Duality, Fundamentality, and Emergence

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Abstract

We argue that dualities offer new possibilities for relating fundamentality, levels, and emergence. Namely, dualities often relate two theories whose hierarchies of levels are inverted relative to each other, and so allow for new fundamentality relations, as well as for epistemic emergence. We find that the direction of emergence typically found in these cases is *opposite* to the direction of emergence followed in the standard accounts. Namely, the standard emergence direction is that of decreasing fundamentality: there is emergence of less fundamental, high-level entities, out of more fundamental, low-level entities. But in cases of duality, a more fundamental entity can emerge out of a less fundamental one. This possibility can be traced back to the existence of different classical limits in quantum field theories and string theories.

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Contents

1. Introduction

In this paper we discuss the bearing of duality, which is a significant phenomenon in theoretical physics, on the notions of fundamentality and emergence. How to best construe fundamentality is an important question both in physics and in philosophical reflections about the nature of reality. The approaches of course differ, depending on whether one tries to construe fundamentality as an epistemic or as an ontological notion, and on how these two perspectives are related. There is a lively debate, both in science and in philosophy, about fundamentality. This debate has usually focused on the concepts of reduction and emergence, and on their interrelation.

The focus on reduction in connection with fundamentality, both in philosophy of science and in science, stems from the interest in the question of how to best formulate scientific theories, and of whether certain formulations are more fundamental than others. The idea is that, if two theories are related by reduction, then the reducing theory is more fundamental than the reduced theory, since the latter follows logically from the former: usually under specific assumptions, and generally using bridge principles.

The focus on emergence, on the other hand, goes together with the recognition that, even if high-level theories can be reduced to lower-level ones, the high-level theories have a certain autonomy and independence from the theories from which they emerge. Here, emergence is usually construed as novel and robust behaviour (or properties, or theories) relative to an appropriate comparison class (see Butterfield (2011), Bedau (1997)). ‘Novel’ is usually taken to mean not definable from the comparison class, and ‘robust’ is usually taken to mean ‘the same for various choices of, or assumptions about, the comparison class’ (Butterfield (2011, p. 921)).

In this paper, we will consider both the emergence of one theory from another, and the emergence of entities in two descriptions related by duality. In the case of the emergence of entities, we will use the standard way of identifying what is elementary and what is more fundamental, in physics. In the case of emergence of theories, we will take fundamentality to mean exactness and-or applicability of description to a given situation, so that a more fine-grained theory will be more fundamental than a coarse-grained theory, and a theory that is mathematically under control will be more fundamental than one that is not under control in a certain regime.

Moreover, we will make use of the standard distinction between ontological and epistemic emergence, according to the kind of novelty, see for example Bedau (1997), Guay and Sartenaer (2016), De Haro (2018). Roughly, the difference lies in whether the novelty characterising emergence is novelty in the world or only novelty in the description. In this paper, we will mostly deal with epistemic emergence.

A second aspect of emergence that we will consider is the fact that, since emergence entails novelty (or autonomy), it can therefore provide new grounds for fundamentality: so that the resulting notion of fundamentality is relative to the level or to the description one is considering.

Given our usage of fundamentality and emergence, it might be natural to connect with the recent philosophical discussion of 'grounding'.[2] Nevertheless, since, in this paper, the notions of fundamentality and emergence (and, therefore, the notions of levels and priority) will be mostly epistemic, our discussion will not be directly connected to grounding, although there are interesting parallels. An explicit connection has been made, in the context of an *ontological* understanding of both emergence and fundamentality, in De Haro (2018), to which we refer for details.

The discussion of fundamentality and emergence generally depends on one's construal of the notion of a 'level'. This is a core underlying concept which needs to be clarified. And as with the associated notions of fundamentality and emergence, levels can be construed in several ways, depending on which kind of unit one is considering: theories, entities or properties.[3] Here we will characterise levels according to the physical scales involved, as labelled by a parameter—a length scale, for example, or an energy scale. A level (and its hierarchical position) is then defined by a given range of values of the parameter on the scale: in terms of energy, for example, a level L_i will be considered to be "coarser" than another level, L_j, if L_i corresponds to a lower range of energy values.[4]

Given these characterisations of fundamentality, emergence, and levels (and we will say more about them in section 2), we can summarise this paper's aim. Namely, the impact, on the above debates, of another concept which is becoming a basic ingredient in recent theoretical physics, that is duality.[5] A duality is, roughly speaking, a relation of formal equivalence between different theories in physics. Thus it is a specific kind of inter-theoretic relation. In the cases we will consider, this relation will be between theoretical descriptions taken at very different scales (energy, or another significant parameter in the theory). In other words, duality is a formal relation, but it invites a discussion of physical equivalence (as the recent philosophical literature on dualities reflects, see footnote 4). As we will see, as a relation which, although formal, has significant interpretative consequences, duality offers a new perspective on fundamentality and the interconnected notions of emergence and levels.

In this paper, in order to analyse the implications of duality for fundamentality and emergence, we

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[2] The literature on the subject is already quite extensive. See, for example, Wilson (2014) and references therein.

[3] For a recent survey of the notion of a level, see List (2017).

[4] See, for example, Castellani (2002: p. 258).

[5] For a conceptual introduction to dualities, see De Haro et al. (2016). In this paper, we will take our notion of duality from De Haro (2016) and De Haro and Butterfield (2017). See also the contributions to the special issue on dualities, Castellani and Rickles (2017).

will focus on case studies from quantum field theory and string theory which are particularly relevant for the issues at stake. The structure of the paper is as follows. In section 2, we introduce the notions of duality, fundamentality, and emergence. Section 3 contains our case studies: generalized electric-magnetic duality and gauge-gravity duality. Section 4 concludes.

2. Duality, Fundamentality, and Emergence

In this section, we introduce the notions of duality (section 2.1), fundamentality and priority (section 2.2) and emergence, of the weak, epistemic kind (section 2.3).

2.1. Dualities in physics

The idea of duality has been at the centre of many important developments in the theoretical physics of the last 50 years. In fundamental physics, the notion of duality has been applied to very different kinds of theories. First, there is the dual resonance model of the late sixties, from which early string theory originated. Successively, one of the most important developments of this idea was the generalization, proposed by Claus Montonen and David Olive in 1977, of electromagnetic duality in the framework of quantum field theory. This was later extended to the context of string theory, where dualities have also spawned recent developments in fundamental physics, offering a window into non-perturbative physics, and motivating both the M theory conjecture and gauge-gravity duality (see section 3.3).

