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Verisimilitude and strongly semantic information

Gustavo Cevolani*
University of Bologna
Department of Philosophy
gustavo.cevolani@unibo.it

ABSTRACT
In *The Philosophy of Information*, Luciano Floridi presents a theory of "strongly semantic information", based on the idea that "information encapsulates truth" (the so-called "veridicality thesis"). Starting with Popper, philosophers of science have developed different explications of the notion of verisimilitude or truthlikeness, construed as a combination of truth and information. Thus, the theory of strongly semantic information and the theory of verisimilitude are intimately tied. Yet, with few exceptions, this link has virtually pass unnoticed. In this paper, we briefly survey both theories and offer a critical comparison of strongly semantic information and related notions, like truth, verisimilitude, and partial truth.

KEYWORDS
Strongly semantic information, verisimilitude, truthlikeness, partial truth

1. *Introduction*

In his *Logik der Forschung*, Popper (1934) introduced to philosophy of science the notion of the information content of a theory or hypothesis. The systematic study of this concept was then developed by Rudolf Carnap, Yehoshua Bar-Hillel, Jaakko Hintikka, Isaac Levi, and others. Later, in an effort to provide an epistemological basis to his falsificationist methodology, Popper (1963) also introduced the concept of verisimilitude or truthlikeness of scientific theories and hypotheses, construed as a combination of truth and information. A statement is highly verisimilar when it is both highly informative (it says many things about the target domain) and highly accurate (many of those things are true, or approximately true). A number of scholars, among which Pavel Tichý, Risto Hilpinen, Graham Oddie, Ilkka Niiniluoto, and Theo A. F. Kuipers, developed different post-Popperian theories of verisimilitude, in order to avoid some logical problems encountered by Popper's explication of this notion.

According to the classical theory of semantic information (Carnap and

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Bar-Hillel 1952), a logically false or contradictory statement is maximally informative, a fact that Carnap and Bar-Hillel themselves acknowledged as "counterintuitive". Some scholars have found this consequence of the theory plainly unacceptable: Floridi (2004), for instance, dubbed it "the Bar-Hillel-Carnap semantic Paradox (BCP)". Accordingly, Floridi (2004, 2011b) has proposed a theory of "strongly semantic" information, based on the idea that "information encapsulates truth", i.e., that a statement has to be "truthful" or "veridical" about a target domain in order to count as an item of information at all.

Without entering in the debate about this "veridicality thesis", in the present paper we focus on the link between Floridi's notion of strongly semantic information and verisimilitude, which has virtually passed unnoticed in the literature (a recent exception is D'Alfonso 2011; cf. also Frické 1997). In particular, we try to disentangle the conceptual relations between the notions of semantic information, truth, and verisimilitude, and we offer three different precise explications of the intuition that "information encapsulates truth". To this purpose, we first introduce a simple logical framework to define and compare those notions (Section 1.1); then, we proceed as follows. In Section 2, we present the classical theory of semantic information, and in Section 3 the post-Popperian theories of verisimilitude. We then consider Floridi's theory of strongly semantic information (Section 4); this notion is critically examined and compared with the related notions of truth, verisimilitude and partial truth in Section 5. As a result, we obtain three different interpretations of the veridicality thesis, whose plausibility is briefly discussed. Some general remarks (Section 6) conclude our discussion.

1.1. *Preliminary notions*

Since the theories of semantic information, of verisimilitude, and of strongly semantic information are often phrased in quite different terms, it will be useful to introduce a common logical framework that will be used to define and compare those notions.

For the sake of simplicity, we will consider a finite propositional language $\mathbb{L}_n$ with n (logically independent) atomic sentences (or "atoms") $a_1, \ldots, a_n$. A literal or "basic sentence" (Carnap 1950, p. 67) of $\mathbb{L}_n$ is either an atomic sentence a_i or its negation $\neg a_i$; a literal will be denoted by "$\pm a_i$", where $\pm$ is either empty or "$\neg$". Following established terminology (e.g., Hintikka 1973, p. 152), a conjunction of n literals, one for each atomic sentence, is called a "constituent" of $\mathbb{L}_n$: each constituent C has thus the form $\pm a_1 \wedge \ldots \wedge \pm a_n$. One can check that the set $\mathbb{C}$ of the constituents of $\mathbb{L}_n$ contains exactly $q = 2^n$ elements, and that only one constituent in $\mathbb{C}$ is *true*. When $\mathbb{L}_n$ is used to describe a given domain $\mathbb{U}$ ("the world"), each constituent provides the most complete description of a possible state of affairs of $\mathbb{U}$ (or of a "possible world").

The following generalization of the standard notion of constituent will be useful later. A quasi-constituent (Oddie 1986, p. 86) or *conjunctive statement* is a conjunction of k literals corresponding to k different atomic sentences. A conjunctive statement, or *c-statement* for short, has thus the form: $\pm a_{i_1} \wedge \ldots \wedge \pm a_{i_k}$, where $0 \leq k \leq n$.[1] If $k = 0$, then the c-statement is tautological; if $k = n$ then the c-statement is a constituent. One can check that the set of the quasi-constituents of $\mathbb{L}_n$ contains 3^n members (including the 2^n constituents of $\mathbb{L}_n$). Whereas constituents are complete descriptions of a possible world, a quasi-constituent can be construed as a partial or incomplete description of a possible state of affairs of $\mathbb{U}$.

