in the proof.

And yet, it appears that we cannot appeal to the reduction picture to explain how we can justifiably accept a principle like *conjunction elimination.* Eventually, it seems, we must reach a point at which such a reduction is no longer possible. Eventually, that is, we must reach *basic* logical principles.[7] As

[5]Elsewhere, Dummett calls this a "proof-theoretic justification of the first grade" [Dummett(1991), 188].

[6]Of course, I might also employ resources in my argument that are not themselves logical principles. I might apply a tableaux procedure in which I perform various operations on the formulae of the principle (e.g. I operate on a disjunction by splitting into into two branches, etc.). But here, again, I've reduced the question whether the principle is valid to the question of whether those operations track validity. Or, I might employ resources that are not simply syntactic; I might avail myself of a theory of truth-in-an-interpretation and prove metatheoretically that the succedent is true in any interpretation in which the antecedent is true. Here I've reduced the question of the justification of that principle to the justification of the metatheoretic resources employed in this semantic proof.

[7]The reasoning here, typically left implicit by advocates of the reduction picture, is essentially that of Agrippa's trilemma, famous from the regress problem in epistemology. According to this reasoning, our beliefs must be supported either by infinite sequences of reasons, sequences of reasons which are ultimately circular, or sequences of reasons that terminate in foundational beliefs. The argument for thinking that there must be basic logical principles ultimately relies on the thought that the overall structure of logical principles cannot be cir-

Charles Parsons nicely puts it, the justificatory buck has to stop somewhere [Parsons(2008), Ch. 9]. But how are such principles justified? As Frege puts it in the first volume of *Basic Laws of Arithmetic*, for example: "As to the question, why and with what right we acknowledge a logical law to be true, logic can respond only by reducing it to logical laws. Where this is not possible, it can give no answer" [Frege(2013), XVII]. Frege is here describing the reduction picture of justification by proof, and pointing out that, at the level of basic principles, this sort of justification is no longer possible. But, if logic "can give no answer" as to the justification of such principles—they cannot be justified by proof—how *can* they be justified?

2.2 Basic Logical Principles and Logical Systems

The reduction picture of proof and justification leads us to the thought that at some point we must employ *basic* logical principles in our proofs.[8] But what is a *basic* logical principle? Paul Boghossian's formulation of the question provides a useful starting point. "[S]uppose that we are working within a system in which *modus ponens*... is the only underived rule of inference. My question is this: is it so much as *possible* for us to be justified in supposing that *modus ponens* is a valid rule of inference, necessarily truth-preserving in all its applications?" [Boghossian(2000), 229] Boghossian uses the expressions 'underived', 'fundamental', and 'basic' interchangeably in his discussion. For a variety of reasons, prominent among which is a desire that we not unduly privilege syntactic notions of proof, I employ 'basic' throughout. The question that Boghossian asks in the above passage certainly seems to be a deep and serious one; it appears to be a question that is neither easily answered nor dismissed. I propose to take this appearance at face value. A constraint on a proper understanding of 'system' and 'basic' in this context is, accordingly, that this question turn out to be deep, important, and not easily answered.

First, what is a basic principle? Notice that Boghossian's formulation of the question suggests that basic principles are system-relative. Boghossian asks us to suppose that we are employing "*a system* in which *modus ponens* is the sole underived rule of inference." The supposition is *not* that *modus ponens* is the sole basic principle, period. As a first pass, let's say that a basic principle in a system is something along the lines of what Frege called a "primitive truth"; something which is employed in proofs but "for which no proof is given in our system, and for which no proof is needed" [Frege(1979), 205].

But now, what is a system? Let's answer this question by first considering

cular or infinitely descending. Thanks to an anonymous reviewer for helping me to clarify this point.

[8]This is a central idea in Frege's logicist project, and I think its influence has extended well beyond the bounds of logicism. Frege's idea was that all proof-chains will eventually terminate in unprovable "primitive truths", and we can determine the status of the theorem proved (i.e. whether it is analytic, synthetic *a priori*, or synthetic *a posteriori*) by determining the statuses of those primitive truths. Arithmetic was to turn out to be analytic, on this view, because the primitive truths from which all theorems of arithmetic were to be proven are analytic (i.e. are definitions and logical laws) [Frege(1950), §§3–4].

an answer that won't work. We might suppose that a system consists simply of a particular set of syntactically specified proof rules. For example, our system might comprise Gentzen's rules for NK. What counts as proof in the system is, accordingly, specified by reference to the rules for syntactic transformation of the system, which rules constitute the system's basic principles. The problem is that, on this conception of a system, the question of how it is possible for us to justifiably accept the basic principles of the system is not deep or interesting at all; indeed it admits of a variety of easy and perfectly adequate answers. Suppose, to fix ideas, that the basic principles of our system are just Gentzen's NK rules, among whose basic principles is $\rightarrow$E, i.e. *modus ponens*. Of course, since *modus ponens* is basic in our system, we cannot justify it by proof-theoretically reducing it to other principles in the system[9], but this is unproblematic since we can easily prove the validity of *modus ponens* from within another set of syntactically specified proof rules. We could, for instance, employ a different natural deduction system in which the basic principles include

$$A \vee B, \neg A \Rightarrow B \textit{ (Disjunctive Syllogism)} \tag{7}$$

$$A \rightarrow B \Rightarrow \neg A \vee B \textit{ (Material Implication)} \tag{8}$$

and

$$A \Rightarrow \neg\neg A \textit{ (Double Negation)} \tag{9}$$

but not *modus ponens*. In that system, it would be easy to prove the validity of *modus ponens*. Or, we could easily prove the validity of *modus ponens* in a tableaux system in which the operation of branching a conditional is taken as basic. Still another idea is to give a semantics for the formulae of our syntax-language (i.e. interpret them over some, typically set-theoretic, domain) and prove (in the metatheory) that *modus ponens* is sound with respect to the given semantics.

