Duality and 'particle' democracy

Elena Castellani*

1 Introduction

Among the significant philosophical issues raised by the central role of physical dualities in recent fundamental research, one has a specific ontological flavor, as it concerns 'dual entities', i.e. entities exchanged by a duality mapping between theories. In particular, the type of duality that is known as *weak/strong duality*,[1] or *S-duality* according to current terminology,[2] seems to imply new surprising features from an ontological point of view.

Weak/strong duality has become a basic ingredient in field and string theories, especially since the 1990s (see Polchinski, this issue). In general terms, it is described as an equivalence map between two different theories of the same physics, such that the weak coupling regime of one theory is mapped to the strong coupling regime of the other theory. Hence the special interest in this form of duality, seen as a new tool for getting information on physical quantities in the case of large values of the coupling constant (where the usual perturbative methods fail) by exploiting the results obtained in the weak coupling regime of the dual description.

This duality is usually accompanied by a novel, puzzling feature: the fact that under this kind of duality it often happens that what is viewed as 'elementary' in one description gets mapped to what is viewed as 'composite' in the dual description. To use the words of Ashoke Sen – one of the physicists who significantly contributed in the 1990s to extend dualities to the string setting – "the classification of particles into elementary and composite loses significance as it depends on which particular theory we use to describe the system" (Sen, 2001, p. 3). What does this mean? At first sight, this interchanging role of elementary and composite seems to have strong implications for reductionism

*Department of Humanities and Philosophy, University of Florence, via Bolognese 52, 50139, Firenze, Italy. E-mail: elena.castellani@unifi.it

[1] Or, indifferently, strong/weak duality.

[2] The term "S-duality" started to be used in connection with the first proposals for extending the weak/strong duality conjecture from the case of supersymmetric four dimensional Yang-Mills theories to the context of superstring theory (first of all, Font, Ibanez, Lüst and Quevedo (1990)). The name was "a historical accident", to quote Harvey (1996, p. 30): it was introduced, for reasons of practicality, to indicate the discrete symmetry group $SL(2, \mathbf{Z})$ of the ten-dimensional heterotic string theory compactified to four dimensions. More details can be found, for example, in Schwarz (1996, p. 3).

and fundamentality issues. According to Sen, for example, it implies a radical change in our understanding of the ultimate constituents of matter "by bringing in a sort of democracy between all particles, elementary and composite" (Sen, 1999, p. 1642). Another leading string theorist, Leonard Susskind, goes further: in the section significantly entitled "The End of Reductionism" of his contribution to a *Foundations of Physics*'s 2013 special issue on "Forty Years of String Theory", he gives a clearly anti-reductionist reading of the apparent ontological ambiguity connected with weak/strong duality (Susskind, 2013, pp.177-178).[3]

Philosophers, on their side, cannot draw such quick conclusions. If they want to go this route and discuss weak/strong dualities in relation to fundamentality and reductionism, it is their task to address some basic issues before: first of all, how to understand fundamentality, whether to ascribe it to objects or just to structures, and how to substantiate the link between elementary and fundamental.[4] This is not the route followed in this paper. The stance adopted is rather to avoid a literal reading of the elementary/composite interchange and, on this basis, to avoid mixing the question of its meaning with the question of physical fundamentality. The attitude is analogous to the one shared in this volume about how to understand apparently puzzling features such as the interchange of tiny and huge dimensions connected with T-duality in string theory[5], or the duality of dimension under the AdS/CFT (gauge/gravity) correspondence.[6] The underlying idea is that, what the dual descriptions do not agree upon, should not be attributed a real physical significance.[7] In fact, this means nothing else than saying that the physics (including its ontology) remains the same under the duality. What changes, is just the way of looking at it.

This paper elaborates a bit on this shared view on dualities, in the specific case of weak/strong duality and related 'elementary'/'composite' correspondence. In order to have a better informed view on the meaning of this correspondence, Section 2 is devoted to examining the history of weak/strong duality by following the main developments of the idea of electric-magnetic duality (EM duality) from which it originates – from the origin of EM duality with

[3]He concludes the section in the following way: "I could go on and on, taking you on a tour of the space of string theories, and show you how everything is mutable, nothing being more elementary than anything else. Personally, I would bet that this kind of anti-reductionist behavior is true in any consistent synthesis of quantum mechanics and gravity" (Susskind, 2013, p. 178).

[4]A discussion of the metaphysical implications of weak/strong duality, especially in regard to the fundamentality question, is provided in McKenzie, this issue. Concerning the ontological significance of dualities, a common attitude in previous philosophical literature has been to envisage some form of ontological structural realism (in short, the thesis that "all that there is, is structure") as the only viable option for escaping the antirealist conclusions apparently implied by the elementary/composite ambiguity. This has been usually discussed in connection with the issue of theoretical equivalence and, in particular, the question as to whether the equivalence between dual theories should be read as an instance of underdetermination of scientific theory by empirical evidence. See Rickles, this issue, for an updated discussion of this point. Previous references are Dawid (2007), Rickles (2011) and Matsubara (2013).

[5]See Huggett, this issue.

[6]See de Haro, this issue.

[7]In this sense some authors propose to view duality as a 'gauge'. This is discussed by Rickles, this issue, and in the contribution of de Haro, Teh and Butterfield, this issue.

Dirac's theory of magnetic monopoles and its successive generalizations in the context of (Abelian and non-Abelian) field theory, to arrive at its first extension to string theory. The aim is to clarify, in the light of this history, the nature of the correspondence between 'dual particles'. This analysis is then used as evidential basis for discussing, in Section 3, the philosophical implications of weak/strong duality.

2 Electric-magnetic duality and its generalizations

"Electromagnetic duality is an idea with a long pedigree that addresses a number of old questions in theoretical physics, for example: Why does space-time possess four dimensions? Why is electric charge quantised? What is the origin of mass? What is the internal structure of the elementary particles? How are quarks confined?". These are the introductory remarks on EM duality by David Olive, in his contribution to the collective volume on *Duality and Supersymmetric Theories* (Olive and West, 1999). He then continues by pointing out how the "old idea of electromagnetic duality" could be considerably enhanced in the light of crucial and apparently unrelated developments in the quantum field theory of the last forty years, such as "unified gauge theories with Higgs, supersymmetry, instanton theory, the theory of solitons, the idea of integrable quantum field theories as deformations of conformally invariant QFTs", with the bonus of obtaining "a compelling framework of ideas within which these apparently disparate developments become much more unified" (Olive, 1999, p. 62).

This section will try to highlight some of the key moments and notions of the fascinating history of the electric-magnetic duality idea in field theory, setting the basis for its successive extension to supergravity and string theory.

2.1 First steps

As is well known, the idea of a close similarity between electricity and magnetism, going back to Ampère and Faraday, was first made more precise with Maxwell's formulation of his famous equations for a unified theory of electric and magnetic fields.