When $\mathbb{L}_n$ is used to describe a given domain $\mathbb{U}$, the true constituent, denoted by "$C_\star$", can be identified with "the (whole) truth" about $\mathbb{U}$ in $\mathbb{L}_n$, i.e., with the most comprehensive true description of the actual state of affairs expressible within $\mathbb{L}_n$. Note that $C_\star$ is the logically strongest true statement, and entails all true statements in $\mathbb{L}_n$. Conversely, $\neg C_\star$, the negation of the true constituent, is the weakest false statement of $\mathbb{L}_n$, since it does not entail any non-tautological true statements in $\mathbb{L}_n$ (cf. Niiniluoto 2003, p. 28). Whereas $C_\star$ represents the complete truth in $\mathbb{L}_n$, "the complete falsity" can be identified with the strongest false statement in $\mathbb{L}_n$, to be denoted by "$C_\dagger$". One can check that $C_\dagger$ is the constituent consisting in the conjunction of the negations of the literals of $C_\star$, whereas $\neg C_\star$ is the disjunction of such negations.[2]

It is well known (e.g., Hintikka 1973) that any consistent (non-contradictory) sentence A of $\mathbb{L}_n$ can be expressed as a disjunction of constituents, i.e., in its so-called normal disjunctive form, as follows:

$$A \equiv \bigvee_{C_i \in \mathbb{R}_A} C_i, \tag{1}$$

where $\mathbb{R}_A$ is the the "range" of A (Carnap 1950, p. 79), i.e., the set of constituents entailing A. Intuitively, the constituents in the range of A describe the possible worlds in which A is true. Denoting with "$\top$" and "$\bot$" a generic tautology and a generic contradiction of $\mathbb{L}_n$, respectively, one can check that $\mathbb{R}_\top = \mathbb{C}$ and $\mathbb{R}_\bot = \varnothing$. Thus, A is "factual" or "contingent" if and only if $\varnothing \neq \mathbb{R}_A \neq \mathbb{C}$. Moreover, A is true just in case $C_\star \in \mathbb{R}_A$, and false oth-

[1] A quasi-constituent has been called "descriptive statement" (or D-statements) by Kuipers (1982) and "conjunctive theory" (or c-theory) by Cevolani, Crupi, and Festa (2011).

[2] Cevolani, Crupi, and Festa (2011) call $C_\dagger$ the "specular" of $C_\star$. Cf. Niiniluoto (2003, pp. 28, 30, and 35, note 4).

erwise. If A (logically) entails B – or B is a (logical) consequence of A, in symbols $A \vDash B$ – then $\mathbb{R}_A \subseteq \mathbb{R}_B$, and vice versa. Thus, the range of logically stronger statements is a subset of the range of weaker ones; this is in agreement with the intuition that stronger statements excludes more possibilities than weaker ones.

2. *Semantic information and content*

As Floridi (2011b, p. 81) notes, "[i]nformation is notoriously a polymorphic phenomenon and a polysemantic concept so, as an *explicandum*, it can be associated with several explanations" (cf. also Hintikka 1970). Indeed, a number of explications of the pre-systematic notion of information have been developed by mathematicians, computer scientists, and philosophers.

A first distinction which is commonly drawn is that between "semantic" and "syntactic" information, the latter being the subject of information theory – also known as the "(mathematical) theory of communication", the "theory (of transmission) of information" (Carnap and Bar-Hillel 1952), or the "statistical theory of information" (Hintikka 1970). This theory, developed by Claude Shannon in 1948, studies information in terms of the statistical rarity of a message, i.e., of the relative stable frequency in which a message is transmitted through a communication channel. This notion of information "is purely syntactical, independent of the semantic content or meaning of a message" (Niiniluoto 1987, p. 151). Starting with Popper (1934), philosophers have instead been concerned with the notion of semantic information, on which we will focus in the following.

The classical philosophical theory of (semantic) information was created by Carnap and Bar-Hillel (1952), and then further elaborated and systematized in particular by Hintikka.[3] In Popper's view, science aims at true and highly informative theories about its domain of inquiry. Accordingly, scientists should aim at highly falsifiable hypotheses, i.e., hypotheses which exclude many (empirical) possibilities and than have a great amount of (empirical) content. In other words, the more possibilities a statement (theory, hypothesis) A excludes, the more A is informative and hence falsifiable.[4] In

[3] See Hintikka (1968, 1970, 1984) and Hintikka and Pietarinen (1966). For a compact presentation of this theory see Niiniluoto (1987, Section 4.5) and Niiniluoto (2011, Section 5). See also Kuipers (2006, Section 2).

[4] Popper (1934, pp. 103–104) defined two notions of informative content of A. The first is the "logical content" of A, defined as the class of its non-tautological logical consequences: $\{X : A \vDash X \text{ and } \nvDash X\}$. The second is the *empirical* content of A, defined as the class of its "potential falsifiers", i.e., of the basic empirical sentences X which are logically incompatible with A. In the fifties, Popper accepted a definition of content essentially identical to Carnap's definitions (2) and (3) below (see for instance appendixes 7, 8, and 9 to the English (1959) edition of his *Logik der Forschung*).

agreement with this Popperian intuition, Carnap (1942, Section 23, p. 151) defines the content of A as the class of constituents excluded by A, i.e., as the range of the negation of A:

$$\mathrm{Cont}(A) \overset{\mathrm{def}}{=} \mathbb{C} \setminus \mathbb{R}_A = \mathbb{R}_{\neg A} \tag{2}$$

One can check that definition (2) fulfills the following condition of adequacy, proposed by Carnap and Bar-Hillel (1952, p. 7) for any adequate definition of information:

(INF1) $\mathrm{Cont}(A) \supseteq \mathrm{Cont}(B)$ if and only if $A \vDash B$

According to definition (2), the contents of most statements turn out to be incomparable. In order to compare any two statements (within a given language) with respect to their informativeness, one need to introduce a quantitative notion of the *amount* or *degree of content* of A. To this purpose, Carnap and Bar-Hillel (1952, p. 15) propose the following content measure:

$$\mathrm{cont}(A) \overset{\mathrm{def}}{=} 1 - P(A) = P(\neg A) \tag{3}$$

where $P(A)$ is a regular measure of the prior probability of A.[5] Levi (1967, pp. 69–70) defines the "uniform" content measure cont_U as the number of possibilities excluded by A divided by the total number of possible worlds:

$$\mathrm{cont}_U(A) \overset{\mathrm{def}}{=} \frac{|\mathrm{Cont}(A)|}{|\mathbb{C}|} = 1 - \frac{|\mathbb{R}_A|}{|\mathbb{C}|} \tag{4}$$

The name "uniform" is due to the fact (Niiniluoto 1987, p. 152) that Levi's definition is a special case of Carnap and Bar-Hillel's one, since (3) is reduced to (4) when P assigns the same degree of probability $\frac{1}{|\mathbb{C}|} = \frac{1}{q}$ to each constituent. Thus, A is highly informative when it excludes many possible state of affairs, i.e., when is highly improbable.[6] One can prove that the quantitative definition (3) coheres with the comparative definition (2) of content in the following sense (Carnap and Bar-Hillel 1952, p. 12):

(INF2) If $\mathrm{Cont}(A) \supseteq \mathrm{Cont}(B)$ then $\mathrm{cont}(A) \geq \mathrm{cont}(B)$.