Common to these approaches is the idea that we can "step out" of the system whose basic principles we are considering and, from outside the system, prove the validity of basic principles of the system. But this tactic seems somehow cheap—too easy—and this suggests that we've failed to capture the depth and importance of the real problem. One worries that, in the midst of all this marvelous mathematics, the point has been missed. To be sure, the problem is *not* with the mathematics, but with the too-narrow conception of a system that we've been entertaining. One way to bring this out is to note that, even when we step outside of the syntactically specified proof system to prove results that cannot be proven from within that system itself, we appear to be employing resources not relevantly different from those which we are supposed to be justifying. As has been pointed out numerous times in the literature, when we ascend to the metatheory to prove the soundness of *modus ponens*, for

[9] Caveat: you might suppose, inspired by the work of [Prawitz(2006)], that *modus ponens can* be reduced, in the system we are considering, to a principle of conditionalization. That may be so, but then of course the issue we are considering here will simply arise for that latter principle. So I will ignore this possibility for now.

instance, we simply appear to be employing *modus ponens*, or a principle not relevantly different from it, in our metatheoretic proof.[10] This suggests that the conception of a system as syntactically specified proof rules is too narrow to capture the force of Boghossian's question concerning the justification of basic logical principles. What we need, it seems, is to understand a system of logical principles as something within which, and by use of the resources of, *all* proving (hence, justifying, in the sense relevant to logic) is to take place. A system should thus be conceived more expansively; it is the sort of thing that Frege attempted to construct with his *begriffsschrift* and develop in his *Basic Laws of Arithmetic*. In more contemporary terms, we can think of a system of logical principles as the deductive closure of a class of basic principles.[11]

Armed with this expansive conception of a system, let's consider our guiding question concerning the justification of our system's basic principles; those principles which are in our system because we rely on them in constructing proofs, but which are not themselves susceptible to proof within our system.[12] First, how do know that our system even *has* basic principles? Now, it clearly isn't true that, in fact, *modus ponens* is the sole basic principle in our system. If it were, our system would be grossly impoverished, since we accept many proofs that rely on principles independent of *modus ponens*. Rather, in Boghossian's formulation of the question, *modus ponens* is a placeholder for *whatever the basic principles in our system are.* And as to the question why we should suppose that our system has any basic principles at all, the essential idea issues from the truism that we can't prove everything. As Zermelo puts it, "not everything can be proved, but every proof in turn presupposes unproved principles" [Zermelo(1908), 187]. The basic principles of a system are just those principles which are in the system—they are employed in or presupposed by proofs—but which are not themselves susceptible to non-trivial proof in the system. Principles that have this status are, in my terminology, *systematically basic.*

Importantly, this reason for thinking that there must be systematically basic principles in a system also implies that these systematically basic principles are sufficient to prove everything else (i.e. all the non-basic principles) in the

[10] Dag Prawitz puts the point nicely: "Tarski's definition... involves exactly [this] kind of circularity... Whether e.g. a sentence $\exists x \neg P(x)$ follows logically from a sentence $\neg\forall x P(x)$ depends according to this definition on whether $\exists x \neg P(x)$ is true in each model (D, S) in which $\neg\forall x P(x)$ is true. And this again is the same as to ask whether there is an element e in D that does not belong to S whenever it is not the case that every e in D belongs to S, i.e. we are essentially back to the question whether $\exists x \neg P(x)$ follows from $\neg\forall x P(x)$" [Prawitz(1974), 68].

[11] This also agrees with Dummett's characterization of basic principles of a logical system [Dummett(1991), 190–191].

[12] There's an additional question we should ask, though due to limited space I won't address it in any detail here. This question is: Are we in any sense working in a system; in an all-encompassing scheme within which all proving takes place? One reason for thinking that we are is that the central goal of studies in the foundations of logic and mathematics is to *systematize* or *rationally reconstruct* these disciplines. Such a project, naturally associated with Frege, aims to show that our successful logical and mathematical practice can be rationally reconstructed so as to constitute a system in the desired sense, and it is via this rational reconstruction that we can best understand how and why our practices are successful. I will have a bit more to say about this idea at the end of this paper.

system.[13] The reason for this is that every principle in a system is there via its role in proof; a principle is in the system just in case it is either provable in the system or employed in a proof in the system. Hence, if a principle is non-basic in the system, it gets into the system only by its being provable in the system; i.e. provable using no resources beyond the systematically basic principles. Hence, any non-basic principle in the system is provable in the system from the systematically basic principles. I abbreviate this idea by saying that the systematically basic principles of a system must be *adequate* for that system.[14]

2.3 From Systematically to Epistemically Basic

In the preceding sections, I argued that the question of the justification of basic logical principles is generated by the idea that, in any system of logical principles, there must be principles that are systematically basic (and thus adequate, in the above sense) in that system. The problem is then to explain how these principles—and in particular, the basic principles of *our* system—can be justified. Now, why should we worry about this question? That is, why is it a *problem*? Boghossian gives a clear, and I think widely-accepted, answer. He writes that "saying that we cannot justify our fundamental rules of inference is extremely problematic. For if a claim to the effect that [*modus ponens*] is truth-preserving is not justifiable, then... neither is our use of [*modus ponens*]. If, however, our use of [*modus ponens*] is unjustifiable, then so is anything based on it, and that would appear to include any belief whose justification is deductive" [Boghossian(2000), 235]. As I stressed above, *modus ponens* is here just a synecdoche for the basic logical principles, *whatever they are.* In light of this, we can more explicitly express the generality of the last sentence of the above quotation as follows: If our use of basic logical principles is unjustifiable, then so is anything based on their use, and that would appear to include any belief whose justification is deductive. The idea here is that if the systematically basic principles of our system are not justified, then neither is any result proven through their use. To borrow an expression from Crispin Wright, the worry is that if our systematically basic principles have an epistemic status weaker than justification, then this status will "leach" upward into the superstructure of principles built upon them [Wright(2004)]. That is, principles obtained through the use of basic principles can have an epistemic status no stronger than that enjoyed by the basic principles.[15] Consequently, if non-basic principles can be justified only by proof, then they are unjustified if the systematically basic principles employed in those proofs are unjustified. Thus, since our systematically basic

