Individually-Rational Collective Choice

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Abstract

There is a collection of exogenously given socially-feasible sets, and, for each one of them, each individual in a group chooses from an individually-feasible set. The fact that the product of the individually-feasible sets is larger than the socially-feasible set notwithstanding, there arises no conflict between individual choices. Assuming that individual preferences are random, I characterize rationalizable collective choices.

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Suppose that a menu of meal choices, say "beef or chicken," is offered to the passengers in an airplane. It seems reasonable to assume that each passenger makes her own choice based only on what she feels like eating, without giving any consideration as to whether there will be enough of both choices to attend the demands of all the passengers in the flight. Quite likely, if all the passengers chose to order the same meal, there would not be enough to serve all of them.[1] However, it is also likely that in many occasions the choices of the passengers are such that everyone can be served her own choice.

Three features of this situation are important:

1. a group of individuals face a collective choice problem where the decision corresponds to a vector of multiple dimensions;

2. each individual chooses some of the dimensions of the vector, considering only an individual feasibility constraint;

3. the set that is collectively feasible is smaller than the Cartesian product of the individually-feasible sets.

Indeed, as Debreu (1952, p. 886) has pointed out,

> "[i]n a wide class of social systems each agent has a range of actions among which he selects one. His choice is not, however, entirely free and the actions of all the other agents determine the subset to which his selection is restricted... and each [agent] tries by choosing his action in his restricting subset to bring about the best outcome according to his own preferences."

In the example, suppose that there are I passengers in the plane who want to eat. Denote by 0 the decision to order beef and by 1 the decision to order chicken. Then, for each individual the feasible set is $\{0, 1\}$ and the collective choice is a vector $x = (x_i)_{i=1}^{I} \in \{0, 1\}^I$. However, if the numbers of meals available are X^C of chicken and X^B of beef, then the socially-feasible set is just $\{x \in \{0,1\}^I : I - X^B \leq \sum_i x_i \leq X^C\}$, which may very well be a proper subset of $\{0,1\}^I$.

If one observes a choice situation like this, and the collective choice is socially feasible, one can argue at least two explanations. The first one is that sufficiently many members of the group did actually take into account the collective constraints and were able to accommodate them by their own choices. An alternative explanation, however, does not require dropping the assumption of individual rationality: if individual preferences are such that the profile of individually-rational choices lies within the feasible set, then no individual needs to consider the collective constraints when making her own choice.

This paper considers a situation in which there is a family of collectively-feasible sets (collective budgets), and in which each individual chooses from their projections into her own choice set. The paper studies the joint distribution of random preferences that can explain, via individual rationality, probabilistic distributions of collective choices over collective budgets. It is motivated by the observation, by Mathematical Psychologists, that a correct explanation of human choice has to take into account the random nature of individual preferences. The problem of collective choice, studied in this setting, stresses the fact that individual preferences need not be independent.

The results are based on McFadden and Richter (1990), where the finite, individual random choice problem is characterized. As McFadden and Richter claim, their result is more general than the application they are explicit about. I consider the more general case of collective choice over not-necessarily-finite domains, and impose the assumptions that are necessary to make this case compatible with the condition derived by them. When this more general setting is considered, however, an unpleasant feature of the McFadden-Richter solution makes itself evident: their result requires the analyst to specify the finite family of (profiles of) preferences that will be allowed to have a positive probability in the rationalization of the observed data. Although this may be acceptable when dealing with finite domains, when one can use the family of all possible orders, it is quite restrictive in the infinite case, as it narrows the concept of rationality: the fact that a data set appears at odds with rationality may be due solely to the family of preferences allowed, and need not mean that there do not exist families of preferences (and distributions over them) that are able to explain the data via rationality. I overcome this difficulty by combining standard revealed preference theory and the McFadden-Richter condition, so as to weaken the rationalizability requirement to just the existence of a family of preferences (controlling only its cardinality) and a probability distribution that are able to exactly explain the data. I also show that, regardless of the cardinality allowed, there exist data sets which cannot be explained by individual rationality under random preferences.

1 Stochastic collective choices

Suppose that there is a finite set of decision makers, denoted by $\mathcal{I} = \{1, \ldots, I\}$. Each decision maker chooses from an individual choice set: individual i chooses from the nonempty set X_i. The result of individual choices is a collective choice; the collective choice set is the Cartesian product of all the individual choice sets, $X = \times_{i \in \mathcal{I}} X_i$.

In individual-choice theory, a budget is a nonempty subset of a choice set. Here, a collective budget is a nonempty subset of the collective choice set, $B \subseteq X$.

Suppose that one observes a nonempty family of collective budgets $\mathcal{B}$.[2] Endow each budget $B \in \mathcal{B}$ with a σ-algebra Σ_B, and suppose that a probability measure $\gamma_B : \Sigma_B \to [0, 1]$ has been observed for each B.

A **stochastic collective choice** is $\{\mathcal{I}, (X_i)_{i \in \mathcal{I}}, \mathcal{B}, (\Sigma_B, \gamma_B)_{B \in \mathcal{B}}\}$. All this information is assumed to be observed data. For a budget B, the σ-algebra Σ_B is determined by how fine the observation of collective choices is; if $C \in$

Σ_B, then C is measurable at budget B, and $\gamma_B(C)$ is understood as the observed probability that the collective choice made from budget B lies in C.

Throughout the paper, I maintain the assumption that the following condition holds:

Assumption 1. *$\mathcal{B}$ is finite, and for each $B \in \mathcal{B}$, Σ_B is finite.*

2 Strong rationalizability

For any set $Z \subseteq X$, denote by Z_i the projection of Z into X_i.