[5] It is worth noting that the theory of semantic information traditionally defines two kinds of information: the *substantive information* or *information content* of a statement and its *unexpectedness* or *surprise value* (Hintikka 1968, p. 313; cf. Kuipers 2006, p. 865). For our purposes, it will be sufficient to consider the former concept, i.e., cont. As a second *explicatum* for the amount of information of A, Carnap and Bar-Hillel (1952) proposed the following definition of the *unexpectedness* or *surprise value* $\mathrm{inf}(A)$ of A: $\mathrm{inf}(A) = -\log P(A)$, which is formally identical to the standard definition used in information theory.

[6] Definition (3) is a formal *explicatum* of Popper's thesis that the information conveyed by A must be inversely related to the probability of A – a fundamental intuition that philosophers of information call "the Inverse Relationship Principle (IRP)" (Floridi 2011b, p. 130). Cf. Popper 1934, in particular sections 34 and 35 and appendix IX (published in 1954 and reprinted in the 1959 English edition), p. 411 and note 8 to this page.

Some relevant consequences of definitions (2) and (3) are worth noting. First, in accordance with (INF1) and (INF2), logically stronger statements yield more information than weaker ones:

$$\text{If } A \vDash B \text{ then } \mathrm{Cont}(A) \supseteq \mathrm{Cont}(B) \text{ and } \mathrm{cont}(A) \geq \mathrm{cont}(B) \tag{5}$$

Tautologies are the weakest statements of $\mathbb{L}_n$, excluding no possibilities at all; hence, a tautology $\top$ conveys no (factual) information and has the minimum degree of content:

$$\mathrm{Cont}(\top) = \varnothing \text{ and } \mathrm{cont}(\top) = 0 \tag{6}$$

Conversely, contradictions are incompatible with all possible state of affairs, hence receive the maximum degree of content:

$$\mathrm{Cont}(\bot) = \mathbb{C} \text{ and } \mathrm{cont}(\bot) = 1 \tag{7}$$

The degree of content of any factual statement is greater than 0 and smaller than 1. As already Carnap and Bar-Hillel (1952, pp. 7–8) pointed out, the fact that a contradiction is maximally informative may be counterintuitive: "It might perhaps, at first, seems strange that a self-contradictory sentence, hence one which no ideal receiver would accept, is regarded as carrying with it the most inclusive information. It should, however, be emphasized that semantic information is here not meant as implying truth. A false sentence which happens to say much is thereby highly informative in our sense. Whether the information it carries is true or false, scientifically valuable or not, and so forth, does not concern us. A self-contradictory sentence asserts too much; it is too informative to be true." Still, if truth and information are analyzed as independent concepts, then (7) is a perfectly acceptable consequence of the corresponding definitions (cf. Floridi 2011b, p. 109). However, some scholars find this separation between truth and information not only at variance with the ordinary conception of information, but also unacceptable on the analytical level. Floridi (2004, p. 198), for instance, calls (7) "[w]ith a little hyperbole [...] the Bar-Hillel-Carnap semantic Paradox (BCP)", and argues that an appropriate definition of informativeness should avoid it, thus recovering the intuitive connection between truth and information. Since this connection plays a central role also in the theory of verisimilitude, we will first discuss this notion before coming back to Floridi's proposal in Section 4.

3. *Truth, content and verisimilitude*

Popper (1963, Ch. 10) introduced the concept of *verisimilitude* or *truthlikeness* of scientific theories and hypotheses in order to provide an epistemological basis to his falsificationist methodology proposed in Popper (1934).[7] Popper claimed that the main cognitive goal of science is truth-approximation and that scientific progress consists in devising new theories which are closer to the truth than preceding ones. In an effort to ground this theoretical framework, Popper proposed a formal definition of verisimilitude, according to which a theory is more verisimilar than another if the former entails more true sentences and less false sentences than the latter. Notwithstanding its intuitive appeal, Popper's definition was shown to be untenable by Tichý (1974) and Miller (1974), who independently proved that, according to this definition, a false theory can never be closer to the truth than another (true or false) theory. The Tichý-Miller theorem opened the way to the post-Popperian approaches to verisimilitude, emerged since 1975. Such approaches escape the strictures pointed out by Tichý and Miller, allowing for a comparison of at least some false theories with regard to their closeness to the truth.[8]

In general terms, a statement is highly verisimilar, or "close to the whole truth", if it says many things about the target domain and if many of those things are true. Thus, verisimilitude can be construed as a "mixture of truth and information" (Oddie 1986, p. 12), or of "truth and content" (Popper 1963, p. 236). In fact, an appropriate measure of the verisimilitude of a theory must depend on both its informativeness or content (how much the theory says) and its accuracy (how much of what the theory says is in fact true). Intuitively, it is easy to see that neither content nor accuracy alone is sufficient to define verisimilitude. As an example, let us suppose that $C_\star \equiv a_1 \wedge \ldots \wedge a_n$ is "the truth" about $\mathbb{U}$ in $\mathbb{L}_n$. Then, statements a_1 and $\neg a_2$ are equally informative, in that both make a single claim about $C_\star$ – but only the former is true, and hence more verisimilar than the latter. On the other hand, a_1 and $a_1 \wedge a_2$ are equally accurate, since both are true – but the latter is more informative, and hence also more verisimilar than the former.