[13]It is important to clarify that I am *not* claiming here that the systematically basic principles of a given logic must be *complete* for that logic in the usual metatheoretic sense. This is because I am not assuming that systematically basic principles must be finitely axiomatizable.

[14]With this word choice I aim to call to mind Tarski's criterion of *material adequacy.* Essentially the idea here is that a set of principles is adequate when it produces all the results that we want it to. In the case of systematically basic principles, what we want is that they underwrite (or maybe better, comprise) the entire logical system. As Frege put it, the basic laws of his system were to contain the whole of arithmetic, as a seed contains a plant.

[15]We'll have a bit more to say on this issue at the end of this paper.

principles are adequate for our system, if they were unjustified, then everything that we accept on the basis of logical inference would be unjustified too.[16]

I should emphasize here that the skeptical worry driving Boghossian is not my concern here. The trouble, as I see it, is not to determine *that* our systematically basic principles are justified, but rather to explain *how* they are justified.[17] And this challenge is daunting because the explanation for the justification of non-basic logical principles that we have been working with comes via the reduction picture. To wit, non-basic principles are justified because their validity can be established by cogent sequences of inferences, by means of which they are "reduced" (ultimately) to basic principles. But for an inference to be cogent, the principles on which it depends must also be justified. Hence, if we have no explanation of the justification of basic logical principles, then our proposed explanation of the justification of non-basic logical principles is no explanation at all.

Now we appear to be in a position adequately to appreciate the seriousness of our guiding question. Even in logic, we can't prove everything that we accept. And yet, the best sense of justification we have available for logic is articulated in terms of proof. So, how can systematically basic principles, those principles that we accept without proof, be justified? A natural idea here is to hold that systematically basic principles must be justified in some direct and immediate, non-inferential, way. This proposal, in my terminology, is that our systematically basic principles are also *epistemically basic*.

We are thus left in the following situation. In large part *because* our systematically basic principles must be adequate, in order to understand how any logical principles at all can be justified, we must understand how systematically basic principles are justified. Furthermore, because these principles are systematically basic, it appears that they can be justified only by epistemically immediate, non-inferential, means. That is, it appears that systematically basic principles can be justified only if they are also epistemically basic. Put another way, because of the special role that systematically basic principles play in our practices of proof, they seem to demand a special epistemology, and thereby pose a challenge to anti-exceptionalism about logic.

In the remainder of this paper, I will argue that this challenge is illusory.

[16] [Boghossian(2000), 235] and [Hale(2002), 280], to cite two examples, explicitly note this as a motivation for working out an epistemology for basic logical principles.

[17] Here I share Goodman's general attitude. Concerning a related point, he claims that "[a] philosophic problem is a call to provide an adequate explanation in terms of an acceptable basis." And, "[t]he disproportionate emphasis put on the problem has resulted in a gross exaggeration of the consequences of the failure to solve it. Lack of a general theory of goodness does not turn vice into virtue; and lack of a general theory of significance does not turn empty verbiage into illuminating discourse" [Goodman(1983), 31]. Similarly, lacking an account of the epistemology of logic does not deprive us of justification for accepting logical principles. That said, there is a possibility of a skeptical worry creeping in here. If, despite our best efforts, we find ourselves unable to explain how logical principles can be justified, one possible explanation for this state of affairs is that logical principles cannot be justified after all. As should become clear by the end of this paper, I don't think this degree of skepticism is warranted in this case. Thanks to an anonymous referee for this journal for pressing me to clarify this point.

While the foregoing considerations make it appear that systematically basic principles must be justified in some special way—some way that is different from the way in which non-basic principles, and scientific principles more generally are justified—I will endeavor to show that this is not the case. In fact, I will argue that the existence of systematically basic principles actually motivates an abductive account of their justification, according to which they are justified by virtue of the role that they play in the best available logical system. Far from posing a challenge to anti-exceptionalism about logic, systematically basic principles actually provide additional support for the idea that logical principles generally are justified in same way that other scientific principles are. To make this case, I begin, in the next section, by investigating the idea that some logical principles could be *epistemically* basic.

3 Systematic Adequacy and the Epistemically Basic

What makes a principle *epistemically* basic? Candidate answers to this question abound in the literature, but for my purposes here, it will suffice to briefly note a few of them. Some, for instance [BonJour(1998)], hold that epistemically basic principles are those that can be justified directly by rational insight. On another view, such as that advocated by [Peacocke(1998)], epistemically basic principles are justified in virtue of conditions for concept possession. Still others take a naturalistic approach. [Maddy(2002), Maddy(2007)] holds that epistemically basic principles are in some way built into the operation of our primitive cognitive mechanisms. Or, according to Schechter's view [Schechter(2010), Schechter(2013), Schechter(2018)], epistemically basic principles are those such that reasoning in accord with them is a trait that our ancestors were selected for.[18]

