Title
Challenges for emergent gravity

Permalink

Journal

ISSN

Author
Carlip, Steven

Publication Date

DOI

Copyright Information

Challenges for Emergent Gravity

S. Carlip*

Department of Physics
University of California
Davis, CA 95616
USA

Abstract

The idea of gravity as an "emergent" phenomenon has gained popularity in recent years. I discuss some of the obstacles that any such model must overcome in order to agree with the observational underpinnings of general relativity.

---

*email: carlip@physics.ucdavis.edu

Over the past few years, the idea of gravity as an "emergent" phenomenon has become increasingly popular. Emergence is seldom sharply defined, and the notion has been described as "vague and contentious" [1], but the basic picture is that gravity, and perhaps space or spacetime themselves, are collective manifestations of very different underlying degrees of freedom. Such proposals are not new—Wheeler wrote about pregeometry as early as 1963 [2], and Finkelstein introduced a version of causal set theory in 1969 [3]—but historically they were marginal aspects of research in quantum gravity, whose most noted proponents were often outsiders to the field (e.g., [4–6]). More recently, though, such mainstream approaches as string theory matrix models [7], the AdS/CFT correspondence [8, 9], loop quantum cosmology [10], and quantum Regge calculus [11] have all been described as "emergent."

Given the enormous difficulty of quantizing general relativity, it is natural to consider the possibility that we have simply been trying to quantize the wrong degrees of freedom [12, 13]. But general relativity has an enormous body of observational support, and models of emergence face formidable challenges in reproducing these successes. The goal of this article is to explain some of these challenges.

This is by no means a comprehensive survey of emergent gravity. The specific models I refer to have been chosen primarily as exemplars for particular issues, and most of the ideas are well known to those working in the field, although they have not been collected in one place. For broader reviews, see [14] or articles in [15].

I should also start with a cautionary note. In one sense, *any* quantum theory of general relativity will have "emergent" aspects. All observables (in Dirac's sense) are necessarily nonlocal [16, 17], so locality must be an emergent property. Moreover, for asymptotically anti-de Sitter spacetimes, and perhaps for asymptotically flat spacetimes as well, the algebra of observables is isomorphic to an algebra of operators at an asymptotic boundary [18], so the bulk physics can itself be viewed as emergent from a boundary theory. Such a picture seems contrary to the spirit of emergence, since the boundary degrees of freedom are merely asymptotic values of ordinary bulk degrees of freedom, but I do not know how to make this distinction precise.

1   Classifying emergent models

My aim is to describe obstacles facing all models of emergent gravity, rather than focusing on details of any particular approach. Still, it will be useful to distinguish two broad categories of models that face rather different challenges.

The first category, which I will call type I, comprises models in which the fundamental degrees of freedom live in some sort of "environment": a medium, a lattice, a pre-existing space or spacetime, or the like. Examples include analog models based on fluid flows and similar phenomena [19–22], gravity-like excitations near the Fermi point [6, 23], quantum Hall effect edge states [24], deformations in an elastic solid [25], and spin systems on a fixed lattice [26, 27]. The background environment can have fairly minimal structure; in "pregeometric" models, for example, composite gravitons appear in what is initially a purely topological manifold [28–32]. Type I models must typically decouple the environment from observable quantities quite strongly to reproduce observation.

The second category, which I will call type II, comprises models in which space or spacetime are themselves emergent. Examples include graph-based models such as quantum graphity [33] and DUCTs [34], group field theory [35, 36], certain matrix models [37], Wheeler's notion of "it from bit" [38], and other attempts to model the Universe as a quantum computer [39, 40].

Arguably, some versions of the AdS/CFT correspondence [8,41,42]—those in which the conformal field theory is considered primary and the bulk spacetime emergent—fall into this category; the CFT lives in some lower dimensional, nondynamical space, but points in that space need not have any relationship with points in our bulk spacetime, so the latter might be considered emergent. In type II models, before even asking about contact with observation, one must typically work quite hard to find observables that display the large scale existence of spacetime at all.

The distinction I am making is similar, although not quite identical, to Bain's differentiation between emergence from a "spatiotemporal structure" and a "non-spatiotemporal reality" [43]. It is not a sharp one. Causal set models [44, 45], for instance, contain elements of a pre-existing spacetime in the form of points and their causal relations, but the continuum is emergent. If the conformal field theory is viewed as primary in the AdS/CFT correspondence, our bulk spacetime might be considered emergent, but if the correspondence is a true duality, the bulk spacetime is present from the start. The hypothesis of spontaneous dimensional reduction at short distances [46,47], the dynamical generation of extra dimensions [48], and some "CFT primary" forms of the AdS/CFT correspondence [42] suggest another possibility: some dimensions may be emergent while others are not.

I am omitting a third category, models in which spacetime, the metric, and diffeomorphism invariance are present, but the *dynamics* of gravity is emergent. The archetype is Sakharov's induced gravity [49, 50], in which the Einstein-Hilbert action first appears as a counterterm in the matter action.* From the point of view of effective field theory [51], the distinction between such models and general relativity is marginal: an effective action includes all possible terms, and it is not clear that one can distinguish their origins. Also in this category are models in which the dynamics of the metric is determined by thermodynamics [52–54], or as a consistency condition for the propagation of other fields [55]. While these are certainly interesting, they fail my (perhaps narrow) criteria for emergence unless the metric is absent as a fundamental field.

2 Gravity as a metric theory

Before proceeding further, it is worthwhile to review the physical basis for treating gravity as a metric theory. As first discussed by Weyl and later developed by Ehlers, Pirani, and Schild [56], the observations of local Lorentz invariance and the universality of free fall allow one to construct a metric description of gravity. Briefly summarized, the argument goes as follows [56,57] (for a somewhat different approach, see section 2.3 of [58]):

1. **Local Lorentz invariance** implies the existence of a field of light cones, which establishes a causal structure and a topology. The light cones also determine a conformal structure— an equivalence class of metrics that differ only by local rescalings—for which paths of light rays are null geodesics.

2. **The equivalence principle**, specifically the universality of free fall, determines a set of preferred paths in spacetime, the trajectories of freely falling structureless objects. Such a projective structure, in turn, determines an equivalence class of affine connections for which these paths are geodesics.

---

*In some Sakharov-inspired models, the metric appears only as a collective field. I consider such models to be emergent; the examples I am aware of are type I. Note that because the metric is a functional of other fields, the resulting field equations may differ from those of general relativity even if an Einstein-Hilbert action is induced [13].