At this point, let us characterise dualities more precisely. While a symmetry is a relation within a *single* theory (e.g. an automorphism of the space state and-or the set of quantities of the theory), a duality is a relation between *different theories*.[6] Indeed, dualities can obtain between very different-looking theories, such as a theory of gravity and a quantum field theory, in different dimensions (see section 3.3). As we said in the introduction, a duality is a case of formal equivalence (sometimes also called ‘theoretical equivalence’). Physicists tend to construe duality as an *isomorphism* of theories: the rough idea is that there is a duality if two theories make the same predictions for all the physical quantities that one can write down in the theory. De Haro (2016) and De Haro and Butterfield (2017) have developed a schema for dualities based on a conception of a theory as a set of states, a set of quantities, and a dynamics (plus some additional properties, such as symmetries). On this conception of theory, a duality is an appropriate isomorphism between the sets of states, quantities, and dynamics of the two theories.[7] Thus duality turns out to be a matter of different, isomorphic representations of a theory; where a theory is seen as a triple, comprising states, quantities, and dynamics. As shown in the papers just mentioned, a number of examples of duality in string theory and in quantum field theory can indeed be formulated in such terms.

A well-known example of a duality is the position-momentum duality in basic quantum mechanics. Although this is an elementary example, and one does not expect it to have the interesting properties of the dualities one finds in string theory and in quantum field theory, it is nevertheless illustrative of the general notion of duality just introduced. Namely, in basic quantum mechanics one starts with an algebra of operators, that is usually the Heisenberg algebra for position and momenta:

$$[x,p] = i\hbar\,. \tag{1}$$

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[6] Or between different descriptions of the same theory in case the duality is a self-duality. In such a case, the duality is a symmetry in the usual sense of model theory on the semantic conception of theories.

[7] More precisely, we define a duality as an isomorphism between two models of a single theory, where ‘model’ is here understood not in the usual sense, but as a mathematical *representation* of the theory (i.e. as a homomorphism from the theory to a mathematical structure that does the representing). What we here call the ‘single theory’ is a bare theory, and the originally-given two dual theories are now called ‘models’.

Notice that the third formula in Eq. (1) underlies Heisenberg's uncertainty principle for position and momentum.

Recall the notion of a theory, above, as a triple of states, quantities, and dynamics. In quantum mechanics, the states are taken to live in a Hilbert space. The quantities of the theory are the appropriate operators constructed from x and p, satisfying the conditions in Eq. (1). Finally, the dynamics, that is the time evolution of the states, is described by the Schrödinger equation.

Now the above algebra of operators, Eq. (1), can be represented in two different ways, depending on whether one takes the position, or the momentum, to have a well-defined value: which is in agreement with Heisenberg's principle. Accordingly, the wave-function that one constructs can be either a function of the position, or a function of the momentum. The transformation that relates these two representations of the wave-functions is the Fourier transformation.[8] Leaving the problem of measurement aside: one can show that all the quantities, i.e. all the matrix elements of all the quantities constructed from x and p, can be calculated either as functions of the position basis or as functions of the momentum. The Fourier transformation relates the two, for all the matrix elements. Thus the Fourier transformation is a duality in the above sense, albeit a very simple one.

Although the position-momentum duality just discussed is not a case of weak/strong coupling duality, we can draw a useful analogy with the weak/strong coupling dualities that we will introduce in section 3. In fact, the Fourier transformation has the property of "inverting the uncertainties" of the states, since it maps a well-localised wave-function to an ill-localised one. Namely, take a wave packet with spread Δx. This means that the position of the particle is known with uncertainty Δx. Let us also assume that the particle is well-localized, i.e. that Δx is very small, compared to a relevant length scale. Then it follows from the Fourier transformation of the wave-function (alternatively, it follows from Heisenberg's uncertainty principle) that the uncertainty in the particle's momentum will at least be of order $\hbar/2\Delta x$, which is very large if Δx is small. Thus the Fourier transformation maps a localised particle to a delocalised one.

Informally speaking, we can say that 'having a well-defined position' is a typical particle property, while 'having a well-defined momentum' is a typical wave-like property.[9] This can also be seen as a *duality between particle and wave-like properties,*[10] and it traditionally goes under the name of 'wave-particle duality'.

Just to give an idea, the analogy between the situation just described and the weak/strong coupling duality that we will discuss goes as follows. In the more sophisticated cases of dualities in string theory and quantum field theory, when one theory is weakly coupled, so that the classical approximation holds good (as in the case of the well-localised particle), the other theory is strongly coupled, i.e. it is highly interacting and quantum (as in the case where the momentum is ill-defined, so that the description in terms of a single momentum value is not valid).

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[8] A Fourier transformation is a mathematical technique widely used for waves, e.g. to decompose sound waves (which are described as the vibrations of air in space, i.e. the oscillations are functions of the position in space) into elementary frequencies (so that the oscillation function is now not a function of the position, but of the frequencies or, equivalently, of the momenta: see also footnote 6). The Fourier transformation relates the two descriptions.

[9] A wave is typically characterised by its wavelength, λ. But by de Broglie's relation between the momentum and the wavelength of the particle, viz. $\lambda = h/p$, a wave can also be characterised by its momentum. This is the reason why we say that dependence on the momentum is a typical wave-like property since, against the background of de Broglie's relation, it is also dependence on the wavelength.

[10] We are being informal here, since the aim of the example is only to illustrate a duality that is analogous to the case weak/strong coupling duality. The notion of wave-particle duality of course depends not only on Heisenberg's uncertainty principle, but also on whether the behaviour is "particle-like" or "wave-like", on detection with a measurement apparatus.

2.2. Fundamentality and priority

In the Introduction, we discussed the notion of fundamentality by focusing on its relation with reduction and emergence. In this section, we relate fundamentality to priority and the notion of levels, especially in the context of modern physics. As we said, these two notions are among the basic conceptual tools in recent discussions on grounding.

In particular, metaphysical priority has recently been much discussed under the heading of 'grounding'. The discussion is often about what grounding is: whether it is a general relation, or whether it is to be understood in terms of other special relations, such as causality, metaphysical explanation, etc.[11] Grounding is often also seen as nomological or explanatory priority. Whether nomological fundamentality and explanatory fundamentality are epistemic or ontological notions of course depends on one's view about, say, the laws of nature, the kind of explanations one is considering, and how those explanations are intended. In this paper, we will consider both fundamentality and emergence epistemically. Therefore, although the question of metaphysical priority *is* related to our story, we nevertheless set this interesting question aside, in this contribution.

Here, we consider the relation between fundamentality and priority in the following way: a theory or an entity is said to be more *fundamental* than another if it has *priority* with respect to the other theory or entity (and we will define priority of entities in the usual way in physics, through mereology, i.e. elementary entities will be regarded as more fundamental than composite ones). And we will speak of a *hierarchy of levels,* ordered in the way levels are defined above, i.e. according to a given scale.