Thus, verisimilitude is a "mixture" of two ingredients, truth and content. If truth were the only ingredient, then all truths, including tautologies, would be equally (and maximally) verisimilar; and conversely, if only content were relevant, then a plain contradiction, being maximally informative, would be closer to the truth than any other theory (cf. formula (7) in Section 2). On the contrary, as Oddie (1986, p. x) notes, "the truthlikeness of a proposition depends not on the quantity of its information… but on the

[7] In this paper, we use as synonymous terms like "verisimilitude", "truthlikeness" and "approximation (or closeness, or similarity) to the truth".

[8] The main post-Popperian theories of verisimilitude have been developed by Oddie (1986), Niiniluoto (1987), Kuipers (1987, 2000), and Schurz and Weingartner (1987, 2010). An excellent survey of these theories can be found in Niiniluoto (1998).

quality of its information". In sum, devising highly verisimilar theories is a "game of excluding falsity and preserving truth" (Niiniluoto 1999, p. 73).

The so called "similarity approach" to verisimilitude (Hilpinen 1976; Niiniluoto 1987; Oddie 1986; Tichý 1974) is based on the idea that an appropriate measure of the verisimilitude $Vs(A)$ of a statement A should express the similarity between A and "the truth" $C_\star$ or, equivalently, the closeness of A to $C_\star$. More specifically, the basic intuition underlying the similarity approach is that $Vs(A)$ is defined as an inverse function of the distances between the disjuncts C_i of $A \equiv \bigvee_{C_i \in \mathbb{R}_A} C_i$ and $C_\star$. This means that the verisimilitude of A expresses the closeness of the possible worlds admitted by A to the actual state of affairs.

Niiniluoto (1987, Ch. 6) has shown how to define different kinds of measures of the distance between A and $C_\star$. First, the distance $\Delta(C_i, C_j)$ between two constituents C_i and C_j is identified with the number of the differences in the $\pm$-signs between C_i and C_j, divided by n; i.e., with the number of literals on which C_i and C_j *disagree*, divided by the total number of atomic sentences.[9] Second, the distance $\Delta(A, C)$ between statement A and a generic constituent C is defined as a function of the distances $\Delta(C_i, C)$ between the disjuncts C_i of A and C. For instance, given a constituent C, the minimum distance of A from C is defined as the distance from C of the *closest* constituent entailing A, as follows:

$$\Delta_{min}(A, C) \stackrel{\text{def}}{=} \min_{C_i \in \mathbb{R}_A} \Delta(C_i, C). \tag{8}$$

Similarly, the maximum distance of A from C is defined as the distance from C of the *farthest* constituent entailing A, as follows:

$$\Delta_{max}(A, C) \stackrel{\text{def}}{=} \max_{C_i \in \mathbb{R}_A} \Delta(C_i, C). \tag{9}$$

The "min-max" distance function is defined as a weighted combination of the two measures just introduced, as follows:

$$\Delta^{\gamma}_{mm}(A, C) \stackrel{\text{def}}{=} \gamma\Delta_{min}(A, C) + (1-\gamma)\Delta_{max}(A, C) \tag{10}$$

with $0 < \gamma < 1$. Note that Δ_{min}, Δ_{max}, and Δ^{γ}_{mm} are normalized, i.e., vary between 0 and 1. Thus, three measures of the similarity of A to C can be immediately defined as follows:

$$\begin{aligned} \mathrm{sim}_{min}(A, C) &\stackrel{\text{def}}{=} 1 - \Delta_{min}(A, C) \\ \mathrm{sim}_{max}(A, C) &\stackrel{\text{def}}{=} 1 - \Delta_{max}(A, C) \\ \mathrm{sim}^{\gamma}_{mm}(A, C) &\stackrel{\text{def}}{=} 1 - \Delta^{\gamma}_{mm}(A, C) \end{aligned} \tag{11}$$

[9] This is the so called Hamming distance between constituents. Note that $0 \leq \Delta(C_i, C_j) \leq 1$ and $\Delta(C_i, C_j) = 0$ iff $i = j$.

Niiniluoto (1987, pp. 218–222) has convincingly argued that neither sim_{min} nor sim_{max} can serve as a basis for an adequate definition of closeness to the whole truth. Instead, the (degree of) verisimilitude of A can be defined as:[10]

$$Vs^{\gamma}_{mm}(A) \ \stackrel{\text{def}}{=}\ \mathrm{sim}^{\gamma}_{mm}(A, C_\star). \tag{12}$$

Some properties of definition (12) are worth noting. First, Vs^{γ}_{mm} allows that false statements can be closer to the truth than other false statements, in agreement with the basic requirement of all post-Popperian accounts of verisimilitude mentioned above. Moreover, it may well happen that a true statement is too weak or "cautious" to be verisimilar, whereas a very informative or "bold" statement may be highly verisimilar, although false. In other words:

(VS1) If A is true and B is false, it may be that $Vs^{\gamma}_{mm}(A) < Vs^{\gamma}_{mm}(B)$.

In particular, a tautology is true, but has empty content, i.e., it does not convey any factual information about the world. Accordingly, many false but informative theories are more verisimilar than tautologies. Since verisimilitude is a combination of truth and content, if A and B are both true, and A entails B, then A is also more verisimilar than B:

(VS2) If A and B are true, and $A \vDash B$, then $Vs^{\gamma}_{mm}(A) \geq Vs^{\gamma}_{mm}(B)$.

In other words, among true statements, verisimilitude co-varies with logical strength. This condition, however, should not hold among false statements, since logically stronger falsehoods may well lead us farther from the truth: if A and B are both false, the more verisimilar theory will be the one implying less – or less serious – falsehoods. Thus:

(VS2) If A and B are false, and $A \vDash B$, it may be that $Vs^{\gamma}_{mm}(A) < Vs^{\gamma}_{mm}(B)$.