3. **Compatibility** of these two structures—the observation that the trajectories of freely falling massive objects lie within the light cones but can "chase" light arbitrarily closely—fixes a Weyl structure, an equivalence class of conformal metrics and affine connections such that

$$\nabla_a g_{bc} = A_a g_{bc} \qquad \text{for some vector field } A. \tag{2.1}$$

As Einstein first noted, though (see [59]), unless A_a is of the form $\partial_a \varphi$ for some scalar φ, eqn. (2.1) implies that lengths change under parallel transport. This would lead to a "second clock effect," a dependence of the rate of a clock on its history. The observation that this is not the case in our Universe suggests a further condition:

4. **The absence of a "second clock effect"** implies that the vector A can be eliminated by a Weyl transformation, picking out a unique representative of the conformal class of metrics. This requirement can be replaced by a number of others: for instance, that neighboring clocks remain synchronized [56] or that matter waves follow geodesics in the short wavelength limit [60].

Together, these observations imply that motion in a gravitational field can be described as geodesic motion in a Lorentzian spacetime, with a metric providing a full description of the field. To go beyond this kinematic setting and obtain the dynamics of general relativity, more is needed. One avenue—certainly not the only one—is this:

5. **The absence of nondynamical background structures** implies "general covariance" in the sense commonly used by physicists [61]: gravity should be described by diffeomorphism-invariant expressions involving only the metric and other dynamical fields. This is a new assumption: while Lorentz invariance restricts nondynamical objects, it does not eliminate them. A flat background metric, for instance, is allowed by Lorentz invariance, but permits Nordstrøm's conformally flat theory and Rosen's bimetric theory (see [58]); a nondynamical volume element, also allowed by Lorentz invariance, permits unimodular gravity [62].

We must still address the possibility of additional dynamical fields. We can, of course, eliminate these by fiat—in a scalar-tensor theory, for instance, we can call the purely metric piece of the interaction "gravity" and relegate the scalar to the status of an extra "fifth force." Alternatively, note that our kinematic assumptions led to a picture in which the response of matter to the gravitational field depended solely on the metric. We might ask that the reciprocal response of the gravitational field to matter also occur solely through the metric, with no added fields whose sole function is to mediate between the two (as in, for instance, TeVeS [63]). That is,

6. **The decoupling of any nonmetric degrees of freedom** from the dynamics of gravity implies that the gravitational effective action should depend on the metric alone.

7. **The methods of effective field theory** then tell us how to formulate the action [51]. At the scales at which a metric description applies, the effective action will include all possible local, diffeomorphism-invariant functions of the metric that are not excluded by other symmetries. At low energies, this effective action will coincide with the Einstein-Hilbert action, albeit with a cosmological constant whose magnitude remains a mystery. One caveat remains: the derivative expansion that defines the effective action works only if the metric varies slowly compared to the Planck scale. This is certainly true observationally, but at a deeper level it is a further mystery.

These arguments do not require that the metric be a *fundamental* degree of freedom. Rather, they describe the setting in which an effective metric description might naturally emerge. On the one hand, this provides hope for emergent gravity: only a few steps are needed to obtain a model that agrees with observation. On the other hand, though, none of these steps can be avoided, and this places severe restrictions on such models. Most of the pitfalls I describe below arise, directly or indirectly, from these requirements.

3   Challenges for models of emergent gravity

The preceding section established a set of conditions that a model of emergent gravity should meet to reproduce general relativity. While such a criterion may be too strong at cosmological or sub-millimeter distances, general relativity is extremely well established at intermediate scales [58, 64], and any disagreements are strongly constrained by observation. Let us now try to understand the extent to which these conditions create obstacles for emergent gravity.

3.1   Lorentz invariance

We start with Type I models, in which the fundamental degrees of freedom live in a space or spacetime environment. The basic problem is clear: unless that background is itself Lorentz invariant, it must decouple from observable quantities strongly enough to match observations. Since experimental limits on Lorentz violation are very strong [65], this is a severe restriction.

Perhaps surprisingly, we have simple examples in which such a decoupling occurs, at least to lowest order. In many analog models—models in which curved spacetimes are mimicked by phenomena such as fluid flows—small perturbations in the flow satisfy Lorentz invariant equations, with an effective "speed of light" determined by properties of the medium [19, 20]. Quite generally, linearization of a field theory around a nontrivial background leads to an effective Lorentzian metric [66], whose signature is fixed by the hyperbolicity of the partial differential equations. At higher orders, Lorentz violations reappear [67, 68], and can serve as constraints on such models.

Problems arise, though, as soon as more than one kind of excitation can occur. In that case, distinct excitations typically have distinct Lorentz invariances, with different effective metrics and speeds of light [14, 69]. Even this is a special case; in general, the Lorentzian geometry becomes Finslerian, losing contact with the desired physics. One can sometimes recover a single metric by imposing a discrete symmetry on the fundamental fields [70, 71], but this step seems rather artificial. Note that it is *not* enough to simply claim that the excitations are all perturbations of the same medium: even in an ordinary elastic solid, longitudinal and transverse waves travel at different speeds.

One solution is to postulate that the "environment" is itself Lorentz invariant. If this environment is a spacetime, this begs the question: as in the EPS construction of section 2, invariance implies the existence of a conformal class of metrics, and one must explain why these are not already dynamical. Models such as [72] in which the background has no metric avoid this problem, and Lorentz invariance may be inserted by hand as a gauge symmetry. Relating this symmetry to spacetime Lorentz invariance requires a fairly elaborate scenario, though: one must ensure the emergence of a nondegenerate soldering form, a tetrad whose two-index structure "solders" the fibers in which the gauge group acts to the spacetime [73].

If the background is discrete, one must work harder. As Dowker has emphasized [74], Lorentz invariance requires a radical nonlocality on a lattice, in the sense that each point has infinitely

many nearest neighbors [44, 75]. Causal set theories [45, 74, 76] can achieve statistical Lorentz invariance with a suitable "sprinkling" of spacetime points, but this becomes much harder if one starts with a more complex discrete structure. Causal dynamical triangulations [11] may recover Lorentz invariance as an average over noninvariant simplicial complexes, but this is not certain; the continuum limit of this model may be a Hořava-Lifshitz theory with a preferred time slicing [77, 78].

One may look instead for models in which Lorentz invariance is only recovered at large distances. After all, ordinary lattice quantum field theory is constructed on a lattice that is not even rotationally invariant, but by tuning parameters to a second order phase transition one can send correlation lengths to infinity, wiping out any memory of the underlying lattice. It is not clear that a similar procedure exists for Lorentz invariance, though, where the correlations must respect the light cone structure. One possible ingredient could be an emergent supersymmetry, which can suppress Lorentz violation [79].

Alternatively, Lorentz invariance could appear as a low energy symmetry under the renormalization group flow [80]. Models are known in which different "speeds of light" flow to a single value at an infrared fixed point [81], but this flow is typically only logarithmic in energy, requiring enormous initial energy scales or delicate fine tuning to meet observational constraints.

