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# NO SWITCHBACKS: RETHINKING ASPIRATION-BASED DYNAMICS IN THE ULTIMATUM GAME

ABSTRACT. Aspiration-based evolutionary dynamics have recently been used to model the evolution of fair play in the ultimatum game showing that in-credible threats to reject low offers persist in equilibrium. We focus on two extensions of this analysis: we experimentally test whether assumptions about agent motivations (aspiration levels) and the structure of the game (binary strategy space) reflect actual play, and we examine the problematic assumption embedded in the standard replicator dynamic that unhappy agents who switch strategies may return to a rejected strategy without exploring other options. We find that the resulting "no switchback" dynamic predicts the evolution of play better than the standard dynamic and that aspirations are a significant motivator for our participants. In the process, we also construct and analyze a variant of the ultimatum game in which players can adopt conditional (on their induced aspirations) strategies.

KEY WORDS: Aspirations, Experiment, Learning, Replicator dynamics, Ultimatum game

JEL CODES: C78, C91



## 1. INTRODUCTION

Almost two decades have passed since Güth et al. (1982) first documented a now familiar pattern in ultimatum game experiments—"fair" offers are more common, and "unfair" ones rejected more often, than is consistent with subgame perfection.[1] Evolutionary game theorists would later find this pattern to be less anomalous than their predecessors, however. In an influential paper, Binmore, Gale, and Samuelson (1995) (BGS) would show that when the shares of proposers and responders committed to pure strategies in a miniature Ultimatum Game (MUG) evolve on the basis of "replicator dynamics" (RD),

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there are two stable outcomes.[2] The first of these corresponds to the subgame perfect equilibrium—no proposers are fair, and all of their offers, fair or not, are accepted—but in the second, all proposers are fair, and a substantial (but indeterminate) number of responders would reject unfair offers. No less important, BGS were able to rationalize RD as a form of social evolution based on ‘‘aspiration-based’’ learning.[3]

Our own contribution follows from three observations about these results. First, while the experimental evidence is consistent with the presence of considerable fairness, there is less fairness than the second RD equilibrium implies, with or without decision errors.[4] This echoes the previous work of van Huyck et al. (1995), who found that the RD did not predict the observed behavior in two person ‘‘divide the dollar’’ games. Second, and on a related note, the binary choice version of the ultimatum game in BGS differs from that which experimental subjects typically play. And third, there is a possible lacuna in the BGS treatment of ‘‘disenchanted’’ players, who are sometimes assumed to ‘‘switch back’’ to their original strategies, no matter how disappointing these have proven. We find that these observations are connected: the amended dynamics described in Section 2 are more consistent with the new evidence presented in Section 3, based on an experimental design in which aspiration levels are either assumed to be present or induced. In anticipation of concerns that the induction of aspirations should alter the predicted evolution of play, we also consider an extension of MUG in which conditional (on these aspirations) strategies are available to both proposers and responders, and find that the results, though somewhat different, lend further support to our modified dynamics. Furthermore, our empirical results support the use of simple aspiration-based learning as a plausible basis for social evolution, in contrast to the recent emphasis on rules-based approaches—see, for example, Stahl (2001) or Costa-Gomes and Weizsacker (2001).[5]

It will be useful, however, to first review the treatment of MUG in BGS. There are two populations, proposers and responders, the members of which are matched at random each period to play the normal form game:



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| | Accept | Reject |
|---|---|---|
| Fair | 2,2 | 2,2 |
| Selfish | 3,1 | 0,0 |

in which proposers must decide whether to offer a fair (equal) division of a pie of size 4 or demand most of it, and it is assumed that fair offers are never rejected.[6] Let the shares of fair and selfish proposers be denoted $s_F^P$ and $s_S^P$, the shares of responders who accept and reject unfair offers $s_A^R$ and $s_R^R$, and suppose that time is marked in discrete intervals of length $\Delta$. Suppose, too, that each period, a fraction $\Delta$ of proposers and responders evaluate their current performance, and that this evaluation is based on a comparison of their current payoff with some "aspiration," the value of which is drawn from a uniform distribution over $[a_{\mathrm{L},}\ a_{\mathrm{H}}]$, where in this particular framework, $a_L \geqslant 0$ and $a_H \leqslant 4$. When a proposer's payoff exceeds her aspiration, for example, she retains her current strategy, but when it falls short, she is assumed to "change" it, where the likelihoods that strategies are adopted are equal to their current shares in the population. (This also assumes, of course, that the proposer either observes the composition of her own population or perhaps samples and imitates.) We use quotation marks because these changes are sometimes more nominal than real: when all of the proposers are fair, for example, even the disenchanted are assumed to remain so.

It follows, therefore, that the shares of fair proposers will evolve as

$$s_F^P(t+\Delta) = s_F^P(t) - \Delta p_F^P(t) + s_F^P(t)\left[\Delta p_F^P(t) s_F^P(t) + \Delta p_S^P(t) s_S^P(t)\right],$$

where $p_F^P(t)$ $(P_S^P(t))$ is the likelihood that a fair (selfish) proposer falls short of her aspiration in period $t$. The second term on the right-hand side is the number of fair proposers who become disenchanted in the current period, and the third is the product of the total number of unsatisfied proposers, fair and unfair, and the current share of fair proposers, or the number of "new" fair proposers. Since $p_F^P(t) = (a_H - \pi_F^P(t))/(a_H - a_L)$



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and $p_S^P(t) = (a_H - \pi_S^p(t))/(a_H - a_L)$, where $\pi_F^P(t)$ and $\pi_S^P(t)$ are the current payoffs to fair and selfish proposers, it follows that:

$$\frac{s_F^P(t+\Delta) - s_F^P(t)}{\Delta} = \left(\frac{1}{a_H - a_L}\right) s_F^P(t)\left(\pi_F^P(t) - \overline{\pi}^P(t)\right), \tag{1.1}$$

where $\overline{\pi}^P = s_F^P(t)\pi_F^P(t) + s_S^P(t)\pi_S^P(t)$ is the mean payoff for all proposers.[7] Likewise, for responders, we have

$$\frac{s_A^R(t+\Delta) - s_A^R(t)}{\Delta} = \left(\frac{1}{a_H - a_L}\right) s_A^R(t)(\pi_A^R(t) - \overline{\pi}^R(t)). \tag{1.2}$$

As $\Delta \to 0$, (1.1) and (1.2) comprise a scaled version of the continuous time RD

$$\dot{s}_F^P(t) = \left(\frac{1}{a_H - a_L}\right) s_F^P(t)\left(\pi_F^P(t) - \overline{\pi}^P(t)\right),$$
$$\dot{s}_A^R(t) = \left(\frac{1}{a_H - a_L}\right) s_A^R(t)\left(\pi_A^R(t) - \overline{\pi}^R(t)\right).$$

The particular form of the RD in this case is

$$\dot{s}_F^P(t) = \left(\frac{1}{4}\right) s_F^P(t)\left(1 - s_F^P(t)\right)\left(2 - 3s_A^R(t)\right),$$
$$\dot{s}_A^R(t) = \left(\frac{1}{4}\right) s_A^R(t)\left(1 - s_A^R(t)\right)\left(1 - s_F^P(t)\right) \tag{1.3}$$

for $a_L = 0$ and $a_H = 4$.

Plot 1 illustrates the two stable outcomes under (1.3): $(s_F^P(t) = 0, s_A^R(t) = 1)$ is locally asymptotically stable, and the connected set $(s_F^P = 1, 0 \leq s_A^R \leq 2/3 - \varepsilon)$ is Liapunov stable.

## 2. A MODIFIED ASPIRATION MODEL

We introduce two modifications to the treatment of social evolution in BGS. First, those with unrealized aspirations are now required to adopt new strategies: the disenchanted cannot return or "switch back" to their initial choices, no matter how common these are. (This does not preclude switches and, if and



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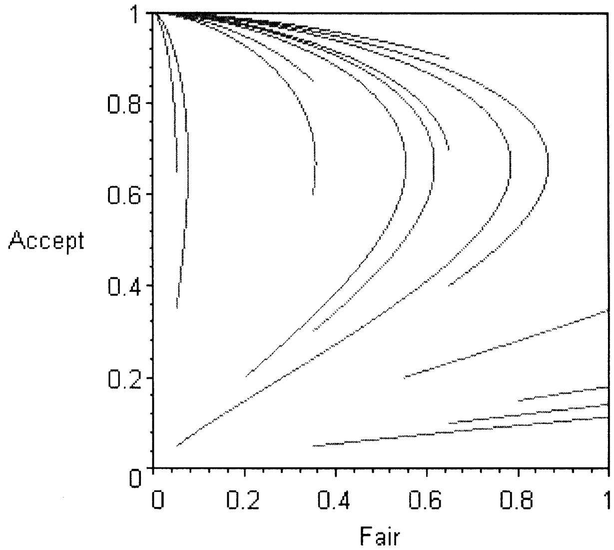