However, combining the notion of priority with a hierarchy based on an ordering of levels is not as straightforward as it could seem. Indeed, identifying fundamentality and priority with a given ordering, from finer-grained to coarser-grained levels, may be too limited. Our aim in this paper is to show that this identification, when combined with duality, is ultimately not tenable, and that there is indeed a more articulated connection between the level structure and fundamentality. Namely, one can envisage three kinds of connections between priority and levels:

(i) The priority relation connects two different levels, according to the hierarchy of levels.
(ii) The priority relation obtains within the same level.
(iii) The priority relation and the hierarchy of the levels are disconnected from each other, i.e. two levels are related according to a given hierarchy of levels, but the priority relation—in case there is one—does not follow that hierarchy.

Although (ii) and (iii) may look similar at this stage, we will see the difference in section 3, where it will become clear that option (iii) is in fact a special combination of (i) and (ii), which is made possible by the introduction of duality.

As we said, we will discuss these three options only from the point of view of physics. We start with the first option, (i). This is the option usually endorsed in the philosophy of science literature in connection with reduction: what is at a finer-grained level is viewed as more fundamental than what is at a coarser-grained level. This is the sense according to which, for example, particle physics is considered a more fundamental description (because it is more "fine-grained") of the physical world than condensed matter physics, condensed matter physics more fundamental than chemistry, chemistry more fundamental than biology, and so on proceeding in the hierarchy based on the distance scale.

In particle physics, in particular, the idea that the physical description and its degree of fundamentality depend on the scale—based on the fact that at different scale ranges we can have remarkably different

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[11] For a discussion of grounding in the context of emergence and duality, see De Haro (2018a).

physics—has found an explicit realization in the so-called effective field theory approach. In general terms, an "effective theory" is a theory which "effectively" captures what is physically relevant in a given domain. More precisely, it is a convenient, appropriate description of the relevant physics in a given region of the parameter space of the physical world.[12]

In the framework of quantum field theory (QFT), in terms of which the Standard Model of so-called elementary particles is formulated, this general idea could be implemented in a precise, fruitful way. This is due to given characteristics of the local quantum field description, allowing for applying the concept of the renormalization group (or RG: for more on this, see section 3.1) to represent the variation of the effective physical description as the scale changes. Regarding the question of fundamentality in physics, current QFTs are now viewed as effective field theories (EFTs), that is approximate theories describing those particles that are actually relevant at the range of energies considered. This has motivated the following kind of scenario, from the viewpoint of the EFT approach: a level structure of theories, each one being a (low-energy) approximation of higher-energy, more fundamental theory, connected with each other by means of the RG equations (and the matching conditions at the level boundary). What is of special interest, here, is that this approach offers a precise, technical way of characterising the degrees of fundamentality and emergence: the passage from the finer-grained theories at higher energies (i.e. more fundamental) to the coarser-grained theories (less fundamental) at lower energies is described in terms of the RG framework and marks the direction in which theories emerge one from the other.[13] This corresponds to the situation of option (i) (see above).

A key point in the EFT approach just discussed is the separation of the physics at the chosen energy scale from the physics at much higher energies: an EFT describes the physics relevant at a given regime and this low-energy description is largely independent of the high-energy theory. In this sense one can say that the low-energy theory is emergent with respect to the high-energy one.

In fact, both the decoupling between what happens at a high-energy level and what happens at a low-energy level, and the emergence of new properties and behaviours at different levels of physical reality, have been notoriously used by the Nobel Prize P. W. Anderson, in his 1972 seminal article entitled "More Is Different", for arguing against the view that high-energy physics is more fundamental than condensed matter physics (i.e. arguing against (i)).

This leads to option (ii). In fact, those who endorse this option with respect to the priority relation and its use in defining fundamentality, argue as follows: because of the facts of emergence and decoupling, fundamentality is not to be based on the position in the level hierarchy, but it is to be discussed at the same level, i.e. each and every level, considered on its own. Accordingly, it must be based on another characteristic of the description: for example, part-whole relationships, or considering certain entities within the given level as being more fundamental.

Here we will focus on the third option, (iii). This is not a commonly discussed case, and we will argue that its relevance is suggested by dualities. To this aim, we will discuss two case studies, both involving weak/strong coupling duality. This kind of duality provides interesting examples of what we can characterize as weak epistemic emergence, which we discuss in the next section.

2.3. Weak epistemic emergence

As we noted in the Introduction, there are different kinds of emergence, depending on various factors: in particular, the kind of novelty that one considers (for a survey, See Guay and Sartenaer (2016)). For our purposes, the epistemic vs. ontological, and the weak vs. strong emergence distinctions will

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[12] See for example Georgi (1997, p. 88). On the concept of a parameter space, see section 3.1 below.

[13] According to the 'general idea of one theory T_1 being emergent from another T_2 if, in a certain part of T_2's domain of application, the results of T_2 are well approximated by those of T_1. See, on this point, Butterfield and Isham, 2000, p. 57.

be the most relevant.

With respect to the first distinction: since in this paper we will concentrate on epistemic emergence, we will set ontological emergence aside. For what regards epistemic emergence, it generally arises when there are epistemic limitations to the description of systems. More precisely, the novelty is, in this case, not to be found in the world (which is the case of ontological emergence) but in our description of the world, i.e. in the theory (and in how the theory describes the world). Epistemic emergence is therefore usually associated with lack of predictability, unexplainability, etc. In our examples, we will point out a slightly different type of epistemic emergence, though connected with this one, based on the fact that the emergent features are dependent on the type of description chosen, that is, they are "context-dependent". We will show the sense in which the two types of emergence can be related.

The second distinction, weak vs. strong epistemic emergence, refers roughly speaking to whether emergence is only in practice, or also in principle. For example, the properties of chaotic systems are often seen as a case of emergence, because they cannot be predicted from the theory. However, this lack of predictability is a problem of practice; it is not that the theory does not contain the information required to make the predictions: it is just that the calculations cannot be carried out in practice. The origin of emergence in our examples will of course have nothing to do with chaos; nevertheless, the analogy may be helpful.

We agree with much of the philosophical literature in thinking that epistemic emergence is a genuine form of emergence. More precisely, we agree with Bedau (1997) that weak emergence is not to be dismissed as merely "subjective" or as referring only to human factors: it rather has to do with algorithmic complexity, in our case with the complexity and the applicability of the mathematical description. Indeed, we will find that, in cases in which a description ceases to be valid, a novel description emerges, through duality, describing new entities or new theories.

