Tributes 49

Logically Speaking

A Festschrift for Marie Duží

Editors
Pavel Materna
Bjørn Jespersen

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Tributes Series Editor
Dov Gabbay dov.gabbay@kcl.ac.uk

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edited by

Pavel Materna

Bjørn Jespersen

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Contents

What Topic for *off-topic* in WK3?

Massimiliano Carrara, Filippo Mancini and Wei Zhu
FISPPA Department, University of Padua, Italy.

Abstract

Beall [1] proposes to read the middle-value of Weak Kleene logic as *off-topic*. This interpretation has recently drawn some attention: for instance, Francez [5] has pointed out that Beall's interpretation does not meet some important requirements to count as a truth value. Moreover, Beall is silent about what a topic (or a subject matter) is. But arguably, *what is a topic?* is a crucial question, and an answer is really important to fully understand his proposal. Thus, our goal here is to help to remedy this deficiency, and show how Beall's interpretation of Weak Kleene truth-values interacts with the notion of topic. To do that, we formalize his motivating ideas and draw some consequences from them.

Keywords: weak Kleene logic; topic; interpretation of the value u in weak Kleene systems; off-topic; subject matter

Introduction

In the field of many-valued logics, Weak Kleene logic (WK3) is a greatly underdeveloped subject compared to its strong counterpart — i.e. K3. Despite the several attempts to provide a complete characterization for these system (e.g., [2], [6], and [8]), the problem of giving a philosophically sound interpretation for the semantic non-classical value, **u**,[1] still persists. Some different interpretations are

[1] **u** means *undefined* as in [8]. This value might also be referred to as the *third value*, the *middle value*, or 0.5.

now available, such as *nonsense*, *meaninglessness*, and *undefined*.[2] Among them, a new proposal by Beall [1] suggests to read **u** as *off-topic*. Thus, a proposition that obtains this value should be regarded as being off-topic.

Such an interpretation has recently drawn some attention and it has proved useful — e.g. it is adopted by Carrara and Zhu [3] to distinguish two kinds of computational errors. However, it still has some problem and lacks some explication. For example, Francez [5] has pointed out that **u** as *off-topic* does not satisfy the pre-theoretic understanding of a truth-value in a truth-functional logic.[3] If Francez's criticism is valid, a characterization of topic (or a subject matter) is needed to evaluate pro and cons of Beall's proposal. But Beall [1] is silent about what a topic is.[4] Thus, our goal here is to help to remedy this deficiency, and show how Beall's interpretation of WK3 truth-values could interact with the notion of topic.

This paper is divided into two sections. In §1 we introduce WK3. In §2 we elaborate on Beall's notion of topic and draw some consequences about how topics work according to his conception.

1 WK3

Let us briefly introduce WK3. Its language is the standard propositional language, L. Given a nonempty countable set $\mathsf{Var} = \{p, q, r, \dots\}$ of atomic propositions, the language is defined by the following Backus-Naur Form:

$$\Phi_L ::= p \mid \neg\phi \mid \phi \vee \psi \mid \phi \wedge \psi \mid \phi \supset \psi$$

[2] For a survey on new interpretations of **u** see Ciuni and Carrara [4].

[3] Specifically, Francez [5] claims that any notion that aspires to qualify as an interpretation of a truth-value has to satisfy certain requirements, and that Beall's interpretation of **u** as *off-topic* does not do that.

[4] "Topic" and "subject matter" are just synonyms, and throughout the paper we will use them interchangeably.

We use $\phi, \psi, \gamma, \delta \dots$ to denote arbitrary formulas, $p, q, r, \dots$ for atomic formulas, and $\Gamma, \Phi, \Psi, \Sigma, \dots$ for sets of formulas. Propositional variables are interpreted by a valuation function $V_a : \mathsf{Var} \longmapsto \{\mathbf{t}, \mathbf{u}, \mathbf{f}\}$ that assigns one out of three values to each $p \in \mathsf{Var}$. The valuation extends to arbitrary formulas according to the following definition:

Definition 1.1 (Valuation)**.** A valuation $V : \Phi_L \longmapsto \{\mathbf{t}, \mathbf{u}, \mathbf{f}\}$ is the unique extension of a mapping $V_a : \mathsf{Var} \longmapsto \{\mathbf{t}, \mathbf{u}, \mathbf{f}\}$ that is induced by the tables from Table 1.