*Plot 1:* MUG under the standard replicator dynamics

when there is disappointment in future rounds, switchbacks.) With just two strategies available to the members of each population, the transition function is a simple one, and its information requirements minimal: fair proposers who fall short of their aspirations must become selfish ones, for example, and do not need to know the composition of either population. In discrete time, the proportions of fair and selfish proposers will therefore evolve as

$$s_F^P(t+\Delta) = \left(1 - \Delta p_F^P(t)\right) s_F^P(t) + \Delta p_S^P(t) s_S^P(t),$$
$$s_S^P(t+\Delta) = \left(1 - \Delta p_S^P(t)\right) s_S^P(t) + \Delta p_F^P(t) s_F^P(t).$$

It follows that $\sum_j s_j^P(t+\Delta) = \sum_j s_j^P(t)$, so that $\sum_j s_j^P(0) = 1 \to \sum_j s_j^P(t) = 1$ for each t—that is, population shares will never "wander off the unit square"—so that we can substitute $1 - s_F^P(t)$ for $s_S^P(t)$ and limit attention to the first of these

$$s_F^P(t+\Delta) - s_F^P(t) = -\Delta p_F^P(t) s_F^P(t) + \Delta p_S^P(t)\left(1 - s_F^P(t)\right).$$

Likewise, for responders, we have



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$$s_A^R(t+\Delta) - s_A^R(t) = -\Delta p_A^R(t) s_A^R(t) + \Delta p_R^R(t)\left(1 - s_A^R(t)\right).$$

Combining these and letting $\Delta \to 0$ produces

$$\dot{s}_F^P(t) = -p_F^P(t) s_F^P(t) + p_S^P(t)\left(1 - s_F^P(t)\right),$$
$$\dot{s}_A^R(t) = -p_A^R(t) s_A^R(t) + p_R^R(t)\left(1 - s_A^R(t)\right). \quad (2.1)$$

These constitute the ‘‘no switchback’’ dynamics (NSD) for MUG.

The connections between standard notions of evolutionary equilibrium and the stable rest points of evolutionary dynamics, a characteristic feature of the RD, vanish under the NSD. For example, if the proposers who make selfish offers and the responders who turn down these offers are ever dissatisfied, the shares that correspond to the perfect equilibrium of MUG will not even be a rest point under NSD, let alone a stable one. Furthermore, this condition will (almost) never be satisfied: if more than a small subset of the responder population aspires to more than one, for example, the proportion of those who reject selfish offers must soon rise. For similar reasons, the set of locally stable states in which no proposer is selfish and two thirds or fewer of responders would agree to an unequal split, a subset of the Nash equilibria of MUG, will not be an attractor either. However, to the extent that the experimental evidence is consistent with limit points composed of strictly mixed populations, dynamics that lead to equilibria in the interior of the state space are desirable.

We are not the first, of course, to suggest that non-Nash outcomes can be stable. Drawing on the work of McKelvey and Palfrey (1995), for example, Chen et al. (1995) define a variant of the quantal response equilibrium, the ‘‘boundedly rational Nash equilibrium’’ (BRNE), in ‘‘which the strategy of each player is a vector of discrete choice probabilities which is a random choice (modified multinomial logit) best response to the choice probabilities of the remaining players.’’[8] Chen et al. show that all finite games have BRNEs and that under broad



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conditions, fictitious play will converge to a unique BRNE. As shown below, the stable rest point of the NSD corresponds to a BRNE of MUG in which proposers and responders are both "more rational" than consistent with, for example, the Luce (1959) notion of probabilistic choice.

As these observations hint, the distribution of aspiration levels matters more under NSD. Under the alternative RD, for example, as $a_H$ rises—that is, as the numbers of proposers and responders who fall short of their respective aspirations increases—the pace of evolution is affected, but its character is not. That is, the solution orbits are the same, but velocities on these orbits are not. Under the NSD, on the other hand, this increase would push the interior limit point(s) to (1/2, 1/2), for intuitive reasons: in discrete time, $\Delta s_F^P(t)$ fair proposers, all of those who evaluate their performance in a particular period, will become selfish, while all $\Delta s_S^P(t)$ of the selfish ones who self-evaluate will become fair, and these flows will not offset one another unless $\Delta s_F^P(t) = \Delta s_S^P(t) = (1/2)$.

This leads to our second modification. BGS (87) mention differences in the distribution of aspiration levels as a natural extension of their model, but also note, in effect, that with switchback, it is the basins of attraction, not the attractors themselves, that are affected. We shall allow for differences in the (still uniform, however) distribution, too, but because the limit points of the NSD are sensitive to these, a selection criterion is called for. The levels induced in our subjects, for example, were consistent with the requirement that no one is bound to be satisfied or dissatisfied in all possible states of the world. In more practical terms, we suppose that proposers draw, or have drawn for them, from $U$[0, 3], and responders from $U$/[0, 2].

It follows that under these conditions, $p_F^P(t) = (1/3)$, $p_S^P(t) = 1 - s_A^R(t)$, $p_A^R(t) = (1/2)(1 - s_F^P(t))$, and $p_R^R(t) = 1 - s_F^P(t)$. One third of the fair proposers who reconsider their situation in a particular period, for example, will become selfish, no matter what the characteristics of the responder population. This is the expected result: fair proposers receive 2 for certain, and with a uniform distribution of aspirations between 0 and 3, one third



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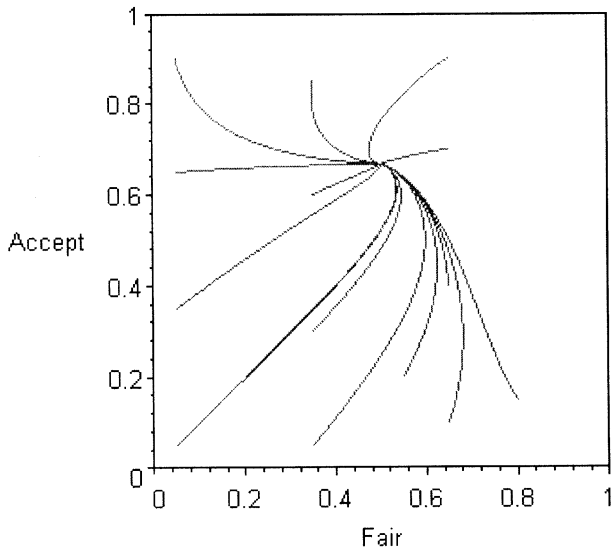


*Plot 2:* MUG under the no switchback dynamics

will not be satisfied with this. For similar reasons, the observation that while responders' "likelihood of disappointment" varies with the number of fair proposers, the likelihood that those who turn down unequal splits is twice that of those who do not is also more or less intuitive.

Substitution for the $p_j^{i\prime}s$ and $\pi_j^{i\prime}s$ in (2.1) leads, after further simplification, to the particular NSD for this model

$$\dot{s}_F^P(t) = -(1/3)s_F^P(t) + \left(1 - s_F^P(t)\right)\left(1 - s_A^R(t)\right),$$
$$\dot{s}_A^R(t) = \left(1 - s_F^P(t)\right)\left(1 - \frac{3}{2}s_A^R(t)\right). \tag{2.2}$$

The associated phase diagram is depicted in Plot 2. There is a single, asymptotically stable, equilibrium, $(s_F^P = 1/2, s_A^R = 2/3)$, in which half of the offers are fair, and two thirds of all unfair offers are accepted.[9] This prediction is sharper than that obtained under the RD and more consistent with the experimental evidence (Roth, 1995). It is also a more "turbulent" equilibrium, another characteristic of the experimental data: one third of all proposers, fair and selfish, switch each period, as do half of the responders who reject unfair offers and one quarter of the responders who do not.[10] We observe, too, that this equilibrium is invariant with respect to common affine transformations, so that the conversion of experimental monetary units into dollars, or the use of rewards for participation, have no effect, provided the endpoints of the distributions of aspirations are also suitably transformed.

If these proportions are instead (re)interpreted as mixed strategy profiles for a one shot version of MUG, this equilibrium corresponds to a BRNE in which responders' "degree of rationality" $\mu_R$ is ln 2/ln 1.5, but proposers' $\mu_P$ is indeterminate.[11] On the continuum of possible $\mu$-values, 0 is associated with equal choice probabilities, 1, with Luce's notion of probabilistic choice, and $\infty$, with "full rationality," from which we conclude that responders and, for reasons outlined in the footnote, proposers are more rational than, for example, probabilistic choosers would be. It is tempting, therefore, to view the NSD as another selection mechanism for BRNEs.

Last, and in anticipation of some of our experimental results, observe that initial states "close" to the northeast corner of state space ($s_F^P = 1, s_A^R = 1$) are not "pulled across the top," to the point corresponding to the subgame perfect equilibrium, as in BGS, but rather into the interior of the space, consistent with the behavior we observed (this statement anticipates Section 3). Additionally, because the dynamics assume an infinitely large population of bargainers, but our experiments were run with a modest number of participants in each role, it is plausible to expect cycles towards or around an equilibrium because games with finitely many agents may not be able to follow the theoretical paths to equilibrium. For example, notice that under the NSD, populations that find themselves in the southwest quadrant of the phase space move quickly to the northeast quadrant then to the west as the number of fair offers falls in a population of mostly accepters. Fewer fair offers then cause fewer acceptances on the way to equilibrium. In a finite population, this last transition may not be possible because it would require the "right" number of agents to change their behavior. Consider the case of 5 bargaining paris. If one person on either side changes his or her behavior, the population dis-



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tribution changes by 20% meaning that, if the population found itself to the northwest of the equilibrium and one responder switches from accept to reject, the population would overshoot the equilibrium and find itself in a situation in which the dynamics will send it back to the northwest quadrant. This implies that in experimental populaitons, the realization of our NSD model may be cycles in the northwestern quadrant of the strategy space.