3. Two Case Studies

In this section, we will introduce the two case studies that we will use to analyse the bearing of duality on fundamentality and emergence. The first case study, illustrated in section 3.2, is "generalized electric-magnetic duality". The second case study, discussed in section 3.3, is "gauge-gravity duality". Since both case studies are examples of weak/strong coupling duality, we will first give, in section 3.1, a short introduction to weak/strong coupling duality in the framework of perturbation theory in quantum field theory.

3.1. Weak/strong coupling duality and perturbation theory

Weak/strong coupling duality has become a basic ingredient in fundamental physics, especially since the 1990s. In general terms, weak/strong coupling duality is a duality such that the weak coupling regime of one theory is mapped to the strong coupling regime of the other theory. The special interest in this form of duality stems from the fact that it is seen as a new tool for getting information on physical quantities in the case of large values of the coupling constant, where the usual perturbative methods fail,[14] by exploiting the results obtained in the weak coupling regime of the dual description.

Let us unpack some of the notions used above, especially: 'couplings' (or coupling parameters), and

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[14] "Failure of perturbative methods" here means that the expansion in section 3.1 (below) does not converge, because g is not small (in a weaker sense, it means that one has to take into account an infinite number of terms in this expansion, which in practice is often impossible to do). This makes dualities particularly interesting and useful in the context of quantum field theory and string theory, since we usually know only the perturbative part of a theory, that is its 'weak coupling' regime. Dualities thus can be used to relate what is still unknown to what is known.

'perturbation theory'. A coupling is, roughly, a parameter characterising the strength of a force. Thus Newton's constant, G_{N}, is the coupling parameter of the gravitational force, and the spring constant, k, is the parameter characterising the strength of Hooke's law, viz. the coupling of the spring force. In Maxwell's theory, the electric charge, e, plays the role of the coupling.

We will also consider another important parameter, Planck's constant $\hbar$;[15] it is the dimensionful parameter that typically indicates the importance of quantum effects (in other words, quantum effects are large or small compared to this parameter). Although Planck's constant is strictly speaking not a coupling constant in the way just described (it does not characterise the strength of a force, but rather the importance of quantum effects), we will see that it plays much the same role as the coupling constants do.

Let us write the coupling constant as g, for whatever force is present in the problem. Since the coupling constant characterises the strength of the force, an expansion of the physical quantities around the point $g = 0$ is an expansion subject to the assumption that the force is weak, and so that the interactions are small:

$$Q(g) = Q(0) + g\, Q_1 + g^2 Q_2 + \cdots \,, \tag{1}$$

where $Q(g)$ is the quantity of interest, as a function of the coupling. The above expansion is called the 'perturbative expansion' of the theory, i.e. it is an asymptotic expansion for small interactions, or weak coupling.[16] In quantum field theories, where the interactions are of a quantum mechanical nature from the start, the above expansion turns out to *coincide* with the expansion in $\hbar$, as we will discuss in section 3.1.3. So, the first term is the classical contribution, and the sub-leading terms are quantum corrections. [17]

An important ingredient of quantum field theories is the so-called 'flow of the couplings'. Namely, unlike ordinary quantum mechanics where the coupling g is a constant, the coupling in quantum field theory is a function of the momentum, k, i.e. $g = g(k)$, where k is like energy (see the notion of a level in the Introduction). This has to do with the effects of *renormalization,* namely the basic fact that—due to the infinite number of particles that are assumed to be present in quantum field theory—the self-interactions and mutual interactions of fields give rise to new terms that have to be taken into account in the interactions of the theory. We cannot go in detail into this here: for a philosophical review, see Butterfield and Bouatta (2015); for a brief discussion, close to our second case study where we will discuss renormalization, see Dieks et al. (2015: p. 207). In fact, the coupling constant $g(k)$ satisfies an equation, the renormalization group equation, which fixes the dependence of the coupling on k. This equation describes the 'flow' of the coupling constants (if there are more than one) in their parameter space. We will get back to this notion in section 3.3. We now turn to illustrating our two case studies.

3.2. Generalized electric-magnetic duality

Electric-magnetic duality (EM duality) represents the first form of duality to be explicitly applied in twentieth century fundamental physics. The idea that there is a substantial symmetry between electricity and magnetism is an old one, dating back to the 19th century where it played a role in

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[15] The constant $\hbar = h/2\pi$, where h is Planck's constant, is called the *reduced* Planck constant. For simplicity, we will continue to call it Planck's constant.

[16] We will now not enter into the details of whether this expansion converges. This is obviously an important issue. However, in theories with dualities it is usually a good assumption (modulo technicalities), because the duality ensures that the regimes of both small and large g are under control. In those cases, the difficulty will be not the convergence, but the fact that one needs to take into account an infinite number of terms (see footnote 13).

[17] This coincides with the celebrated *Feynman diagram expansion*, which may be familiar to some readers.

Faraday's discovery of electromagnetic induction and was first made more precise with Maxwell's formulation of his famous equations regulating the behaviour of electric and magnetic fields. In its contemporary form, its origin and first developments are due to P. A.M. Dirac's famous paper (1931 and 1948) on his "theory of magnetic poles". In fact, the very idea of weak/strong duality stems from Dirac's seminal work and its successive generalizations in the context of field and string theory.

From the viewpoint of the issue at stake here—namely the significance of duality in the discussion of fundamentality and emergence—EM duality in its generalised form is particularly interesting because of the following novel feature. The weak/strong coupling nature of the duality manifests itself in the fact that under EM duality it often happens that what is viewed as "elementary" in one description gets mapped to what is viewed as "composite" in the dual description: as we will illustrate in sections 3.2.1-3.2.3 below. This interchange between what is 'fundamental' and what is 'composite' could, at first sight, be taken to suggest an ontological, relative notion of fundamentality. But this reading is too quick. Actually, what this case seems to best suggest is a form of epistemic relative fundamentality or "representational fundamentality" (as argued in Castellani, 2017).

In what follows, we will enter into some details of the generalized EM duality case study, in order to identify those specific features that illustrate the option (iii) mentioned in section 3, in the relationship between duality, emergence and fundamentality. We will structure this brief overview of the main features of the EM duality according to its actual historical development. In section 3.2.1, we will discuss the classical formulation of EM duality in the context of Mawxell's electromagnetic theory. In section 3.2.2, we will discuss the extension of EM duality to the quantum context with Dirac's "Theory of Magnetic Poles". In section 3.2.3, we will discuss the generalization of EM duality, within the framework of quantum field theory.