If one assumes that decision makers act noncooperatively, then for each budget B and each measurable subset $C \in \Sigma_B$, observed probability $\gamma_B(C)$ is understood as the probability that if each player i chooses x_i from her individually-feasible set, B_i, then the collective choice $(x_i)_{i\in\mathcal{I}}$ lies in C.

Individuals are assumed to care only about their own decisions, so a preference relation for individual i is a binary relation over X_i. For each individual i, let $\mathcal{R}_i = (R_{i,1}, R_{i,2}, \ldots, R_{i,S})$ be a finite sequence of preference relations.

Let $\mathcal{R}$ be the set that contains the profiles of preferences conformed by the individual sequences:

$$\mathcal{R} = \{(R_{1,1}, \ldots, R_{I,1}), \ldots, (R_{1,S}, \ldots, R_{I,S})\}.$$

It is convenient that individually-rational choices be uniquely defined for all observations in the data, and for all the preferences relations under consideration, so the following condition is assumed in this section:

Assumption 2. *For every individual i and every $s \in \{1, \ldots, S\}$, relation $R_{i,s}$ is a weak order over X_i such that, for every observed budget $B \in \mathcal{B}$, $\{x \in B_i : xR_{i,s}x' \text{ for all } x' \in B_i\}$ is a singleton set.*

Under the previous condition, one can further denote by $\arg\max_{B_i} R_{i,s}$ the (unique) maximizer of preferences $R_{i,s}$, when restricted to B_i.

The concept of collective rationality, for a given family of profile preferences $\mathcal{R}$, can now be stated:

Definition 1. *A stochastic collective choice is $\mathcal{R}$-**rationalizable** if there exists a probability measure over the set preferences $\mathcal{R}$, that explains the observed data via individually-rational, noncooperative choices: there exists $\delta : \mathcal{P}(\mathcal{R}) \to [0, 1]$, a probability measure, such that*

$$\delta(\{R \in \mathcal{R} : (\arg\max_{B_i} R_i)_{i \in \mathcal{I}} \in C\}) = \gamma_B(C),$$

for every observed budget $B \in \mathcal{B}$, and every measurable subset $C \in \Sigma_B$.

Collective rationalizability cannot always be studied by immediate application of tools of individual choice theory. To see this, consider the following example:

Example 1. *Suppose that there are two individuals, with choice sets $X_1 = X_2 = \{0, 1\}$. Suppose that only one collective budget, $B = \{(0, 1), (1, 0)\}$, has been observed (so $\mathcal{B} = \{B\}$). Let the observed probabilities be $\gamma_B((0, 1)) = \gamma_B((1, 0)) = 1/2$.*

If one considers the information available for one individual only, the observation is that she chooses each available alternative with equal probability, which can be rationalized by (and only by) assuming that, for her, the two possible orders over $\{0, 1\}$ occur with equal probability, $1/2$. This individual analysis, however, does not suffice to explain the collective data: when collectively choosing from B, the product of individual probabilities would place probability $1/4$ on each of the choices $(0, 0)$ and $(1, 1)$, contradicting the data.

The following characterization of $\mathcal{R}$-rationalizability is derived from McFadden and Richter (1990).[3] Define the binary function $\alpha : \mathcal{R} \times \bigcup_{B \in \mathcal{B}}(\{B\} \times$

$\Sigma_B) \to \{0,1\}$ as follows:

$$\alpha(R,B,C) = \begin{cases} 1, & \text{if } (\arg\max_{B_i} R_i)_{i\in\mathcal{I}} \in C; \\ 0, & \text{otherwise.} \end{cases}$$

Theorem 1. *A stochastic collective choice is $\mathcal{R}$-rationalizable if, and only if, for every finite sequence of pairs of (observed) budgets and associated measurable subsets, $(B_k, C_k)_{k=1}^K$ such that $C_k \in \Sigma_{B_k}$ for all k, there exists a postulated profile of preferences, $R \in \mathcal{R}$, such that*

$$\sum_{k=1}^K \gamma_{B_k}(C_k) \leq \sum_{k=1}^K \alpha(R, B_k, C_k).$$

Proof. Define the ternary relation Γ on the set $\mathcal{R} \times \mathcal{B} \times \bigcup_{B\in\mathcal{B}} \Sigma_B$ as follows: say that $\Gamma(R,B,C)$ if, and only if, $C \in \Sigma_B$ and $(\arg\max_{B_i} R_i)_{i\in\mathcal{I}} \in C$. By construction, $\Gamma(R,B,C)$ implies $C \in \Sigma_B$ and $\neg\Gamma(R,B,B\setminus C)$. Also, the collective choice is $\mathcal{R}$-rationalizable if, and only if, there exists a probability measure $\delta : \mathcal{P}(\mathcal{R}) \to [0,1]$ such that $\gamma_B(C) = \delta(\{R \in \mathcal{R} : \Gamma(R,B,C)\})$ for all $B \in \mathcal{B}$ and all $C \in \Sigma_B$. And, by construction, $\alpha(R,B,C) = 1$ if, and only if, $\Gamma(R,B,C)$. Although the choice sets X_i need not be finite, since $\mathcal{B}$ is finite and each Σ_B is finite, it then follows from McFadden and Richter (1990), theorem 2 and footnote 30, that the collective choice is $\mathcal{R}$-rationalizable if, and only if, for every finite sequence $(B_k, C_k)_{k=1}^K$ in $\bigcup_{B\in\mathcal{B}}(\{B\} \times \Sigma_B)$, it is true that $\sum_{k=1}^K \gamma_{B_k}(C_k) \leq \max_{R\in\mathcal{R}} \sum_{k=1}^K \alpha(R, B_k, C_k)$. □