Intuition suggests that the introduction of some "decision noise" should not have much effect on our already turbulent equilibrium. To verify this, suppose that a fraction $\theta^P$ of proposers, and $\theta^R$ of responders, commit self-evaluation errors—that is, a share $\theta^P$ of proposers, both fair and unfair, who should be satisfied conclude otherwise, and then switch, and that the same share who should be dissatisfied fail to do so, and likewise for responders. In general terms, the modified NSD are

$$\begin{aligned}\dot{s}_F^P(t) &= -(1-\theta^P)p_F^P(t) + \theta^P(1-p_F^P(t))s_F^P(t) \\ &\quad + (1-\theta^P)p_S^P(t) + \theta^P(1-p_S^P(t))(1-s_F^P(t)), \\ \dot{s}_A^R(t) &= -(1-\theta^R)p_A^R(t) + \theta^R(1-p_A^R(t))s_A^R(t) \\ &\quad + (1-\theta^R)p_R^R(t) + \theta^R(1-p_R^R(t))(1-s_A^R(t)). \qquad (2.3)\end{aligned}$$

The effects of such noise on the equilibrium shares $s_F^P$ and $s_A^R$ are recorded in Table I. The introduction of minimal noise

TABLE I

The effect of decision noise on the NSD equilibrium

| | $\theta^R$ | | | |
|---|---|---|---|---|
| | 0 | 0.01 | 0.10 | 0.25 |
| $\theta^P$ | | | | |
| 0 | 0.500,0.667 | 0.503,0.662 | 0.531,0.623 | 0.566,0.565 |
| 0.01 | 0.500,0.667 | 0.503,0.662 | 0.530,0.623 | 0.564,0.565 |
| 0.10 | 0.500,0.667 | 0.503,0.662 | 0.522,0.624 | 0.549,0.567 |
| 0.25 | 0.500,0.667 | 0.501,0.662 | 0.512,0.624 | 0.528,0.569 |

Note: $\theta^{P(R)}$ is the amount of proposer (responder) noise.



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($\theta^P = 0.01, \theta^R = 0.01$) has almost no effect on the (still stable) equilibrium: the share of fair proposers rises, from 50% to 50.3% and that of responders who reject unfair offers falls, from 66.7% to 66.2%. Since the rest point is hyperbolic,[12] such "persistence" is more or less expected. The surprise, perhaps, is that as the level of noise in both populations increases a substantial amount, to, say, 10%, the share of fair proposers rises just a little more, to 52.2%, while the proportion of responders who reject unfair offers falls, also a little bit, to 62.4%. In more general terms, the equilibrium share $s_F^P(s_A^R)$ is a decreasing (increasing) function of $\theta^P$, and an increasing (decreasing) function of $\theta^R$ with, in a loose sense, responder noise the more decisive influence. There is perhaps a loose parallel here to BGS, who find that responders must be "noisier" than proposers for the perfect equilibrium not to become the unique limit point.

## 3. EXPERIMENTAL EVIDENCE

To examine whether the standard model of aspiration-based social learning developed in BGS or the current model based on the no switchback principle best describes behavior in MUG, we ran eight experimental sessions in two treatments. In the first treatment, *no aspirations*, participants played the simple binary choice version of MUG. In the second treatment, *induced aspirations*, we induced aspirations in our participants using a protocol similar to Siegel and Fouraker (1960). Ninety-six students, representing various majors, were recruited from the undergraduate population at Middlebury College. On average, our participants earned $12.88, including a $5 show-up fee. The experiment was computerized with payoffs stated in terms of experimental monetary units (EMUs), that were translated into cash at the end of the experiment. Proposers were asked to choose between a 'selfish' proposal, 3EMUs for the proposer and 1EMU for the responder, and a 'fair' proposal 2EMUs for each player. Responders were then given the opportunity to accept or reject the proposal.



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Because we are interested in the limit point of a social learning process, we were careful to take precautions to prevent any possible endgame effects. We hypothesized that subjects might disregard the history of play near the end of a session, especially in the induced aspiration treatment, if they had no chance of meeting their aspirations. The instructions therefore stated that the experiment would proceed for as many rounds as time permitted. An hour and a half was allocated for each session, but after piloting the procedures in an informal setting, we discovered by debriefing participants that many lost interest after round 25. With this in mind, each session ran for 20 rounds, which took about an hour. Further, participants remained in the same role for the entire experiment, but were randomly reassigned a new partner after each round.

### 3.1. *The no aspirations sessions*

Table II summarizes the starting and ending states for each session. All four of the no aspiration sessions start in the interior of the strategy space and, taken together, the four sessions provide different initial conditions for the experiment. Just as our phase diagrams sweep the entire strategy space when examining potential paths to equilibrium, the differences in starting states allow us to be confident that our experimental analysis is not limited to local behavior in one region of the unit square. One can also see that the final states vary by session, but, on average, play tends to stay in the interior of the unit square as predicted by the no switch-back model.

The direction of play is better illustrated by plotting each session. In Figure 1 we map the paths taken on the unit square. Numbers indicate the transitions in the evolution of play in chronological order. Clearly, play never starts, ends, or even approaches the subgame perfect equilibrium of MUG. However, we are more interested in whether play proceeds in the direction of the unique perturbation-induced "fair" equilibrium calculated in BGS, or if play remains in the interior of the unit square as predicted by the no switch-back model.



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TABLE II
Summary of play in the experiment

| | No aspirations | | | |
|---|---|---|---|---|
| | Session 1 | Session 2 | Session 3 | Session 4 |
| Start state | 0.20,0.40 | 0.50,0.83 | 0.17,0.67 | 0.57,0.86 |
| End state | 0.80,1.0 | 0.67,0.83 | 0.83,1.0 | 0.57,0.86 |
| Mean state | 0.41,0.68 | 0.71,0.85 | 0.69,0.81 | 0.75,0.89 |
| N | 10 | 12 | 12 | 14 |
| | Induced aspirations | | | |
| | Session 5 | Session 6 | Session 7 | Session 8 |
| Start state | 0.55,0.55 | 0.83,0.83 | 1.0,1.0 | 0.33,0.50 |
| End state | 0.78,0.67 | 0.83,1.0 | 1.0,1.0 | 0.67,0.67 |
| Mean state | 0.77,0.62 | 0.76,0.83 | 0.93,0.95 | 0.62,0.75 |
| $\bar{a}^P$ | 1.22 | 1.54 | 0.60 | 2.41 |
| $\bar{a}^R$ | 1.56 | 0.78 | 1.40 | 1.77 |
| *frac*($a$) | 0.50 | 0.75 | 0.88 | 0.17 |
| N | 18 | 12 | 8 | 12 |

Note: $\bar{a}^{P(R)}$ is the mean proposer (responder) aspiration level induced, and *frac*($a$) is the fraction of participants in a session who reach their aspirations.

In each of the four no aspiration sessions play either remains in the interior of the unit square or moves to a state on the border where everyone offers an equal split and all offers are accepted. However, play never approaches the point $(s_F^P, s_A^R) \approx (1, 2/3)$ predicted by BGS. We conclude that the sort of rational, error-prone behavior described by the perturbed RD does not describe play in this experiment. In addition, although in each session play transits to the upper border of the unit square, indicating that some responders accept the selfish offer, play is never dragged across the top to the subgame perfect equilibrium either. Instead, the majority of play cycles counterclockwise in the interior of the strategy space as we suggested is consisent with a model of NSD in a finite (and not large) population of bargainers.



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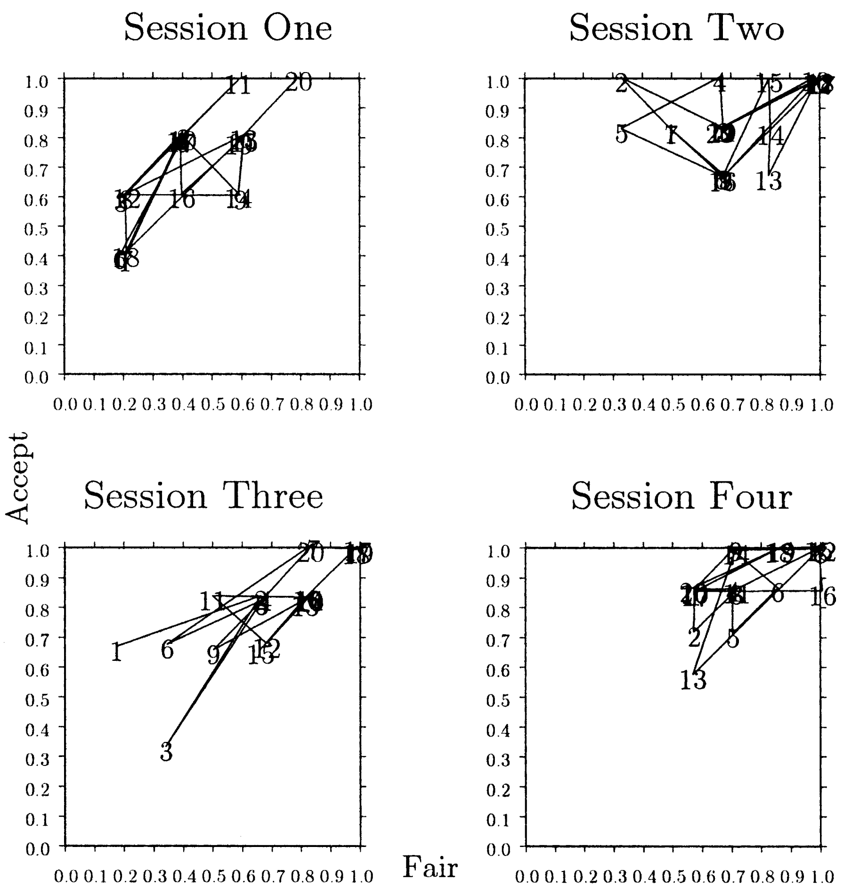


*Figure 1.* The evolution of play in the no aspirations treatment.