For type II models, Lorentz invariance may be less contrived. By a century-old argument [82], the existence of inertial frames, isotropy of space, and the relativity principle are enough to imply Lorentz transformations with some (perhaps infinite) "speed of light." For type I models, the presence of a background typically violates either isotropy or relativity, but this need not be the case for for type II models. The effective speed of light might then be determined dynamically, for example from the Lieb-Robinson limit on the speed of information propagation [83] or from a group structure already present in the fundamental degrees of freedom [84]. But while isotropy may be natural, the relativity principle is more problematic. The Lorentz group is noncompact, so transformations must relate inertial frames that are arbitrarily "distant" (although it is again conceivable that an emergent compact supersymmetry could help). This noncompactness also makes it difficult to achieve Lorentz invariance by averaging over noninvariant configurations, since the integral over boosts diverges, though there has been some work on defining a group average [85].

Of course, we have not experimentally tested Lorentz invariance up to infinite boost. But even violations at very high energies can feed back into quantum field theory through loop effects and lead to drastic consequences at low energies [86], although there are proposals for avoiding this problem (e.g., [87]). Small violations of Lorentz invariance also lead to problems with black hole thermodynamics: unless black holes simply do not exist in the underlying theory, such effects generically violate the generalized second law of thermodynamics, allowing perpetual motion machines [88, 89].

3.2 Principle of equivalence

The principle of equivalence takes a number of different guises, not all exactly equivalent. For our purposes, the most relevant version is the universality of free fall, with its implication that all forms of matter couple to gravity with equal strength. As Feynman emphasized [90], this universality implies a spin two graviton: energy falls with the same acceleration as mass, and the unique Lorentz covariant combination of mass and energy density, the stress-energy tensor, couples to spin two. A model of emergent gravity must thus ensure that

5

1. only one massless spin two field is relevant;

2. this field couples with equal strength to all matter;

3. any spin zero or spin one components of the interaction are absent or strongly suppressed. (In some models [24], higher spin interactions must also be suppressed.)

The principle of equivalence is extremely well tested from millimeter to Solar System distances [58, 91], so while the very short and long distance behavior may differ, these requirements are quite strong.

In type I models, the would-be gravitational degrees of freedom typically have no initial connection to the geometry; their role as a metric emerges later. Hence there is no obvious reason to expect only a single massless spin two field. For a large class of models built from field theory fluctuations around a linearized background [69], for instance, many "gravitons" appear, and the imposition of an ad hoc symmetry [70] is the only known way to force universality. In models of composite gravitons, a similar multiplicity of potential metrics occurs [92]. One can argue that if the effective action is invariant under diffeomorphisms and local Lorentz transformations, only one such field will remain massless, with the others acquiring large masses [92]—typically on the order of the Planck mass, although approximate symmetries can make them smaller [93]. The requirement of exact invariance is, of course, a very strong one: strong enough, in fact, to *forbid* more than one massless graviton [94]. But this solution is also problematic, since most models with massive spin two fields are sick, containing negative energy Boulware-Deser ghosts [95]. While a few exceptions exist [96–99], these require a very special form of the action, and it is not at all clear how such a feature would emerge from a more primitive model.

Once one has a single metric, though, a result of Weinberg offers a path for deriving the equivalence principle [100]. The "soft graviton theorem" shows that a Lorentz-invariant, massless spin two particle that can scatter nontrivially must couple universally to a single conserved stress-energy tensor. One must be careful of assumptions here; see the discussion below of the Weinberg-Witten theorem. In particular, Weinberg's result are only relevant if Lorentz invariance has already emerged. But the theorem suggests that if only one metric is present, universal coupling may not be arbitrary, but may be associated with the universality of Lorentz invariance.

For type II models, the primary question comes earlier: does a dynamical spacetime emerge at all? If the fundamental degrees of freedom generate such a spacetime at some scale, a metric description offers a natural way to describe the dynamics. As in ordinary general relativity, one might then expect a single spacetime to have a single metric. On the other hand, if matter emerges from the same degrees of freedom, Weinberg's soft graviton theorem, with its requirement of Lorentz invariance, is the only reason I know to expect universal coupling. Since very few type II models are yet able to describe the coupling of matter to gravity, much less to compare couplings of more than one species of matter, the problem remains almost completely open.

3.3 Self-coupling

One aspect of universal coupling deserves special attention: we observe gravity's coupling to its own energy to occur at the same universal strength as its coupling to matter [58, 101]. This self-coupling implies that the interaction is nonlinear, and, in fact, it can be used to determine the nonlinear terms, giving another route to the Einstein field equations [90, 102–104].

This property places requirements on emergent models beyond the linear approximation. Obtaining the correct linear behavior—a massless spin two excitation, even with the correct

coupling to matter—is not sufficient to show that one has a model with gravity. Indeed, there are known examples (e.g., [24, 26]) in which the correct nonlinear behavior seems to require a good deal of fine tuning.

If one can obtain Lorentz invariance and diffeomorphism invariance, however, these provide some very helpful constraints. As Kraichnan first showed [102], if one starts with a massless spin two field h_{ab} on a manifold with a flat metric η_{ab} and assumes that its field equations can be derived from a Lorentz invariant, diffeomorphism invariant action with no additional background structures, then the action can depend only on the combination $\eta_{ab} + h_{ab}$. This largely determines the form of the nonlinearities to be those of general relativity, thus fixing the self-coupling.

3.4 Diffeomorphism invariance I

Diffeomorphism invariance is a notoriously slippery concept in general relativity [61,105]. The rather heuristic form I will use is the absence of any nondynamical background structure that could define a preferred reference frame. Most type I models have a nondynamical background, so the issue is again one of decoupling. Most type II models do not, but one must show that the emergent spacetime is enough like a smooth manifold for diffeomorphism invariance to make sense at all. Note that while diffeomorphism invariance and Lorentz invariance are conceptually distinct, they are not completely unrelated: in an "already Lorentz invariant" model, the only invariant background structures are a flat metric and a volume element, so the possible forms of diffeomorphism noninvariance are restricted.

As Witten has stressed [9], diffeomorphism invariance also requires the absence of local observables [16,17]. This presents yet another decoupling problem [106]: in type I models, all local observables, including any fundamental stress-energy tensor, must be invisible at the scale at which gravity emerges, while in type II models, the emergent spacetime should probably be free of local observables from the start.