ϕ	$\neg\phi$
t	**f**
u	**u**
f	**t**

$\phi \vee \psi$	**t**	**u**	**f**
t	**t**	**u**	**t**
u	**u**	**u**	**u**
f	**t**	**u**	**f**

$\phi \wedge \psi$	**t**	**u**	**f**
t	**t**	**u**	**f**
u	**u**	**u**	**u**
f	**f**	**u**	**f**

$\phi \supset \psi$	**t**	**u**	**f**
t	**t**	**u**	**f**
u	**u**	**u**	**u**
f	**t**	**u**	**t**

Table 1: Weak tables for logical connectives in L

Table 1 provides the full *weak tables* from Kleene *et al.* [8, §64], that obtain "by supplying [the third value] throughout the row and column headed by [the third value]".[5] Note that in WK3 negation works like in K3, but conjunction and disjunction work differently. Specifically, the interpretation of disjunction is not *max* and that of conjunction is not *min*.[6] The way **u** transmits is usually called *contamination* (or *infection*), since the value propagates from any $\phi \in \Phi_L$ to any construction $k(\phi, \psi)$, independently from the value of ψ (here, k is any

[5]It is clear from Table 1 that $\wedge$ and $\supset$ could be defined from $\neg$ and $\vee$: it is easy to check from Table 1 that $\phi \wedge \psi = \neg(\neg\phi \vee \neg\psi)$ and $\phi \supset \psi = \neg\phi \vee \psi$. We prefer to introduce them all as primitives in order to have a complete overview with Table 1.

[6]It is for this reason that we find appropriate to use **u** rather than 0.5 for the non-classical value, since in WK3 it does not have the behavior of a "middle value".

complex formula made out of some occurrences of both ϕ and ψ and whatever combination of $\vee$, $\wedge$, $\supset$). To better capture the way **u** works in combination with the other truth-values, let us introduce the following definition:

Definition 1.2. For any $\phi \in \Phi_L$, *var* is a mapping from Φ_L to the power set of Var, which can be defined inductively as follows:

$$var(p) = \{p\},$$
$$var(\neg\phi) = var(\phi),$$
$$var(\phi \vee \psi) = var(\phi) \bigcup var(\psi),$$
$$var(\phi \wedge \psi) = var(\phi) \bigcup var(\psi),$$
$$var(\phi \supset \psi) = var(\phi) \bigcup var(\psi).$$

We can extend the above definition concerning *var* as follows:

Definition 1.3. *Let X be a set of sentences. var denotes a mapping from the power set of Φ_L to the power set of* Var*. That is,* $var(X) = \bigcup\{var(\phi) \mid \phi \in X\}$.

Then, the following fact expresses *contamination* very clearly:

Fact 1.1 (Contamination)**.** For all formulas ϕ in L and any valuation V:

$$V(\phi) = \mathbf{u} \quad \textit{iff} \quad V_a(p) = \mathbf{u} \text{ for some } p \in var(\phi)$$

The left-to-right direction is shared by all the most common three-valued logics; the right-to-left direction is clear from Table 1, and it implies that ϕ takes value **u** if some $p \in var(\phi)$ has the value, and no matter what the value of q is for any $q \in var(\psi) \setminus \{p\}$.

The WK3 consequence relation is defined as preservation of the value **t**, so that an important feature of WK3 is that Addition is invalid: $\phi \nvdash_{wk3} \phi \vee \psi$.

2 Beall's *Off-topic* Interpretation

According to Halldén [6] and Bochvar and Bergmann [2], the third value in WK3 is interpreted as *meaningless* or *nonsense*. However, such interpretations seem to suffer from some problems. For example, it is not at all obvious that we can make the conjunction or the disjunction of a meaningless sentence with one with a traditional truth-value. Observing this problem, Beall [1] proposes an alternative interpretation for **u**: i.e. *off-topic*. Thus, *What is a topic?* is a crucial question for his proposal. For depending on how we answer this question we may have different consequences on such a reading. Unfortunately, Beall [1] is silent about that. But he gives some constraints we can use to explore how topics behave and how they relate to the WK3 truth-values.