One might expect that even though the instructions clearly stated that individual choices would never be revealed, players may feel more anonymous in big groups. If anonymity causes more self-interested play, we would expect more greedy proposals and more acceptances in the larger sessions. If this hypothesis is correct, then our large sessions should partially control for unmodeled social factors and provide each model with its best chance of success. We ran regressions on the individual choice data, controlling for individual heterogeneity by including individual random effects, to examine this hypothesis, and the extent to which play was sensitive to the passage of time.

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TABLE III
The determinants of choice

| | No aspirations | | Induced aspirations | |
|---|---|---|---|---|
| | Proposers | Responders | Proposers | Responders |
| $a^{P(R)}$ | | | −0.88*** | 1.39*** |
| | | | −0.22(0.25) | 0.05(0.44) |
| $a^{P(R)} - \pi_{AVG}$ | | | −0.40*** | 0.73*** |
| | | | −0.10(0.14) | 0.03(0.33) |
| Session size | 0.38*** | 0.13 | −0.04 | −0.41*** |
| | 0.10(0.02) | 0.004(0.19) | −0.01(0.04) | −0.01(0.08) |
| Round | 0.08*** | −0.02 | 0.06*** | −0.11*** |
| | 0.02(0.01) | −0.0005(0.02) | 0.02(0.01) | −0.004(0.03) |
| Proposal | | 3.51*** | | 4.87*** |
| | | 0.50(0.39) | | 0.93(0.58) |
| N | 480 | 480 | 500 | 500 |
| Wald $\chi^2$ | 1132 | 86 | 28 | 75 |

Note: The dependent variables are 1 = *fair* for proposers and 1 = *accept* for responders. All regressions are random effects probits, where *,**, and *** denote significance at the 10%, 5% and 1% levels. Marginal effects are reported before (standard errors).

Starting with proposer choices, we see from Table III that the sign of the session size coefficient is the opposite of what we predicted—proposers are 20% more likely to be fair when another bargaining pair is added. Additionally, proposers become more fair over time, but, while the effect is significant, it is also small. This time effect makes sense given informal debriefings we conducted at the end of our sessions in which responders stated they tried to discipline proposers early on by rejecting selfish offers. Apparently, this tacit collusion on the part of responders was somewhat effective. At the same time, responders in the no aspiration treatment seem invariant to the size of the session and the round. Instead, the only factor that seems to matter to them is the size of the offer (Proposal = 1 for the fair offer, 0 otherwise).



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Before moving to the induced aspiration session, we shall set the stage for a discussion of the determinants of strategy switches. Defining a switch for a proposer is straight-forward. For our purposes, a responder switches when she faces the same offer in two consecutive periods and changes her response. In the no aspiration data, we see in Table IV (Equation (1a)) that proposers are 17% more likely to change their strategies between rounds than responders are. This seems like a large difference, but since we do not expect responders to start rejecting fair offers, it is not. In Equation (lb), we see that proposers remain 11% more likely to switch when we control for the fact that all players are less likely to switch as the game progresses (given the differential effect of time on proposers is small and insignificant).

In sum, our no aspiration sessions provide evidence favoring the NSD model of play in MUG. Play tends to start inside the unit square and remain there cycling clockwise in the neighborhood of the no switchback equilibrium. This is contrary to the subgame perfect equilibrium which predicts that play will be dragged to the upper left corner of the unit square and the perturbation induced "fair" equilibrium which predicts evolution towards the *fair* = 1 boundary.

### 3.2. *The induced aspirations sessions*

To be as fair as possible to aspiration-based models, we ran four additional sessions in which we induced aspiration levels in our participants. We accomplished this by modifying the procedures used in Siegel and Fouraker (1960). At the beginning of each session, participants were randomly assigned an aspiration level from an interval that depended on their role in the experiment (recall the above discussion of asymmetric aspiration intervals). Proposer aspiration levels, $a^P$, were drawn from the interval [0,3] and responder aspiration levels, $a^R$, were drawn from [0,2]. This asymmetry is appropriate given responders could never earn more than 2EMUs in a round. To make the aspiration level salient, participants were told that if their average earnings at the end of the experiment met or



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TABLE IV
The determinants of switching

| | No aspirations | | Induced aspirations | |
|---|---|---|---|---|
| | (1a) | (1b) | (2a) | (2b) |
| $a^{P(R)} - \pi_{AVG}$ | | | −0.14** | −0.37 |
| | | | −0.03(0.08) | −0.05(0.23) |
| Proposer | 1.25*** | 0.82* | 1.14*** | 1.73*** |
| | 0.17(0.36) | 0.11(0.49) | 0.18(0.16) | 0.26(0.40) |
| Round | | −0.07** | | 0.02 |
| | | −0.01(0.03) | | 0.003(0.03) |
| $(a^{P(R)} - \pi_{AVG}) \times$ Rnd | | | | 0.01 |
| | | | | 0.002(0.01) |
| $(a^{P(R)} - \pi_{AVG}) \times$ Prop | | | | 0.05 |
| | | | | 0.01(0.21) |
| Round × Proposer | | 0.05 | | −0.05* |
| | | 0.01(0.03) | | −0.01(0.03) |
| N | 734 | 734 | 778 | 778 |
| Wald $\chi^2$ | 12 | 18 | 55 | 66 |

Note: The dependent variable is 1 if (a) proposers switch strategies between rounds or (b) responders switch, given the responder is considering the same offer as last round. All regressions are random effects probits, where *,**, and *** denote significance at the 10%, 5% and 1% levels. Marginal effects are reported before (standard errors).

exceeded their aspiration levels, they would be given the chance to double their earnings.[13] When paying the participants at the end of the experiment, anyone whose average earnings exceeded their aspiration level was given a die to roll. If the die landed with either a 1 or a 2 up, the participant's earnings were doubled.

Some readers will be concerned that the introduction of the lottery could contaminate our results. There is some effect on the predicted evolution of proposer and responder behavior, of course, but there is also some reason to believe that the differences tend to *favor* the NSD. To elaborate, consider a modified MUG in which participants who meet or exceed their induced aspirations or *targets* double their payoffs with prob-



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ability $p$. Because this contamination can be attributed to the (possible) influence of targets on subsequent behavior, we allow for conditional strategies. Proposers, for example, can still be unconditionally fair (that is, extend fair offers whether their targets are high or low) or unconditionally selfish, but can also be fair if the target is high and selfish if it is low and *vice versa*. We shall denote these rules $F/H$ & $F/L$, $S/H$ & $S/L$, $F/H$ & $S/L$ and $S/H$ & $F/L$, and their respective population shares $s_1^P, s_2^P, s_3^P$ and $s_4^P = 1 - s_1^P - s_2^P - s_3^P$. In a similar vein, responders can accept unfair offers under all circumstances ($A/H$ & $A/L$) or no circumstances ($R/H$ & $R/L$), or accept them only if their target is high ($A/H$ & $R/L$) or low ($R/H$ & $A/L$), where the respective population shares are $s_1^R, s_2^R, s_3^R$ and $s_4^R = 1 - s_1^R - s_2^R - s_3^R$.