For type I models, the problem of diffeomorphism invariance parallels the decoupling problem for Lorentz invariance. One case is known in which a weak form of diffeomorphism invariance appears [71]. In this analog model, Nordstrøm gravity emerges at lowest order, but conformal invariance of the matter fields makes the flat background metric unobservable, leaving only a background conformal structure. For other models, useful insights may come from the existing body of work on diffeomorphism invariance on a lattice. While some of this work directly addresses emergent models [107], much of it is in the context of lattice regularization of general relativity [108–113]. In particular, there are interesting ideas for obtaining an invariant lattice action—a "perfect action"—from a Wilsonian coarse-graining of the continuum [108–110, 113], which could point to a new type of emergent model.

A new problem arises in models in which the time evolution of the fundamental degrees of freedom depends on data in a finite region, as is the case in lattice models [34]. Consider two disjoint spatial regions R_1 and R_2 at time t_1, initially evolving independently, and let $I(R_1)$ and $I(R_2)$ be their respective future domains of influence. If these domains overlap at some later time t_2, the relative rate of evolution can matter: the data in R_1 at time t_1, for instance, will change the data in $I(R_1) \cap I(R_2)$ at time t_2, and thus the subsequent evolution of R_2. Using a term from computer science, Wall calls this the "race problem": two independent regions are "racing" toward the intersection $I(R_1) \cap I(R_2)$, and the one that gets there first determines the subsequent evolution. Such behavior, which is known to occur in particular models, clearly breaks diffeomorphism invariance; in an emergent gravity model, it would be interpreted as a failure of Hamiltonian constraints smeared by two different lapses to weakly commute. Avoiding this

7

problem seems very difficult, requiring either an arbitrary division of space into nonoverlapping regions or the imposition of extremely delicate consistency conditions. It can be argued that these consistency conditions appear automatically for a "perfect action," a discrete action that is already invariant under the full diffeomorphism group [108, 114], but it is not clear how such a structure would arise from a more primitive noninvariant theory.

For some type II models, another problem can occur. While such models involve emergent space, some (e.g., [33, 39]) include a time parameter to describe the evolution of the underlying degrees of freedom. In such cases, one must worry about the relationship between this fundamental time and the emergent time in the description of gravity. This is another decoupling problem: the time in which the fundamental degrees of freedom evolve is a background structure, and any coupling to the emergent degrees of freedom would define a preferred time and break diffeomorphism invariance.

3.5 Diffeomorphism invariance II

Diffeomorphism invariance plays another key role in general relativity: it eliminates the spin zero and spin one degrees of freedom, leaving only spin two modes. In the ADM formalism, the spatial metric and its conjugate momentum form six independent canonical pairs (q_{ij}, π^{ij}), but the four diffeomorphism constraints eliminate four pairs. It is crucial that the constraints are first class (i.e., that the commutator of two constraints is itself proportional to the constraints); a first class constraint eliminates two phase space degrees of freedom, while a second class constraint eliminates only one [115]. While the presence of additional degrees of freedom cannot be completely excluded by experiment, spin zero or one components of gravity are very strongly constrained, since, for example, they would imply violations of the principle of equivalence.

This aspect of diffeomorphism invariance presents a particularly strong challenge for emergent models, since an "approximate symmetry" can be qualitatively different from an exact one. Suppose, for instance, that an emergent weak gravitational field can be described at some length scale by a Lorentz invariant spin two field, but without the gauge invariance corresponding to linearized diffeomorphisms. The lowest order action is then the Fierz-Pauli action for a field of mass m. But although linear diffeomorphism invariance is restored in the $m \to 0$ limit, that limit differs from weak field general relativity [116, 117], and gives incorrect predictions for Solar System tests. This van Dam-Veltman-Zakharov (vDVZ) discontinuity arises because an extra scalar mode fails to decouple even in the massless limit. Moreover, as noted earlier, nonlinear extensions of this model typically contain negative energy Boulware-Deser ghosts [95, 96].

As Vainshtein first pointed out [118], the vDVZ discontinuity may indicate a breakdown of weak field perturbation theory: nonlinear effects proportional to inverse powers of m may appear, signaling the onset of a strong coupling regime. The unwanted scalar mode might then be screened, and a different perturbative expansion at short distances might more closely approximate general relativity (see [96] for a review). This mechanism has been confirmed for particular models (e.g., [119–121]), and with a very special choice of action, the Boulware-Deser ghosts may also be banished [97–99]. But if the Vainshtein mechanism applies, it poses a new challenge: since the usual weak field approximation can no longer be trusted, one must work hard to find even a Newtonian limit for emergent gravity.

To a certain extent, this argument can be turned on its head: if one is certain that a massless, Lorentz invariant, purely spin two field has emerged with no lower spin partners, this strongly suggests the presence of diffeomorphism invariance or a similar symmetry. A symmetric rank two tensor field h_{ab} contains components of spin zero, one, and two, and the only known Lorentz

8

covariant way to project out the lower spins is with a gauge invariance. At linear order, the minimal requirement is invariance under "transverse diffeomorphisms," diffeomorphisms generated by those vector fields ξ^a for which $\partial_a \xi^a = 0$ [122–124]. This group may be extended by including the remaining diffeomorphisms, giving full diffeomorphism invariance, or by appending Weyl transformations, yielding "WTDiff" invariance [123]. Contrary to popular folklore, however, neither extension is required for a consistent nonlinear theory; Wald [126] and Heiderich and Unruh [127] have explicitly constructed consistent non-covariant models of interacting massless spin two fields, and the latter contain explicit transverse diffeomorphism invariance.[†]

If one makes the much stronger assumption that the field is sourced by its own stress-energy tensor, then, as noted above, general relativity (and thus full diffeomorphism invariance) will emerge [103, 104], although for the exceptional choice of a WTDiff-invariant linearized action one may instead obtain unimodular gravity [123–125]. The Weinberg-Witten theorem, described below, leads to a similar conclusion, that a pure spin two field with a conserved source must normally be a gauge theory.

If Lorentz invariance is not exact, the Fierz-Pauli action is no longer unique. The problem then becomes more difficult to analyze, although there are some models that appear to avoid both ghosts and the vDVZ discontinuity [128]. But similar issues of extra "gauge" modes appear in other settings, such as lattice models [129]. A basic lesson is that the linear approximation may be quite misleading; one must ensure that any undesirable lower spin modes decouple at the full nonlinear level.

3.6 Diffeomorphism invariance and flat backgrounds

It is worth noting a somewhat subtle technical issue in emergent diffeomorphism invariance. Under an infinitesimal diffeomorphism generated by a vector field ξ, the metric transforms as

$$g_{ab} \rightarrow g_{ab} + \nabla_a \xi_b + \nabla_b \xi_a = g_{ab} + g_{ac} \partial_b \xi^c + g_{bc} \partial_a \xi^c + \xi^c \partial_c g_{ab} \qquad (3.1)$$

The last term is crucial: it reflects the fact that diffeomorphisms "move points," and are not just ordinary pointwise gauge transformations.