3.2.1. Electric-magnetic duality in classical electromagnetism

In Maxwell's theory, there is an evident similarity in the role of electric and magnetic fields. This similarity is complete in the absence of source terms (electric charges and currents), and this is mathematically expressed by the fact that Maxwell's equations do not change in form when the roles of the electric field E and the magnetic field B are exchanged in the following way:

$$D: \quad E \rightarrow B, \quad B \rightarrow -E$$

The transformation D is called a *duality transformation,* and one says that Maxwell's equations are invariant under this duality.[18]

When electric source terms are present, however, the Maxwell equations are no longer invariant under the duality transformation, D. In order to restore the duality of the theory in the presence of source terms, one needs to postulate the existence of magnetic charges beside electric charges and, accordingly, to modify Maxwell's equations. In their new form, these equations are then invariant under the duality transformation D', which at the same time exchanges the roles of the electric and magnetic fields, and of the electric and magnetic sources, as follows:

$$\begin{aligned} D': \quad E &\rightarrow B, & B &\rightarrow -E \\ (e, j_e) &\rightarrow (g, j_g), & (g, j_g) &\rightarrow (-e, -j_e) \end{aligned} \tag{2}$$

Here, (e, j_e) represents the electric charge and electric current, and (g, j_g) the magnetic charge and

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[18] Since the transformation is on the same theory, one says that it is a case of self-duality, i.e. a symmetry.

magnetic current.[19]

There is a problem, however: isolated magnetic charges, i.e. the so-called magnetic monopoles, have never been observed. Breaking a magnet bar in two parts, one obtains two smaller magnets but never an isolated North pole or an isolated South pole. Assuming, nevertheless, the existence of magnetic charges in order to save a perfect duality between electricity and magnetism, leaves this question to be addressed. In fact, the extension of EM duality to the quantum context, as we will see below, allowed Dirac to give the following answer: isolated magnetic poles had never been observed because an enormous amount of energy was needed to produce a particle with a single magnetic pole.

3.2.2. Extension to the quantum context

The extension of EM duality to the quantum context was carried out by Dirac in the two papers (1931, 1948) in which he developed his theory of magnetic monopoles. In this work, Dirac proved that it is possible for a magnetic charge, g, to occur in the presence of an electric charge, e, without disturbing the consistency of the coupling of electromagnetism to quantum mechanics.[20] The condition for this to be possible, known as *Dirac's quantization condition,* is as follows:

$$eg = 2\pi n\hbar c \qquad n = 0, \pm 1, \pm 2, \dots \tag{3}$$

where c is the speed of light. Dirac's condition thus established the existence of an inverse relation between electric and magnetic charge values, with many relevant consequences.[21] In particular, from the viewpoint of interest here, this condition provided the basis for the idea of weak/strong coupling duality. Indeed, by combining Dirac's condition with the fact that EM duality interchanges the roles of electric and magnetic charges, as above (i.e. combining Eqs. (2) and (3)), we obtain the following inverse relations:

$$e \rightarrow g = \frac{2\pi n\hbar c}{e}$$

$$g \rightarrow -e = -\frac{2\pi n\hbar c}{g}$$

This means that, if the charge e is small (i.e. weak coupling), the dual charge g is strong (strong coupling), and vice versa: in other words, in a quantum context EM duality relates weak and strong coupling. However, it is only with the generalization of EM duality to the framework of quantum field theory that the idea of weak/strong coupling duality started to acquire its modern meaning and fruitfulness. Thus, we now turn to this decisive step in the history of EM duality, with a particular focus on the related interchanging role of "elementary" and "composite" between the dual descriptions (which, as we already indicated in section 2.2, will determine what we call fundamental).

3.2.3. Sine-Gordon/Thirring duality and Montonen-Olive conjecture

Historically, the seminal contribution for the generalization of EM duality to the quantum field theories of particle physics was the 1977 work by Montonen and Olive, entitled "Magnetic monopoles

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[19] For more detail on this and the next subsection, we refer the reader to Castellani (2010, 2017).

[20] Turning from the classical to the quantum formulation of electromagnetic theory with magnetic sources posed a consistency problem*:* the electromagnetic vector potential A, playing a central role in coupling electromagnetism to quantum mechanics, is introduced in standard electromagnetism by taking advantage of the absence of magnetic source terms.

[21] First, it provided an explanation of why isolated magnetic poles had never been observed. Second, it explained the quantization of the electric charge: the mere existence of a magnetic charge, g, somewhere in the universe would have implied the quantization of electric charge, since any electric charge should occur in integer multiples of the unit.

as gauge particles?", where they formulated their celebrated EM duality conjecture: that is, in their own words, the conjecture that "there should be two 'dual equivalent' field formulations of the same theory in which electric (Noether) and magnetic (topological) quantum numbers exchange roles" (p. 117).

In order to understand the physical implications of this conjecture, let us take a step back and mention a previous result: namely, the duality between the so-called *sine-Gordon theory* and massive *Thirring model.*[22] This duality, which was firmly established by works of S. Coleman and S. Mandelstam in the mid-1970s, originated from pioneering contributions by T.H.R. Skyrme towards the end of the 1950s and beginning of the 1960s. Let us mention two things in particular: (a) his pioneering idea that a soliton could be interpreted as a quantum particle,[23] and that a dual correspondence could be established between this sort of particle—which is extended, and therefore not considered as elementary—and the familiar elementary particles of quantum field theory; (b) his conjecture that the nucleons (spin 1/2 fermionic states) could emerge as the soliton states of a purely bosonic field theory.

Skyrme's conjecture was confirmed in 1975 by Coleman and Mandelstam's work proving the dual equivalence of the sine-Gordon and massive Thirring models in general terms. From the viewpoint of this paper, we will focus on the following results:

(a) The equivalence was proven to be a weak/strong coupling duality: the weak coupling regime of the sine-Gordon fields corresponds to the strong coupling regime of the massive Thirring model, and *vice-versa*.
(b) This duality implies, in particular, a precise correspondence between the soliton states of the quantized sine-Gordon theory and the elementary particle states of the dual massive Thirring model.

In other words: by means of the weak/strong coupling duality, the sine-Gordon quantum soliton was proven to be a particle (the "elementary" fermion of the massive Thirring model) in the usual sense of the concept in particle physics.[24] Thus, in the full quantum theory, particles could appear as solitons or as elementary particles, depending on the way the theory was formulated (whether as the quantum sine-Gordon model or as the massive Thirring model): their status was equivalent. Coleman (1975, p. 2096) famously commented on this fact in terms of a situation of democracy among the particles: "Thus, I am led to conjecture a form of duality, or nuclear democracy in the sense of Chew, for this two-dimensional theory."[25]

As we mentioned in the preamble of this section, in the present case study we will take the elementary vs. composite distinction as the mark of fundamentality: a particle is more fundamental if it is elementary, and less fundamental if it is composite or extended.