Thus, among false statements, verisimilitude does not co-vary with logical strength. It is worth noting that most measures developed within the post-Popperian theories of verisimilitude satisfy the three properties above. Indeed, it can be argued that (VS1)–(VS3) should be regarded as adequacy conditions for any appropriate notion of closeness to the truth.[11]

[10] One should note that Niiniluoto's favorite definition of the verisimilitude of A is not based on $\mathrm{sim}^{\gamma}_{mm}$ but on the so called *min-sum distance* function:

$$\Delta^{\gamma\gamma'}_{ms}(A, C) \ \stackrel{\text{def}}{=}\ \gamma\Delta_{min}(A, C) + \gamma'\Delta_{sum}(A, C),$$

with $0 < \gamma, \gamma' \leq 1$, defined as a weighted sum of the minimum distance and the *normalized sum distance* $\Delta_{sum}(A, C)$. The normalized sum distance of A from C is the sum of the distances from C of all the constituents in the range of A *normalized* with respect to the sum of the distances of *all* the elements of $\mathbb{C}$ from C: $\Delta_{sum}(A, C) \stackrel{\text{def}}{=} \sum_{i \in \mathbb{R}_A} \Delta(C_i, C) \Big/ \sum_{C_j \in \mathbb{C}} \Delta(C_j, C)$.

[11] See Niiniluoto (1987, pp. 232 ff.) for a detailed defense of these and other adequacy conditions. It should be

4. *Strongly semantic information*

According to the classical theory of information summarized in Section 2, a contradiction is more informative than any other statement, including contingently true statements. Floridi (2004, 2011b) argues that this paradoxical consequence (BCP) is inevitable within a theory of "weakly" semantic information (TWSI) like that of Carnap and Bar-Hillel (1952). For this reason, any such theory should be rejected in favor of a theory of "strongly" semantic information (TSSI): "The general hypothesis is that BCP indicates that something has gone essentially amiss with TWSI. TWSI is based on a semantic principle that is too weak, namely that truth-values supervene on semantic information. A semantically stronger approach, according to which *information encapsulates truth*, can avoid the paradox and is more in line with the ordinary conception of what generally counts as information (Floridi 2011b, p. 110, italics added)." According to Floridi (2011b, pp. 119 and ff.), the more accurately a statement A describes the actual state of affairs $C_\star$, the more informative A is. Thus, "to develop a clear understanding of semantic information we need to move from likelihood (TWSI) to likeness (TSSI)" (Floridi 2011b, p. 129): information is not primarily improbability, but similarity to the true complete description of the target domain. It follows, in particular, that a contradiction is not informative at all, since it does not yield valuable information about $C_\star$.

To formally state these intuitions, Floridi defines the *degree of discrepancy* (or distance) $\delta(A, C_\star)$ of each statement A in $\mathbb{L}_n$ with respect to the actual state of affairs described by constituent $C_\star$.[12] For true statements, discrepancy coincides with *vacuity*, which is defined as the number of possibilities admitted by A divided by their total number (Floridi 2011b, p. 122):

$$\mathrm{vac}(A) \stackrel{\mathrm{def}}{=} \frac{|\mathbb{R}_A|}{|\mathbb{C}|} \tag{13}$$

Thus, the degree of vacuity is inversely related to degree of content; one may note, in particular, that it is complementary to Levi's notion of informativeness (cf. definition (4) in Section 2):

$$\mathrm{vac}(A) = 1 - \mathrm{cont}_U(A) \tag{14}$$

noted, however, that according to some verisimilitude theorists, notably Oddie (1986), logically stronger true theories are not necessarily more verisimilar than weaker ones, thus violating (VS2).

[12] For the sake of uniformity, and to emphasize the connection between strongly information and verisimilitude, from now on we depart from Floridi's notation and terminology, rephrasing his theory in the terminology introduced in Section 3.

If A is tautological, then its degree of vacuity gets the maximum value:

$$\mathrm{vac}(\top) = 1 \tag{15}$$

If A is false (but not contradictory), then its degree of discrepancy coincides with its *degree of inaccuracy*. As D'Alfonso (2011, Sec. 3.1) points out, this notion is only defined for statements expressed as conjunctions of literals of $\mathbb{L}_n$, i.e., for what we called "conjunctive statements" (quasi-constituents). Given a conjunctive statement A, Floridi (2011b, p. 121) defines the "length" of A as the number k_A of its conjuncts, and f_A as the number of its *false* conjuncts. The degree of inaccuracy of A is then defined as:[13]

$$\mathrm{inacc}(A, C_\star) \stackrel{\mathrm{def}}{=} \frac{f_A}{k_A} \tag{16}$$

In sum, the degree of discrepancy of A is defined as follows (Floridi 2011b, pp. 121–122):

$$\delta(A, C_\star) \stackrel{\mathrm{def}}{=} \begin{cases} \mathrm{vac}(A) & \text{if } A \text{ is true and } A \not\equiv C_\star \\ -\mathrm{inacc}(A, C) & \text{if } A \text{ is false and not contradictory} \end{cases} \tag{17}$$

(The reason for the slightly counterintuitive minus sign before $\mathrm{inacc}(A, C)$ will be clear in a moment.) One should note that, according to definition (13), the true constituent has degree of vacuity $\mathrm{vac}(C_\star) = \frac{1}{|\mathbb{C}|}$. Thus, Floridi (2011b, p. 120) needs to stipulate that the maximally informative true description has the lowest degree of discrepancy:

$$\delta(C_\star, C_\star) \stackrel{\mathrm{def}}{=} 0 \tag{18}$$

The case of contradictory statements is also taken into account by stipulation (*ibidem*):

$$\delta(\bot, C_\star) \stackrel{\mathrm{def}}{=} -1 \tag{19}$$

To sum up, all statements receive a degree of discrepancy, or distance from the actual state of affairs $C_\star$, greater than $\delta(\bot, C_\star) = -1$ and smaller than $\delta(\top, C_\star) = 1$, as displayed in Figure 1. Moreover, the null degree of discrepancy $\delta(C_\star, C_\star)$ represents a threshold in the sense that all false statements receive a *negative* degree of discrepancy and all true statements (excluding $C_\star$ itself) a *positive* degree of discrepancy.