Suppose, however, one expands around a flat metric η_{ab}. Then to lowest order, (3.1) becomes

$$\eta_{ab} \rightarrow \eta_{ab} + \eta_{ac} \partial_b \xi^c + \eta_{bc} \partial_a \xi^c. \qquad (3.2)$$

The crucial derivative is now hidden. This is not uncommon in emergent gravity (for example, [26, 130]), where it may seem natural to build diffeomorphisms out of local gauge transformations. But as a pointwise transformation, (3.2) is not yet a diffeomorphism—it is easy to check, for instance, that the algebra of such transformations is not the algebra of diffeomorphisms—and the nonlinear interactions of general relativity do not automatically appear. As in the preceding section, a reliable demonstration of diffeomorphism invariance requires an expansion to nonlinear order.

3.7 The Weinberg-Witten theorem

In 1980, Weinberg and Witten proved a result that further constrains type I emergent models [131]. The theorem can be stated as follows (see [133, 134] for further discussion):

[†]Wald notes that it may be difficult—in some cases impossible—to couple such models to matter through the standard stress-energy tensor. But in the context of emergent gravity, one cannot simply assume the standard coupling; one should start with the underlying theory and see what coupling emerges.

> Let $T^{\mu\nu}$ be a Lorentz covariant, conserved current. Then no massless spin two field can carry a nonzero charge under the operator $P^\mu = \int d^3x \, T^{\mu 0}$.

In particular, if $T^{\mu\nu}$ is a conserved stress-energy tensor, the theorem asserts that no massless "graviton" can carry energy or momentum.

The Weinberg-Witten theorem uses no detailed properties beyond Lorentz invariance and conservation, and applies to composite as well as elementary fields. Naively, it would seem to rule out any theory, including general relativity, in which the gravitational field carries energy. This cannot be the case, but by seeing how particular models evade the result, we can understand the true limits.

Let us start with general relativity in the weak field approximation. The obvious loophole is that the stress-energy tensor is not conserved, $\partial_\mu T^{\mu\nu} \neq 0$, but only covariantly conserved, $\nabla_\mu T^{\mu\nu} = 0$. But one can always add a gravitational stress-energy pseudotensor to form a conserved current. Contrary to some claims in the literature, the resulting quantity can be fully Lorentz covariant: its definition requires a flat background metric, but this need not lead to any Lorentz violation [132]. The result is not a tensor with respect to general coordinate transformations, but it is not obvious that this is relevant, since the proof of the theorem does not rely explicitly on general covariance.

The real issue is somewhat more subtle. The Weinberg-Witten theorem requires a "pure" spin two field, with no spin zero or spin one admixtures. We may achieve this in two ways:

- We may project out the unphysical helicity zero and one states. But such a projection is only Lorentz covariant up to a gauge transformation, violating one condition of the theorem.

- We may appeal to diffeomorphism invariance to argue that the spin zero and one components of the metric are "pure gauge," and therefore irrelevant. But this argument only works in a gauge covariant formulation. We are caught: the stress-energy pseudotensor is not covariant, while the covariant stress-energy tensor is not conserved.

It is this interaction of gauge invariance and Lorentz invariance that provides the loophole.[‡]

Now, if a model of emergent gravity reproduces general relativity above some length scale L, the same loophole should apply at that scale. The question becomes whether the Weinberg-Witten theorem restricts the model at shorter scales. Possible solutions include [14, 135]

1. Broken Lorentz invariance: in analog models [19] and models in which the graviton is a Goldstone boson for broken Lorentz invariance [14, 133, 136], for instance, the fundamental degrees of freedom are not Lorentz invariant, evading one condition of the theorem.

2. Nonlocality: in Sundrum's "fat graviton" model [137], and arguably the AdS/CFT correspondence, gravitons are nonlocal, and do not couple to a local stress-energy tensor.

3. No spin two fields below L: if spin two fields first emerge at the same scale as general relativity, there is no room for the Weinberg-Witten theorem to apply. For example, in models in which the background manifold is topological [29–32, 72], there may be no nontrivial conserved stress-energy tensor at all at small scales.

[‡]This explains the apparent conflict between the Weinberg-Witten theorem and the soft graviton theorem of section 3.2. The soft graviton theorem is an on-shell result, requiring only Lorentz invariance of the S-matrix; it has no requirement of a local, conserved Lorentz covariant stress-energy tensor.

4. Emergent spacetime: in type II models of emergence, the basic setting of the Weinberg-Witten theorem, spin two excitations in a flat spacetime, is absent at the fundamental level, though one must check carefully at larger scales.

3.8 Where does the emergent theory live?

In section 1, I introduced two general categories of emergent models: type I models, which assume a background "environment," and type II models, in which spacetime itself is emergent. In some ways, type II models are more appealing: if macroscopic gravity is a characteristic of the structure of spacetime, shouldn't the structure emerge with the spacetime itself? But for the same reason, type II models are also much harder to connect to known physics.

Most type I models, on the other hand, present us with a basic question: what determines the environment? For example, in models of gravitons as composite spin two particles in a flat Minkowski space, why is the background spacetime flat? A century ago, this could have passed as an "obvious" assumption. But once we know that curvature can be dynamical, we cannot simply forget that knowledge; we now know that we are secretly postulating a field equation, $R_{abcd} = 0$, for the background. Similarly, for models on a fixed lattice, what fixes the lattice topology and spacing? We know that these features can be dynamical, as they are in Regge calculus; why do they not evolve in these models? In particular, what prevents the back-reaction of the emergent gravitational degrees of freedom on the "fixed" components of the environment?

These are not experimental questions, and the answer could be simply, "That's the way Nature is." But the spirit of the emergent gravity program is to replace general relativity with something more fundamental, and a fixed background seems to be a step in the wrong direction. One interesting attempt to address such questions comes from work on "noiseless subsystems" [138], in which the emergent structure is *defined* by its decoupling from the background, but it remains to be seen whether such a special characteristic can hold for realistic models.

3.9 The usual problems of quantum gravity

Emergent gravity is sometimes advertised as a solution to the problems of quantizing general relativity. This is not an unreasonable hope: the underlying degrees of freedom may be renormalizable, for instance, or may have a discrete structure that provides a natural cutoff. But there is more to quantum gravity than renormalizability, and it is not clear that emergent models can do better than ordinary general relativity in addressing fundamental conceptual problems [139].

For example, a quantum theory of general relativity has no local observables [16, 17], and it is quite difficult to reconstruct a local picture of physics. As long as an emergent model recovers diffeomorphism invariance, this problem will persist, at least at the scale at which an effective gravitational description is possible.