The condition of this theorem is what McFadden and Richter have called the "Axiom of Revealed Stochastic Preference." Its intuition is that events that are likely to happen should happen often. That is, consider the situation 'for each k, if each individual chooses from $B_{i,k}$, then, collectively, they choose an element of C_k,' and suppose that such situation is 'highly likely,' in the sense that the left-hand side of the condition of the theorem, $\sum_k \gamma_{B_k}(C_k)$ is 'high;' then, it should also be true that for at least one of the preferences profiles, it happens that from 'many' of the budgets B_k, the group would choose an element of C_k, which would make the right-hand side of the condition 'high' as well.

For the case of individual choice problems over not-necessarily-finite choice sets, Clark (1996) has shown that rationalizability is equivalent to DeFinetti's Coherence Axiom of Probability, and that this axiom is equivalent to the Axiom of Revealed Stochastic Preference. In the finite case, an alternative characterization of rationalizability at the individual level was given by Falmagne (1978), and refined by Barberá and Pattanaik (1996). McFadden (2005) formalizes the equivalence between the latter condition and the Axiom of Revealed Stochastic Preference.

3 Weak rationalizability

The previous section assumed that a finite family of individual preference relations, and the way in which they form profiles of preferences, were given.

Suppose now that one only knows a nonempty, finite set of states of the world, Ω, and that for each $i \in \mathcal{I}$, one only fixes a class $\mathcal{R}_i$ of binary relations over X_i. Here, I consider only the families defined by the following condition:

Assumption 3. *For every $i \in \mathcal{I}$, $\mathcal{R}_i$ is the family of all weak orders, R_i, over the choice set X_i, such that for each observed budget $B \in \mathcal{B}$, $\{x \in B_i : xR_ix' \text{ for all } x' \in B_i\}$ is a singleton set.*

Weak rationalizability is obtained if one can assign to each state of the world a profile of preferences and a probability which are able to explain the observed probabilities via pure individual rationality:

Definition 2. *A stochastic collective choice is Ω-**rationalizable** if there exist a probability measure over the set of states of the world, and an assignment of preferences to states of the world that explain the observed data via* *individually-rationality: there exist $\delta : \mathcal{P}(\Omega) \to [0,1]$, a probability measure, and $R : \Omega \to \times_{i\in\mathcal{I}}\mathcal{R}_i$ such that*

$$\delta(\{\omega \in \Omega : (\arg\max_{B_i} R_i(\omega))_{i\in\mathcal{I}} \in C\}) = \gamma_B(C),$$

for every observed budget $B \in \mathcal{B}$ and every measurable subset $C \in \Sigma_B$.

The following theorem is the characterization, analogous to theorem 1, for this weaker definition of rationalizability. For every set $Z \subseteq X$, let $1_Z : X \to \{0,1\}$ denote its indicator function.

Theorem 2. *A stochastic collective choice is Ω-rationalizable if, and only if, there exist individually-feasible choices, $x_{i,B,\omega} \in B_i$, for every individual $i \in \mathcal{I}$, every budget $B \in \mathcal{B}$ and every state of nature $\omega \in \Omega$, such that for every finite sequence of pairs of observed budgets and associated measurable subsets, $(B_k, C_k)_{k=1}^K$ such that $C_k \in \Sigma_{B_k}$ for all k, it is true that:*

1. *For every individual and every state, the Congruence Axiom is satisfied: for every $i \in \mathcal{I}$ and every $\omega \in \Omega$, if $x_{i,B_{k+1},\omega} \in B_{i,k}$ for every $k \leq K-1$, then either $x_{i,B_K,\omega} = x_{i,B_1,\omega}$ or $x_{i,B_1,\omega} \notin B_{i,K}$.*

2. *There exists some state of the world $\omega \in \Omega$ such that*

$$\sum_{k=1}^{K} \gamma_{B_k}(C_k) \leq \sum_{k=1}^{K} 1_{C_k}((x_{i,B_k,\omega})_{i\in\mathcal{I}}).$$

Proof. Suppose that a choice is Ω-rationalized by the probability measure $\delta : \mathcal{P}(\Omega) \to [0,1]$ and the function $R : \Omega \to \times_{i\in\mathcal{I}}\mathcal{R}_i$. Define, for each i, each B and each ω, $x_{i,B,\omega} = \arg\max_{B_i} R_i(\omega)$. By construction, $x_{i,B,\omega} \in B_i$. Since the (deterministic) individual choice $(x_{i,B,\omega}, B_i)_{B\in\mathcal{B}}$ is regular-rational, it follows from Richter (1966), theorem 1, that it must satisfy condition 1 of the theorem. Moreover, let $\mathcal{R} = R(\Omega)$ and define $\tilde{\delta} = \mathcal{P}(\mathcal{R}) \to [0,1]; \mathcal{Q} \mapsto \delta(R^{-1}(\mathcal{Q}))$. Function $\tilde{\delta}$ is a probability measure over $\mathcal{R}$, and satisfies that

$\tilde{\delta}(\{R \in \mathcal{R} : (\arg\max_{B_i} R_i)_{i\in\mathcal{I}} \in C\}) = \gamma_B(C)$, for all $B \in \mathcal{B}$ and all $C \in \Sigma_B$. The latter means that function $\tilde{\delta}$ $\mathcal{R}$-rationalizes the collective choice. Since, by construction, $\mathcal{R}$ is finite, it follows from theorem 1 that for every finite sequence $(B_k, C_k)_{k=1}^K$ defined in $\bigcup_{B\in\mathcal{B}}(\{B\} \times \Sigma_B)$, there exists some $\tilde{R} \in \mathcal{R}$ such that

$$\sum_{k=1}^{K} \gamma_{B_k}(C_k) \leq \sum_{k=1}^{K} \alpha(\tilde{R}, B_k, C_k).$$

Let $\omega \in R^{-1}(\tilde{R})$ and notice that $\alpha(R(\omega), B, C) = 1$ is true if, and only if, $1_C((x_{i,B,\omega})_{i\in\mathcal{I}}) = 1$, which proves condition 2.