Given the structure of MUG, we draw a natural distinction between low and high: for proposers, targets between 0 and 2 (resp. 2 and 3) will be considered low (resp. high), but for responders, those between 0 and 1 (resp. 1 and 2) are considered low (resp. high). There are then four sorts of proposer/responder matches

Proposer's Target Low; Responder's Target Low

| | $A/H \& A/L$ | $R/H \& R/L$ | $A/H \& R/L$ | $R/H \& A/L$ |
|---|---|---|---|---|
| $F/H \& F/L$ | $2+2p, 2+2p$ | $2+2p, 2+2p$ | $2+2p, 2+2p$ | $2+2p, 2+2p$ |
| $S/H \& S/L$ | $3+3p, 1+p$ | $0, 0$ | $0, 0$ | $3+3p, 1+p$ |
| $F/H \& S/L$ | $3+3p, 1+p$ | $0, 0$ | $0, 0$ | $3+3p, 1+p$ |
| $S/H \& F/L$ | $2+2p, 2+2p$ | $2+2p, 2+2p$ | $2+2p, 2+2p$ | $2+2p, 2+2p$ |

Proposer's Target Low; Responder's Target High

| | $A/H \& A/L$ | $R/H \& R/L$ | $A/H \& R/L$ | $R/H \& A/L$ |
|---|---|---|---|---|
| $F/H \& F/L$ | $2+2p, 2+2p$ | $2+2p, 2+2p$ | $2+2p, 2+2p$ | $2+2p, 2+2p$ |
| $S/H \& S/L$ | $3+3p, 1$ | $0, 0$ | $3+3p, 1$ | $0, 0$ |
| $F/H \& S/L$ | $3+3p, 1$ | $0, 0$ | $3+3p, 1$ | $0, 0$ |
| $S/H \& F/L$ | $2+2p, 2+2p$ | $2+2p, 2+2p$ | $2+2p, 2+2p$ | $2+2p, 2+2p$ |

Proposer's Target High; Responder's Target Low

| | $A/H \& A/L$ | $R/H \& R/L$ | $A/H \& R/L$ | $R/H \& A/L$ |
|---|---|---|---|---|
| $F/H \& F/L$ | $2, 2+2p$ | $2, 2+2p$ | $2, 2+2p$ | $2, 2+2p$ |
| $S/H \& S/L$ | $3+3p, 1+p$ | $0, 0$ | $0, 0$ | $3+3p, 1+p$ |
| $F/H \& S/L$ | $2, 2+2p$ | $2, 2+2p$ | $2, 2+2p$ | $2, 2+2p$ |
| $S/H \& F/L$ | $3+3p, 1+p$ | $0, 0$ | $0, 0$ | $3+3p, 1+p$ |



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Proposer's Target High; Responder's Target High

| | $A/H \& A/L$ | $R/H \& R/L$ | $A/H \& R/L$ | $R/H \& A/L$ |
|---|---|---|---|---|
| $F/H \& F/L$ | $2, 2+2p$ | $2, 2+2p$ | $2, 2+2p$ | $2, 2+2p$ |
| $S/H \& S/L$ | $3+3p, 1$ | $0, 0$ | $3+3p, 1$ | $0, 0$ |
| $F/H \& S/L$ | $2, 2+2p$ | $2, 2+2p$ | $2, 2+2p$ | $2, 2+2p$ |
| $S/H \& F/L$ | $3+3p, 1$ | $0, 0$ | $3+3p, 1$ | $0, 0$ |

Because the targets are (also) drawn from uniform distributions, the likelihoods of the first and second matches are$1/3 = 2/3 \times 1/2$ each, while the likelihoods of the third and fourth are $1/6 = 1/3 \times 1/2$ each. It is then tedious, but not difficult, to calculate the expected payoffs for all proposer and responder strategies

$$\pi_1^P = 2 + \frac{4}{3}p,$$

$$\pi_2^P = \frac{1}{2}(3+3p)(1 - s_1^R - s_2^R),$$

$$\pi_3^P = \frac{2}{3} + \frac{1}{3}(3+3p)(1 - s_1^R - s_2^R),$$

$$\pi_4^P = \frac{2}{3}(2+2p) + \frac{1}{6}(3+3p)(1 - s_1^R - s_2^R)$$

and

$$\pi_1^R = (2+2p)\left(\frac{2}{3} + \frac{1}{3}s_1^P - \frac{2}{3}s_2^P - \frac{1}{3}s_3^P\right) + (2+p)\left(\frac{1}{6} - \frac{1}{6}s_1^P + \frac{1}{3}s_2^P + \frac{1}{6}s_3^P\right),$$

$$\pi_2^R = (2+2p)\left(\frac{2}{3} + \frac{1}{3}s_1^P - \frac{2}{3}s_2^P - \frac{1}{3}s_3^P\right),$$

$$\pi_3^R = (2+2p)\left(\frac{2}{3} + \frac{1}{3}s_1^P - \frac{2}{3}s_2^P - \frac{1}{3}s_3^P\right) + \left(\frac{1}{6} - \frac{1}{6}s_1^P + \frac{1}{3}s_2^P + \frac{1}{6}s_3^P\right),$$



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$$\pi_4^R = (2+2p)\left(\frac{2}{3}+\frac{1}{3}s_1^P-\frac{2}{3}s_2^P-\frac{1}{3}s_3^P\right) + (1+p)\left(\frac{1}{6}-\frac{1}{6}s_1^P+\frac{1}{3}s_2^P+\frac{1}{6}s_3^P\right).$$

To illustrate, consider the expected payoff $\pi_2^R$ of the responder who unconditionally rejects unfair offers. With probability (1/3), both her own target and the proposer's will be low, in which case she will receive $2 + 2ps_1^P + s_4^P = 1 - s_2^P - s_3^P\%$ of the time (the likelihood that she is matched with a proposer who is fair, either all of the time or conditional on his own low target) and 0 otherwise. With probability (1/3), her target is low but the proposer's is still high, and she once more receives $2 + 2p1 - s_2^P - s_3^P\%$ of the time and 0 otherwise. With probability (1/6), their situations are reversed (that is, the responder's target is high but the proposer's is low) and she receives 2 + 2$p$ with likelihood $s_1^P + s_3^P$, or whenever the proposer is fair, either all of the time or conditional on his own now low target, and 0 otherwise. Last, with probability (1/6), both proposer and responder have high targets, and the responder once more receives (2 + 2$p$) $(s_1^P + s_3^P)$ % of the time. It then follows that:

$$\begin{aligned}\pi_2^R &= \frac{1}{3}(1-s_2^P-s_3^P)(2+2p)+\frac{1}{3}(1-s_2^P-s_3^P)(2+2p)\\ &\quad+\frac{1}{6}(s_1^P+s_3^P)(2+2p)+\frac{1}{6}(s_1^P+s_3^P)(2+2p)\\ &= \frac{2}{3}(1-s_2^P-s_3^P)(2+2p)+\frac{1}{3}(s_1^P+s_3^P)(2+2p)\\ &= \left(\frac{2}{3}+\frac{1}{3}s_1^P-\frac{2}{3}s_2^P-\frac{1}{3}s_3^P\right)(2+2p)\end{aligned}$$

as claimed above. The other derivations follow more or less similar lines.

To ensure that both the RD and NSD remain well defined, however, another distinction is needed, this one between induced aspirations (or as we now call them, targets) and aspirations over the whole of the modified MUG. In particular, we shall assume



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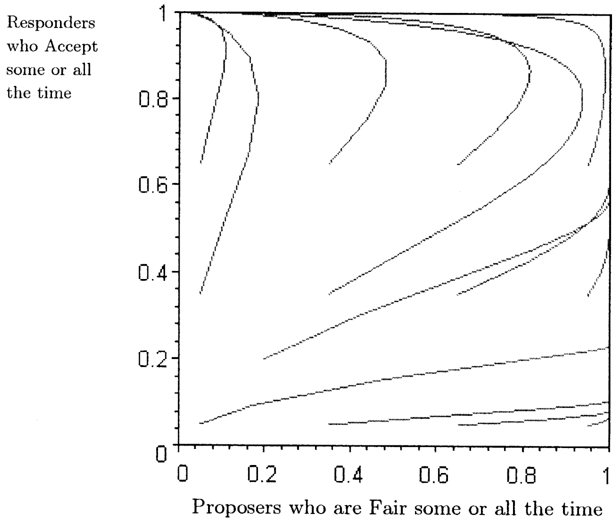
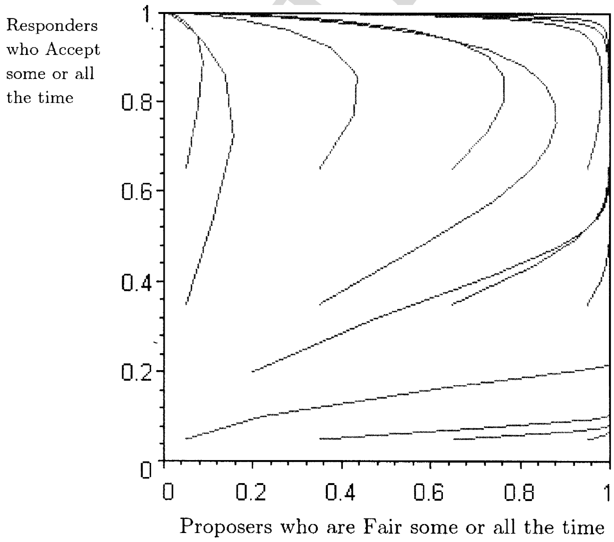
that proposers' (resp. responders') aspirations are drawn from a uniform distribution over [0,3 + 3*p*] (resp. [0,2 + 2*p*]). The form of the (scaled) RD is once more $\dot{s}_j^i = s_j^i(\pi_j^i - \overline{\pi}^i), i = P, R$ and $j = 1, 2, 3$, where $\overline{\pi}^i$ are the population-wide means, but with the increase in dimension, from two to six, its properties are more difficult to adduce. Simulation exercises reveal, however, that *in the absence of drift*, the most important feature of "simple MUG"—that is, the existence of two stable equilibria, one in which all proposers are selfish and all responders accept their unfair offers and another in which all proposers are fair and an indeterminate number of responders would turn down unfair offers at least some of the time—is robust under the RD.