Similarly, many aspects of the infamous "problem of time" [140] will remain. A model that recovers diffeomorphism invariance will have no preferred time coordinate, and will have a Hamiltonian constraint rather than a Hamiltonian. If, on the other hand, the fundamental degrees of freedom have a preferred time that does not completely decouple from the gravitational degrees of freedom, the absence of a Hamiltonian constraint will lead to extraneous degrees of freedom, with the concomitant problems discussed in section 3.5. Even in this case, the Weinberg-Witten theorem will imply the absence of a Lorentz invariant, conserved Hamiltonian at the scale at which gravity emerges.

Causality is problematic as well. A fundamental feature of ordinary quantum field theory is that spacelike separated operators commute. But if the metric—even an emergent one—is subject to quantum fluctuations, there will be no fixed light cones to define spacelike and timelike separation. In some type I models, one might hope that the underlying "environment" defines an absolute causality. This would be radically different from the analogous classical situation, however. Classically, if one treats the gravitational field as a massless spin two field $h_{\mu\nu}$ propagating on a flat background, one finds that the nonlinearities of the action force matter to couple to the full metric $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, hiding all traces of the flat background metric [102,103]. In particular, the support of Greens functions lies within the g light cones, not the η light cones. It is true that the g light cones normally lie inside the η light cones [141], so the "emergent" metric does not violate background causality. But this result depends on special features of classical general relativity, and even there it holds only if matter satisfies the null energy condition, a condition that quantum fluctuations do not obey [142].

4   Where we stand

Einstein gravity is a robust theory, which can be reached from many different starting points. Chapter 17 of the famous textbook by Misner, Thorne, and Wheeler describes six routes to the Einstein field equations [143]; the EPS derivation described in section 2 provides a seventh. This might offer hope that emergent gravity could also lead to the same large scale physics.

As I have tried to show, life is not so easy. Gravity may be an emergent phenomenon, but models of emergent gravity faces formidable obstacles. For all its simplicity, general relativity rests heavily on a few fundamental features—local Lorentz invariance, the principle of equivalence, diffeomorphism invariance and background independence—that are not easy to mock up.

Moreover, these features are intertwined. A local model with a background time, for instance, must lose all traces of its Hamiltonian at the scale at which gravity emerges, or the Weinberg-Witten theorem might force the emergent theory to be Lorentz-violating. A model in which the gravitational field does not couple universally to matter is likely to have no single conserved stress-energy tensor, and thus no suitable gauge invariance for a spin two field. A model whose background environment fails to sufficiently decouple will have problems not only with Lorentz invariance, but with diffeomorphism invariance, the principle of equivalence, and, quite likely, Boulware-Deser ghosts.

These difficulties do not mean that the search for emergent gravity is doomed. But they suggest that current ad hoc approaches are unlikely to succeed. While there is much to be learned from such models, it seems likely that a successful theory of emergent gravity will require some more fundamental principle, as yet unknown, to allow its emergent properties to be organized into a realistic model of spacetime.

References

[1] J. Butterfield and C. J. Isham, in *The arguments of time*, edited by J. Butterfield, Oxford University Press, 1999, arXiv:gr-qc/9901024

[2] J. A. Wheeler, in *Relativity, Groups, and Topology*, edited by B. S. DeWitt and C. DeWitt (Gordon and Breach, New York, 1964)

[3] D. Finkelstein, Phys. Rev. 184 (1969) 1261

[4] R. B. Laughlin and D. Pines, Proc. Nat. Acad. Sci. 97 (2000) 28

[5] R. B. Laughlin, Int. J. Mod. Phys. A18 (2003) 831, arXiv:gr-qc/0302028

[6] G. E. Volovik, Phys. Rept. 351 (2001) 195, arXiv:gr-qc/0005091v3

[7] N. Seiberg, arXiv:hep-th/0601234

[8] G. T. Horowitz and J. Polchinski, in *Approaches to Quantum Gravity*, edited by D. Oriti (Cambridge University Press, 2009), arXiv:gr-qc/0602037

[9] E. Witten, "Emergent Phenomena in Condensed Matter and Particle Physics," Alba Nova colloquium, April 2008, `http://agenda.albanova.se/conferenceDisplay.py?confId=575`

[10] M. Bojowald, in *Encyclopedia of Mathematical Physics*, edited by J.-P. Franccoise, G. L. Naber, and Tsou S. T. (Elsevier, 2006), arXiv:gr-qc/0603110

[11] R. Loll, Class. Quant. Grav. 25 (2008) 114006, arXiv:0711.0273

[12] B. L. Hu, J. Phys. Conf. Ser. 174 (2009) 012015, arXiv:0903.0878

[13] C. Barcelo, M. Visser, and S. Liberati, Int. J. Mod. Phys. D10 (2001) 799, arXiv:gr-qc/0106002

[14] L. Sindoni, SIGMA 8 (2012) 027, arXiv:1110.0686

[15] *Approaches to Quantum Gravity*, edited by D. Oriti (Cambridge University Press, 2009)

[16] C. G. Torre, Phys. Rev. D48 (1993) 2373, arXiv:gr-qc/9306030

[17] S. B. Giddings, D. Marolf, and J. B. Hartle, Phys. Rev. D74 (2006) 064018, arXiv:hep-th/0512200

[18] D. Marolf, Phys. Rev. D79 (2009) 044010, arXiv:0808.2842

[19] C. Barcelo, S. Liberati, and M. Visser, Living Rev. Rel. 8 (2005) 12, arXiv:gr-qc/0505065

[20] W. G. Unruh, Phys. Rev. D51 (1995) 2827

[21] S. Weinfurtner, arXiv:0711.4416

[22] B. L. Hu, Int. J. Theor. Phys. 44 (2005) 1785, arXiv:gr-qc/0503067

[23] G. E. Volovik, Phil. Trans. R. Soc. London A366 (2008) 2935, arXiv:0801.0724

[24] S.-C. Zhang and J. Hu, Science 294 (2001) 823, arXiv:cond-mat/0110572

[25] J. Zaanen and A. J. Beekman, Annals Phys. 327 (2012) 1146, arXiv:1108.2791

[26] Z.-C. Gu and X.-G. Wen, Nucl. Phys. B863 (2012) 90, arXiv:0907.1203

[27] S. Randjbar-Daemi and J. A. Strathdee, ICTP preprint IC/94/312 (1994)

[28] K. Akama, Y. Chikashige, T. Matsuki, and H. Terazawa,

[29] D. Amati and G. Veneziano, Phys. Lett. B105 (1981) 358 Prog. Theor. Phys. 60 (1978) 868