For sufficiency, first fix an individual, i, and a state of the world, ω. Since $(x_{i,B,\omega}, B_i)_{B\in\mathcal{B}}$ satisfies the condition 1 of the theorem, it follows from Richter (1966), theorem 1, that there exists $R_{i,\omega} \in \mathcal{R}_i$ such that for all $B \in \mathcal{B}$, it is true that $\arg\max_{B_i} R^{i,\omega} = x_{i,B,\omega}$. Now, define $\mathcal{R} = \{(R_{i,\omega})_{i\in\mathcal{I}} : \omega \in \Omega\}$. Take a finite sequence $(B_k, C_k)_{k=1}^K$ defined in $\bigcup_{B\in\mathcal{B}}(\{B\} \times \Sigma_B)$. By condition 2, there exists a state $\omega \in \Omega$ such that

$$\sum_{k=1}^{K} \gamma_{B_k}(C_k) \leq \sum_{k=1}^{K} 1_{C_k}((x_{i,B_k,\omega})_{i\in\mathcal{I}}) = \sum_{k=1}^{K} \alpha((R_{i,\omega})_{i\in\mathcal{I}}, B_k, C_k).$$

Since $\mathcal{R}$ is finite, the latter implies, by theorem 1, that there exists a probability measure $\tilde{\delta} : \mathcal{P}(\mathcal{R}) \to [0,1]$ that $\mathcal{R}$-rationalizes the collective choice. Define the functions $R : \Omega \to \times_{i\in\mathcal{I}}\mathcal{R}_i; \omega \mapsto (R^{i,\omega})_{i\in\mathcal{I}}$, and $\delta : \mathcal{P}(\Omega) \to [0,1]; \Phi \mapsto \tilde{\delta}(\{(R^{i,\omega})_{i\in\mathcal{I}} \in \mathcal{R} : \omega \in \Phi\})$. Function δ is a probability measure over Ω. Also, for any $B \in \mathcal{B}$ and any $C \in \Sigma_B$,

$$\begin{aligned}\delta(\{\omega \in \Omega : (\arg\max_{B_i} R_i(\omega))_{i\in\mathcal{I}} \in C\}) &= \tilde{\delta}(\{R \in \mathcal{R} : (\arg\max_{B_i} R_i)_{i\in\mathcal{I}} \in C\}) \\ &= \gamma_B(C),\end{aligned}$$

so functions δ and R Ω-rationalize the collective choice. $\square$

4 Nonrationalizable data sets

The previous subsections characterize stochastic, collective rationalizability. I now show examples of collective choices that are not weakly (and therefore not strongly) rationalizable.

4.1 Regularity

Consider the following data:

Example 2. *There are two individuals, $\mathcal{I} = \{1, 2\}$. Suppose that their choice sets are $X_1 = X_2 = \{1, 2, 3, 4, 5\}$, and let there be two observed collective budgets, $\mathcal{B} = \{\hat{B}, \tilde{B}\}$, where $\hat{B} = \{1, 2\} \times X_2$ and $\tilde{B} = \{1, 2, 3, 4\} \times X_2$. Suppose that the measurable subsets include $\{(1, 1), (1, 2)\} \in \Sigma_{\hat{B}}$, $\{(1, 3), (1, 4), (1, 5)\} \in \Sigma_{\hat{B}}$, $\{1\} \times X_2 \in \Sigma_{\tilde{B}}$, and the observed probabilities are $\gamma_{\hat{B}}(\{(1, 1), (1, 2)\}) = 1/6$, $\gamma_{\hat{B}}(\{(1, 3), (1, 4), (1, 5)\}) = 1/6$, and $\gamma_{\tilde{B}}(\{1\} \times X_2) = 1/2$.*

These data are not Ω-rationalizable, for any Ω, since rationalizability would require functions δ and R such that

$$\begin{aligned}
1/3 &= \gamma_{\hat{B}}(\{(1, 1), (1, 2)\}) + \gamma_{\hat{B}}(\{(1, 3), (1, 4), (1, 5)\}) \\
&= \delta(\{\omega \in \Omega : (\arg\max_{\hat{B}_i} R_i(\omega))_{i\in\mathcal{I}} \in \{(1, 1), (1, 2)\}\}) \\
&\quad +\delta(\{\omega \in \Omega : (\arg\max_{\hat{B}_i} R_i(\omega))_{i\in\mathcal{I}} \in \{(1, 3), (1, 4), (1, 5)\}\}) \\
&= \delta(\{\omega \in \Omega : (\arg\max_{\hat{B}_i} R_i(\omega))_{i\in\mathcal{I}} \in \{1\} \times X_2\}) \\
&= \delta(\{\omega \in \Omega : \arg\max_{\{1,2\}} R_1(\omega) = 1\}) \\
&\geq \delta(\{\omega \in \Omega : \arg\max_{\{1,2,3,4\}} R_1(\omega) = 1\}) \\
&= \delta(\{\omega \in \Omega : (\arg\max_{\tilde{B}_i} R_i(\omega))_{i\in\mathcal{I}} \in \{1\} \times X^2\}) \\
&= \gamma_{\tilde{B}}(\{1\} \times X^2) \\
&= 1/2.
\end{aligned}$$

Indeed, the way in which the collective budgets is given here implies that the restrictions of individual stochastic choice theory apply, and the example violates the Regularity Principle of Block and Marshak (1960).