[13] One can check that, if A is a statement of generic form, there is no simple way to satisfactorily define "the number of erroneous atomic messages" of A, nor its "length", nor obviously their ratio, i.e., $\mathrm{inacc}(A, C)$. For more on this point, see Cevolani (2012).

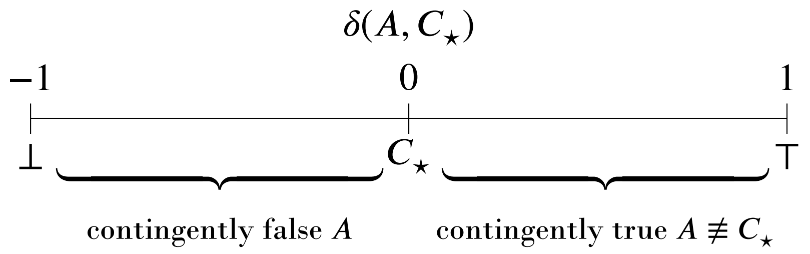

Figure 1: Degrees of discrepancy (distance from the truth $C_\star$) in Floridi's theory.

Finally, the *degree of strongly semantic information* (or strongly semantic content) of A is defined as (Floridi 2011b, p. 123):

$$\text{cont}_S(A, C) \stackrel{\text{def}}{=} 1 - \delta(A, C_\star)^2 \tag{20}$$

Some features of Floridi's definition of informativeness (20) are worth noting. First, one can then check that the cont_S measure is normalized, i.e., $0 \leq \text{cont}_S(A, C) \leq 1$ for any statement A. Second, both tautologies and contradictions have the minimum degree of informativeness (i.e., 0). Third, cont_S fulfills Carnap and Bar-Hillel's adequacy conditions (INF1) and (INF2) only with respect to true statements: among true statements, information covaries with logical strength:

$$\begin{array}{l} \text{If } A \text{ and } B \text{ are true and } A \vDash B \text{ then} \\ \text{cont}_S(A, C) \geq \text{cont}_S(B, C) \end{array} \tag{21}$$

Among false statements, this condition does not hold: logically stronger false statements may be *less* informative than weaker ones.

5. *Strongly semantic information, (partial) truth, and verisimilitude*

Floridi's notion of strongly semantic information (Section 4) appears to be very close to Popper's notion of verisimilitude (Section 3), as the following quotation clearly reveals (Floridi 2011b, pp. 118–119):[14] "two [statements] can both be false and yet significantly more or less distant from the event or state of affairs [$C_\star$] about which they purport to be informative, e.g. 'there are ten people in the library' and 'there are fifty people in the library', when in fact there are nine people in the library. Likewise, two [statements] can both be true and yet deviate more or less significantly from [$C_\star$], e.g. 'there is someone in the library' vs. 'there are 9 or 10 people in the library'. This

[14] Here, although Floridi cites Popper (1934), he's probably referring to Popper (1963).

implies that a falsehood with a very low degree of discrepancy may be pragmatically preferable to a truth with a very high degree of discrepancy (Popper 1934)." Since Floridi explicitly defines informativeness as the degree of similarity or closeness to the true description of the actual state of affairs, strongly semantic information and verisimilitude become virtually indistinguishable notions (cf. also D'Alfonso 2011, p. 66). Thus, the problem arises of what the conceptual and formal relationships between these two notions are.

In answering this question, it is useful to focus first on the relationship between information and truth. Indeed, Floridi's starting point in putting forward his theory of strongly semantic information is the so-called "veridicality thesis" (VT), which raised much debate among philosophers of information (see Floridi 2011b, Ch. 4 and Floridi 2011a for a survey of different positions). According to VT, statement A has to be "truthful" or "veridical" about the target domain in order to qualify as a piece information at all. As noted in Section 2, Carnap and Bar-Hillel defended the opposite claim, according to which information and truth can be construed as independent notions. Without entering into this debate, we wish to note that the thesis that "information encapsulates truth" can be formally phrased in quite different ways. Three of them deserving particular attention are the following:

VT1 A is informative if and only if A is factually true.

VT2 A is informative if and only if A is truthlike.

VT3 A is informative if and only if A is partially true.

(The notion of partial truth will be defined in due time.) In informal presentations, VT is usually equated with VT1. For instance, Floridi (2011b, e.g. p. 105) tends to treat "veridical" and "true" as synonymous terms (or as merely stylistic variants); this implies that the expression "'true information' is simply redundant and 'false information', i.e., misinformation, is merely pseudo-information" (*ibidem*, p. 82). Note that VT1 implies that:

$$\text{If } A \text{ is false, then } A \text{ is uninformative} \tag{22}$$

In fact, if A has to be true in order to be informative, then all false statements would be not informative at all. A simple information measure coherent with VT1 is the following "truth-content" measure:

$$\mathrm{cont}_T(A) \overset{\mathrm{def}}{=} \begin{cases} \mathrm{cont}_U(A) & \text{if } A \text{ is true} \\ 0 & \text{otherwise} \end{cases} \tag{23}$$

Some relevant properties of this measure are the following:

$$\begin{aligned}
&\mathrm{cont}_T(\top) = 0\\
&\mathrm{cont}_T(\bot) = 0\\
&\text{If } A \text{ is false then } \mathrm{cont}_T(A) = 0\\
&\text{If } A \text{ and } B \text{ are true and } A \vDash B \text{ then } \mathrm{cont}_T(A) \geq \mathrm{cont}_T(B)
\end{aligned} \tag{24}$$

Thus, cont_T trivially satisfies (22), and hence avoids BCP. Moreover, it agrees with Floridi's cont_S as far as true statements are concerned, as one can easily check noting that, if A is true, $\mathrm{cont}_S(A) = 1 - \mathrm{vac}(A)^2$ and $\mathrm{cont}_T(A) = 1 - \mathrm{vac}(A)$.