As I explained in the previous section, the problem to which these various approaches might be hoped to provide a solution is that of explaining the justification of *systematically* basic principles; those principles on which we depend in making inferences in a given system, but whose validity we cannot establish by making inferences in that system. Given this description of the task, these approaches might appear initially promising. To simplify exposition of this point, let's posit for the moment that *modus ponens* is systematically basic for us, and that our task is to explain how it is justified. It may look initially plausible that we can just immediately grasp that *modus ponens* is valid. It seems that simply by *understanding modus ponens* we can thereby come to acquire a "direct or immediate, non-discursive" reason for thinking that it is valid [BonJour(1998), 102]. And this seems to provide exactly the sort of explanation that was called for. While it may be that complex logical principles

[18]More accurately, Schechter's view is that reasoning in accord with basic logical principles is a *byproduct* of a trait that our ancestors were selected for, but this subtlety does not make a difference to my overall point here.

are justified by proof, it seems plausible that principles like *modus ponens* are justified in an *immediate* way; we can simply "see" that they are valid by an act of rational insight. Alternatively, *modus ponens* may be justified because our acceptance and application of this principle is a direct result of the workings of our most primitive cognitive mechanisms, or it is a result of traits that our ancestors were selected for [Maddy(2007), Schechter(2013)]. Again, this seems to provide precisely the sort of immediate, non-inferential, justification that was called for. Or, we might suppose that *modus ponens* is simply *constitutive* of the meaning of the conditional, so that simply understanding what one says in making an inference employing *modus ponens* is sufficient to justify it. All of these accounts, that is, provide *prima facie* plausible answers to the following question: "How are basic logical principles justified?"

It might appear strange to group together these naturalistic and *aprioristic* approaches. But they share two common features which, I think, justify this grouping. The first such feature is that all of these approaches presuppose that basic logical principles require a special epistemology. This is clear in the case of the *aprioristic* approaches noted above, but even the naturalistic approaches sketched here treat the epistemology of logic as special. Consider Schechter's account, for example. On this account, basic logical principles are justified because reasoning in accord with those principles is a byproduct of traits that our ancestors were selected for. But, this account is not intended to explain our justification for principles that might plausibly be considered basic in other areas (e.g. conservation principles in physics). Our justification for accepting those principles presumably comes from something other than our evolutionary history. Even on this naturalistic approach, then, basic logical principles are epistemologically exceptional. Thus, it is worth noting, both naturalistic and *aprioristic* accounts of the epistemology of logic, can be exceptionalist, contrary to the way that [Hjortland(2017)] carves up the terrain.

Moreover, these approaches share a second important feature with some anti-exceptionalist approaches to the epistemology of logic, notably that of [Williamson(forthcoming)]. This common feature is that, on these approaches, the status of a principle as epistemically basic is independent of that principle's role in any particular system. What makes a principle epistemically basic, on these approaches, is independent of whether or not that principle is systematically basic. On Williamson's recent account, for example, knowledge of basic principles comes via "normal mathematical processes," which are defined to be "all those ways in which our mathematical knowledge can grow" [Williamson(forthcoming), 2].[19] One way in which we can grow our mathematical knowledge, according to Williamson, is by adopting axioms in such a way that we could not easily have been wrong about them. That is, by adopting axioms in way that is *safe*, epistemologically speaking. He suggests that one such means is to adopt axioms which we find "primitively compelling" where, due to our evolutionary history, we could not have easily been wrong in accepting claims which we find primitively compelling in this way [Williamson(forthcoming), §3].

[19]Williamson is using 'mathematical' here in a broad sense that also encompasses logic.

The important thing to observe here is whether or not we find a principle primitively compelling on the basis of an evolutionary history that renders our acceptance of it safe is independent of what role that principle may play in any particular logical (or mathematical) system.

This common feature, shared by the various exceptionalist approaches described above, as well as Williamson's recent anti-exceptionalist approach, is the source of a serious difficulty. The difficulty is that, if a principle's status as epistemically basic is independent of that principle's status as systematically basic, it is at best highly implausible that it is precisely our systematically basic principles that will turn out to be justified in an epistemically basic way. The two notions of 'basic' simply come apart.

On the one hand, it is clearly possible that principles could be epistemically basic in any of the senses described above despite not being systematically basic. For example, in many systems, *hypothetical syllogism*

$$A \to B,\, B \to C \;\Rightarrow A \to C \tag{10}$$

will be not be systematically basic since it is provable by use of *modus ponens* and conditionalization. But it's hard to see how to motivate the idea that, considered independently of a system, hypothetical syllogism is not epistemically basic while those other principles are. If we find *modus ponens* "primitively compelling" or it is "built in" to our most primitive cognitive apparatus, surely the same could be said about *hypothetical syllogism*.

This difficulty is typically obscured, I believe, by focus on principles—like *modus ponens*—that seem to be plausible candidates for both systematic and epistemic basicness. But, as I just pointed out, there are principles that appear to be just as epistemically basic as *modus ponens* despite not being systematically basic. On the other hand, and more seriously, there are other principles which are systematically basic in some systems, but are of dubious standing as epistemically basic. Principles like Gentzen's ∃E

$$\exists x\phi,\, \phi(x/a) \to A \;\Rightarrow\; A \tag{11}$$[20]

are hardly primitively compelling, and it is hard to imagine what selection pressures could have led to our ancestors being disposed to accept all instances of such reasoning. Perhaps even clearer examples can be found in the foundations of mathematics. The axioms of choice and infinity, while systematically basic in ZFC, are hardly candidates for principles that do not seem to be in need of some further justification. The trouble here is that the class of principles that is epistemically basic in any of the senses canvassed above may be *inadequate* in the sense characterized in the previous section; it may be that not all of the principles that are systematically basic in our system are epistemically basic.[21]

[20]Where, as usual, $\phi(x/a)$ is the result of uniformly substituting a for all free occurrences of x in ϕ, and where a does not occur in A.