4.2 Co-variation

By its construction, the previous example did not exploit any co-variation in individual preferences. Consider now the following data:

Example 3. *There are two individuals,* $\mathcal{I} = \{1, 2\}$*. Suppose that the choice sets are* $X_1 = \{1, 2, 3, 4, 5\}$ *and* $X_2 = [1, 5]$*, and let there be two collective budgets, so* $\mathcal{B} = \{\hat{B}, \tilde{B}\}$*, where*

$$\hat{B} = (\{1\} \times [1, 5]) \cup (\{2\} \times [1, 4]) \cup (\{3\} \times [1, 3]) \cup (\{4\} \times [1, 2]) \cup \{(5, 1)\}$$

$$\tilde{B} = \{(1, 5)\} \cup (\{2\} \times [4, 5]) \cup (\{3\} \times [3, 5]) \cup (\{4\} \times [2, 5]) \cup (\{5\} \times [1, 5]).$$

Define $\hat{C} = (\{1\} \times [4, 5]) \cup \{(2, 4)\}$ *and* $\tilde{C} = X_1 \times \{5\}$*, and suppose that these sets are measurable:* $\hat{C} \in \Sigma_{\hat{B}}$ *and* $\tilde{C} \in \Sigma_{\tilde{B}}$*. Finally, suppose that the observed probabilities are* $\gamma_{\hat{B}}(\hat{C}) = 1/3$ *and* $\gamma_{\tilde{B}}(\tilde{C}) = 1/2$*.*

Since $\hat{B}_1 = \tilde{B}_1 = X^1$ and $\hat{B}_2 = \tilde{B}_2 = X_2$, if functions δ and R Ω-rationalize the data for some Ω, then, for every observed budget, B, and every state of the world with positive probability, $\omega \in \Omega$ such that $\delta(\omega) > 0$, it must be true that

$$\arg\max_{B_1} R_1(\omega) + \arg\max_{B_2} R_2(\omega) = 6.$$

But this implies that

$$\begin{aligned}
1/3 &= \gamma_{\hat{B}}(\hat{C}) \\
&= \delta(\{\omega \in \Omega : (\arg\max_{X_i} R_i(\omega))_{i\in\mathcal{I}} \in \{(2,4),(1,5)\}\}) \\
&\geq \delta(\{\omega \in \Omega : (\arg\max_{X_i} R_i(\omega))_{i\in\mathcal{I}} = (1,5)\}) \\
&= \gamma_{\tilde{B}}(\tilde{C}) \\
&= 1/2.
\end{aligned}$$

This shows that these data are not weakly rationalizable and, therefore, cannot be strongly rationalized. This example illustrates the need for covariation in the joint distribution of individual preferences, which was also observed in example 1. Example 1, however, was collectively rationalizable, while example 3 is not individually rationalizable. An example of data that are individually rationalizable, but not collectively rationalizable is given next:

Example 4. *As before, there are two individuals, $\mathcal{I} = \{1, 2\}$. Their choice sets are $X_1 = X_2 = \{0, 1, 2\}$, and two collective budgets are observed,*

$$\hat{B} = \{(0,0),(0,1),(1,1),(1,2),(2,2),(2,0)\},$$

and

$$\tilde{B} = \{(0,0),(1,0),(1,1),(2,1),(2,2),(0,2)\},$$

so $\mathcal{B} = \{\hat{B}, \tilde{B}\}$. There is perfect measurability, and observed probabilities are $\gamma_{\hat{B}}(\{x\}) = 1/6$ for all $x \in \hat{B}$, and $\gamma_{\tilde{B}}(\{x\}) = 1/6$ for all $x \in \tilde{B}$.

These individual choices can be Ω-rationalized for any Ω with at least three states of nature. To see this, suppose that one wants to explain the behavior of individual i only. Since $\hat{B}_i = \tilde{B}_i = X_i$, one only needs to concentrate on the (common) marginal distribution of observed choices over the whole choice set:

$$\gamma_i(\{0\}) = \gamma_i(\{1\}) = \gamma_i(\{2\}) = 1/3.$$

Let $\Omega_i = \{1, 2, 3\}$, let δ_i be the uniform probability measure over Ω_i, and define the following (individual) strict preference assignment: $R_i(1)$ orders X_i as $0 \succ 1 \succ 2$, $R_i(2)$ as $1 \succ 2 \succ 0$, and $R_i(3)$ as $2 \succ 0 \succ 1$. Then, for every $x \in X_i$,

$$\delta_i(\{\omega \in \Omega_i : \arg\max_{X_i} R_i(\omega) = x\}) = \frac{1}{3} = \gamma_i(\{x\}).$$