Before presenting them, let us make some assumptions and define a simple notation to facilitate the discussion. We assume that topics can be represented by sets.[7] We use bold letters for topics, such as $\mathbf{s}$, $\mathbf{t}$, etc. $\subseteq$ is the inclusion relation between topics, so that $\mathbf{s} \subseteq \mathbf{t}$ expresses that $\mathbf{s}$ is included into (or is a subtopic of) $\mathbf{t}$. Given that, we define a *degenerate* topic as one that is included in every topic.[8] Also, we define the overlap relation between topics as follows: $\mathbf{s} \cap \mathbf{t}$ iff there exists a non-degenerate topic $\mathbf{u}$ such that $\mathbf{u} \subseteq \mathbf{s}$ and $\mathbf{u} \subseteq \mathbf{t}$. Further, it is assumed that every meaningful sentence α comes with a *least* subject matter, represented by $\tau(\alpha)$. $\tau(\alpha)$ is the unique topic which α is about, such that for every topic α is about, $\tau(\alpha)$ is included into it. Thus, we say that α is *exactly* about $\tau(\alpha)$.[9] But α can also be *partly* or *entirely* about other topics: α is entirely about $\mathbf{t}$ iff $\tau(\alpha) \subseteq \mathbf{t}$, whereas α is partly about $\mathbf{t}$ iff $\tau(\alpha) \cap \mathbf{t}$.

[7] This is a natural assumption. For note that topics are represented by sets in all the main approaches to subject matter as discussed by Hawke [7].

[8] The inclusion relation, $\leq$, is usually taken to be reflexive, so that every topic includes itself.

[9] Throughout this paper, when we talk about the topic of a sentence we mean its least topic. In case we want to refer to one of its topics that is not the least one, we will make it clear.

2.1 Beall's Terminology and Motivating Ideas

[1]'s new interpretation starts from setting a terminology concerning a *theory*, T. T is a set of sentences closed under a consequence relation, Cn. That is, $T = Cn(X) = \{\phi \mid X \vdash \phi\}$, where X is a given set of sentences and $\vdash$ is the consequence relation of the logic we are working with. As for WK3, theories are sets of sentences closed under WK3 logical consequence. Then, Beall puts forward the following motivating ideas for his proposal:

> 1. A theory is about all and only what its elements — that is, the claims in the theory — are about.
> 2. Conjunctions, disjunctions and negations are about exactly whatever their respective subsentences are about:
>   (a) Conjunction $\phi \wedge \psi$ is about exactly whatever ϕ and ψ are about.
>   (b) Disjunction $\phi \vee \psi$ is about exactly whatever ϕ and ψ are about.
>   (c) Negation $\neg\phi$ is about exactly whatever ϕ is about.
> 3. Theories in English are rarely about every topic expressible in English.
>
> [1, p. 139]

As for the three WK3 semantic values, Beall proposes to "[...] read the value 1 not simply as *true* but rather as *true and on-topic*, and similarly 0 as *false and on-topic*. Finally, read the third value 0.5 as *off-topic*" [1, p. 140]. Thus, an arbitrary sentence ϕ is either true and on-topic, false and on-topic, or off-topic. And note that since both on-topic and off-topic sentences are arguably meaningful, the problem concerning the conjunction/disjunction of meaningless sentences vanishes.