[21]Gödel's first incompleteness theorem *may* be relevant to this point. If we assume that (1) there are only finitely many (independent) epistemically basic principles, (2) the epistemically basic principles are consistent, and (3) our overall system is sufficiently strong, then Gödel's theorem implies that the epistemically basic principles are not adequate with respect to our system.

Reflecting on various independence and incompleteness results from the last century or so of research in logic and mathematics makes the issue clearer, I think. Developments in mathematics and logic have revealed the need for new foundational principles in order to secure various results. And the primary motivation for adopting these principles is not that they seem particularly compelling, considered on their own, but rather that we seem to *need* them in order to prove theorems that we think we ought to be able to prove. As an example, this appears to have been Frege's motivation for adopting the ill-fated Basic Law V. In the afterword to Volume II of [Frege(2013)], Frege addresses the contradiction that Russell discovered to have stemmed from Basic Law V. He writes: "I have never concealed from myself that it is not as obvious as the others nor as obvious as must properly be required of a logical law. Indeed I pointed out this very weakness in the foreword to the first volume, p. VII. I would gladly have dispensed with this foundation if I had known of some substitute for it. Even now, I do not see how arithmetic can be founded scientifically [without it]..." [Frege(2013), 253]. In this passage, Frege claims that he included Basic Law V as a *systematically* basic principle in his system, despite the fact that it did not appear to be *epistemically* basic. Moreover, the reason why he included it was that it seemed to him that his systematically basic principles would be *inadequate* without it. That is, he could not see a way to derive arithmetic from logic without using this principle.

To suppose, contrary to the points just made, that all and only systematically basic principles are epistemically basic, is to adopt a variant of the old Euclidean ideal—and the dream of the likes of Leibniz and Descartes—that there is a firm and certain independently justified base upon which *all* other knowledge can be based by firm and certain deductive inference. The point I am making here is a variation on an old theme in epistemology, namely that there are competing demands on what should count as the "foundations" of knowledge. The pressure to ensure that the foundations are justified—and in particular justified in a system-indepedent and immediate way—demands that they be very limited in scope, but the pressure to ensure that the foundations suffice to justify everything else demands that their scope be expansive. These pressures pull in opposite directions, and the old bugaboo of the foundationalist is that the idea that these demands could jointly be satisfied smacks of a totally implausible pre-established harmony. These competing demands are placed equally on the notion of basic logical principles—via the notions of epistemically basic and systematically basic, respectively—and it is just as hard to see how they could both be met by a single class of principles.[22]

[22] The point raised in this paragraph bears some similarity to Sellars's famous attack on the "myth of the given" [Sellars(1956)]. But to ward off undue confusion, I will point out an important difference. According to Sellars, foundationalism's "basic beliefs", including importantly perceptual beliefs, must be directly accessible in some way. But, because of this, these beliefs must be nonceptualized. But if this is so, they will be incapable of entered into inferential relations, and will thus be isolated from the non-basic beliefs which they are supposed to support. If this is right, the basic beliefs are grossly inadequate. By contrast, the inadequacy I am urging for the epistemically basic principles is not so gross. My claim is not that they are incapable of interacting with other principles—they can be employed to prove

3.1 Against the Core/Periphery Approach

No doubt proponents of the views I've lately been criticizing will be itching to protest that I've been attacking a straw man. After all, I've been holding their views to the impossibly high standard that epistemically basic principles, whatever they turn out to be, must also be adequate for any acceptable logical system. Indeed, some proponents of this sort of view explicitly disavow the idea that the epistemically basic principles are adequate in this sense. Consider, for example, Maddy's view, on which a relatively weak class of logical principles (loosely based on K3, three-valued "strong" Kleene logic) is epistemically basic; being in some sense built in to our primitive cognitive mechanisms. This class of logical principles is insufficient to serve as a systematically adequate basis for all other principles that we accept.[23] In light of this, Maddy proposes that further principles (specifically, principles of two-valued classical logic) can be postulated as epistemically basic on roughly pragmatic grounds. Thus, on this view, there is an inadequate *core* of immediately justified epistemically basic principles, which a system may supplement with additional systematically basic principles at their *periphery*, which are justified in a different way. Against this sort of "core/periphery" approach, I will now argue that if a proposed class of epistemically basic principles is not adequate for our system, the status of these principles as epistemically basic is irrelevant with respect to the problem which motivates their postulation in the first place.

First, recall that the requirement that our systematically basic principles be adequate for our system is necessary to generate the problem to which the notion of epistemically basic principles is supposed to provide a solution. This is, as I explained, the problem of accounting for justification by proof generally, and not just a problem concerning the justification of systematically basic principles. To be sure, the latter might still appear to be a serious problem. Suppose we have delimited a core C of epistemically basic principles that is a proper subclass of our systematically basic principles, and hence is inadequate in our system. It is of course true that, were these epistemically basic principles to turn out to be unjustified, then any principles derived from them would be unjustified too. Since those principles are systematically basic, furthermore, this would appear to undercut the justification of a large swath of principles that we currently take to be valid, even if it doesn't generate a full-blown problem concerning all justification by proof. As I will now endeavor to show, however, this conclusion is mistaken. To the contrary, no especially problematic consequences would result if the principles in C were to turn out to be unjustified.

Consider the proposed inadequate core C of epistemically basic principles. All of the members of C, we can suppose, are justified in a suitably direct and immediate way. Nonetheless, since C is inadequate, there is at least one principle P in our system such that P is neither in C nor provable from C. We can assume,

the validity plenty of other principles—but only that they do not suffice to prove *all* other principles that we accept.