The pseudo phase diagrams in Plots 3 and 4, for example, plot the evolution of the composite shares $(1 - s_2^P)$ and $(1 - s_2^R)$—that is, the proportions of proposers who extend fair offers either some or all of the time and responders prepared to accept unfair offers either some or all of the time—for the two cases $p = 0$ and $p = (1/3)$ and various initial conditions such that $s_1^P(0) = s_3^P(0) = 1 - s_1^P(0) - s_2^P(0) - s_3^P(0)$ $(= s_4^P(0))$ and $s_1^R(0) = s_3^R(0) = 1 - s_1^R(0) - s_2^R(0) - s_3^R(0)(= s_4^R(0))$. Since these initial shares, which amount to a level field for the three varieties of fair proposers and the three sorts of rational responders, do not remain equal, however, it is possible for a particular state $[(1 - s_2^P), (1 - s_2^R)]$ to be reached from different initial conditions, consistent with the observation that our pseudo trajectories sometimes cross.

The no lottery case ($p = 0$) depicted in plot 3 is, in effect, the BGS model. In the unfair equilibrium, it is obvious that no proposer *ever* offers an equal split, and it is not difficult to confirm that no responder *ever* turns down the lopsided offer. It is also not difficult to show that in the fair equilibrium, proposers' fairness is unconditional, but that all four sorts of responders will be present: when $s_2^P(0) = 0.35$ and $s_2^R(0) = 0.65$, for example, the shares of those committed to $A/H$ & $A/L$, $R/H$ & $R/L$, $A/H$ & $R/L$ and $R/H$ & $A/L$ tend toward 33.9%, 36.9%, 15.2% and 14.0%, respectively, but when $s_2^P(0) = 0.65$ and $s_2^R(0) = 0.95$, the same shares now tend toward 6.0%, 89.0%, 3.3% and 1.7%.




*Plot 3:* Evolution of composite shares in enhanced MUG (replicator dynamics, no lottery, no drift).




*Plot 4:* Evolution of composite shares in enhanced MUG (replicator dynamics, one third lottery, no drift).

The surprise, perhaps, is that plot 3 shares these features with Plot 4, in which, consistent with the experiment, one third of those who meet or exceed their targets are “lottery winners.”

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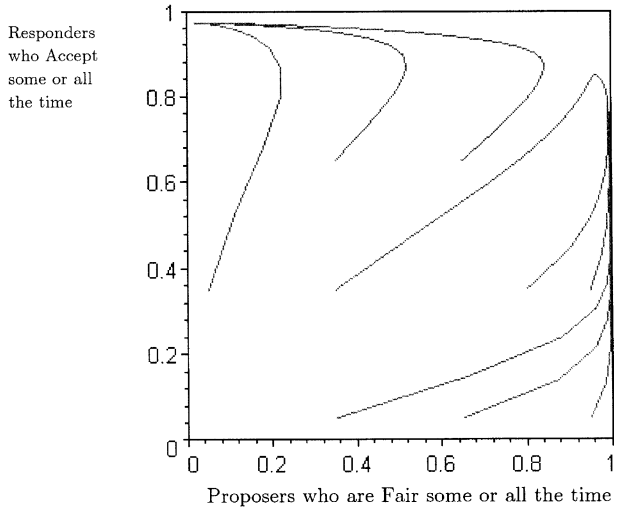


*Plot 5:* Evolution of composite shares in enhanced MUG (replicator dynamics, no lottery, drift).

There are still two stable equilibria, one fair the other selfish, and both proposers and responders choose unconditional behaviors at the unfair point. In the fair continuum, proposers are still fair all of the time, and all four sorts of responders are present. The two diagrams also hint, however, that with the introduction of the lottery, both the continuum of fair equilibria and its basin of attraction become smaller, which implies that fair division should be less common, and more difficult to rationalize.

The differences between the two cases become sharper with the introduction of deterministic noise or drift. To illustrate, Plots 5 and 6 are the equivalent of plots 3 and 4 for the perturbed RD, $\dot{s}^i_j = (1-\theta^i)s^i_j(\pi^i_j - \overline{\pi}^i) + \theta^i(\frac{1}{4} - s^i_j)$, where, in the spirit of BGS, we assume that responders are much noisier than proposers, $\theta^R = 0.1$ and $\theta^P = 0.01$. Consistent with BGS, there are now two asymptotically stable *points* in the no lottery case, an unfair equilibrium in which $s^P_1 = 0.003, s^P_2 = 0.982$, $s^P_3 = 0.010$, and $s^P_4 = 0.005$, and $s^R_1 = 0.867, s^R_2 = 0.027$, $s^R_3 = 0.053$ and $s^R_4 = 0.053$ (almost all proposers extend unfair offers all of the time, and almost all responders would accept such an offer no matter what their aspirations) and a fair

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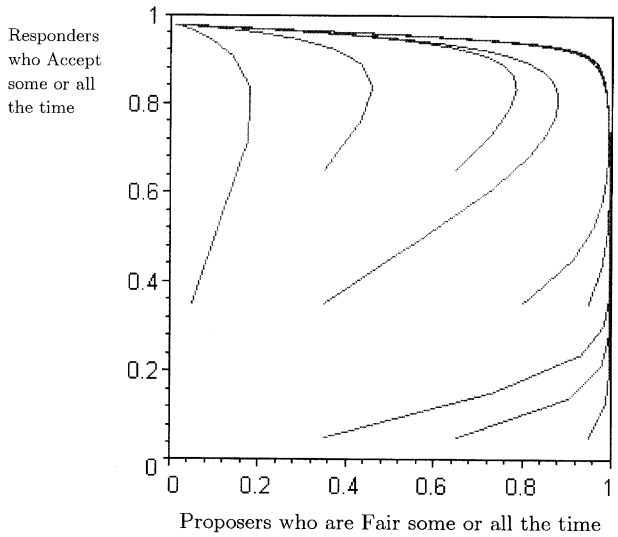


*Plot 6:* Evolution of composite shares in enhanced MUG (replicator dynamics, one third lottery, drift).

equilibrium in which $s_1^P = 0.971, s_2^P = 0.005, s_3^P = 0.008$, and $s_4^P = 0.016$, and $s_1^R = 0.266, s_2^R = 0.237, s_3^R = 0.248$ and $s_4^R = 0.248$ (almost all proposers extend fair offers all of the time, and almost three quarters of responders would turn down an unfair offer at least some of the time, with a third of these prepared to do so under all conditions).

As Plot 6 reveals, however, the introduction of the one-third lottery causes the fair equilibrium to vanish: all paths tend, over time, to an unfair equilibrium in which almost all proposers are once more selfish all the time and almost all responders accept their offers. Furthermore, it is not difficult to show that this result is robust with respect to the choice(s) of initial conditions, and other simulation exercises (not reported here) hint that it is also robust with respect to variations in drift rates and reasonable $p$ values. The fair equilibrium is still absent, for example, when the likelihood that eligible players win the lottery falls to 1 in 5, but (re)-appears when it is 1 in 10.

Our tentative conclusion, then, is that when aspirations are induced and participants who meet or exceed these are re-

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warded, it becomes more difficult for the RD model to rationalize the fairness observed in the lab.

It remains to show, however, that the same cannot be said about the NSD, with or without drift. The increase in the number of behavioral rules or strategies available to both proposers and responders introduces a new complication, however: in "simple MUG," when there were two such rules, the no switchback requirement meant that proposers or responders who were dissatisfied adopted the *other* rule, but there are now *three* alternatives. The specification most consistent with the spirit of BGS, we believe, would assume that the dissatisfied switch to these alternatives in proportion to their relative shares. It is (also) consistent with a modified imitation parable. Under these conditions, and in the absence of drift, the NSD would assume the form:

$$\dot{s}^i_j = -p^i_j s^i_j + s^i_j \sum_{k \neq j} p^i_k \frac{s^i_k}{1 - s^i_k}, \quad i = P, R \quad \text{and} \quad j = 1, 2, 3,$$

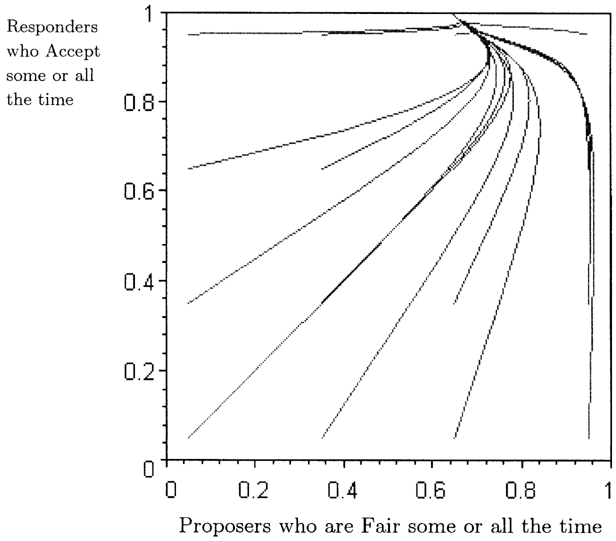
where

$$p^P_j = \frac{(3+3p) - \pi^P_j}{(3+3p)} \quad \text{and} \quad p^R_j = \frac{(2+2p) - \pi^R_j}{(2+2p)}$$