Maxwell's equations display an evident similarity in the role of electric and magnetic fields. In the absence of source terms, the similarity is complete and the equations are invariant under the duality transformation D exchanging the role of the electric field $\vec{E}$ and the magnetic field $\vec{B}$ as follows:

$$D: \quad \vec{E} \to \vec{B}, \quad \vec{B} \to -\vec{E}. \tag{1}$$

Generalizing D to duality rotations parameterized by an arbitrary angle θ and reformulating in terms of the complex vector field $\vec{E} + i\vec{B}$, Maxwell's equations then display the following duality rotation symmetry:

$$\vec{E} + i\vec{B} \to e^{i\theta}(\vec{E} + i\vec{B}). \tag{2}$$

The first natural extension of this duality was to include the presence of charges. For the duality to still obtain, the existence of magnetic charges had to be assumed beside the presence of electric charges. The accordingly modified Maxwell's equations were then invariant under the duality rotation exchanging, at the same time, the role of electric and magnetic fields, and electric and magnetic sources: that is, the duality rotation (2) augmented by the charge transformation

$$q + ig \quad \to \quad e^{i\theta}\,(q + ig) \tag{3}$$

The following natural step was the extension of this EM duality to the quantum context. This was achieved by Dirac (1931, 1948). His theory of magnetic monopoles represented the first attempt to obtain a consistent quantum generalization of EM duality.[8] In particular, Dirac proved that it was possible for a magnetic charge g to occur in the presence of an electric charge q, without disturbing the consistency of the coupling of electromagnetism to quantum mechanics, if the following quantization condition was satisfied:

$$qg = 2\pi n \qquad\qquad n = 0, \pm 1, \pm 2, \ldots \tag{4}$$

(using the unit system $\hbar = c = 1$).

This is the famous *Dirac quantization condition*, establishing an inverse relation between electric and magnetic charge values.[9] It has many important consequences. First, it provided an explanation of why isolated magnetic poles (magnetic monopoles) had never been observed.[10] Second, it had the following striking implication: the mere existence of a magnetic charge g somewhere in the universe would imply the quantization of electric charge, since any electric charge should occur in integer multiples of the unit $2\pi/g$. For Dirac, this was indeed one of the main reasons for interest in his theory of magnetic poles.[11]

[8] Dirac (1931) treated the case of an electrically charged particle moving in a fixed magnetic monopole field. Dirac (1948) is a more general analysis of the relativistic classical and quantum dynamics of a system of moving magnetic monopoles and electric charges in interaction.

[9] For more details on the development of this duality idea from the classical to the quantum context, see for example Olive, 1999, pp. 65-68. On the physical meaning of this inverse relation in association with electric-magnetic duality, see Polchinski, this issue, Section 2.5.

[10] The explanation was based on the great difference between the numerical values for the quantum of electric charge and the quantum of magnetic pole. For example, two one-quantum magnetic poles of opposite sign attract one another with a force $(137/2)^2$ times as great as that between two one-quantum electric charges at the same distance: hence the difficulty of separating poles of opposite sign. Dirac's conclusion was that "this explains why electric charges are easily produced and not magnetic poles" (Dirac, 1948, p. 830).

[11] As he wrote in his 1948 paper (p. 817): "The interest of the theory of magnetic poles is that it forms a natural generalization of the usual electrodynamics and *it leads to the quantization of electricity.* [...] The quantization of electricity is one of the most fundamental and striking features of atomic physics, and there seems to be no explanation for it apart from the theory of poles. This provides some grounds for believing in the existence of these poles."

Much of the following developments in the history of EM duality can be seen as progressive generalizations of Dirac's theory. The next significant stage was originated by the extension of the duality to the quantum field theories of particle physics. A crucial development was its generalization to the non-Abelian case represented by the Yang-Mills gauge theories used for describing weak and strong interactions. Montonen and Olive (1977), presenting their celebrated electric-magnetic duality conjecture, was the seminal contribution in this regard. The developments leading to this result and its successive generalizations were deeply interwoven with other contemporary threads in fundamental physics: namely, the development of the theory of solitons in the quantum field theory setting, the development of supersymmetry, and the intense theoretical activity towards a unification of all fundamental physical forces (grand unification theories and string theory, reinterpreted as a theory of all interactions including gravity).

This intriguing history, however interesting, cannot be analyzed in detail here. In what follows, the focus will be confined to those results and concepts that seem of special relevance for discussing the apparent ontological implications of generalized EM duality. In particular: the interpretation of monopoles as solitons, the role of monopoles in gauge theories and the Montonen-Olive conjecture, including its successive generalizations.

2.2 Monopoles, solitons, and the Montonen-Olive duality

After Dirac's pioneering work, we had to wait until the second half of the sixties for the first development of a quantum field theory of electric and magnetic charges: first, a theory of particles with either electric or magnetic charges (Schwinger, 1966); then, also a theory of particles carrying both electric and magnetic charges, named *dyons* by Schwinger (Zwanziger, 1968; Schwinger, 1969). This led, among other things, to the following generalized quantization condition for the charges (q_1, g_1) and (q_2, g_2) of any pair of dyons:

$$q_1 g_2 - q_2 g_1 = 2\pi n \qquad\qquad n = 0, \pm 1, \pm 2, \ldots. \tag{5}$$

This condition, it is worth noting, is invariant under the duality rotation (3) applied to both the dyons (that is, under the generalized duality rotation in the (q, g)-plane), thus respecting the symmetry.[12]

The successive developments in the theory of magnetic monopoles were considerably enhanced by the interplay of apparently different and independent lines of thought, a feature that remarkably contributed to raise the physicists' interest and confidence in the generalizations of EM duality. Here, we provide a brief overview of the main threads leading to the Montonen-Olive conjecture and its successive generalizations. In particular, we focus on: (Section 2.2.1) the relevance of the concept of soliton to particle physics (starting with Skyme's pioneering work and the quantum equivalence of sine-Gordon and massive Thirring theories); (Section 2.2.2) the interpretation of monopoles as solitons, and their

[12] For details, see for example Olive, 1999, p. 67.

appearance in grand unification theories (related to the issue of charge quantization); (Section 2.2.3) the Montonen-Olive conjecture on EM duality in (a specific case of) non-Abelian gauge theory: in short, the conjecture that "there should be two 'dual equivalent' field formulations of the same theory in which electric (Noether) and magnetic (topological) quantum numbers exchange roles" (Montonen and Olive, 1977, p. 117). A few remarks on the generalization of this conjecture, first to the supersymmetric context, then also to string theory, conclude the Section.

2.2.1 Solitons and particles in field theories

Historically, the first explicit example of a dual interchange of 'Noether charges' and 'topological charges' in quantum field theory was provided in the framework of the quantum equivalence of the so-called *sine-Gordon theory* and *massive Thirring model*. As shown by Coleman (1975) and Mandelstam (1975), the sine-Gordon theory (describing a massless scalar field ϕ in one space and one time dimension) is related by a duality to the so-called massive Thirring model (a two dimensional theory of a massive self-coupled fermion). As will be illustrated below, this duality implies, in particular, that there is a precise correspondence between the soliton states of the quantized sine-Gordon theory and the elementary particle states of the dual massive Thirring model. In which sense this means an interchange between topological and Noether charges, will be discussed in more detail in Section 2.2.2.

The idea that a soliton could be interpreted as a quantum particle, and the related idea that a dual correspondence could be established between this sort of particle and the familiar elementary particles of quantum field theory, were first developed in some pioneering works of T. H. R. Skyrme in the late 1950s and early 1960s. In the framework of a particular type of classical non-linear field theory considered for its relevance to the description of strongly interacting particles, Skyrme found and studied soliton solutions of the classical field equation which could be interpreted as particles (the so-called *Skyrmions*). This interpretation could be fully established by investigating the nature of the quantized solutions, and in particular by showing the existence of particle-like states in the quantum version of the theory.