2.2 Formalizing Beall's Ideas

We can arguably formalize Beall's motivating ideas as follows:[10]

Definition 2.1. *Let T be a WK3 theory and $\tau(T)$ be its topic. The following conditions show how the topics of WK3 sentences and $\tau(T)$ are related.*

1. $\tau(T) = \bigcup\{\tau(\phi) \mid \phi \in T\}$.
2. (a) $\tau(\phi \wedge \psi) = \tau(\phi) \cup \tau(\psi)$.
   (b) $\tau(\phi \vee \psi) = \tau(\phi) \cup \tau(\psi)$.
   (c) $\tau(\neg\phi) = \tau(\phi)$.
3. *for any T in English, there is some topic ζ expressible in English such that $\zeta \notin \tau(T)$.*

These formulations correspond to Beall's three motivating ideas concerning the off-topic interpretation. But the way we formally capture condition 3 requires a comment. Beall claims that "[t]heories in English are *rarely* about every topic expressible in English" (emphasis added). However, our formal translation ignores "rarely", and replaces it with "never". Nonetheless, as explained in §2.3, we do believe that there might be a theory that is about every topic. But since the existence of such a theory does not affect our considerations throughout the paper, we omit "rarely" in our formalization of Beall's condition 3.

For ease of understanding, some examples are presented below.

Example 2.1. *Let $\phi = q \vee r$ and $\psi = \neg p \wedge q$. Then, according to Def. 2.1 the following results follow:*

[10] Note that we use the same notation, $\tau(\dots)$, both for the topic of a sentence and the topic of a theory — i.e. a set of sentences.

1. $\tau(\phi) = \tau(q) \cup \tau(r)$

2. $\tau(\psi) = \tau(p) \cup \tau(q)$

3. $\tau(\phi \vee \psi) = \tau(q) \cup \tau(r) \cup \tau(p)$

4. $\tau(\psi \vee \neg\psi) = \tau(p) \cup \tau(q)$

5. $\tau(\phi \wedge \neg\phi) = \tau(q) \cup \tau(r)$

These examples immediately follow from Def. 2.1. We note that what a classical tautology $\psi \vee \neg\psi$ is about is what its atomic components (p, q) are about, although neither p nor q is about **tautology**. Following this result, we can derive two further outcomes. First, a classical tautology might be off-topic of a theory about **tautology**. Suppose a theory $T^{\star}$'s topic is about **tautology**. A claim like "The Moon is made of green cheese or the Moon is not made of green cheese" is off-topic, because this claim is about the **Moon** and **green cheese**, but not about **tautology**. Second, classical tautologies are not neutral to topics. In classical propositional logic, a tautology is true for all possible truth-value assignments to its atomic components. In weak Kleene logics, a tautology can be off-topic. That is, a tautology can be either true and on-topic or off-topic, but cannot be false and on-topic. Similar result holds for a contradiction: a contradiction can be either false and on-topic or off-topic, but cannot be true and on-topic.

From Def. 2.1 we can derive also the following results.

Corollary 2.1. *For any* $\phi \in \Phi_L$, $\tau(\phi) = \bigcup\{\tau(p) \mid p \in var(\phi)\}$.

Proof. We can prove it by induction.

1. Let ϕ be an atomic sentence p. Then $\tau(\phi) = \tau(p)$.

2. Let $\phi = \neg\psi$ and $\tau(\psi) = \bigcup\{\tau(p) \mid p \in var(\psi)\}$. Since $var(\phi) = var(\neg\psi) = var(\psi)$, we have $\tau(\neg\psi) = \tau(\psi) = \bigcup\{\tau(p) \mid p \in var(\phi)\}$.

3. Let $\phi = \gamma \wedge \delta$, $\tau(\gamma) = \bigcup\{\tau(p) \mid p \in var(\gamma)\}$, and $\tau(\delta) = \bigcup\{\tau(q) \mid q \in var(\delta)\}$. Since $var(\phi) = var(\gamma) \cup var(\delta)$, we can derive $\tau(\phi) = \tau(\gamma) \cup \tau(\delta) = \bigcup\{\tau(r) \mid r \in (var(\gamma) \cup var(\delta))\}$. That is, $\tau(\phi) = \bigcup\{\tau(r) \mid r \in var(\phi)\}$.

4. For $\phi = \gamma \vee \delta$, we can prove that $\tau(\phi) = \bigcup\{\tau(r) \mid r \in var(\phi)\}$ in the same way as above.