Collective Ω-rationalizability, however, is impossible regardless of Ω, for if functions δ and R Ω-rationalize the data for some Ω, then

$$\begin{aligned}
1/6 &= \gamma_{\hat{B}}(\{(0,1)\}) \\
&= \delta(\{\omega \in \Omega : (\arg\max_{\hat{B}_i} R_i(\omega))_{i=1}^{2} = (0,1)\} \\
&= \delta(\{\omega \in \Omega : (\arg\max_{X_i} R_i(\omega))_{i=1}^{2} = (0,1)\} \\
&= \delta(\{\omega \in \Omega : (\arg\max_{\tilde{B}_i} R_i(\omega))_{i=1}^{2} = (0,1)\} \\
&\leq \delta(\{\omega \in \Omega : (\arg\max_{\tilde{B}_i} R_i(\omega))_{i=1}^{2} \notin \hat{B}\} \\
&= 1 - \delta(\{\omega \in \Omega : (\arg\max_{\tilde{B}_i} R_i(\omega))_{i=1}^{2} \in \hat{B}\} \\
&= 1 - \gamma_{\tilde{B}}(\tilde{B}) \\
&= 0.
\end{aligned}$$

4.3 Consumers

In economics, a prominent decision problem is the choice of consumption bundles, at exogenous prices, under a budget constraint. For the individual demand problem under random preferences, Bandyopadhyay et Al. (1999) extend the weak axiom of revealed preference. Earlier, Hildenbrand (1971) derived properties on the expected demand of an individual consumer with random preferences, and, for collective problems, obtained asymptotic properties as the number of consumers increases, under independence assumptions. Here, I apply the results above to the case of consumers, and illustrate the importance of preference co-variation.

Suppose that there are a finite number, L, of consumption goods, so $X_i = \mathbb{R}^L_+$ for each individual i. Prices, p, are vectors of L positive numbers, one price for each commodity. Individual i is endowed with a bundle of commodities, e_i, which, for simplicity, is assumed to contain positive amounts of all commodities. Individual preferences are restricted to $\mathcal{R}_i = \mathcal{R}$, the class of all relations representable by continuous, strictly monotone, strongly quasiconcave utility functions.

There is finite a set of data, $D \subseteq \mathbb{R}^L_{++} \times (\mathbb{R}^L_{++})^I$, of prices, p, and profiles of individual endowments of commodities, $e = (e_i)_{i \in \mathcal{I}}$. Individuals face constraints in the usual form of individual budgets: given $(p, e) \in D$, each individual i chooses from the standard budget $B(p, e_i) = \{x \in \mathbb{R}^L_+ : p \cdot x \leq p \cdot e_i\}$. Social feasibility, however, must take into account the aggregate endowment of commodities: given (p, e), the collective constraint is

$$B(p,e) = \left\{ (x_i)_{i \in \mathcal{I}} \in \times_{i \in \mathcal{I}} B(p, e_i) : \sum_{i \in \mathcal{I}} x_i = \sum_{i \in \mathcal{I}} e_i \right\}.$$

For each $(p, e) \in D$, $B(p, e)$ is endowed with a finite σ-algebra, $\Sigma_{p,e}$, and a probability measure $\gamma_{p,e} : \Sigma_{p,e} \to [0, 1]$ is assumed to have been observed. Suppose that $\Sigma_{p,e} = \Sigma_{p',e'}$ and $\gamma_{p,e} = \gamma_{p',e'}$, whenever $B(p, e) = B(p', e')$.

A **stochastic collective demand** is $\{\mathcal{I}, D, (\Sigma_{p,e}, \gamma_{p,e})_{(p,e) \in D}\}$. It is Ω-rationalizable if there exist a probability measure δ over Ω, and an assignment of preferences to states of the world, R, that explain the observed data via individually-rationality: for every observed collective budget $(p, e) \in D$, and every measurable subset $C \in \Sigma_{p,e}$,

$$\delta(\{\omega \in \Omega : (\arg \max_{B(p,e_i)} R_i(\omega))_{i\in\mathcal{I}} \in C\}) = \gamma_{p,e}(C).$$

Since, in this case, individual budgets may be larger than the projection of the collective budgets, theorem 2 does not apply immediately.

Theorem 3. *A stochastic collective demand is Ω-rationalizable if, and only if, there exist individual state-contingent demands, $x_{i,(p,e),\omega}$, and real numbers $\lambda_{i,(p,e),\omega} > 0$ and $V_{i,(p,e),\omega}$, for each individual i, each observation $(p,e) \in D$, and each state $\omega \in \Omega$, such that:*

1. *For every state of the world, aggregate feasibility is observed: for all $(p,e) \in D$ and all $\omega \in \Omega$, it is true that $\sum_{i\in\mathcal{I}} x_{i,(p,e),\omega} = \sum_{i\in\mathcal{I}} e_i$.*
2. *For each individual and each state of the world, Walras's law and Afriat inequalities are satisfied: for all $i \in \mathcal{I}$ and all $\omega \in \Omega$, it is true that, for all $(p,e),(p',e') \in D$, $p \cdot x_{i,(p,e),\omega} = p \cdot e_i$ and*

   $$V_{i,(p',e'),\omega} \geq V_{i,(p,e),\omega} + \lambda_{i,(p,e),\omega} p \cdot (x_{i,(p',e'),\omega} - x_{i,(p,e),\omega}),$$

   with strict inequality if $x^{i,(p,e),\omega} \neq x^{i,(p',e'),\omega}$.
3. *For every finite sequence of observed data and measurable sets, $((p_k,e_k),C_k)_{k=1}^K$ such that $C_k \in \Sigma_{p_k,e_k}$ at all k, there exists a state of the world $\omega \in \Omega$ such that*

   $$\sum_{k=1}^K \gamma_{p_k,e_k}(C_k) \leq \sum_{k=1}^K 1_{C_k}((x_{i,(p_k,e_k),\omega})_{i\in\mathcal{I}}).$$