The exact equivalence between the two theories, i.e. result (b) above, is worth stressing. For it means that the fermionic state of the Thirring model *is already there in the sine-Gordon theory*, and vice-versa: a bosonic state of the sine-Gordon theory is already there in the massive Thirring model (see De Haro and Butterfield (2017: section 5.2.2)). The fermionic state is non-perturbative (i.e. not visible at weak coupling) in the sine-Gordon theory; as is the bosonic state not visible in the weakly coupled Thirring model. But the states are there nevertheless. In this sense, there is no ontological emergence,

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[22] These are two field theories in one space and one time dimension, describing, respectively, a massless scalar field ψ (with interaction density proportional to $\cos \beta\psi$) and a massive self-coupled fermionic field. See Castellani (2017, section 2.2.1).

[23] Solitons are extended solutions of classical non-linear field equations, so called by Zabusky and Kruskal (1965) to indicate humps of energy propagating and interacting without distortion. They were first discovered in nineteenth century hydrodynamics in the form of 'solitary water waves', whence their name.

[24] That is, structureless particles arising from the quantization of the wave-like excitations of the fields.

[25] On the idea of nuclear democracy in Chew's S-Matrix approach in the 1960s, according to which no hadron was more fundamental than the other, see in particular Cushing (1990). Comments on this can be found in Castellani (2017, section 3.2).

because the two theories describe exactly the same states, quantities, and dynamics (though, as we will argue later, there is epistemic emergence).

Sine-Gordon/Thirring duality was the first explicit example of a weak/strong duality with a corresponding dual interchange of elementary particles and solitonic particles in the framework of a quantum field theory. It was therefore natural to try to extend these ideas to the more realistic case of a physical space-time of three space and one time dimensions. This was proposed by Montonen and Olive in their 1977 work, in terms of a generalization of Dirac's EM duality in the context of a unified quantum field theory of weak and electromagnetic interactions.

Just as for Dirac's theory, the duality considered by Montonen and Olive is a case of self-duality: the same theory has two equivalent dual descriptions. What is of particular interest, here, is the kind of situation that this generalized EM duality represents: a quantum field theory describing both particles with "electric" charge e, and particles with "magnetic" charge g,[26] which can have two different classical limits, depending on which coupling (charge)—e or g—is kept fixed while taking the classical limit $\hbar \to 0$.[27] Accordingly, there are two possible scenarios, corresponding to the two dual descriptions of the same quantum theory:

(1) If the "magnetic" coupling g is kept fixed, then, from Eq. (3), the classical limit, $\hbar \to 0$, corresponds to weak electric coupling, viz. $e \to 0$: in this case, the electrically charged particles play the role of elementary particles, and the magnetically charged particles of solitons.
(2) If the "electric" coupling e is kept fixed, then, from Eq. (3), the classical limit, $\hbar \to 0$, corresponds to weak magnetic coupling, viz. $g \to 0$: in this case, the magnetically charged particles play the role of elementary particles, and the electrically charged particles of solitons.

The particles, whether electrically or magnetically charged, are all present in the complete quantum theory. In this sense, they all are equally "fundamental", from an ontological point of view. What the duality implies, however, has rather to do with their different modes of appearance when considering the different classical limits of the quantum theory (the dual perspectives). They interchangeably play the role of "elementary" (i.e. "fundamental") or "solitonic" particles, depending on the perspective under which the theory is considered.

3.2.4. Fundamentality and emergence in weak/strong coupling duality

In this section, we discuss the conclusions for fundamentality, priority, and emergence that one may take from the cases of weak/strong duality in QFT just illustrated: namely, sine-Gordon/massive Thirring duality, and generalized EM duality. As seen, these are two cases in which a weak/strong coupling duality is accompanied by an interchange of the elementarity vs. compositeness of the particles in the quantum theory. We can summarise our findings as follows:

(A) In the case of sine-Gordon/massive Thirring model duality, we have two different quantum theories—a bosonic field theory vs. a massive fermionic theory—which are related by a weak/strong coupling duality, such that an elementary particle in one theory becomes a soliton state in the other.

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[26] See for example Sen (1999, Section 2); Polchinski (2017, p. 7). Electric and magnetic are here to be intended in a generalized sense. For a more detailed treatment, we refer to Castellani (2017, section 2.2.3).

[27] Planck's constant $\hbar$ is of course a dimensionful constant of nature, and we cannot change its value. What we have in mind here is that we consider a sequence of semi-classical solutions of the theory, with successively larger values of the typical action in the solution (in comparison with $\hbar$) while we keep the couplings (including $\hbar$) fixed. Taking the $\hbar \to 0$ thus involves comparing different physical systems. This is the case also if one takes e.g. $e/\sqrt{\hbar}$ as one's expansion parameter. Also, notice that in the quantum field theory literature, e is not measured in Coulombs, because it has been divided by the square root of the vacuum permittivity. This is the reason why $e/\sqrt{\hbar}$ can be taken to be a dimensionless parameter, and electric and magnetic charges can be related through Eq. (3).

Regarding fundamentality: as said above, the entities of the two theories are ontologically equally fundamental, since all the states and operators (for both particles and solitons) are already there in the two theories. Therefore, there is no ontological emergence, as already discussed. However, we can also look at the different roles that the bosonic and fermionic particles play in the two dual *descriptions*: in one description, the bosonic particles are the elementary particles, while the fermions only emerge as solitons in the high-energy limit. In the dual description, it is the reverse. In other words: being elementary or being a soliton is purely a matter of the convenience of the description, i.e. it depends on the specific fields one is working with, and the relation between the two pictures is like a (admittedly, very complicated) change of variables. Thus, this kind of emergence is only weakly epistemic. If we take the elementary particles to be more fundamental than the composite ones in a given description, then the notion of fundamentality is relative to that description, and—like emergence—fundamentality in this example is relative, from an epistemic point of view.

(B) In the case of generalized EM duality (Montonen-Olive duality), priority and emergence take place within the same theoretical context, since it is a case of self-duality (i.e. the duality map does not take us out of the theory). Nevertheless, we can reach the same conclusions as in (A). Namely, in one description the electric particles are elementary (and the magnetic particles are then solitons or "composite"), while in the other description it is the opposite.[28]

3.3. Gauge-gravity duality

Around 1995, the discovery of string dualities and of D-branes (which are extended, non-perturbative objects in string theory) motivated the idea of the existence of a relation between gravity theories and gauge theories. The microscopic entropy counting of Strominger and Vafa (1996) for extremal black holes, which is seen as one of the successes of string theory, vindicated this relation between gauge theory and gravity: for the entropy of a black hole (the gravitational object *par excellence*) is calculated by counting microstates in an associated gauge theory. In 1997, Maldacena took this relationship a step further, by relating string theory in anti-de Sitter space, or AdS (i.e. a space with a negative cosmological constant)[29] to a quantum field theory (QFT) which is scale invariant, i.e. a gauge theory.[30] This is called 'gauge-gravity duality'.