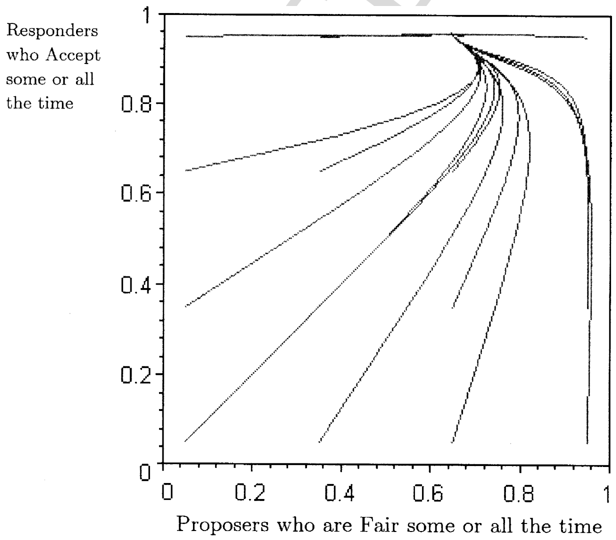
are the likelihoods that proposers and responders find themselves disappointed. If, as before, it is further assumed that a proportion $\theta^P$ of proposers and $\theta^R$ and responders will be dissatisfied despite the fact that their aspirations have been met or satisfied when aspirations have not been met, the perturbed NSD will have the form:

$$\dot{s}^i_j = -[(1-\theta^P)p^i_j + \theta^P(1-p^i_j)]s^i_j + s^i_j \sum_{k \neq j} [(1-\theta^P)p^i_k + \theta^P(1-p^i_k)] \frac{s^i_k}{1-s^i_k}.$$

Plots 7–9 depict the evolution of the same composite shares in the cases where there is (a) no lottery and no noise, (b) a one third lottery and no drift, and (c) a one third lottery and drift of size $\theta^P = 0.01$ and $\theta^R = 0.10$. Each features one stable rest point, and all are in some sense close to one another: between




*Plot 7:* Evolution of composite shares in enhanced MUG (no switchback dynamics, no lottery, no drift).




*Plot 8:* Evolution of composite shares in enhanced MUG (no switchback dynamics, one third lottery, no drift).

60% and 70% of proposers are fair at least some of the time, and between 0% and 10% of responders would turn down an unfair offer all of the time. The observation that decision noise has so little effect comes as no surprise: the rest points of the

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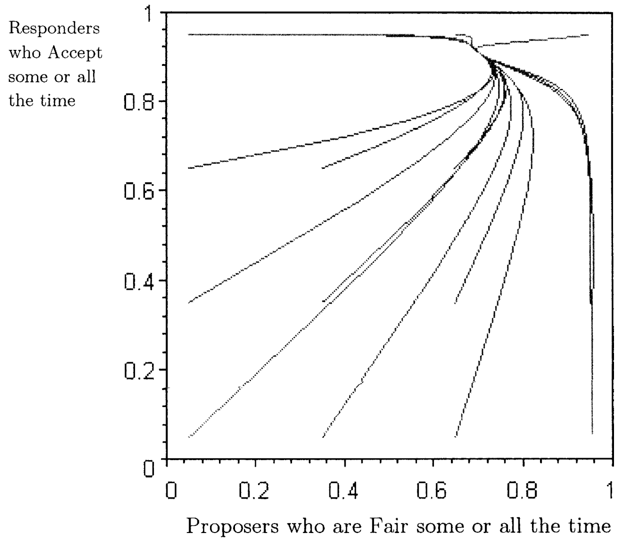


*Plot 9:* Evolution of composite shares in enhanced MUG (no switchback dynamics, one third lottery, drift).

NSD are, for the reasons described earlier, more turbulent than those of the RD, so that (a little) more turbulence is almost inconsequential.

The surprise, perhaps, is that the lottery itself does not matter more: in equilibrium, the share of proposers who are fair some or all of the time is 64.3% when $p = 0$ and 64.1% when $p = (1/3)$, while the shares of responders who would accept an unfair offer some or all of the time are 99.9% and 96.0%, respectively. These numbers obscure some important, if subtle, differences, however. With the addition of the lottery, the share of proposers who are fair *only* when their target is high, for example, decreases a substantial amount, from 28.5% to 20.4%, while the share of those who are fair *only* when their target is low increases an almost equal amount, from 21.6% to 29.6%. Because proposers are more likely to draw a low target than a high one, the number of fair offers will increase in the presence of the lottery, consistent with the intuition that selfish behavior then becomes riskier for high target proposers. The effects of the lottery on the responder population are less pronounced: the proportions of those who would turn unfair offers

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all of the time or turn them down only for high targets each increase 3–4%, while the proportions of those who would accept unfair offers all the time or turn them down only for low targets each decrease more or less the same amount.

To some extent, the similarities in the composite shares reflect the fact that an increase in $p$ will have two effects on the likelihoods of disappointment that work in opposite directions. On the one hand, for a particular value of $\pi_j^i$, the likelihood increases because mean aspirations have also increased: the right endpoint of the distribution of aspirations is an increasing function of $p$, while the left remains fixed, at 0. On the other hand, all of the $\pi_j^i$'s are themselves increasing (or at least non-decreasing) functions of $p$—that is, the expected payoff to *all* strategies rise, or at least do not fall, with the likelihood that eligible players are lottery winners—and this causes the likelihood of disappointment to fall. Given the structure of MUG, and the artificial, and somewhat problematic, assumption that aspirations are drawn from a *uniform* distribution, these effects will often be close in absolute size.

This should not detract from our main result, however, which is that behavior in MUG experiments, with or without induced aspirations, is easier to rationalize with the NSD than the RD.

Return to Table II which also reports summary statistics from the induced aspiration sessions. As in the sessions without aspirations, the four with aspirations start, and for the most part, cycle within the unit square. Interestingly, aspiration levels and the act of meeting one's aspiration appear to correlate with average play in the experiment which is evidence that our aspiration-inducement procedure was successful. More specifically, in accordance with subgame perfect play, higher proposer aspiration levels tend to reduce the number of fair offers and high responder aspirations appear to yield more acceptances. Participants also seem to respond to the size of the session.[14] Large sessions tend to stay closer to the center of the unit square while our smallest session, 3, remains close to the all fair, all accept vertex. We analyze these observations in more detail below.



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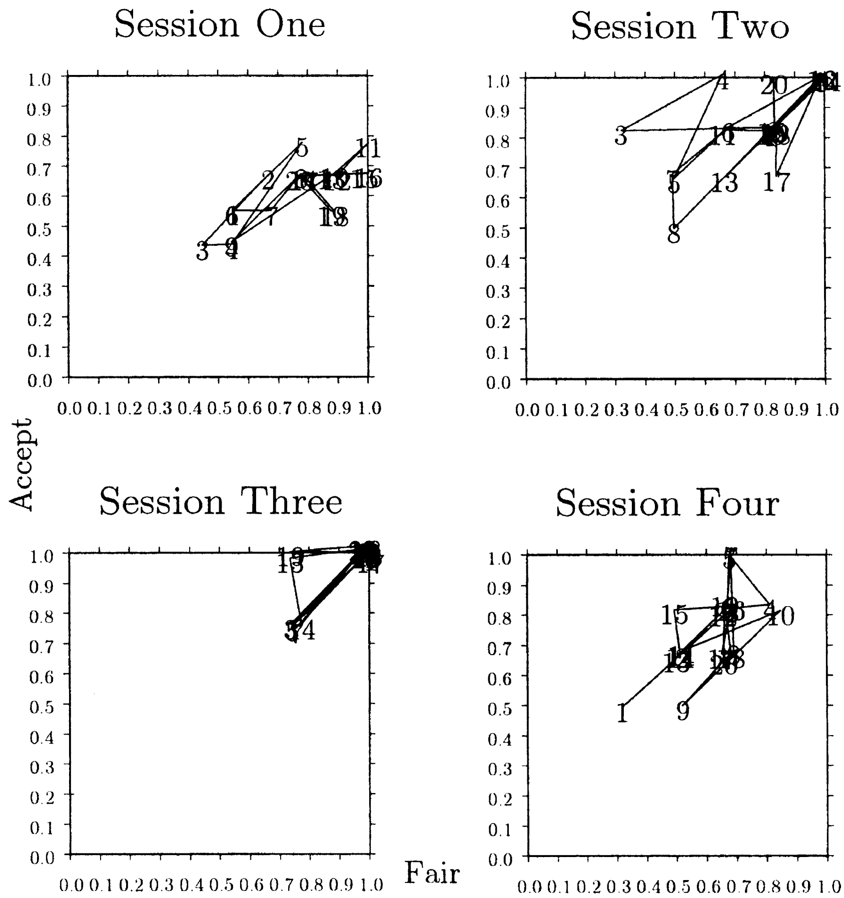


*Figure 2.* The evolution of play in the induced aspirations treatment.

Figure 2 looks very similar to Figure 1. As with the no aspiration games (with the exception of rounds 15 and 16 in session 1 which approach the fair BGS equilibrium), play either remains in the interior of the unit square or moves to a state on the border where everyone offers an equal split and all offers are accepted (sessions two and three). As in the first four session, the majority of play cycles in the northeast quadrant of the strategy space.

The econometrics of the induced aspiration sessions are more interesting because we can directly test whether aspirations actually play a role. If the aspiration levels we induced

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were salient, one should expect (as shown by Siegel and Fouraker (1960)) that they will tend to crowd out other-regarding feelings and therefore retard the evolution of play towards the all fair, all accept vertex. If our hypotheses are correct, then our large sessions with high induced aspirations provide the aspiration-based model with its best chance of success.