At first sight, particle-like solitons like the Skyrmions were very different from the elementary particles of quantum field theory, i.e. structureless particles arising from the quantization of the wave-like excitations of the fields. To begin with, solitons appear at the classical level: they are solutions of classical non-linear field equations. Morevover, they are endowed with an extended (though finite) structure. First discovered in nineteenth century hydrodynamics in the form of 'solitary water waves',[13] they were named *solitons* by Zabusky and

[13]The story is that this kind of wave was first noticed by a Scottish engineer and naval architect, J. Scott Russell, in 1834, while he was observing the motion of a boat rapidly drawn along a narrow channel by a pair of horses. When the boat suddenly stopped, he made the following observation: the mass of water which the boat had put in motion "accumulated round the prow of the vessel in a state of violent agitation, then suddenly leaving it behind, rolled

Kruskal (1965) to indicate humps of energy propagating and interacting without distortion.[14] In that they were characterized as non-dispersive localized packets of energy, solitons could be seen as a sort of particles, though *extended* and solutions to *classical* non-linear *wave* equations. It was natural then to investigate, in the context of quantum field theory, the relation between these sorts of particles and the familiar elementary particles, themselves localized packets of energy (though structureless). This was achieved by studying the correspondence between the soliton solutions of the classical field theory considered and the extended-particle states of the quantized version of that theory.

As said, a seminal work in this direction was due to Skyrme. The Skyrmions emerged from a model of nuclear interactions where the spin 1/2 nucleons were considered to move in a non-linear, classical pion field (a "mesonic fluid"). The particular non-linear theory for the scalar pion field studied by Skyrme allowed also soliton solutions. Searching for the counterparts of these solutions in the quantized version of the theory, Skyrme arrived at the remarkable conjecture that the nucleons (spin 1/2 fermionic states) could emerge as the soliton states of the purely bosonic field theory. This insight came from studying, for simplicity reasons, a toy model of a two dimensional relativistic scalar field, which was in fact the sine-Gordon model. Skyrme's idea was that, in the complete quantum theory, the soliton solution of the sine-Gordon equation could be interpreted equivalently as a particle in the familiar sense of a quantum excitation of a field. This required the construction of a new field operator (of which the soliton states were quantum excitations), which Skyrme found to be fermionic and related in a non-local way to the original field (Skyrme, 1961).[15] He concluded that the soliton modes of the sine-Gordon equation were fermions, interacting through a four-fermion interaction (that is, an interaction of Thirring-model type).

Skyrme's conclusion was a conjecture. It was rigorously confirmed some years later by Coleman (1975) and Mandelstam (1975): their works proved the dual equivalence of sine-Gordon and massive Thirring models in general terms, by studying directly the quantum theory. Recall that the sine-Gordon model describes a massless scalar field ϕ in $1+1$ space-time dimensions with interaction density proportional to $\cos\beta\phi$ (where β is a real parameter), while the massive Thirring model is the theory of a two-dimensional self-coupled fermion field ψ, with interaction of the form $\frac{1}{2}g\overline{\psi}\gamma^{\mu}\psi\overline{\psi}\gamma_{\mu}\psi$ (where g is the coupling constant). Coleman found that the parameter β of the sine-Gordon equation and the coupling constant g of the Thirring model were related, in the following

forward with great velocity, assuming the form of a large solitary elevation, a rounded, smooth and well-defined heap of water, which continued its course along the channel apparently without change of form or diminution of speed".

[14]More precisely, the name *soliton* was coined, after "solitary" and in analogy with familiar particles such as the photon, electron, and so on, to indicate "solitary-wave pulses .. reappearing virtually unaffected in size or shape", that is, without losing their identity, after (nonlinearly) interacting (Zabusky and Kruskal, 1965, p. 240).

[15]More precisely, the new quantum field was related to the original sine-Gordon field by what was later recognized as a "vertex operator construction". A very detailed analysis of Skyrme's model and how it originated and developed is provided by Makhankov *et. al.* (1993).

way:

$$\frac{\beta^2\hbar}{4\pi} = \frac{1}{1+g\hbar/\pi} \tag{6}$$

By using (6) and suitably adjusting other parameters, Coleman could then demonstrate the quantum equivalence of the sine-Gordon and massive Thirring models. Moreover, it followed that this equivalence was a *weak/strong duality*: the weak coupling of the sine-Gordon fields ($\beta << 1$) corresponds to the strong coupling limit of the massive Thirring model ($g >> 1$). Note that in the quantum sine-Gordon theory the relevant parameter is not β but $\beta^2\hbar$. It follows that the small-$\hbar$ approximation is necessarily also a small-β approximation; or, in other words, that weak coupling (small-β limit) corresponds to the classical field limit (small-$\hbar$ limit).[16]

Coleman (1975) obtained his results by comparing the two perturbation series for the sine-Gordon and massive Thirring cases. The successive step by Mandelstam (1975) was to provide a simpler re-derivation of Coleman's results without using perturbation theory. Mandelstam indeed showed that operators for the creation and annihilation of bare solitons could be constructed "fairly simply" from sine-Gordon operators and that they satisfied the commutation relations and field equations of the massive Thirring model.

The sine-Gordon quantum soliton was thus proved to be a *particle* (the 'fundamental' fermion of the massive Thirring model) in the usual sense that the concept has in particle physics. This means that, in the full quantum theory, particles could appear as solitons or as quantum excitations depending on the way the theory was formulated: their status was equivalent. Quoting Coleman (1975, p. 2096): "Thus, I am led to conjecture a form of duality, or *nuclear democracy in the sense of Chew*,[17] for this two-dimensional theory. A single theory has two equally valid descriptions in terms of Lagrangian field theory: the massive Thirring model and the quantum sine-Gordon equation."

At this point, the natural step was to try to extend these ideas to the more realistic case of a physical space-time of three space and one time dimensions. This proved to be not so straightforward,[18] but it did result very fruitful from the viewpoint of theoretical progress. Different ideas and research threads - electric-magnetic duality, monopoles as topological solitons, charge quantization, unification of gauge theories, quark confinement, early string theory, supersymmetry – all converged in this creative theoretical activity, leading to remarkable advances in fundamental physics.[19] In what follows, we will focus on some seminal developments in the monopole idea in the 1970s, starting with the interpreta-

[16] For a discussion of this point in the general context of quantum field theory, see Polchinski, this issue, Section 1.

[17] The emphasis is mine. On Chew's S- matrix program and the bootstrap idea leading to "nuclear democracy", see Section 3.2.

[18] "Speculation on extending these ideas to four dimensions is left as an excercise for the reader" was the challenging conclusive remark of Coleman in his 1975 paper (p. 2096).

[19] A very effective overview of these different threads and their convergence is provided in Olive (1999). The seminal paper in this respect is Goddard and Olive (1978).

tion of monopoles as topological solitons in the context of spontaneously broken field theories.

2.2.2 Monopoles as topological solitons

Solitons can be of different types and origins. In particular, they can be topological, as are the solitons of interest in relation to the physical dualities we are discussing. This means that, in contrast with the familiar particles of quantum field theory, the 'soliton particles' considered have a topological structure. Such a structure is represented by a conserved quantity, the 'topological charge', which is related to their behavior at spatial infinity and to which they owe their stability. In the quantized theory, it becomes a conserved quantum number characterizing the soliton state.