[30] R. Floreanini and R. Percacci, Mod. Phys. Lett. A5 (1990) 2247

[31] C. Wetterich, Phys. Rev. D70 (2004) 105004, arXiv:hep-th/0307145

[32] J. Alfaro, D. Espriu, and D. Puigdomènech, arXiv:1201.4697

[33] T. Konopka, F. Markopoulou, and S. Severini, Phys. Rev. D77 (2008) 104029, arXiv:0801.0861

[34] A. C. Wall, arXiv:1201.2489

[35] D. Oriti, in *Approaches to Quantum Gravity*, edited by D. Oriti (Cambridge University Press, 2009), arXiv:gr-qc/0607032

[36] D. Oriti, PoS QG-PH (2007) 030, arXiv:0710.3276

[37] H. Steinacker, Class. Quant Grav. 27 (2010) 133001, arXiv:1003.4134

[38] J. A. Wheeler, in *Complexity, entropy, and the physics of information*, edited by W. H. Zurek (Addison-Wesley, 1990)

[39] S. Lloyd, arXiv:quant-ph/0501135

[40] F. Markopoulou, arXiv:1201.3398

[41] D. Berenstein, JHEP 0601 (2006) 125, arXiv:hep-th/0507203

[42] R. de Mello Koch and J. Murugan, in *Foundations of Space and Time*, edited by G. Ellis, J. Murugan, and A. Weltman (Cambridge University Press, 2012), arXiv:0911.4817

[43] J. Bain, `http://ls.poly.edu/~jbain/papers/EmergenceQG.pdf`, to appear in STHMP

[44] L. Bombelli, J. Lee, D. Meyer, and R. D. Sorkin. Phys. Rev. Lett. 60, (1988) 656

[45] R. D. Sorkin, arXiv:gr-qc/0309009

[46] S. Carlip, in *Foundations of Space and Time*, edited by G. Ellis, J. Murugan, and A. Weltman (Cambridge University Press, 2012), arXiv:1009.1136.

[47] S. Carlip, in *The Planck Scale Symposium*, Proc. of the XXV Max Born Symposium, AIP Conf. Proc. 1196 (2009), arXiv:0909.3329

[48] N. Arkani-Hamed, A. G. Cohen, and H. Georgi, Phys. Rev. Lett. 86 (2001) 4757, arXiv:hep-th/0104005

[49] A. D. Sakharov, Sov. Phys. Dokl. 12 (1968) 1040; reprinted in Gen. Rel. Grav. 32 (2000) 365

[50] S. L. Adler, Rev. Mod. Phys. 54 (1982) 729

[51] C. P. Burgess, Living Rev. Rel. 7 (2004) 5, arXiv:gr-qc/0311082

[52] T. Jacobson, Phys. Rev. Lett. 75 (1995) 1260, arXiv:gr-qc/9504004

[53] T. Padmanabhan, Rept. Prog. Phys. 73 (2010) 046901, arXiv:0911.5004

[54] T. Padmanabhan, J. Phys. Conf. Ser. 306 (2011) 012001, arXiv:1012.4476

[55] T. Padmanabhan, Int. J. Mod. Phys. D13 (2004) 2293, arXiv:gr-qc/0408051

[56] J. Ehlers, F. A. E. Pirani, and A. Schild, in *General Relativity: Papers in Honor of J. L. Synge*, edited by L. O'Raifeartaigh (Clarendon, 1972)

[57] N. M. J. Woodhouse, J. Math. Phys. 14 (1973) 495

[58] C. M. Will, *Theory and experiment in gravitational physics* (Cambridge University Press, 1993)

[59] H. F. M. Goenner, Living Rev. Rel. 7 (2004), 2

[60] J. Audretsch and C. Lämmerzahl, J. Math. Phys. 32 (1991) 2099

[61] J. L. Anderson, *Principles of Relativity Physics* (Academic Press, 1967)

[62] W. G. Unruh, Phys. Rev. D40 (1989) 1048

[63] J. D. Bekenstein, Phys. Rev. D70 (2004) 083509, arXiv:astro-ph/0403694

[64] C. M. Will, Living Rev. Rel. 9 (2005) 3, arXiv:gr-qc/0510072

[65] D. Mattingly, Living Rev. Rel. 8 (2005) 5, arXiv:gr-qc/0502097

[66] C. Barcelo, S. Liberati, and M. Visser, Class. Quant. Grav. 18 (2001) 3595, arXiv:gr-qc/0104001

[67] S. Liberati, arXiv:1203.4105

[68] C.-Y. Lin and D.-S. Lee, arXiv:1205.0133

[69] C. Barcelo, S. Liberati, and M. Visser, Class. Quant. Grav. 19 (2002) 2961, arXiv:gr-qc/0111059

[70] L. Sindoni, Phys. Rev. D83 (2011) 024022, arXiv:1011.4411

[71] F. Girelli, S. Liberati, and L. Sindoni, Phys. Rev. D79 (2009) 044019, arXiv:0806.4239

[72] C. Wetterich, arXiv:1201.6505

[73] R. Percacci, Phys. Lett. B144 (1984) 37

[74] F. Dowker, J. Phys. Conf. Ser. 306 (2011) 012016

[75] C. Moore, Phys. Rev. Lett. 60 (1988) 655

[76] L. Bombelli, J. Henson, and R. D. Sorkin, Mod. Phys. Lett. A24 (2009) 2579, arXiv:gr-qc/0605006

[77] J. Ambjorn, A. Görlich, S. Jordan, J. Jurkiewicz, and R. Loll, Phys. Lett. B690 (2010) 413, arXiv:1002.3298

[78] C. Anderson, S. Carlip, J. H. Cooperman, P. Hořava, R. Kommu, and P. R. Zulkowski, Phys. Rev. D85 (2012) 044027, arXiv:1111.6634

[79] S. G. Nibbelink and M. Pospelov, Phys .Rev. Lett. 94 (2005) 081601, arXiv:hep-ph/0404271

[80] S. Chadha and H. B. Nielsen, Nucl. Phys. B217 (1983) 125

[81] M. M. Anber and J. F. Donoghue, Phys. Rev. D83 (2011) 105027, arXiv:1102.0789

[82] V. Baccetti, K. Tate, and M. Visser, JHEP 1205 (2012) 119, arXiv:1112.1466

[83] A. Hamma, F. Markopoulou, I. Premont-Schwarz, and S. Severini, Phys. Rev. Lett. 102 (2009) 017204, arXiv:0808.2495

[84] F. Girelli, E. R. Livine, and D. Oriti, Phys. Rev. D81 (2010) 024015, arXiv:0903.3475

[85] A. Gomberoff and D. Marolf, Int. J. Mod. Phys. D8 (1999) 519, arXiv:gr-qc/9902069

[86] J. Collins, A. Perez, D. Sudarsky, L. Urrutia, and H. Vucetich, Phys. Rev. Lett. 93 (2004) 191301, arXiv:gr-qc/0403053