3.3.1. A weak/strong coupling duality

Gauge-gravity duality is a case of weak/strong coupling duality similar to the ones described above: for when the coupling of the bulk gravity theory is weak (viz. far away from the centre of the bulk) the gauge theory is strongly coupled. This can be seen as follows: both theories have two parameters in terms of which one can do a perturbative expansion (recall section 3.1). In the gravity theory, we have Newton's constant, G_{N}, and the radius of curvature of the AdS space, ℓ (Newton's constant is proportional to the string length, α').[31] On the other hand, in the gauge theory we have the coupling constant, g (which determines the strength of the interactions), and the rank of the gauge group, N (the number of colours in the theory; for Quantum Chronodynamics this would be $N = 3$). These parameters are related between the two theories as follows (see De Haro et al. (2017: section 4.1.2)):

$$\frac{G_{\mathrm{N}}}{\ell^3} = \frac{\pi}{2N^2} \tag{4}$$

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[28] For more detail on this case and the successive extension of the idea of generalized EM duality to the context of string theory, see Castellani (2017, section 2).

[29] The space is actually only required to be *asymptotically, locally* AdS, rather than pure AdS.

[30] The QFT does not actually need to need to be exactly scale invariant. It is sufficient that it has a conformal fixed point.

[31] These parameters can be written alternatively in string theory language, in terms of the string length (squared), α', and the string coupling, g_s. The string length determines how a string differs from a point particle, and the string coupling determines the perturbative expansion of the string theory. The expressions given in Eqs. (4) and (5) are for a five-dimensional gravity theory, which is the original example considered by Maldacena (1997).

$$\frac{\ell^4}{\alpha'^2} = g^2 N\,, \tag{5}$$

where the parameters on the left-hand side are those of the gravitational theory, and the parameters on the right-hand side are those of the gauge theory.

Gravity is weak if Newton's constant, G_N, is much smaller than the radius of curvature of the AdS space, ℓ, so that $\frac{G_N}{\ell^3} \ll 1$. Eq. (4) above then implies that the number of colours has to be large, i.e. $N \gg 1$. Also, quantum corrections will be suppressed if the radius of curvature of the space is much larger than the string length, $\frac{\ell^4}{\alpha'^2} \gg 1$ (so that the effects of the finite string length cannot be seen, and we basically deal with a point-particle theory), so that Eq. (5) gives $g^2 N \gg 1$. Now it was argued by 't Hooft (1974) that, in a gauge theory with N colours, the natural coupling constant is not g, but rather the combination $g^2 N$. In other words, when $g^2 N \gg 1$ perturbation theory in the gauge theory breaks down, because the theory is strongly coupled. This is why the weak-gravity, semi-classical regime of the string theory corresponds to a strongly coupled gauge theory. The converse of this statement is of course also true: if the gauge theory is taken to be weakly coupled, so that $g^2 N \ll 1$, then the semi-classical (gravity) approximation to the string theory cannot be trusted, because an infinite tower of string corrections will give non-zero contributions.

The way gauge-gravity duality is a weak/coupling duality, just discussed, is similar to the example of electric-magnetic duality: the coupling constants of the two theories are inversely proportional to each other. However, gauge-gravity duality brings in a new element, in that the coupling constants do not have fixed values on the two sides, but can vary according to the details of a specific physical situation. We will not go into details here (see Dieks et al. (2015, p. 207)): but, roughly, we can say that the gauge theory coupling is a function of momentum, $g(k)$, while the string theory coupling is a function of the position in the AdS space. The region in which the gravity approximation is valid (i.e. the region where the gravity coupling is weak) is the region far away from the centre of the AdS space (if there is e.g. a black hole at the centre of AdS, the curvature will be strong). So, weak coupling requires large distances, far away from the centre, on the gravity side. But, as we saw above, weak gravity coupling is dual to strong coupling in the gauge theory, which happens at high momenta (and hence high energies).

We can summarise this discussion by saying that large distances (weak coupling) in the gravity theory correspond to high energies, i.e. small distances (strong coupling) in the dual gauge theory, and vice-versa. Thus this is analogous, although distinct from, the Fourier transformation example discussed in section 2.1.

One new element of gauge-gravity duality is the fact that the motion from the boundary towards the centre of the space, in the gravity theory, increases the gravitational coupling of the theory because the curvature radius increases. The dual of this inward motion, in the gauge theory, is motion from the UV towards the IR, i.e. towards low energies. This 'motion', which is interpreted in terms of spatial variation in the gravity theory but in terms of energy variation in the gauge theory, is called the 'renormalization group flow' of the gauge theory.

3.3.2. Fundamentality and emergence in gauge-gravity duality

Let us now take stock of what we have found, and reframe it in the language of levels. Recall, from section 2.1, our definition of levels, where the different levels are distinguished by the value of a parameter. In the gauge theory, the parameter is the momentum scale, which is dual to the radial direction in the gravity theory. Thus, we naturally get the diagram in Figure 1, where the vertical direction corresponds to the 'motion' discussed above (the RG flow), while the horizontal direction is the duality map.

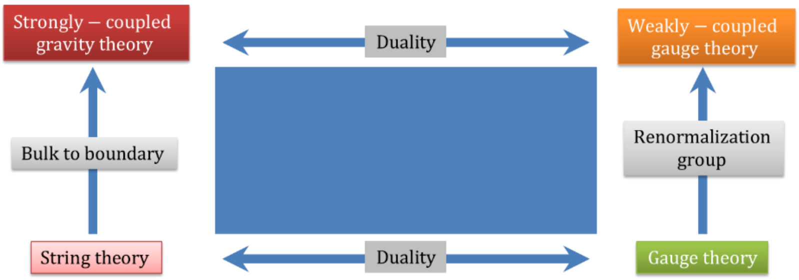

Figure 1. Duality relations vs. renormalization group flow.

In Figure 1, we have a horizontal relation between two theories which are dual, where the duality inverts the values of the couplings, according to Eqs. $(4)-(5)$. But, in addition, we also have a vertical direction, which corresponds to the 'motion' in coupling space: radial motion on the gravity side, vs. the renormalization group flow on the gauge theory side. At each level (i.e. each value of the couplings) in the vertical direction, we have a pair of theories that are dual to each other.