Returning to Table III and beginning with proposers, we see that the sign on the session size coefficient is in the predicted direction, larger groups yield fewer fair offers, but the effect is insignificant. However, proposers react strongly to the level of their aspirations. A unit increase in a proposer's aspiration level reduces the likelihood of a fair offer by 22%, even controlling for the deviation between a proposer's current average payoff and their aspiration level $(a^P - \pi_{AVG})$. Notice that proposers are also sensitive to the distance between their aspiration levels and their current average payoffs. Specifically, proposers appear to try to make up ground by choosing the unfair offer more often when their average payoffs fall below their aspiration levels. Lastly, proposers in the induced aspiration treatment mimic the behavior of proposers in the no aspiration treatment with respect to time. We conclude that proposers are driven by the absolute level of their aspirations, as well as the payoff implications of these aspirations (i.e. the deviation between aspirations and average payoffs).

The anonymity of a session does affect the choices of responders. Contrary to our predictions about increased self-interest in large groups, responders are significantly more likely to reject an offer of given size in such groups. This suggests that anonymity triggers more, not less, spite, a result similar to Bolton and Zwick (1995). Further, responders are more likely to accept each offer when they draw high aspiration levels. Similar to proposers, the deviation of a responder's current average payoff and the aspiration level works in the hypothesized direction (higher deviations make responders more likely to accept), and is a significant influence.

We end our discussion of the experiment by noting that aspiration-based models of social evolution make specific predictions about switching behavior that we can test using our



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data. We would expect players to be more likely to change strategies when their average payoffs falls below their aspiration levels. The results in Table IV assess this prediction. Equation (2a) confirms that aspirations cause players to switch strategies. More specifically, unhappy players (i.e. $a^{P(R)} - \pi_{AVG} < 0$) are more likely to switch than players who have met or surpassed their aspiration levels. Notice that the aspiration deviation is significant even controlling for the fact that proposers are more likely to switch strategies (a result that is common to both treatments). In Equation (2b) we add all the interactions to fully control for the difference in switching behavior between proposers and responders. Under these restrictions, the aspiration deviation effect abates and we conclude that, while aspirations tend to influence switching behavior in the hypothesized direction, the effect is not robust. However, this very specific test should be viewed together with the results of Table III which suggest that aspirations are important determinants of choice.

## 4. CONCLUDING REMARKS

Our purpose was twofold in this paper. First, we were interested in developing a model of the evolution of play in the ultimatum game that was based on the assumption that dissatisfied players switched strategies for certain, and required that players draw aspirations from the set of available game payoffs. Our hope was that such a model would predict outcomes better than the standard aspiration-based replicator dynamic. Second, to assess the success or failure of our modifications to the standard evolutionary dynamic, we were also interested in running an experiment designed to replicate the conditions necessary for an aspiration-based model to predict; namely, we decided to run a binary choice version of the game.

Concerning our first objective, we find that a model of social evolution wherein agents abandon strategies that produce payoffs falling short of their aspirations for sure results in a unique asymptotically stable attractor much closer to the center of the strategy space than equilibria under the standard (noisy)



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dynamic. This result is noticeably more consistent with existing experimental results. That is, in most repeated versions of the ultimatum game, each period generates both fair and selfish offers and selfish offers are rejected with non-vanishing probability (e.g. Prasnikar and Roth, 1992). Further, if we allow for asymmetries in the distribution of aspirations that are role-dependent, our equilibrium moves even closer to actual play and produces cyclical paths to equilibrium that qualitatively match what we see in the lab. When the model is extended to allow for the adoption of conditional (on the induced aspiration) strategies, the differences between the RD and NSD become more pronounced, and tend to favor the latter.

We summarize the results of our experiment as follows. Play in our eight sessions remains in the interior of the unit square contrary to the predictions of earlier models of fairness in the ultimatum game. Regression analysis (Table III) suggests that our aspiration manipulation was successful. In our experiment induced aspirations have the predicted effect of pushing play in the direction of the subgame perfect equilibrium (i.e. fewer fair offers and more acceptances), but these forces are not strong enough so that the subgame perfect equilibrium was realized in any session. Instead, group size tends to attenuate the effect of aspiration on responders (i.e. responders are emboldened to reject in larger, more anonymous settings). The end result is best viewed in Figure 2 — controlling for aspiration levels and group size, the no switchback dynamic is a better predictor of the evolution of play than either the subgame perfect equilibrium or the connected set of equilibria in which all offers are fair. Lastly, our experiment indicates that aspiration-based models are a sensible way to think about social evolution: our second set or regressions (Table IV) provides tentative evidence that players make strategic choices based on deviations from induced aspirations.

These results suggest two future directions for research in this area. First, from an experimental point of view, we were surprised by the magnitude of the effect of induced aspirations on the experimental outcomes. We speculate that inducing aspirations in other well understood game environments (e.g. public goods, or common pool resources) will also yield interesting



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results tractable by evolutionary models. Second, we are encouraged by our theoretical results which indicate that tailoring the standard story of social evolution to better fit a given situation yields results more consistent with observed behavior. Other manipulations are obvious, but we will mention one we feel is particularly interesting. We suspect that an even better way to think about aspirations is that they evolve with the history of play, as in Karandikar et al. (1998). In future work, we plan to explore the implications of endogenous aspirations without switchbacks, and hope to report our results in the near future.

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## NOTES

1. In the ultimatum game, the first mover or “proposer” offers a division of some finite “pie” to the second mover or “responder,” who either accepts or rejects this offer. An accepted division is then implemented, but a rejected one leaves both with nothing.
2. Explaining laboratory behavior using evolutionary and other dynamics, (best response, for example) has also been taken up by van Huyck et al. (1994), Friedman (1996), and Carpenter (2002) among others.
3. BGS (69) caution readers not to “place too much significance on the particular value of the equilibrium offer ... [since] ... different specifications ... can give different results.” Despite this, their rationalization for the RD remains both an appealing, and influential, one.
4. With more or less comparable noise in the two populations, the outcome in which all proposers are selfish, and no responder turns down a selfish offer becomes the unique rest point. When responders are noisier, there is a second stable rest point, in which “almost all” proposers are fair. For more details, see BGS.
5. This said, the aspirations we induce are, by current theoretical standards, simple ones. We do not allow these aspirations to evolve over time, for example, or consider peer influence. For an overview of recent developments (see Bendor et al., 2000).

6. As it, turns our, 43 of the 693 (6%) of the fair offers we observed in our experiment were rejected. We should note, however, that 40 of these 43 occurred during one session, and that three disenchanted responders with high induced aspirations were responsible. Dan Goldman, a student and participant in the experiment, later identified two possible reasons for the rejection of fair offers: ‘‘spite’’ on the part of those who would never realize their aspirations, and a preoccupation with relative outcomes on the part of those well above their aspirations.
7. Since it is well known the vector field is invariant under RD, we do not consider the behavior of $s_S^P = 1 - s_F^P$.
8. We thank Larry Samuelson for bringing this connection to our attention.
9. The trace of the relevant Jacobian, evaluated at this point, is equal to $-17/12 < 0$, the determinant is $(1/2) > 0$, and since $(17/12)^2 > 4(1/2)$, the eigenvalues are negative and unequal, so that the rest point is locally asymptotically stable.
10. In the sequential bargaining experiment elaborated on in Carpenter (2002), 66% of first movers change their offers from period to period. This fraction seems even larger given the central tendency of offers was not significantly different from period to period. It should be noted, however, that the turbulence can be ‘‘tuned down’’ in our model if we assumed that proposers and responders evaluate their situation less frequently.
11. Letting the mixed strategies be $(s_F^P(t), 1 - s_A^R(t))$ and $(s_A^R(t), 1 - s_A^R(t))$ the two conditions for a BRNE are: $s_F^P = ((2^{\mu}P)/(2^{\mu}P + (3s_A^R)^{\mu P}))$ and $s_A^R = ((1 + s_F^P)^{\mu R})/((1 + s_F^P)^{\mu R} + (2s_F^P)^{\mu R}))$ where $\mu_P$ and $\mu_R$ are the aforementioned degrees of rationality. For $(s_F^P = 1/2, s_A^R = 2/3)$, these will be satisfied for $\mu_R = \ln 2/\ln 1.5$ and all $\mu_P$. The value of $\mu_P$ is indeterminate because when $s_A^R = 2/3$, the expected values of fair and selfish offers are equal and there is no premium for more rational behavior. Suppose, however, that responders sometimes tremble when confronted with a fair offer, and let the expected outcome under (*fair*, *reject*) be $(2 - \delta, 2 - \delta)$. It is then not difficult to show that as $\delta \to 0, s_F^P \to 1/2, s_A^R \to 2/3$, $\mu_R \to \ln 2/\ln 1.5$, *but* $\mu_p \to 3$.
12. That is, the relevant Jacobian has no zero or purely imaginary eigenvalues. For details, see, for example, Glendenning (1994).
13. Participants saw both their current average payoff and their (nonchanging) aspiration level in each round.
14. Friedman (1996) also mentions group size effects on the convergence to ‘‘behavioral equilibria.’’



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