[87] M. Pospelov and Y. Shang, Phys. Rev. D85 (2012) 105001, arXiv:1010.5249

[88] S. L. Dubovsky and S. M. Sibiryakov, Phys. Lett. B638 (2006) 509, arXiv:hep-th/0603158

[89] C. Eling, B. Z. Foster, T. Jacobson, and A. C. Wall, Phys. Rev. D75 (2007) 101502, arXiv:hep-th/0702124

[90] R. P. Feynman, F. B. Morinigo, and W. G. Wagner, *The Feynman Lectures on Gravitation* (Addison-Wesley, 1995)

[91] E. G. Adelberger, J. H. Gundlach, B. R. Heckel, S. Hoedl, and S. Schlamminger, Prog. Part. Nucl. Phys. 62 (2009) 102

[92] C. Wetterich, Phys. Lett. B712 (2012) 126, arXiv:1203.5214

[93] T. Damour and I. I. Kogan, Phys. Rev. D66 (2002) 104024, arXiv:hep-th/0206042

[94] N. Boulanger, T. Damour, L. Gualtieri, and M. Henneaux, Nucl. Phys. B597 (2001) 127, arXiv:hep-th/0007220

[95] D. G. Boulware and S. Deser, Phys. Rev. D6 (1972) 3368

[96] K. Hinterbichler, Rev. Mod. Phys. 84 (2012) 671, arXiv:1105.3735

[97] C. de Rham, G. Gabadadze, and A. J. Tolley. Phys. Rev. Lett. 106 (2011) 231101, arXiv:1011.1232

[98] S. F. Hassan and R. A. Rosen, JHEP 1204 (2012) 123, arXiv:1111.2070

[99] K. Hinterbichler and R. A. Rosen, arXiv:1203.5783

[100] S. Weinberg, Phys. Rev. 135 (1964) B1049

[101] J. G. Williams, S. G. Turyshev, and D. H. Boggs, Int. J. Mod. Phys. D18 (2009) 1129, arXiv:gr-qc/0507083

[102] R. H. Kraichnan, Phys. Rev. 98 (1955) 1118

[103] S. Deser, Gen. Rel. Grav. 1 (1970) 9, arXiv:gr-qc/0411023

[104] S. Deser, Gen. Rel. Grav. 42 (2010) 641, arXiv:0910.2975

[105] D. Giulini, Lect. Notes Phys. 721 (2007) 105, arXiv:gr-qc/0603087

[106] H. Elvang and J. Polchinski, arXiv:hep-th/0209104

[107] C. Wetterich, Phys. Rev. D85 (2012) 104017, arXiv:1110.1539

[108] B. Bahr and B. Dittrich, Phys. Rev. D80 (2009) 124030, arXiv:0907.4323

[109] B. Bahr, B. Dittrich, and S. Steinhaus, Phys. Rev. D83 (2011) 105026, arXiv:1101.4775

[110] B. Dittrich, arXiv:1201.3840

[111] R. Gambini and J. Pullin, Phys. Rev. Lett. 90 (2003) 021301, arXiv:gr-qc/0206055

[112] B. Bahr, R. Gambini, and J. Pullin, SIGMA 8 (2012) 002, arXiv:1111.1879

[113] A. Kempf, Phys. Rev. Lett. 92 (2004) 221301, arXiv:gr-qc/0310035

[114] B. Bahr and B. Dittrich, Class. Quant. Grav. 26 (2009) 225011, arXiv:0905.1670

[115] M. Henneaux and C. Teitelboim, *Quantization of Gauge Systems* (Princeton University Press, 1992)

[116] H. van Dam and M. J. G. Veltman, Nucl. Phys. B22 (1970) 397

[117] V. I. Zakharov, JETP Lett. 12 (1970) 312

[118] A. I. Vainshtein, Phys. Lett. B39 (1972) 393

[119] E. Babichev, C. Deffayet, and R. Ziour, Phys. Rev. Lett. 103 (2009) 201102, arXiv:0907.4103

[120] N. Kaloper, A. Padilla, and N. Tanahashi, JHEP 1110 (2011) 148, arXiv:1106.4827

[121] A. De Felice, R. Kase, and S. Tsujikawa, Phys. Rev. D85 (2012) 044059, arXiv:1111.5090

[122] J. J. van der Bij, H. van Dam, and Y. J. Ng, Physica 116A (1982) 307

[123] E. Alvarez, D. Blas, J. Garriga, and E. Verdaguer, Nucl. Phys. B756 (2006) 148, arXiv:hep-th/0606019

[124] D. Blas, J. Phys. A 40 (2007) 6965, arXiv: hep-th/0701049

[125] E. Alvarez, arXiv:1204.6162

[126] R. M. Wald, Phys. Rev. D33 (1986) 3613

[127] K. R. Heiderich and W. G. Unruh, Phys. Rev. D42 (1990) 2057

[128] D. Comelli, M. Crisostomi, F. Nesti, and L. Pilo, arXiv:1204.1027

[129] B. Dittrich, Adv. Sci. Lett. 2 (2009) 151, arXiv:0810.3594

[130] C. Xu and P. Hořava, Phys. Rev. D81 (2010) 104033, arXiv:1003.0009

[131] S. Weinberg and E. Witten, Phys. Lett. 96B (1980) 59

[132] S. Weinberg, *Gravitation and Cosmology* (John Wiley & Sons, 1972)

[133] A. Jenkins, arXiv:hep-th/0607239

[134] F. Loebbert, Annalen Phys. 17 (2008) 803

[135] A. Jenkins, Int. J. Mod. Phys. D18 (2009) 2249, arXiv:0904.0453

[136] P. Kraus and E. T. Tomboulis, Phys. Rev. D66 (2002) 045015, arXiv:hep-th/0203221

[137] R. Sundrum, Phys. Rev. D69 (2004) 044014, hep-th/0306106

[138] D. W. Kribs and F. Markopoulou, arXiv:gr-qc/0510052

[139] S. Carlip, Rept. Prog. Phys. 64 (2001) 885, arXiv:gr-qc/0108040

[140] K. Kuchař, in *General relativity and relativistic astrophysics*, Proc. of the 4th Canadian conference, edited by G. Kunstatter, D. E. Vincent, and J. G. Williams (World Scientific, 1992)

[141] M. Visser, B. Bassett, and S. Liberati, Nucl. Phys. Proc. Suppl. 88 (2000) 267, arXiv:gr-qc/9810026

[142] C. J. Fewster, L. H. Ford, and T. A. Roman, Phys. Rev. D81 (2010) 121901, arXiv:1004.0179

[143] C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (W. H. Freeman and Company, 1973)