These topological quantum numbers are very different in origin from the familiar Noether charges associated with manifest symmetries of the theory's Lagrangian. In fact, the topological charges characterizing soliton solutions arise as boundary conditions, their conservation is due to the condition that the energy be finite and it holds independently of the equations of motion. Moreover, in many cases, they are closely related to what is known as "spontaneous symmetry breaking".

A simple example is provided by the way in which solitons emerge in the framework of the one spatial-dimension sine-Gordon theory seen in Section 2.2.1, in connection with the vacuum degeneracy of the theory and the resulting spontaneous symmetry breaking of its discrete symmetry.

The sine-Gordon equation

$$\frac{\partial^2\phi}{\partial t^2} - \frac{\partial^2\phi}{\partial x^2} = -\alpha \sin\beta\phi. \tag{7}$$

has an infinite number of vacuum solutions $\phi_n = \frac{2\pi n}{\beta}$ $(n = 0, \pm 1, \pm 2, \dots)$, all with the same minimum energy (equal to zero).[20] The field equation (as well as the Lagrangian) is invariant under the transformations $\phi \to \phi' = \phi + \frac{2\pi n}{\beta}$, but this discrete symmetry is not respected by the vacuum solutions. Hence, after one of these vacua is chosen, the symmetry is broken and this is called a "spontaneous symmetry breaking".[21]

[20] The sine-Gordon equation may be derived from the Lagrangian density $\mathcal{L} = \frac{1}{2}\left\{\left(\frac{\partial\phi}{\partial t}\right)^2 - \left(\frac{\partial\phi}{\partial x}\right)^2\right\} - V(\phi)$, where $V(\phi) = \frac{\alpha}{\beta}(1 - \cos\beta\phi)$. The energy vanishes at the absolute minima of $V(\phi)$, which are $\phi_n = \frac{2\pi n}{\beta}$. Note that by expanding the Lagrangian density in powers of a coupling constant $\lambda = \alpha\beta^2$, the theory can be seen as describing a scalar field of 'mass' $m = \sqrt{\alpha}\beta$ and a non-polynomial self-interaction (for $\lambda \to 0$, we recover the familiar free Klein-Gordon equation, whence, by the way, the name "sine-Gordon equation"). For the soliton solutions, however, we must consider the full Lagrangian and solutions which are non-perturbative in λ. For details, see for example Rajaraman, 1982, Chapter 2.

[21] The symmetry, however, is recovered in the ensemble of all vacuum solutions or 'ground states': any two degenerate ground states are transformed into one another under the action of the discrete symmetry group. In this sense the symmetry is said to be 'hidden'.

In correspondence to this situation of spontaneous symmetry breaking, the quantized theory has two types of particles: the particles in the familiar sense of the quanta of the fluctuations of the field ϕ about any of the vacua; and the particles corresponding to the soliton solutions. In the case considered (a single scalar field in one spatial-dimension), the soliton solutions are configurations of the field ϕ that connect two successive vacua (corresponding, respectively, to the solution's values at spatial infinity, i.e. at $x = \pm\infty$).[22] Accordingly, the space of all finite-energy configurations can be divided into sectors topologically unconnected, characterized by two integer indices (corresponding to the values of the field at $x = \pm\infty$), the difference of which

$$\phi(\infty) - \phi(-\infty) = \frac{2\pi N}{\beta} \tag{8}$$

gives an integer N. This number is called the *topological charge*, and it can be regarded as a quantum number characterizing the state of the system.[23]

The connection among the existence of topological solitons, the topology of the space boundary and the theory's spontaneous symmetry breaking,[24] above described in the case of the one spatial-dimension sine-Gordon theory, is in fact a general feature, holding also for higher spatial-dimension cases. In generalizing it to the realistic case of three space dimensions, however, a general result known as Derrick's theorem implies a further condition: in order to have solitons, systems with more than just scalar fields must be considered.[25] It was found, in particular, that topological soliton solutions could be obtained in the case of gauge theories with scalar fields: that is, when gauge fields (spin-1 fields) were introduced in addition to the scalar fields (the theory's global continuous symmetry being replaced by a local symmetry).[26]

[22]In order that the energy be finite, the field must tend to a minimum of the potential at every point on spatial infinity (at every instant t).

[23]In the case considered, the solution going from $\phi_0 = 0$ to $\phi_1 = \frac{2\pi}{\beta}$ (or, equivalently, from $\frac{2\pi}{\beta}$ to $\frac{4\pi}{\beta}$, etc.), corresponds to $N = 1$ and represents the soliton. The other solution, corresponding to $N = -1$, represents the antisoliton. (These solutions are usually called 'kink' ($N = 1$) and 'antikink' ($N = -1$) in the physics literature; see however Rajamaran [1982, pp. 21-22 and 38] for a finer distinction between a kink solitary wave and a genuine soliton.) Solutions characterized by different topological charges cannot be transformed into each other by any continuous deformation: they belong to different topological sectors, and a field configuration from one sector cannot be deformed continuously into another of a different sector without violating the requirement of finiteness of the energy (the change would require an infinite amount of energy). In particular, a field configuration from any one sector stays within it as time evolves, since time evolution is a continuous transformation. For more details, see Rajamaran, 1982, Sections 2.4 and 2.5.

[24]That is, the fact that the existence of this type of solitons requires the theory to have degenerate vacua and a nontrivial map from the spatial boundary to the set of these vacua.

[25]Derrick 1964's result shows that, with scalar fields alone, there exist no stable time-independent solutions of finite energy of nonlinear equations in three spatial dimensions. This implies that if we want soliton solutions in the realistic $(3 + 1)$ dimensional case, we must consider systems with higher-spin fields in addition to the scalar fields.

[26]A seminal paper illustrating the generalization of the results obtained in one space and one time dimension to a physical space-time of three space and one time dimensions is Goddard and Olive (1978). See, in particular, Sections 3.4 and 7.1 for an illustration of the role of

Historically, this result was discussed and used in the case of four space-time dimensions by 't Hooft (1974) and Polyakov (1974) in searching for magnetic monopoles. They independently showed that a certain spontaneously broken non-Abelian gauge theory in four dimensional space-time, known as the *Georgi-Glashow model*, possessed topological soliton solutions and these solitons could be physically interpreted as monopoles (with the topological charge related to the magnetic charge).[27] Note that, in contrast with the Dirac monopole requiring a point magnetic source to be put in by hand in the theory, the so-called 't Hooft-Polyakov monopole was naturally contained in the theory, in that it arises as a soliton solution of the field equation. This means, in particular, that its properties (including its mass) are calculable from the theory.[28]

The Georgi-Glashow model was proposed in 1972 as a unified gauge theory of weak and electromagnetic interactions which, in contrast to other unified models such as Weinberg's, didn't imply the appearance of neutral lepton currents (not yet observed at the time). It was based on the symmetry gauge group $SO(3)$, spontaneously broken to a subgroup $U(1)$ (identified with the electromagnetic gauge group) by the so-called Higgs mechanism.[29] For finding magnetic monopoles, the model had the desired feature that the gauge group $SO(3)$ is a compact group, a condition required for the possibility of constructing monopole solutions when the symmetry is appropriately spontaneously broken.[30] What is of special relevance, here, is that this is closely connected with the condition for the quantization of electric charge, provided for by the structure of the unified theory's gauge group. In fact, the quantization of electric charge and the mathematical concept of the compactness of the gauge group

the dimension of space-time in determining which theories are capable of supporting stable finite-energy time-independent solutions.