However, it is doubtful that supporters of VT would regard VT1 as an adequate explication of the veridicality thesis, not least because (22) appears at least as counter-intuitive as BCP. For instance, Floridi's own measure cont_S assigns a positive degree of strongly semantic information to most false statements. According to cont_S, false statements are the more informative the more accurate they are about the target domain: a false statement yielding more truths than falsehoods will be quite informative. This suggests that VT could be interpreted as VT2, i.e., as saying that A has to be verisimilar in order to be informative. To our knowledge, VT2 was first proposed and defended by Frické (1997), in a paper which, although passed virtually unnoticed among philosophers of information, seems to anticipate some central intuitions underlying the debate about the veridicality thesis: "With true statements, verisimilitude increases with specificity and comprehensiveness, so that a highly specific and comprehensive statement will have high verisimilitude; such statements also seem to be very informative. With false statements, verisimilitude is intended to capture what truth they contain; if false statements can convey information, and the view taken here is that they can, it might be about those aspects of reality to which they approximate. Verisimilitude and a concept of information appear to be co-extensive (Frické 1997, p. 882). " Frické (1997, pp. 855, Sec. 4, p. 891) discusses and rejects both VT1 and the classical thesis that truth and information are independent concepts, and proposes to use the similarity measures developed by verisimilitude theorists for defining quantitative notions of semantic information. Following this strategy, in agreement with VT2, a notion of informativeness may be defined with the help of Niiniluoto's verisimilitude minmax measure:

$$\mathrm{cont}^{\gamma}_{VS}(A) \stackrel{\mathrm{def}}{=} Vs^{\gamma}_{mm}(A) \tag{25}$$

Some relevant properties of this measure are the following (cf. Niiniluoto 1987, p. 223):

$$\begin{aligned}
&\mathrm{cont}^{\gamma}_{VS}(\top) = \gamma > 0\\
&\text{If } A \text{ and } B \text{ are true and } A \vDash B \text{ then } \mathrm{cont}^{\gamma}_{VS}(A) \geq \mathrm{cont}_{VS}(B)
\end{aligned} \tag{26}$$

Note that, since $\mathrm{cont}^{\gamma}_{VS}$ is undefined for contradictions, one could stipulate that their degree of information is 0, in agreement with Floridi's (19), thus avoiding BCP.

It seems that VT2 captures some, but not all, of the intuitions underlying Floridi's conception of strongly semantic information. The close link between verisimilitude and *strongly* semantic information has been pointed out for instance by Bremer and Cohnitz (2004, p. 90), but has been first discussed in full details in a recent paper by D'Alfonso (2011). D'Alfonso seems to endorse VT2, and applies Niiniluoto's and Oddie's measures of similarity, as well as a new measure of his own, to quantify strongly semantic information. However, one may note that verisimilitude and strongly semantic information differ under important respects. In particular, $\mathrm{cont}^{\gamma}_{VS}$ assigns to tautologies a positive degree of informativeness, in contrast with both the classical theory of semantic information and Floridi's theory. This is natural in the case of verisimilitude since tautologies, when conceived as answers to a cognitive problem, correspond to suspending the judgement, which is better than accepting strong falsehoods.

A part from this problem, which may be solved by stipulation (as suggested by cf. D'Alfonso 2011, p. 73), the relevant point is that Floridi's favored measure cont_S can not serve as a measure of the verisimilitude of false statements. This becomes evident considering the case of *completely false* c-statements. A c-statement A is completely false when it is a conjunction of false literals (Cevolani, Crupi, and Festa 2011, Sec. 2). In this case, it is easy to check that $\mathrm{inacc}(A, C) = 1$ and hence $\mathrm{cont}_S(A) = 0$ (cf. definitions 16 and 20). This means that, were cont_S to be used as a measure of verisimilitude, it would assigns to all completely false c-statements the lowest degree or verisimilitude, on a par with contradictions. For instance, assuming that $C_\star \equiv a_1 \wedge a_2 \wedge a_3$ is the truth in $\mathbb{L}_3$ (cf. Table 1), statements $\neg a_1$, $\neg a_1 \wedge \neg a_2$, and $\neg a_1 \wedge \neg a_2 \wedge \neg a_3$ would be all equally verisimilar according to cont_S, although intuitively their verisimilitude is quite different. On the contrary, one can check that Vs^{γ}_{mm} correctly discriminate these three cases, yielding $Vs^{\gamma}_{mm}(\neg a_1) > Vs^{\gamma}_{mm}(\neg a_1 \wedge \neg a_2) > Vs^{\gamma}_{mm}(\neg a_1 \wedge \neg a_2 \wedge \neg a_3)$. In sum, whereas verisimilitude is a combination of truth and content, strongly semantic information (as defined by Floridi) corresponds to informative content for true statements and to mere *accuracy* for false (c-)statements. This implies that, somehow paradoxically, cont_S is insensitive to content, as far as complete falsehoods are concerned.