□

Corollary 2.2. $\tau(T) = \bigcup\{\tau(p) \mid p \in var(T)\}$.

This corollary follows from the Def. 2.1 and Corollary 2.1. It shows that what a theory T is about boils down to the union of what the atomic components of each claims in T are about. Moreover, even if Beall does not mention what an arbitrary set (i.e. not necessarily a theory) is about, we buy the following very plausible definition:

Definition 2.2. *Let X be a set of sentences. Such a set is about all and only what its elements are about. That is, $\tau(X) = \bigcup\{\tau(\phi) \mid \phi \in X\}$.*

Thus, the following corollary follows from Def. 2.2:

Corollary 2.3. *Let X be a set of sentences. Then $\tau(X) = \bigcup\{\tau(p) \mid p \in var(X)\}$.*

By virtue of Corollary 2.3 and Def. 2.1, we can derive the following results:

Corollary 2.4. *For any $\phi, \psi \in \Phi_L$, $\tau(k(\phi, \psi)) = \tau(\{\phi, \psi\})$.*

Proof. By Def. 3.1, $\tau(k(\phi, \psi)) = \tau(\phi) \cup \tau(\psi)$. According to Def. 2.2, $\tau(\{\phi, \psi\}) = \tau(\phi) \cup \tau(\psi)$. Hence, $\tau(k(\phi, \psi)) = \tau(\{\phi, \psi\})$. □

Corollary 2.5. *For any $\phi, \psi \in \Phi_L$, $\tau(k(\phi, \psi)) = \tau(var(\phi)) \cup \tau(var(\psi))$.*

Proof. According to Corollary 2.4, $\tau(k(\phi,\psi)) = \tau(\{\phi,\psi\})$. By virtue of Corollary 2.3, we can derive $\tau(k(\phi,\psi)) = \bigcup\{\tau(p) \mid p \in var(\{\phi,\psi\})\}$. According to Def. 1.2, $var(\{\phi,\psi\}) = var(\phi) \cup var(\psi)$. Hence, $\tau(k(\phi,\psi)) = \bigcup\{\tau(p) \mid p \in (var(\phi) \cup var(\psi))\} = \bigcup\{\tau(p) \mid p \in var(\phi)\} \cup \bigcup\{\tau(q) \mid q \in var(\psi)\}$. Since $var(\phi) = var(var(\phi))$ and $var(\psi) = var(var(\psi))$, we can derive $\tau(k(\phi,\psi)) = \tau(var(\phi)) \cup \tau(var(\psi))$ by Def. 2.2. □

Corollary 2.6. *For any $\phi \in \Phi_L$ and WK3 theory T, $\tau(T) = \bigcup\{\tau(var(\phi)) \mid \phi \in T\}$.*

Proof. From Def. 2.2 and $var(\phi) = var(var(\phi))$, we can derive $\tau(\phi) = \tau(var(\phi))$. Since Def. 2.1 claims that $\tau(T) = \bigcup\{\tau(\phi) \mid \phi \in T\}$, we can derive $\tau(T) = \bigcup\{\tau(var(\phi)) \mid \phi \in T\}$ by substituting $\tau(\phi)$ with $\tau(var(\phi))$. □

Lemma 2.1. *For any $p \in \Phi_L$ and WK3 theory T, if $var(p) \subseteq var(T)$, then $\tau(p) \subseteq \tau(T)$.*

Proof. According to Def. 1.3, $var(T) = \bigcup\{var(\phi) \mid \phi \in T\}$, $var(var(\phi)) = \bigcup\{var(p) \mid p \in var(\phi)\}$. Then $var(T) = \bigcup\{var(p) \mid p \in var(T)\}$. If $var(p) \subset var(T)$, then $p \in var(T)$. Since $\tau(T) = \bigcup\{\tau(p) \mid p \in var(T)\}$, then $\tau(p) \subseteq \tau(T)$. □

However, the result does not hold in the opposite direction. That is, if $\tau(p) \subseteq \tau(T)$, it might not be the case that $var(p) \subseteq var(T)$. To understand this point, consider the following counterexample.