[27]The work by 't Hooft and Polyakov was inspired by 1973 work of Nielsen and Olesen in the framework of early string theory. On the grounds that quantized magnetic flux lines in a superconductor behave very much like the string, it was suggested that the string could be interpreted as a soliton built of magnetic flux lines, arising in a non-Abelian spontaneously broken gauge theory. This string, being built of magnetic lines of force, could only have magnetic charges as end points, which would be confined on the basis of a mechanism analogous to the Meissner effect in superconductivity. This was the basis also for suggesting a way of addressing the problem of the confinement of quarks, by interpreting them as monopoles.

[28]On this and other significant differences between Dirac monopoles and 't Hooft-Polyakov monopoles, see for example Goddard and Olive, 1978, Section 4.5.

[29]In standard terminology, the Georgi-Glashow model is a unified Yang-Mills–Higgs theory with gauge group $SO(3)$, or, equivalently, with gauge group $SU(2)$. This non-Abelian group is associated with a triplet of gauge fields, known as the Yang-Mills fields (from the famous work on non-Abelian gauge field theory by Yang and Mills in 1954), self-coupled to form a non-linear system. In the model proposed by Georgi and Glashow for a unified theory of weak and electromagnetic interactions, the Yang-Mills fields are coupled to a triplet of scalar fields (Higgs fields) leading to the Higgs mechanism (in short, the mechanism by means of which a scalar field, the Higgs field, fails to vanish in the vacuum, with the consequence that mass is produced without spoiling any of the advantages of gauge symmetry, such as renormalizability).

[30]To put it a bit more precisely by quoting 't Hooft (1974, p. 276), "in all those gauge theories in which the electromagnetic group U(1) is taken to be a subgroup of a larger group with a compact covering group, like SU(2) or SU(3), genuine magnetic monopoles can be created as regular solutions of the field equations."

are intimately related, as highlighted in a seminal paper by Yang (1970).[31]

A new perspective on Dirac's relation between electric charge quantization and the existence of magnetic monopoles was thus provided.[32]

The Georgi-Glashow model was soon ruled out by experimental results (weak neutral currents were discovered in 1973). The experimentally correct model for a unified theory of weak and electromagnetic interactions was found to be the *Weinberg-Salam model*, based on the gauge group $SU(2)$ x $U(1)$. But this group didn't satisfy the condition of having a compact covering group and therefore the model couldn't allow for monopoles to appear.[33] This possibility, however, was recovered in what are known as *grand unification theories*: that is, theories of unified strong and electroweak interactions, based on compact gauge groups – such as, for example, $SU(5)$ – spontaneously broken to the gauge group $SU(3)$ x $SU(2)$ x $U(1)$ of the Standard Model for elementary particles.[34]

Thus, the ideas of charge quantization, monopoles as topological solitons, grand unification theory and spontaneous symmetry breaking were found to be intimately connected.[35] This considerably enhanced the interest in EM duality and provided, at the same time, the conceptual framework in which investigating a four space-time dimensional analogue of the duality between Noether and topological charges found in the two space-time dimensional case of the sine-Gordon/Thirring duality. The seminal work in this respect is Montonen and Olive (1977), to which we now turn.

[31] A Lie group (as gauge groups are) is said to be compact if its parameters vary over a closed interval. Quoting Yang (1970, p. 2360): "For Lie groups, compactness is a property of the global structure of the group, which has a determining influence on the nature of the representations of the group. It is, in fact, through its influence on the representations that the compactness of the gauge group has a bearing on the quantization of charges". In short, if the different charge e_i's of different fields ψ_i's are not commensurate with each other (electric charge is not quantized), the space-time-independent gauge transformation on charged fields $\psi_i \rightarrow \psi_i' = \psi_i exp(ie_i\alpha)$ is different for all real values of α, and the gauge group must be defined so as to include all real values α. Hence, the group is not compact. On the other hand, if all different charges are integral multiples of a universal unit of charge e, it is easy to show that the gauge group is compact (the phase factors may all be parameterized by a finite range of values of α, e.g., between 0 and 2π).

[32] This point is emphasized by many authors in the literature on magnetic monopoles. For example Olive (2001, p. 5) makes the following comment: "The first conclusion is that Dirac's explanation of charge quantisation is triumphantly vindicated. At first sight it seemed as if the idea of unification provided an alternative explanation, avoiding monopoles, but this was illusory as magnetic monopoles were indeed lurking hidden in the theory, disguised as solitons".

[33] See fn. 30.

[34] For more details on this connection between charge quantization and the existence of monopoles in unified gauge theories with compact gauge groups, see for example Weinberg, 1996, Section 23.3.

[35] In this respect, Goldhaber (1983, p. 9) speaks about a "golden triangle": "we have found that among the trio of concepts, charge quantization, spontaneously broken non-Abelian gauge theories, and magnetic monopoles, each suggests the other two. While some of the links appear more heuristic than others, all of the connections are so close that one is tempted to view the trio as three aspects of a single phenomenon".

2.2.3 The Montonen-Olive conjecture

As we have seen, in the mid-seventies it was known that monopole solutions could exist in a unified Yang-Mills-Higgs theory if the gauge group was spontaneously broken to a residual subgroup containing an explicit $U(1)$ factor (identified with the electromagnetic gauge group). At about the same time, Julia and Zee succeeded in extending this result by showing that a $SU(2)$ gauge theory with a Higgs triplet admitted also solutions with both magnetic and electric charges ("dyons", according to the terminology introduced by Schwinger in 1969, as seen in Section 2.2). Still in 1975, an exact analytic solution for both the 't Hooft-Polyakov monopole and the Julia-Zee dyon was found by Prasad and Sommerfield, but in the special case where the Higgs self-coupling is zero, i.e. where the Higgs field is massless. Soon after, the existence and stability of such kind of solutions was thoroughly clarified by Bogomol'nyii, and it became usual to indicate them as "Bogomol'nyi-Prasad-Sommerfield (BPS) solutions".[36]

In view of these results and in analogy with the case of sine-Gordon/Thirring duality, it was natural to investigate the possibility of a dual version of the Yang-Mills theory, with quantum field operators creating magnetic monopoles rather than the original gauge particles,[37] just as the Thirring model provided the quantum field theory of the sine-Gordon solitons (now seen as created by massive Thirring quantum fields).

This was the issue addressed by Montonen and Olive in their 1977 seminal paper. More precisely, their declared aim was to present evidence for the following conjecture: "when quantized, the magnetic monopole solutions constructed by 't Hooft and Polyakov, as modified by Prasad, Sommerfield and Bogomol'nyi, form a gauge triplet with the photon, corresponding to a Lagrangian similar to the original Georgi-Glashow one, but with magnetic replacing electric charge" (p. 117).[38]

Thus, a perfect duality in the treatment of electric (Noether) and magnetic (topological) charges was suggested: when proved, this meant a successful generalization of Dirac's electric-magnetic duality to its quantum field theoretical form. Note that also in this case the duality is a self-duality, that is, the dual description (the description with the roles of electricity and magnetism reversed) is formally the same as the original one.[39] Indeed, the dual quantum field the-

[36] In his 1976 paper "Stability of Classical Solutions", Bogomolnyi showed that the field equations for topological defects can be reduced to first-order provided the coupling constant of its potential takes certain values, namely in what has become known as the Bogomolnyi-Prasad-Somerfield (BPS) limit.