Thus we have, in Figure 1, two directions for which we can ask the emergence and fundamentality questions, as in section 1.1: viz. the vertical and the horizontal directions. Indeed, the theories on Figure 1's bottom are here the exact, non-perturbative theories, and (under the assumption of an exact gauge-gravity duality) they are dual to each other. These are the fundamental theories, and the top-row theories are effective theories, and therefore less fundamental than these. Thus fundamentality increases in the downward vertical direction, following the levels, but not in the horizontal direction. Emergence in the vertical direction was discussed in detail in Dieks et al. (2015: p. 210) and De Haro (2015: pp. 118-120), with the conclusion that there *is* ontological emergence in this direction. For example, on the gravity side we have Einstein's theory of general relativity with specific matter fields emerging in the low-energy limit of the underlying string theory. So far, we have the ordinary picture in effective field theories, option (i) of section 1.1.

To get option (iii), we need to change the picture slightly, so that the theories on the bottom row are the weakly coupled string theory (i.e. the semi-classical gravity theory) and the strongly coupled gauge theory which is dual to it. In that case, the duality relation relates a weakly coupled theory to a strongly coupled theory. For this duality, there is a slight difficulty in identifying what is composite/solitonic and what is elementary, because we lack good descriptions of the strongly coupled theories. Nevertheless, we can still identify the weakly coupled theory as the more fundamental description, in the innocuous sense that it is the description in which calculations can be done reliably: and we can identify its strongly coupled dual as the less fundamental one, since that description is out of control, when the coupling is weak in the other theory.[32]

Having defined fundamentality in this example, let us now examine emergence in the horizontal direction, i.e. along the duality. The question is whether *duality* can give rise to emergence. Notice that regarding ontological emergence, Dieks et al. (2015: p. 209) and De Haro (2015: p. 118) concluded that there cannot be any, because the two theories are exactly dual (i.e. equivalent), and therefore there can be no novelty, and so no emergence of one theory from the other.[33] This is because, since the two theories are exactly equivalent, one description cannot be more fundamental than the other.

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[32] It is also very likely that there is a story about what is elementary and what is component in each description, like in the cases discussed in section 3.2: but we will set this issue aside.

[33] This verdict is subject to a specific interpretation, namely a so-called *internal interpretation.* Notice that from the mere presence of a duality, which is a formal relation, one cannot make a verdict about ontological emergence. To that end, one needs to consider the interpretation of the two theories, which in Dieks et al. (2014) and De Haro (2015) was done for internal interpretations.

But this verdict can be modified when we consider emergence in an epistemic sense, i.e. as novelty of description (rather than novelty of reference), and fundamentality not as a property of the full theory, but as a property of the particular description one is dealing with. In this sense, one can indeed say that the weakly-coupled gravity description emerges from the strongly coupled gauge theory. Namely, imagine that one is working within the strongly coupled gauge theory, and unable to do any calculations. And assume that one then stumbles upon the duality, which comes down to a change of variables (in this case, an exceedingly complicated change of variables!), that allows one to reformulate the theory as a semi-classical gravity theory. In this case, the gravity theory (and the objects within it) are indeed epistemically emergent.

4. Discussion and conclusions

The initial question motivating our contribution was the way in which the notions of fundamentality, emergence and duality can be intertwined, and how this connection can shed new light on fundamentality. More precisely, the novel feature is based on duality: how dualities are applied in contemporary physics, in particular the weak/string coupling duality, and the implications of this kind of duality for the philosophical discussion of fundamentality. Our starting point was to consider the three ways, (i) to (iii), in which defining fundamentality on the grounds of priority relations and interrelationships between levels, when using a level framework, can be articulated.

While (i) and (ii) are commonly discussed in the literature on fundamentality in physics, duality suggests (iii) as a new way to construe the relation between levels and fundamentality. We illustrated this in the two cases taken from quantum field theory (sine-Gordon/massive Thirring duality, and generalized EM duality) and in the case of gauge-gravity duality. The conclusion was similar in all the cases: while there is no difference in the fundamentality at the ontological level between two dual descriptions, the physical entities and the theories which are described *can* play a more or less fundamental role, depending on the description chosen. And as such, there is emergence of a more fundamental description, because more elementary (in the mereological sense, for entities; in the sense of being weakly coupled and amenable for calculation, for theories), out of a strongly coupled description (of a composite or extended object, or of a theory in which no calculations can *prima facie* be done). Notice that the direction of emergence is here *opposite* to that of priority. Ordinarily, it is the composite entities that emerge, often by mereological composition, at higher levels. Here, duality decouples mereology from levels, and we get that it is the *simpler description*, i.e. the elementary one, rather than the composite one, which emerges.

This is possible because our notions of fundamentality and emergence are epistemic. What is fundamental is not fixed once and for all by the ontology, but depends upon the description. And so, a more fundamental description can emerge within a strongly coupled theory. And this is made possible by the fact that quantum field theories can have more than one classical limit. In general, we expect that each classical limit will have its own emergent entities, which are more fundamental in that regime of parameters.

Let us make slightly more precise in what sense emergence is here epistemic, as discussed in section 2.3. Recall that, following Bedau (1997), we can characterise novelty of description in terms of “lack of predictability” and “algorithmic complexity”. In other words, there is novelty of description when one finds entities that one could not have been predicted from the lower-level description. Alternatively, the situation of interest is too complex to be described by the lower-level theory, and one needs to change description in order to be able to make predictions. In the cases of weak/strong coupling duality discussed in this paper, although the words ‘prediction’ and ‘algorithmic complexity’ do not have the right connotations, the idea is still the same one: namely, physicists have a description (of an entity, or a theory) that is strongly coupled, and within which they cannot do calculations in perturbation theory. But then an alternative description is found, often as a change of variables, which does allow to do perturbative calculations. Namely, the perturbation expansion is around a different point: not the point of strong coupling, but the point of weak coupling. And it is that description, and the objects within it, that are epistemically emergent. Thus, epistemic emergence is tied to the ability of doing calculations in practice within a given description—which is a case of algorithmic complexity, in the sense of Bedau (1997). Here, emergence is also weak, because it does not refer to the impossibility of doing calculations in perturbation theory in principle, but to the impossibility of doing them in practice, i.e. of taking into account an infinite number of terms. This is why new methods need to be used, and those methods are non-perturbative.

Let us underline, as a final remark, that the kind of weak epistemic emergence found here is tightly connected with the notion of perturbation theory (see section 3.1). This is also the reason why we found no emergence in the cases of classical electromagnetism (section 3.2.1) and Dirac's quantization of it (section 3.2.2), where there are no perturbative expansions or perturbative duality, but only an exact one. It is only in the more sophisticated quantum field theories and string theories that we get sufficient complexity to allow this kind of emergence.

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