[23] It is insufficient to derive the validity of even fairly simple and uncontroversial logical principles such as *modus tollens*.

for simplicity, that P is systematically basic.[24] Now, since P is in the system we are working with, it is taken to be valid. Furthermore, since we accept P even though it is neither in nor provable from C, there must be some reason why we continue to accept it. But what could this reason be? Since P is not epistemically basic, we don't accept P because we grasp it by rational insight, believe it on the basis of the operations of primitive cognitive mechanisms, etc. What else is left? Notice that, since P is arbitrary, the task here is to find a way to account for our acceptance of *all* of the systematically basic principles not in our privileged class C of epistemically basic principles. To ensure that this is accomplished, it seems that the reason why we accept systematically basic principles must be, at least in part, *simply that* they are systematically basic. This is because our systematically basic principles may have nothing more substantive in common than that they are all systematically basic. Remember how we characterized systematically basic principles: They are those principles, *whatever they are*, which are employed in proofs in a system, but which are not themselves reducible to other principles in the system.

In light of this, it seems that the only available option is to hold that systematically basic principles are justified simply *because* they are systematically basic. In next section, I will explain this option in more detail and attempt to make it appear more plausible. For the moment, I just want to point out that core/periphery accounts of the epistemology of logic face a dilemma at this juncture. If they do not accept this sort of justification, then they will have failed to account for the justification of systematically basic logical principles outside of the epistemically basic core, and thereby have failed to account for the epistemology of basic logical principles generally. On the other hand, if they do accept this sort of justification for systematically basic principles outside of C, then showing that the individual principles in C are justified in some epistemically immediate way has no real point. Remember the motivation for this project: If the principles in C were unjustified, then, because of the reduction picture, nothing based on them would be justified either, and this, presumably, would include a large swath of principles that we currently accept. Once the option described above is on the table, however, the issue looks far less pressing. Even if it were to turn out that *none* of the principles in C are in fact immediately justified in a system-independent way, we could still account for their justification by appealing to their systematically basic role in our logical system. In other words, if it turns out that systematically basic principles can be justified simply *because* of the basic role that they play, then it doesn't matter whether these principles are, or could be, justified for some other reason. If the justification of systematically basic principles can be accounted for simply by virtue of the fact that they are systematically basic, then we need not bother ourselves with trying to find any further special epistemic properties that these principles might enjoy.[25]

[24] This assumption engenders no loss of generality. For, if P is not systematically basic, the issue we are about to discuss will nevertheless arise for whichever principle(s) P is ultimately based on, and these cannot be in or based on principles in C.

[25] To be clear, this sort of account might not be adequate for other explanatory and justifi-

To be clear, the problem described here does not stem from the fact that the inadequate core class C of epistemically basic principles is specified naturalistically, as in Maddy's account. Rather, the problem stems from the core/periphery approach generally, in which there is an inadequate core of epistemically basic principles justified in a direct and system-independent way, with additional systematically basic principles outside the core whose justification stems from the systematic role that they play. A variety of non-naturalistic views of this sort exist. For example, [Hale(2002)] argues that there is a core of logical principles whose soundness we cannot rationally doubt, and can thus justifiably accept without proof. But this core—consisting of a handful of principles governing the conditional and the universal quantifier—is inadequate with respect to the overall class of principles that we accept. Hence, in this case too, an additional source of justification must be postulated to cover the rest of the systematically basic principles. Since this additional source must be sufficient to account for justification of all systematically basic principles, it is of course also sufficient to account for the positive epistemic status of the principles in the inadequate core. Thus, the special epistemic status enjoyed by the core principles turns out to be unnecessary, and hence unmotivated by the problem concerning justification by proof, which was supposed to be the motivation for their postulation in the first place.

4 Abductivism and Basic Logical Principles

We seem to be left in the following predicament. We have good reason to suspect that principles that play a systematically basic role for us will not in general share any particular property that would make them epistemically basic, considered independently of their systematic role. In logic and mathematics, we choose systematically basic principles not because they seem obvious, but because they seem necessary. Nonetheless, our systematically basic principles must be capable of being justified in some direct, non-inferential way. To find our way out of this predicament, let's remind ourselves of the role that systematically basic logical principles are supposed to play.

We begin our investigations into logic with an overall body of theorizing that includes a generally successful practice of making and accepting deductive inferences. A central task in logical theory is to systematize this practice; to organize and unify the logical principles which our practices epistemically commit us to accepting, and thereby to clarify our epistemic commitments and our understanding of our own theorizing. In constructing such a system, it is in-

catory aims. For example, if the task is to explain why we find some logical principles to be *obvious*, a core/periphery approach might be appropriate. Similarly, if our task is to bolster the justification of basic logical principles as much as possible, we might accomplish this by finding multiple, independent sources of justification for basic logical principles. Perhaps they get some justification simply for being systematically basic, and additional justification due do some other special property that some of them enjoy. Thanks to an anonymous referee for this journal for raising this interesting issue. See also [Shapiro(2009)] for some important additional issues related to, but distinct from, those considered here.

evitable that some principles will be basic in it, in the sense that they will have no non-trivial proof in the system. Our task in the epistemology of logic is to account for the justification of those principles. This task is daunting because the special, basic role of these principles seems to demand a special account of their epistemology. And, as I argued above, given that the point of these systematically basic principles is to systematize our deductive practices, we have no reason to suppose that these principles will also be epistemically basic, especially considered independently of their systematic role. That is, whatever we pick as the epistemically special characteristic to figure in our explanation, it seems extremely unlikely that this characteristic will be shared by all and only the principles that play the special systematic role; i.e. the principles whose epistemology we are trying to explain.