[37] That is, the electrically charged particles, now occurring as solitons in the dual description.

[38] The reason for choosing the Georgi-Glashow model in the special BPS limit for substantiate their conjecture was that this model provided the simplest Lagrangian with monopoles occurring as solitons. A very clear reconstruction of the physical ideas and results motivating the Montonen-Olive paper, including the preceding conjecture for the structure of the dual gauge group by Goddard, Nuyts and Olive (considering a more general context with a less restrictive exact symmetry group), is provided in Olive, 1999. An almost contemporary and very detailed illustration of all these developments is Goddard and Olive, 1978.

[39] In this sense, there is a difference with the case of the sine-Gordon-Thirring duality, where the Lagrangians of the two dual model are different. See, for a comment on this point,

ory of the monopole solitons considered by Montonen and Olive was based upon exactly the same Lagrangian as the original Georgi-Glashow one (though with the coupling strength inversely related to the original 'electric' coupling). In the original description, "the heavy gauge particles carry the $U(1)$ electric charge, which is a Noether charge, while the monopole solitons carry magnetic charge, which is a topological charge", to use their own words. In the equivalent dual field theory, "the fundamental monopoles fields play the role of the heavy gauge particles, with the magnetic charge being now the Noether charge" (Montonen and Olive, p. 117).

Although Montonen and Olive were unable to prove their conjecture, they could present some important evidence in its favour: namely, the fact that the classical properties of the monopole in the dual field theory were precisely related to the corresponding properties of the heavy gauge particle, as predicted.[40] The real problem, as Montonen and Olive pointed out, was with calculating the quantum properties of the monopole. In particular, the following issues had to be addressed: how could the magnetic monopole soliton possess unit spin (in order to be seen as a heavy gauge particle), given the spherical symmetry of the classical solution? How to calculate the quantum corrections to the classical properties in order to avoid that they could vitiate the mass formula? And how to include the dyon states and how this would affect the conjectured duality?[41]

In fact, all these problems could find a consistent solution by introducing supersymmetry into the picture. The first step in this direction was due to Witten and Olive (1978): they showed that in a supersymmetric extension of the Georgi-Glashow model – the $N = 2$ supersymmetric version – there were no quantum corrections to the classical mass spectrum and the calculation of the exact masses could be obtained.[42] The problem related to the monopole's unit spin remained for this $N = 2$ case, but soon after a solution to it was obtained for the case of $N = 4$ supersymmetry: Osborn (1979) was able to show that when spontaneous symmetry breaking is imposed in the $N = 4$ supersymmetric

Montonen and Olive, 1977, p. 117.

[40] In particular, the properties of the particle spectrum (the particle masses, charges and spins) in the BPS limit of the $SU(2)$ theory supported the idea that the charged gauge particles might be solitons in a dual version of the theory. By applying the duality transformation reversing the role of 'electric' and 'magnetic' components (which could be seen as described by $\pi/2$ rotations in a complex plane of charges $q + ig$), the entries for the gauge particles and the monopoles were interchanged, but the overall spectrum of masses and charges remained the same. Further evidence was supplied by the calculation of the long-range forces between monopoles, which turned out to be precisely analogous to the forces between the charged gauge particles. For details, see Montonen and Olive, 1977, p. 118, and Goddard and Oliver, 1978, p. 1429.

[41] See Montonen and Olive, 1977, p. 119-120. A detailed discussion of these points is offered in Olive, 1999, pp. 76 ff.

[42] $N = 2$ supersymmetry means that the theory has two types of supersymmetry transformations (two copies of the basic algebra). Due to supersymmetry, the BPS bound turned out not to be violated by the quantum corrections (because of the mutual cancellation of the bosonic and fermionic loop diagrams). Moreover, most of the remarkable results associated with the BPS limit could be understood, afterwards, as consequences of supersymmetry. For details on the reasons why supersymmetry helps answer this and the other questions raised by Montonen-Olive conjecture, see in particular Olive, 1999, pp. 82 ff.

gauge theory, the spins of the topological monopole states are identical to those of the massive gauge particles.

Although in the immediately following years a number of important results were obtained,[43] the real interest in the Montonen-Olive conjecture and the properties of monopoles in supersymmetric Yang-Mills theory exploded only in the early 1990s. This was due to fascinating theoretical achievements and significant progress in understanding non-perturbative dynamics which were reached at that time, also thanks to the virtuous interplay of developments in supersymmetric field theory and string theory – a major research field after the so-called first superstring revolution in 1984.[44] In the years 1992-94 in particular, seminal works by Seiberg, Witten, Sen and Schwarz revitalized the interest in the supersymmetric generalization of Montonen-Olive duality and further motivated its extension to the string setting. Two especially influential papers were Seiberg and Witten (1994) and Sen (1994). On the one side, Seiberg and Witten (1994) showed that also for the case of $N = 2$ supersymmetric $SU(2)$ Yang-Mills theory a version of Montonen-Olive duality could hold. This duality, however, exchanged monopoles with quarks (fractional charges), in contrast to the case of $N = 4$ theory, where the duality exchanged monopoles with gauge particles (integer charges). This version of Montonen-Olive duality, mapping the quarks ('electric' degrees of freedom) which are strongly coupled to weakly coupled monopoles ('magnetic' degrees of freedom), could thus be used to obtain a weakly coupled effective description of QCD.[45] On the other side, Sen (1994) was able to present several evidences, including solving the dyon problem, for the generalized Montonen-Olive duality for the case of $N = 4$ supersymmetric Yang-Mills theory. More precisely, the theory he considered was the ten-dimensional heterotic string theory compactified to four dimensions on a six dimensional torus (the toroidal compactification of the heterotic string theory possesses a local $N = 4$ supersymmetry in four dimensions) and he could obtain strong (theoretical) evidence for the strong-weak coupling duality of this string theory in four dimensions by studying its low energy effective field theory.[46]

3 Discussing particle democracy

In the light of the preceding analysis of the first developments of weak/strong duality, we now turn to the philosophical discussion originated by the apparent

[43] See Olive, 1999, pp. 84-89.

[44] For the developments of string theory between its birth in the late 1960s as a theory of strong forces and its outburst in the early 1980s as a unified theory of all interactions including gravity, see Cappelli, Castellani, Colomo and Di Vecchia, 2012.

[45] In more detail, Seiberg and Witten used the duality to compute exact low-energy effective Lagrangians and BPS soliton spectra of $N = 2$ supersymmetric QCD. Moreover, with a suitable perturbation, quark confinement could be described by monopole condensation, thus "giving for the first time a real relativistic field theory model in which confinement of charge was explained in this long-suspected fashion" (p. 25).

[46] For details and the relevance of the results obtained by Sen at that time, see Olive (1999), Section 6. The following developments of weak/strong duality in the framework of string theory are described by Polchinski in his contribution to this issue.

ontological implications of the dual correspondence between 'elementary' and 'composite' particles. The specific aim of this concluding section is to show how the historical insight, beside its intrinsic interest, is also helpful (not to say necessary) for clarifying the philosophical issue at stake. This will be done in three steps: we start with some reflections on the 'elementary'/'composite' divide (Section 3.1), then consider an historical analogy case (Section 3.2) and, finally, draw some conclusions on the 'democracy' issue (Section 3.3).