Proof. The argument is similar to the one given for theorem 2, invoking Matzkin and Richter (1991), theorem 2, instead of Richter (1966). □

Now, consider the following data:

Example 5. *There are two consumers, $\mathcal{I} = \{1,2\}$, and two commodities, $L = 2$. There are two observations of prices and endowments: $\mathcal{D} = \{(\tilde{p},\tilde{e}),(\hat{p},\hat{e})\}$, where $\tilde{p} = (1,2)$, $\tilde{e}_1 = (1,2)$, $\tilde{e}_2 = (5/3,2/3)$, $\hat{p} = (2,1)$, $\hat{e}_1 = (2,1)$, and $\hat{e}_2 = (2/3,5/3)$. Define the set $\hat{C}$ as*

$$\{(x_1,x_2) \in \mathbb{R}^4_+ : x_{1,1}+2x_{1,2} \in [4.5,5.5], 2x_{1,1}+x_{1,2} = 5, \text{ and } x_1+x_2 = (8/3,8/3))\}$$

and let $\tilde{C} = \{((5/3,5/3),(1,1))\}$.[4] *Suppose further that these sets are measurable, so that $\hat{C} \in \Sigma_{\hat{p},\hat{e}}$ and $\tilde{C} \in \Sigma_{\tilde{p},\tilde{e}}$,*[5] *and assume that the observed probabilities are $\gamma_{\hat{p},\hat{e}}(\hat{C}) = 1/3$ and $\gamma_{\tilde{p},\tilde{e}}(\tilde{C}) = 1/2$.*

Suppose that for some set Ω of states of nature, the data is Ω-rationalized by the probability measure δ over Ω, and the preference assignment function R, which maps into $\mathcal{R}^2$. Fix any state $\bar{\omega}$ with positive probability, and such that

$$(\arg\max_{B(\tilde{p},\tilde{e}_i)} R_i(\bar{\omega}))_{i=1}^2 \in \tilde{C},$$

and denote $\tilde{x}_i = \arg\max_{B(\tilde{p},\tilde{e}_i)} R_i(\bar{\omega})$ and $\hat{x}_i = \arg\max_{B(\hat{p},\hat{e}_i)} R_i(\bar{\omega})$, for each i. Since each $R_i(\bar{\omega}) \in \mathcal{R}$, it follows from the Weak Axiom of Revealed Preferences that $\hat{x}_{i,2} \geq \tilde{x}_{i,2}$ for both individuals. But since $\tilde{x}_{1,2} + \tilde{x}_{2,2} = 8/3 = \hat{e}_{1,2} + \hat{e}_{2,2}$, it follows that $\hat{x}_{i,2} = \tilde{x}_{i,2}$, and then, by Walras's law, that $\hat{x}_i = \tilde{x}_i$ for both individuals. Then,

$$(\hat{x}_i)_{i=1}^2 = (\arg\max_{B(\hat{p},\hat{e}_i)} R_i(\bar{\omega}))_{i=1}^2 \in \hat{C}.$$

The latter implies that

$$\begin{aligned} 1/2 &= \gamma_{\tilde{p},\tilde{e}}(\tilde{C}) \\ &= \delta(\{\omega \in \Omega : (\arg \max_{B(\tilde{p},\tilde{e}_i)} R_i(\omega))_{i=1}^2 \in \tilde{C}\}) \\ &\leq \delta(\{\omega \in \Omega : (\arg \max_{B(\hat{p},\hat{e}_i)} R_i(\omega))_{i=1}^2 \in \hat{C}\}) \\ &= \gamma_{\hat{p},\hat{e}}(\hat{C}) \\ &= 1/3, \end{aligned}$$

an obvious contradiction.

5 Concluding remarks:

This paper studied situations in which individuals choose from their own choice sets, subject only to their own constraints, and no conflict arises in spite of the fact that the set of socially feasible choices may be smaller than the product of the individually feasible sets. For this to occur, individual preferences cannot be distributed across individuals in an arbitrary manner. The problem is simple when only one collective choice situation is faced or when individual preferences are assumed to be invariant. A more interesting situation arises when there is a sequence of exogenously given social constraints and individual preferences are allowed to change randomly. In this case, if one has observed probabilistic distributions of collective choices over the socially feasible sets, one can only maintain the hypothesis of individual rationality under the assumption that preferences, however random, are not independent across individuals. The alternative would be to assume that some individuals take into account social feasibility, which amounts to dropping the usual assumption of individual rationality.

Collective choices are characterized in terms of the way in which individual preferences must co-vary in order to explain observed distributions of choices via individual rationality. Two definitions of rationalizability were considered. The first one assumed that one is given the profiles of preferences that are allowed in the rationalization, and the problem reduces to assigning probabilities to those profiles. The main result here is that a condition defined by McFadden and Richter (1990), called the Axiom of Revealed Stochastic Preferences, characterizes rationalizability. This definition, however, appears to be too strong in the sense that lack of rationalizability may be due to the set of profiles of preferences and not to the observed stochastic choice. This leads to the second, weaker definition of rationalizability, in which one is given a set of states of the world, and the problem requires assigning to each one of them a profile of preferences, within certain classes, and a probability. Rationalizability in this case is characterized by a combination of the Axiom of Stochastic Revealed Preferences and several instances of the Congruence Axiom (or the Strong Axiom of Revealed Preferences) – as many as there are states of the world. It is finally shown that there exist collective choices that cannot be rationalized in either sense.