This problem is obscured by the fact that, as I explained above, 'basic' is used to mean both *systematically fundamental* and *epistemically immediate*. What I have been pointing out here is that these notions just come apart in foundational work in logic and mathematics. And this is not a new insight. Russell nicely made this point himself, in 1907: "The usual mathematical method of laying down certain premises and proceeding to deduce their consequences, though it is the right method of exposition, does not, except in the more advanced portions, give the order of knowledge. This has been concealed by the fact that the propositions traditionally taken as premises are for the most part obvious, with the fortunate exception of the axiom of parallels. But when we push analysis further, and get to more ultimate premises, the obviousness becomes less, and the analogy with the procedure of other sciences becomes more visible" [Russell(1973), 282].[26] Russell's point, with which I agree, is that looking for something epistemically special about our systematically basic principles is a mistake; no matter what we find, it won't be up to the explanatory task that we're trying to put it to.

So how *should* we account for the justification of systematically basic principles, then? Russell's formulation gives a clue: by analogy with the justification of basic principles in other sciences. That is, Russell is here suggesting a kind of anti-exceptionalism about logic.[27] But what does this look like? In the previous section, I suggested that systematically basic principles might be justified *simply because* they play this role. But this formulation leaves out an important qualification: the principles in question must be systematically basic in an overall successful logical system; one that performs at least as well as any available alternative. That is, the way forward is to turn to an *abductivist* account of the epistemology of logic. On such an account, logical systems are justified by virtue of providing the best overall systematization of their target phenomena,

[26] [Williamson(2016)] also discusses this paper, and draws conclusions from it that are similar to those that I draw here. See also the extremely helpful commentary on this paper in [Irvine(1989)].

[27] See [Williamson(2016)] for more discussion of this idea. Note, too, that while Russell's proposed methodology is broadly abductivist, he also holds that logical principles can be justified *a priori*. Once again, this shows that the exceptionalist/anti-exceptionalist distinction cuts across the naturalist/*a priorist* distinction, contrary to the claims of [Hjortland(2017)].

viz., the body of inferences that comprises an accepted deductive practice.[28] The logical principles that comprise such systems are, in turn, justified because of the role that they play in those systems. Consequently, on such an account, there is nothing epistemically special about systematically basic principles, but rather, basic and non-basic principles alike are justified by their inclusion in an overall successful logical system.[29]

Notice that to accept this abductivist picture is, in part, to reject the reduction picture of inferential justification that I discussed in §2. So what is the relationship between justification and proof, on the abductivist account? The answer is that to infer one claim from others that one accepts, and to accept it on the basis of having so inferred it, is to give it a place in one's overall system. With respect to logical and mathematical principles, the point of inferential justification by proof on this systematic picture is quite close to what it would be on the traditional reduction picture. Shapiro puts the point nicely: "By deriving a hitherto assumed proposition, or a working proposition, from others, we see what is involved in accepting or rejecting it. . . . The project of systematizing, as endorsed by Frege so enthusiastically, makes sense from the holistic perspective; indeed, system is what drives the enterprise" [Shapiro(2009), 204].

This is almost, but not exactly, the traditional foundational picture. The crucial difference is that, while the system is foundational in *structure*, the justification enjoyed by the principles in the system does not have its *source* in the basic principles and then transmit, via proof, to non-basic principles.[30] This also shows, contrary to Wright's worry, that the epistemic status of basic principles does not "leach" upward into the principles which are derived from them. This worry, we are now in a position to see, is an artifact of the reduction picture. Justification does not transmit, as on the reduction picture, from the basic principles to those proved from them. Instead, I have urged, justification of the logical principles in a system is engendered by the virtues of that system as a whole.

In sum, while it might appear that the special role of systematically basic logical principles makes inevitable the need for a special epistemology to explain our justification for accepting them, this appearance is deceiving. In fact, as I've argued, the reasons that motivate thinking that there *are* systematically basic principles actually undercut, rather than motivate, an exceptionalist epistemology for those principles. Our logical systems have basic principles, but our

[28]With this formulation, I aim to call to mind Goodman's point that general principles of inference and specific inferences are both justified by being brought into "equilibrium" with one another [Goodman(1983)]. But this general idea leaves a lot of work to be done in specifying the details. Most notably, an important question remains: what exactly *are* the inferences to be systematized? At the very least, I think, they include inferences contained in our most successful deductive practice, that of mathematical proof. But they may also include vernacular inferences whose apparent validity we would like to account for. See [Martin and Hjortland(2021)] for detailed and helpful discussion of this issue.

[29][Shapiro(2009), Sher(2013), Williamson(2017)] motivate their own accounts by appeal to similar considerations.

[30]See [Carlson(2015)] for further discussion of the structure/source distinction and its relationship to the justification of logical systems.

justification for accepting them, must, like our justification for accepting basic principles of other sciences, be abductive in character.[31]

References

[Boghossian(2000)] Boghossian P (2000) Knowledge of Logic. In: Boghossian P, Peacocke C (eds) New Essays on the a Priori, Oxford University Press, New York, pp 229–254

[BonJour(1998)] BonJour L (1998) In Defense of Pure Reason: A Rationalist Account of A Priori Justification. Cambridge University Press

[Carlson(2015)] Carlson M (2015) Logic and the Structure of the Web of Belief. Journal for the History of Analytical Philosophy 3(5):1–29

[Corcoran and Hamid(2016)] Corcoran J, Hamid IS (2016) Schema. In: Zalta EN (ed) The Stanford Encyclopedia of Philosophy, Fall 2016 edn, Metaphysics Research Lab, Stanford University

[Dummett(1978)] Dummett M (1978) The Justification of Deduction. In: Truth and Other Enigmas, Harvard University Press, Cambridge, pp 290–318