3.1 Elementary/composite

The usual way of presenting what appears to be an intriguing ontological issue raised by weak/strong dualities is in terms of the interchange (associated with this duality type) between what is elementary and what is composite. This corresponds to how the dual mapping between particles is typically described in plain terms by the experts. Sen (1999), for example, uses the following language: "we see that under duality, the elementary particles of the first theory gets mapped to the composite particles of the second theory and vice versa. In other words, the same particle may be considered elementary in one description and composite in the other description. Thus, in theories possessing dual descriptions, the question of whether a given particle is elementary or composite has different answers depending on which description we use for the theory" (p. 1637).

While it is clear what is meant by "elementary particles", here – i.e., those particles that are associated with the quantum excitations of the fields –, the terminology "composite particle" can indicate different things, depending on the field theory and the duality considered. In general, as seen in Section 2, the dual correspondence associated with weak/strong duality is between elementary particles and solitons (Skyrmions and magnetic monopoles, in the cases considered above). Now solitons, though extended, are not necessarily composed of other particles (in the sense in which neutrons or protons are composed of quarks, say). They can be bound states, but also just 'extended' topological solutions (such as the magnetic monopoles discussed in Section 2.2.3).

The impression is that the elementary/composite terminology can be misleading, especially when discussing philosophical implications of weak/strong duality. In fact, speaking in terms of 'elementary vs composite' is easily suggestive of a connection with such issues as fundamentality and reductionism (see Section 1). Such a suggestion is far less natural, however, when the dual correspondence is presented in terms of an interchange between Noether and topological charges, for example.. As we have seen in Section 2.2.3, the (generalized) electric-magnetic duality implies that what appear as an 'electrically' charged elementary particle (a Noether charge) in the original formulation of the theory (the 'electric' formulation), will appear as a 'magnetically' charged soliton in the alternative (dual or 'magnetic') formulation of the same theory.[47] This is very different from a 'composition' correspondence in the ontological

[47] We ignore the dyon solutions here, for simplicity sake.

sense. Both kinds of particles, either 'electric' or 'magnetic' charges, are present on a par in the complete quantum theory: they are all parts of its ontology.

Of course, electric-magnetic duality is a special case of weak/strong duality, however historically significant. Many different examples of a dual correspondence can be found in the extension of weak/strong duality to string theory (see Polchinski, this issue). When focusing on the 'particles' of quantum field theories, however, this is undoubtedly the relevant kind of duality to discuss. But before turning to the very issue of particle democracy, let us make another historical detour and consider an instructive analogy case.

3.2 An historical analogy: DHS duality and nuclear democracy

In his 1975 work on the dual equivalence between the two-dimensional sine-Gordon and massive Thirring theories, Coleman compares this form of duality with "nuclear democracy in the sense of Chew" (as mentioned in Section 2.2.1). This surely inspired successive views about the relationships between duality and particle democracy, especially since the 1990s by Sen and others.[48] Before putting this particle-democracy view under the lens, it can be useful to go back to Coleman's own source of inspiration. In fact, there is a historical precedent for connecting duality with the idea of particle democracy: the case of the so-called *Dolen-Horn-Schmid* (DHS) *duality*, introduced in the context of the S-matrix approach to describe the physics of strong interactions in the 1960s. DHS duality was the first duality type of string theory: also known as the "dual bootstrap", it was at the core of the so-called *dual resonance models* from which string theory was born between the late sixties and the early seventies.[49]

The S-matrix approach was motivated by the difficulties arising in a field theoretic description of strong interactions. Inspired by earlier work of Heisenberg, it aimed at determining the relevant observable physical quantities – the scattering amplitudes (the elements of the S-matrix) – on the grounds of general principles such as unitarity, analyticity and symmetry, together with a minimal number of additional assumptions. One of these was the "duality principle" proposed by Dolen, Horn and Schmid in 1967. To find a formula for a scattering amplitude that realized the DHS duality in a simple and clear way was the great result obtained by Veneziano in 1968, giving rise to the very intense model-building activity known as the *Dual Theory* and from which string theory was born.

The DHS duality principle was the assumption, suggested by experimental data, that the contributions from resonance intermediate states and from particle exchange each formed a complete representation of the scattering pro-

[48]In the context of string theory, the concept of 'particle' democracy is generalized to strings and 'branes' ('brane democracy').

[49]Detailed accounts of this duality and its meaning in the context of the first developments of string theory are provided in Cappelli, Castellani, Colomo and Di Vecchia, 2012, Parts I and II. The S-matrix program pursued by Chew and his collaborators has been thoroughly investigated from a historical and philosophical point of view in Cushing, 1990.

cess; therefore, they should not be added to one another in order to obtain the total amplitude. In terms of the Mandelstam's variables s and t and in the framework of the so-called Regge theory, to enter in some more detail, what the duality principle established was the following relation between a low-energy and a high-energy description of the hadronic scattering amplitude $A(s,t)$: the low-energy description in terms of direct-channel (s-channel) resonance poles and the high-energy description in terms of the exchange of Regge poles in the crossed-channel (t-channel) could each be obtained from the other by analytic continuation.[50]

The duality principle was thus seen to represent an effective implementation of two inter-related ideas, defended in particular by Geoffrey Chew and his school in the 1960s: the idea of *nuclear democracy*, according to which no hadron is more fundamental than the others; and the *bootstrap idea*, that is the idea of a self-consistent hadronic structure in which the entire ensemble of hadrons provided the forces (by hadron exchange) making their own existence (as intermediate states) possible.[51] The real meaning of this duality, however, emerged clearly only with the string interpretation of the dual models: in fact, the duality principle was soon proved to be a direct consequence of the conformal symmetry of the string amplitudes.[52]

Summing up, there are undoubtedly some similarities between this duality case and the weak/strong dualities discussed in the previous Section: first of all, the interchange of low-energy and high-energy descriptions and the connection with conformal symmetry.[53] However, the contexts are so different that the comparison can hardly be taken to signify more than a suggestive analogy.[54] The differences also regard the dual correspondence between particles in the two cases. To this point, and more specifically to the meaning of the particle democracy associated with weak/strong duality are devoted the following concluding remarks.

3.3 Concluding remarks

In the framework of the dual theory of strong interactions, the idea of nuclear democracy had a clear ontological flavor: all hadrons were seen as composed of the other hadrons (bootstrap), and therefore all hadrons were considered as equal from the point of view of fundamentality. In that context, 'composed of' was clearly intended in the sense of ontological composition.

[50]See, for details, Cappelli *et al.*, Part II.

[51]On the bootstrap idea in Chew's program, see in particular Cushing, 1990, Chapter 6.

[52]On this point, and, more generally, on the group of conformal (or angle-preserving) transformations in the plane, see Cappelli *et al.*, Chapter 10, Sections 10.2.1 and 10.3.

[53]The relation between weak/strong duality and conformal symmetry is a very important one and the topic would deserve a detailed analysis by itself. For what regards the relevance of this relation in the case of the Montonen-Olive duality in $N = 4$ supersymmetric Yang-Mills theory (displaying conformal symmetry), see in particular Olive, 1999, pp. 90 ff.

[54]Without counting the fact that hadrons are not elementary particles, but this is not the relevant point, here: the point is whether duality is associated with a form of democracy (and in which sense) among the particles of the theory.