In terms of the introductory example, consider the case of an airline that wants to test the hypothesis that its passengers on some particular route are systematically rationed in their meal choices. Suppose that the airline offers any two of a finite set of meal alternatives, and suppose, for simplicity, that the number of passengers on this route is always the same I, and all of them always eat.[6] Suppose that, over a long number of flights, the airline has estimated the probabilities of different feasible meal allocations, for every possible combination of alternatives.[7] Fix a set of demographic profiles of passengers for this flight. The airline wants to test whether there exist a probabilistic distribution over demographic profiles and an assignment of preferences to demographic profiles such that all the observed distributions of meal servings are explained by the theoretical probabilities.[8] Theorem 2 provides a (strongest) test for this hypothesis: the hypothesis is not rejected if, and only if, one can define a feasible allocation for every possible menu and every demographic profile, such that: (i) for every seat on the airplane, and for every demographic profile, the postulated choice is rationalizable using standard revealed preference arguments across menus; and (ii) along any finite sequence of menus and feasible allocations, observed probabilities should accumulate no more rapidly than the number of times that the allocation given by the sequence coincides with the postulated allocation, for at least one demographic profile.[9]

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Notes

[1]In fact, the in-flight menu of Cathay Pacific for June 2006 states that they "apologize if, due to previous passenger selection, [the choice of a passenger] is not available."

[2]For any set Z, denote by $\mathcal{P}(Z)$ its power set. Then, $\mathcal{B} \subseteq \mathcal{P}(X) \setminus \{\varnothing\}$.

[3]If, for all observed budgets $B \in \mathcal{B}$, it is true that $B = \times_{i\in\mathcal{I}} B_i$, and if all the binary relations $R_{i,s}$ are representable, then theorem 1 follows directly from Theorem 2 in McFadden and Richter (1990), since maximizing individual relations over individual domains is then equivalent to maximizing their sum over the Cartesian product of those domains.

[4]In the case of two subindexes, the first denotes the consumer and the second the commodity.

[5]By construction, $\hat{C} \subseteq B(\hat{p}, \hat{e})$ and $\tilde{C} \subseteq B(\tilde{p}, \tilde{e})$.

[6]I am not assuming that it is always the same people in the flight, as will be clear below. Suppose that the number of passengers is I, and denote by $\mathcal{X}$ the set of meal alternatives used by the airline. Suppose that for any $(\tilde{x}, \hat{x}) \in \mathcal{X}^2$, $\tilde{x} \neq \hat{x}$, the airline prepares $K_{\tilde{x},\hat{x}}(\tilde{x})$ servings of meal $\tilde{x}$, and $K_{\tilde{x},\hat{x}}(\hat{x})$ of meal $\hat{x}$, satisfying that $K_{\tilde{x},\hat{x}}(\tilde{x}) + K_{\tilde{x},\hat{x}}(\hat{x}) \geq I$. For consistency, $K_{\tilde{x},\hat{x}}(\hat{x}) = K_{\hat{x},\tilde{x}}(\tilde{x})$.

[7]For each $(\tilde{x}, \hat{x}) \in \mathcal{X}^2$, and for each $x \in \{\tilde{x}, \hat{x}\}^I$ such that $\|\{i \in \mathcal{I} : x_i = \tilde{x}\}\| \leq K_{\tilde{x},\hat{x}}(\tilde{x})$ and $\|\{i \in \mathcal{I} : x_i = \hat{x}\}\| \leq K_{\tilde{x},\hat{x}}(\hat{x})$, the observed probability of allocation x of meals is $\gamma_{\tilde{x},\hat{x}}(x)$, when the menu $(\tilde{x}, \hat{x})$ is available.

[8]Enumerate the seats on the airplane $1, \ldots, I$. Let Ω be a set of demographic profiles of passengers for this flight. Let $\mathcal{R}$ be the set of orders over $\mathcal{X}$. The hypothesis is that there exist functions $\delta : \mathcal{P}(\Omega) \to [0, 1]$ and $R : \Omega \to \mathcal{R}^I$ such that, for all pairs of meals $\tilde{x}$ and $\hat{x}$, $\delta(\{\omega \in \Omega : (\arg\max_{\{\tilde{x},\hat{x}\}} R_i(\omega))_{i\in\mathcal{I}} = x\}) = \gamma_{\tilde{x},\hat{x}}(x)$.

[9]There must exist $x_{(\tilde{x},\hat{x}),\omega} \in \{\tilde{x}, \hat{x}\}^I$, feasible, for every menu $(\tilde{x}, \hat{x})$ and every demographic profile ω, such that for every finite sequence $(\tilde{x}_k, \hat{x}_k, x_k)_{k=1}^K$, such that x_k and is feasible for menu $(\tilde{x}, \hat{x})$, one has that: (i) for every seat i and every demographic profile ω, if $x_{i,(\tilde{x}_k,\hat{x}_k),\omega} \in \{\tilde{x}_{k-1}, \hat{x}_{k-1}\}$ for every $k \leq K - 1$, then either $x_{i,(\tilde{x}_K,\hat{x}_K),\omega} = x_{i,(\tilde{x}_1,\hat{x}_1),\omega}$ or $x_{i,(\tilde{x}_1,\hat{x}_1),\omega} \notin \{\tilde{x}_K, \hat{x}_K\}$; and (ii) there exists some demographic profile ω such that $\sum_{k=1}^K \gamma_{(\tilde{x}_k,\hat{x}_k)}(x_k) \leq \sum_{k=1}^K 1_{\{x_k\}}((x_{(\tilde{x}_k,\hat{x}_k)\},\omega})$.

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