[Dummett(1991)] Dummett M (1991) The Logical Basis of Metaphysics. Harvard University Press

[Frege(1950)] Frege G (1950) The Foundations of Arithmetic. Blackwell

[Frege(1979)] Frege G (1979) Posthumous Writings. University Of Chicago Press

[Frege(2013)] Frege G (2013) Basic Laws of Arithmetic. Oxford University Press UK

[Goodman(1983)] Goodman N (1983) Fact, Fiction, and Forecast, 4th edn. Harvard University Press

[Hale(2002)] Hale B (2002) Basic Logical Knowledge. Royal Institute of Philosophy Supplement 51:279–304

[Hjortland(2017)] Hjortland OT (2017) Anti-exceptionalism about logic. Philosophical Studies 174(3):631–658, DOI 10.1007/s11098-016-0701-8

[31] This paper had a lengthy gestation period, and far more people helped me wrestle with the ideas in it than I can adequately acknowledge in this space. I presented earlier versions of the material in this paper at the Eastern APA, the Society for Exact Philosophy, and at Smith College, Wabash College, and the University of New Mexico, and I am grateful to audiences on all of these occasions for helpful questions and comments. Moreover, I am grateful to several anonymous referees for this journal whose careful and astute suggestions made this paper much stronger. Finally, I owe special debts of gratitude to Joshua Schechter, Gary Ebbs, and David McCarty for their patient and helpful comments on earlier versions of this paper and the ideas it contains.

[Irvine(1989)] Irvine AD (1989) Epistemic logicism & Russell's regressive method. Philosophical Studies 55(3):303–327, DOI 10.1007/BF00355328, URL http://www.springerlink.com/content/w514177107372325/

[Maddy(2002)] Maddy P (2002) A Naturalistic Look at Logic. Proceedings and Addresses of the American Philosophical Association 76(2):61–90, URL http://www.jstor.org/stable/3218629

[Maddy(2007)] Maddy P (2007) Second philosophy: a naturalistic method. Oxford University Press, Oxford

[Martin and Hjortland(2021)] Martin B, Hjortland O (2021) Logical predictivism. Journal of Philosophical Logic 50(2):285–318, DOI 10.1007/s10992-020-09566-5

[Parsons(2008)] Parsons C (2008) Mathematical Thought and Its Objects. Cambridge University Press

[Peacocke(1998)] Peacocke C (1998) Implicit Conceptions, Understanding and Rationality. Philosophical Issues Vol. 9, Concepts:43–88

[Prawitz(1974)] Prawitz D (1974) On the Idea of a General Proof Theory. Synthese 27(1/2):63–77, URL http://www.jstor.org/stable/20114906

[Prawitz(2006)] Prawitz D (2006) Natural Deduction: A Proof-Theoretical Survey. Dover Publications

[Russell(1973)] Russell B (1973) The regressive method of discovering the premises of mathematics. In: Lackey D (ed) Essays in Analysis, George Braziller, Inc., New York, pp 272–283

[Russell(2020)] Russell G (2020) Logic isn't normative. Inquiry: An Interdisciplinary Journal of Philosophy 63(3-4):371–388, DOI 10.1080/0020174x.2017.1372305

[Schechter(2010)] Schechter J (2010) The reliability challenge and the epistemology of logic. Philosophical Perspectives 24(1):437–464, DOI 10.1111/j.1520-8583.2010.00199.x

[Schechter(2013)] Schechter J (2013) Could evolution explain our reliability about logic? In: Gendler TS, Hawthorne J (eds) Oxford Studies in Epistemology 4, Oxford University Press, p 214

[Schechter(2018)] Schechter J (2018) Is there a reliability challenge for logic? Philosophical Issues 28(1):325–347, DOI 10.1111/phis.12128

[Sellars(1956)] Sellars WS (1956) Empiricism and the philosophy of mind. Minnesota Studies in the Philosophy of Science 1:253–329

[Shapiro(2009)] Shapiro S (2009) We Hold These Truths to be Self-Evident: But What Do We Mean by That? The Review of Symbolic Logic 2(01):175–207

[Sher(2013)] Sher G (2013) The foundational problem of logic. Bulletin of Symbolic Logic 19(2):145–198, DOI 10.2178/bsl.1902010, URL http://projecteuclid.org/euclid.bsl/1368716899

[Williamson(2016)] Williamson T (2016) Abductive philosophy. Philosophical Forum 47(3-4):263–280, DOI 10.1111/phil.12122

[Williamson(2017)] Williamson T (2017) Semantic paradoxes and abductive methodology. In: Armour-Garb B (ed) Reflections on the Liar, Oxford University Press, pp 325–346

[Williamson(forthcoming)] Williamson T (forthcoming) Absolute provability and safe knowledge of axioms. In: Horsten L, Welch P (eds) The Limits of Mathematical Knowledge, Oxford University Press

[Wright(2002)] Wright C (2002) (Anti-)Sceptics Simple and Subtle: G.E. Moore and John McDowell. Philosophy and Phenomenological Research 65(2), DOI 10.1111/j.1933-1592.2002.tb00205.x, URL http://onlinelibrary.wiley.com/doi/10.1111/j.1933-1592.2002.tb00205.x/abstract

[Wright(2004)] Wright C (2004) Warrant for Nothing (and Foundations for Free?). Proceedings of the Aristotelian Society, Supplementary Volumes 78:167–245, URL http://www.jstor.org/stable/4106950, bibtex: wright2004warrant

[Zermelo(1908)] Zermelo E (1908) A new proof of the possibility of a well-ordering. In: van Heijenoort J (ed) From Frege to Gödel: A Sourcebook in Mathematical Logic, 1879–1931, Harvard University Press