This is not the case, however, for the Montonen-Olive duality and its generalization. As we have seen in Section 2.2.3, all particles – 'electric' charges, 'magnetic' charges, dyons – exist on an equal footing in the complete quantum theory. In this sense, they all are equally 'fundamental' from an ontological point of view. What the duality specifically implies, here, regards not mutual composition of the particles as rather their different modes of appearance when considering the different classical limits of the quantum theory, i.e. the dual perspectives. As seen, the particles interchangeably play the role of Noether charges or topological charges depending on the perspective under which the theory is considered. The *democracy* associated with the duality results from this kind of modality in their appearance: we could say that it is a "representational" or "functional" democracy, rather than an ontological one.[55]

What is, then, the philosophical lesson to be drawn from this sort of particle democracy? Apparently, the following: the different characterizations of the particles of the theory – as elementary particles or as solitons, in the case considered – should not be taken too literally. As seen in Section 2, these characterizations strictly depend on the particular formulations of the theory in certain regions of its parameter space. And this – although supporting a deflating conclusion as far as ontological speculations are concerned – is a very interesting and fruitful result, as showed by the history of generalized electric-magnetic duality and the successive developments of weak/strong duality in the framework of supersymmetric field and string theory.[56]

Acknowledgements

Many thanks to the participants in the 2014 Florence workshop on dualities for helpful discussions and suggestions. I am very grateful to Eric Curiel, Richard Dawid, Dean Rickles, Nic Teh and Johanna Wolf for precious support and feedback. A special thank to Jeremy Butterfield and Sebastian de Haro, and to the reviewers.

4 References

Cappelli, A., Castellani, E., Colomo, F. and Di Vecchia, P. (2012). *The Birth of String Theory.* Cambridge: Cambridge University Press.

Coleman, S. (1975). Quantum sine-Gordon equation as the massive Thirring model. *Phys. Rev., D11 (8)*, 2088-2097.

Cushing, J. T. (1990). *Theory Construction and Selection in Modern Physics: The S-Matrix.* Cambridge: Cambridge University Press.

Dawid, R. (2007). Scientific Realism in the Age of String Theory. *Physics and Philosophy, 11*, 1863-7388.

[55]I am grateful to Erik Curiel for suggesting the "representional democracy" terminology.
[56]See Polchinski, this issue, for a clear illustration of these developments.

Dirac, P. A. M. (1931). Quantised Singularities in the Electromagnetic Field. *Proc. Roy. Soc. Lond., A 133*, 60-72.

Dirac, P. A. M. (1948). The Theory of Magnetic Poles. *Phys. Rev., 74*, 817-830.

Font, A., Ibanez, L., Lüst, D., and Quevedo, F. (1990). Strong-weak coupling duality and non-perturbative effects in string theory. *Phys. Lett., B 249*, 35-43.

Georgi, H., and Glashow, S. L. Unified Weak and Electromagnetic Interactions without Neutral Currents. *Phys. Rev. Lett., 28*, 1494-1497.

Goddard, P., and Olive, D. (1978). Magnetic monopoles in gauge field theories. *Rep. Prog. Phys., 41*, 1357-1437.

Goldhaber, A. S. (1983). Monopoles and gauge theories. In: R. A. Carrigan Jr., and W. P. Trower (Eds.), *Magnetic Monopoles* (pp. 1-14). New York and London: Plenum Press.

Goldhaber, A. S. (2000). Hairs on the Unicorn: Fine Structure of Monopoles and Other Solitons. In: R. MacKenzie, M.B. Paranjape, and WJ. Zakrzewski (Eds.), *Solitons. Properties, Dynamics, Interactions, Applications* (pp. 45-56). Springer.

't Hooft, G. (1974). Magnetic Monopoles in Unified Gauge Theorie. *Nucl. Phys., B 79*, 276-284.

Makhankov, V. G., Rybakov, Y. P. , and Sanyuk, V. I. (1993). *The Skyrme Model. Fundamentals, Methods, Applications.* Berlin-Heidelberg: Springer.

Mandelstam, S. (1975). Soliton operators for the quantized sine-Gordon equation. *Phys. Rev., D 11*, 3026-3030.

Matsubara, K. (2013). Realism, underdetermination and string theory dualities. *Synthese, 190 (3)*, 471-489.

Montonen, C., and Olive, D.I. (1977). Magnetic monopoles as gauge particles? *Phys. Lett., B 72*, 117-120.

Olive, D. I. (1999). Introduction to Duality. In: D. I. Olive and P. West, *Duality and supersymmetric theories* (pp. 62-94). Cambridge: Cambridge University Press.

Olive. D. I., and West, P. (1999). Particle Physics and Fundamental Theory: Introduction and Guide to *Duality and Supersymmetric Theories.* In: D. I. Olive and P. West, *Duality and supersymmetric theories* (pp. 1-20). Cambridge:

Cambridge University Press.

Olive, D. I. (2001). The Quantisation of Charges. Lecture at the Symposium "One hundred years of h", Pavia, 2000. Available at arXiv: hep-th/0104063.

Osborn, H. (1979). Topological charges for $N = 4$ supersymmetric gauge theories and monopoles of spin 1. *Phys. Lett., B 83*, 321-326.

Polyakov, A. M. (1974). Particle spectrum in quantum field theory. *JETP Lett 20*, 194-195.

Rajaraman, R. (1982). *Solitons and Instantons. An Introduction to Solitons and Instantons in Quantum Field Theory.* Amsterdam: North-Holland.

Rickles, D. (2011). A philosopher looks at string dualities. *Studies in History and Philosophy of ModernPhysics, 42(1)*, 54-67.

Seiberg, N. and Witten, E. (1994). Electric-magnetic duality, monopole condensation, and confinement in $N = 2$ supersymmetric Yang-Mills theory. *Nucl. Phys. B 426*, 19-52.

Sen, A. (1994). Strong-Weak Coupling Duality in Four Dimensional String Theory. *Int. J. Mod. Phys. A9*, 3707-3750.

Sen, A. (1999). Duality symmetries in string theory. *Current Science, 77 (12)*,1635-1645. Available at www.hri.res.in/sen.

Sen, A. (2001). Recent Developments in Superstring Theory. *Nucl. Phys. Proc. Suppl. 94*, 35-48. Available at arXiv:hep-lat/0011073.
Skyrme, T.H.R.' (1961). Particle States of Quantized Meson Fields. *Proc. Roy. Soc. London, A 262*, 237-245.

Susskind, L. (2013).String theory. *Foundations of physics, 43(1)*, 174181.

Schwinger J. (1966). Magnetic Charge and Quantum Field Theory. *Phys. Rev., 144 (4)*, 1087-1093.

Schwinger J. (1969). A magnetic model of matter. *Science, 165*, 757-761.

Weinberg, S. (1996). *The Quantum Theory of Fields*, Vol. II. Cambridge: Cambridge UniversityPress.

Witten, E., and Olive, D. (1978). Supersymmetry algebras that include topological charges. *Phys. Lett., B 78*, 97-101.

Yang, C. N. (1970). Charge Quantization, Compactness of the Gauge Group, and Flux Quantization. *Phys. Rev. D, 1(8)*, 2360.

Zabusky, N. J., and Kruskal, M. D. (1965). Interaction of "solitons" in a collisionless plasma and the recurrence of initial states. *Phys. Rev. Lett., 15 (6)*, 240-243.

Zwanziger, D. (1968). Quantum Field Theory of Particles with Both Electric and Magnetic Charges. *Phys. Rev., 176 (5)*, 1489-1495.