Example 2.2. *For any $r, q \in \Phi_L$ and WK3 theory T, let $var(r) \nsubseteq var(T)$, $var(q) \subseteq var(T)$, and $\tau(r) = \tau(q) \subseteq \tau(T)$. Therefore, even though $\tau(r) \subseteq \tau(T)$, $var(r) \nsubseteq var(T)$.*

This counterexample is possible because τ is not necessarily bijective. As a justification, consider the following line of reasoning. Suppose that if $\tau(r) \subseteq \tau(T)$, then $var(r) \subseteq var(T)$. In that case, we get that whatever sentence is in the theory, it is also on-topic; and that whatever sentence is not in the theory, it is also off-topic. But then, Beall's reading of the truth value 0 as false-and-on-topic is not available anymore. In other words, allowing for the bijection results in a conflict between Beall's conception of topic and his reading of the $WK3$ truth values.

To sum up: by Beall's ideas and the way WK3 works we get that (1) the topic of a sentence is completely determined by (is the union of the topics of) its propositional variables, and (2) the topic of a theory is completely determined by (is the union of the topics of) the propositional variables of its sentences. Moreover, we get also the following important result:

Theorem 2.1. *For any $\phi \in \Phi_L$ and WK3 theory T, if $var(\phi) \subseteq var(T)$, then $\tau(\phi) \subseteq \tau(T)$.*

Proof. We can prove it by induction.

1. If ϕ is an atomic sentence, this theorem holds for ϕ by virture of Lemma 2.1.
2. If $\phi = \neg\psi$ and this theorem holds for $\neg\psi$. We can derive that this theorem holds for ϕ, because $var(\neg\psi) = var(\psi)$.
3. If $\phi = \gamma \vee \delta$, and this theorem holds for γ and δ. We can derive this theorem holds for ϕ, because $var(\gamma \vee \delta) = var(\gamma) \cup var(\delta)$.
4. We can prove this holds for $\phi = \gamma \wedge \delta$ in the same way.

□

By virtue of Thm. 2.1, we can clarify Beall's on-topic/off-topic interpretation in the following way.

Corollary 2.7. *Let T be a WK3 theory and $\tau(T)$ be its topic. For any $\phi \in \Phi_L$,*

1. *ϕ is on-topic iff $\tau(\phi) \subseteq \tau(T)$. But note that this does not guarantee that $\phi \in T$. However, if $\phi \in T$, by Def. 2.1, it is definitively on-topic.*

2. ϕ is off-topic iff $\tau(\phi) \nsubseteq \tau(T)$. This suffices to say that $\phi \notin T$.

Finally, we can note that such an interpretation fits Beall's conditions as well as WK3 semantics. To see this, let's conjoin two propositional variables, p and q, to get $p \wedge q$. Suppose that both are on-topic, i.e. $\tau(p) \subseteq \tau(T)$ and $\tau(q) \subseteq \tau(T)$. According to 2(a), $\tau(p \wedge q) = \tau(p) \cup \tau(q)$. Thus, $\tau(p \wedge q) \subseteq \tau(T)$, that is $p \wedge q$ is on-topic, which is in line with WK3 semantics. Now, suppose that at least one of the conjuncts is off-topic, say q. Thus, $\tau(q) \nsubseteq \tau(T)$. Therefore, $\tau(p \wedge q) \nsubseteq \tau(T)$, which is also in line with WK3 semantics. Alternatively, we might also be tempted to consider the following different interpretation: for p to be on-topic means that $\tau(p) \cap \tau(T) \neq \varnothing$, whereas to be off-topic means that $\tau(p) \cap \tau(T) = \varnothing$. For instance, this is exactly what [7, p. 700] suggests: "[t]o say that a claim is *somewhat* on-topic is to say that its subject matter overlaps with the discourse topic".[11] However, such an interpretation is not compatible with Beall's constraints 1-3. For condition 2(a) clashes with WK3 semantics. To see this, suppose that $\tau(p) \cap \tau(T) \neq \varnothing$ but $\tau(q) \cap \tau(T) = \varnothing$. Thus, since $\tau(p \wedge q) = \tau(p) \cup \tau(q)$, it follows that $\tau(p \wedge q) \cap \tau(T) \neq \varnothing$ — i.e. $p \wedge q$ is on-topic. This contradicts WK3 semantics — namely, contamination. Moreover, our observations match Beall [1, fn. 5]: "[a]n alternative account might explore 'partially off-topic', but I do not see this as delivering a natural interpretation of WK3". Here, Beall is suggesting to distinguish two notions: *off-topic* and *partially off-topic*. The latter might be legitimately taken to correspond to the alternative reading in terms of overlap between topics that we rejected — as indeed he does.

2.3 Some remarks about Beall's condition 3

As we anticipated in the previous section, some comments are required about Beall's condition 3: "[t]heories in English are rarely about every topic expressible in English". Now, we believe there are two possible ways to read such a condition: (3a) for Beall there is at least one theory the topic of which is a degenerate topic, so that it is included

[11] Here, the discourse topic is what we call the topic of reference.

in every topic; (3b) for Beall there is at least one theory the topic of which overlaps with every topic. Thus, (3a) reads the aboutness relation in Beall's condition 3 as *entire aboutness*, whereas (3b) reads it as *partial aboutness*. Now, we claim that (3a) should be rejected, based on the following reasons. According to the Corollary 2.6 above, the topic of a theory is the union of the topics of every atomic component of every sentence in that theory. Now, assume (3a) is the correct interpretation. Also, let us exclude the case $T = \varnothing$ due to vacuity. Thus, every theory has at least one propositional variable as a member. Therefore, the only way for a theory to have a degenerate topic is that there is only one propositional variable in the language, which is absolutely implausible. Because of that, we can dismiss (3a). Then, let us try (3b). There are two ways for a theory to have a topic which overlaps with every topic: either to have a degenerate topic, or to have a topic that includes every topic — i.e. a universal topic. Since the first option is rejected by the previous considerations, we are left with the second one. Let us try to elaborate a little on that and use some formalization. According to this reading, a universal theory T_U — i.e. a theory the topic of which is universal — can be defined in the following way:

Definition 2.3. *Let x be a variable ranging over sentences and set of sentences of the language. Thus, T_U is a universal theory iff $\forall x$ $\tau(x) \subseteq \tau(T_U)$. Also, we call the topic $\tau(T_U)$ of a universal theory, T_U, a universal topic.*

Then, we may wonder whether such a universal theory represents a coherent notion, as Beall seems to hold by using the word "rarely". We claim it is, based on the following considerations. We aim at showing that universality does not implies triviality — i.e. that a universal theory is not necessarily trivial.[12] To see that, consider that a universal topic must include the topic of every sentence of the language. Thus, for any $\phi \in \Phi_L$, $\tau(\phi) \subseteq \tau(T_U)$. However, as we

[12]Recall that a trivial theory is a theory that makes every sentence a theorem — i.e. everything is provable in a trivial theory. In other words, a trivial theory has any sentence expressible in the language as a member.

explained in Corollary 2.7, from that we cannot conclude that $\phi \in T_U$. Thus, we cannot infer that T_U is trivial. Therefore, a universal theory appears to be a coherent and available notion.[13]

Conclusion

In this paper we have formalized some of Beall's ideas about the notion of topic and drawn some facts from them, to see what kind of topic is the best one for his off/on-topic reading of the WK3 truth-values. The result is that, from Beall's perspective, for a claim ϕ to be on-topic means that $\tau(\phi) \subseteq \tau(T)$, where T is the WK3 theory at stake; whereas, for ϕ to be off-topic means that $\tau(\phi) \nsubseteq \tau(T)$. This is in line with WK3 semantics.

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[13] An interesting question we were asked by a referee is whether a universal theory is *ipso facto* a theory of everything. Arguably, the answer depends on how a theory of everything is defined. A good strategy might be to take a theory of everything as a theory stating all the truths about every object in the ontological domain. Given that, we cannot find a straightforward way to argue that being a universal theory implies being a theory of everything. But such an issue certainly requires a further and deeper investigation.

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