A third version of VT somehow related to VT2, and apparently also close to Floridi's intuitions, is VT3, according to which A has to be partially true in order to be informative. The intuition is that A has to convey at least *some* (non-tautological) true information about the target domain in order to count as a piece of information at all. According to Hilpinen (1976), the degree of partial truth of A measures the amount of *information about the truth* conveyed by a (true or false) statement A (see also Niiniluoto 1987, Sec. 5.4 and 6.1). An adequate definition of the degree of partial truth is arguably the following (Niiniluoto 1987, pp. 218–220):

$$\mathrm{cont}_{PT}(A) \stackrel{\mathrm{def}}{=} 1 - \Delta_{max}(A, C_{\star}) \tag{27}$$

where Δ_{max} is the max-distance measure introduced in Section 3. One can check that:

$$\begin{aligned}&\mathrm{cont}_{PT}(\top) = 0\\&\text{If } A \vDash B \text{ then } \mathrm{cont}_{PT}(A) \geq \mathrm{cont}_{PT}(B)\end{aligned} \tag{28}$$

Again, one can stipulate that $\mathrm{cont}_{PT}(\bot) = 0$, thus avoiding BCP (an argument to this effect was already proposed by Hilpinen 1976, p. 30). When VT3 is based on cont_{PT}, the degree of partial truth of true statements depend on their content, whereas for false statements it depends on how much information about the truth they convey (Niiniluoto 1987, p. 176). In particular, both $\mathrm{cont}_{PT}(A)$ and $\mathrm{cont}_{S}(A)$ imply that "bad falsities" are completely uninformative:[15]

$$\begin{aligned}&\mathrm{cont}_{PT}(\neg C_{\star}) = 0\\&\mathrm{cont}_{PT}(C_{\dagger}) = \mathrm{cont}_{S}(C_{\dagger}) = 0\\&\text{If } A \text{ is a completely false c-statement then}\\&\mathrm{cont}_{PT}(A) = \mathrm{cont}_{S}(A) = 0\end{aligned} \tag{29}$$

This is coherent with VT3, since it may be argued that completely false (conjunctive) statements are not "veridical" at all, in the sense that they convey no true factual information about the world (cf. Hilpinen 1976 and Niiniluoto 1987, p. 201). The relations between Floridi's measure cont_{S} and measures cont_{T}, $\mathrm{cont}^{\gamma}_{VS}$, cont_{PT} can be better appreciated when these four measures are compared with respect to the 27 c-statements of $\mathbb{L}_3$, as illustrated in Table 1.

6. *Concluding remarks*

In the preceding section, we tried to disentangle the conceptual relations between the notions of semantic information, truth, verisimilitude, and partial truth. To this purpose, we formally compared Floridi's concept of strongly semantic information, on the one hand, and three different notions of information, on the other. These latter notions were inspired by three different

[15] Note that $\mathrm{cont}_{S}(\neg C_{\star})$ is undefined, since $\neg C_{\star}$ is not a c-statement.

	A	$\mathrm{cont}_S(A)$	$\mathrm{cont}_T(A)$	$\mathrm{cont}_{VS}^{\frac{1}{2}}(A)$	$\mathrm{cont}_{PT}(A)$
$C_\star =$ 1.	$a_1 \wedge a_2 \wedge a_3$	1	1	1	1
2.	$a_1 \wedge a_2$	0.94	0.75	0.83	0.6
3.	$a_1 \wedge a_2 \wedge \neg a_3$	0.88	0	0.66	0.6
4.	$a_1 \wedge a_3$	0.94	0.75	0.83	0.6
5.	a_1	0.75	0.5	0.66	0.3
6.	$a_1 \wedge \neg a_3$	0.75	0	0.5	0.3
7.	$a_1 \wedge \neg a_2 \wedge a_3$	0.88	0	0.66	0.3
8.	$a_1 \wedge \neg a_2$	0.75	0	0.5	0.3
9.	$a_1 \wedge \neg a_2 \wedge \neg a_3$	0.55	0	0.33	0.3
10.	$a_2 \wedge a_3$	0.94	0.75	0.83	0.6
11.	a_2	0.75	0.5	0.66	0.3
12.	$a_2 \wedge \neg a_3$	0.75	0	0.5	0.3
13.	a_3	0.75	0.5	0.66	0.3
14.	$\top$	0	0	0.5	0
15.	$\neg a_3$	0	0	0.66	0
16.	$\neg a_2 \wedge a_3$	0.75	0	0.5	0.3
17.	$\neg a_2$	0	0	0.33	0
18.	$\neg a_2 \wedge \neg a_3$	0	0	0.16	0
19.	$\neg a_1 \wedge a_2 \wedge a_3$	0.88	0	0.66	0.6
20.	$\neg a_1 \wedge a_2$	0.75	0	0.5	0.3
21.	$\neg a_1 \wedge a_2 \wedge \neg a_3$	0.88	0	0.33	0.3
22.	$\neg a_1 \wedge a_3$	0.75	0	0.5	0.3
23.	$\neg a_1$	0	0	0.33	0
24.	$\neg a_1 \wedge \neg a_3$	0	0	0.16	0
25.	$\neg a_1 \wedge \neg a_2 \wedge a_3$	0.55	0	0.33	0.3
26.	$\neg a_1 \wedge \neg a_2$	0	0	0.16	0
27.	$\neg a_1 \wedge \neg a_2 \wedge \neg a_3$	0	0	0	0

Table 1: Comparison of Floridi's measure of strongly semantic information cont_S and the measures of information as truth-content (cont_T), as verisimilitude ($\mathrm{cont}_{VS}^{\gamma}$, with $\gamma = \frac{1}{2}$), and as partial truth (cont_{PT}).

interpretations of the so-called "veridicality thesis" VT, according to which "information encapsulates truth".

According to the first (VT1), a statement is informative only if it is true. Although Floridi (2004, 2011b) seems occasionally ready to accept VT1, the consequence that false statements are plainly uninformative is violated by his own information measure cont_S.[16] Thus, it seems safe to conclude that VT1 should be excluded as an interpretation of VT.

According to a different thesis (VT2), proposed by Frické (1997) and endorsed by D'Alfonso (2011), (strongly) semantic information may be identified with verisimilitude or truthlikeness. Notwithstanding its possible merits, VT2 is not supported by Floridi's own theory. In particular, Floridi's measure can not be used as a verisimilitude measure since it can not discriminate among false statements with very different degrees of closeness to the truth. On the other hand, verisimilitude measures violates the requirement that tautologies yield no information, a condition virtually accepted by all philosophers of information.[17]

Finally, a third reading of VT was proposed (VT3), according to which an informative statement is a partially true one. The notion of partial truth has been rigorously defined by Hilpinen (1976) and Niiniluoto (1987), and has many aspects in common with Floridi's notion of strongly semantic information. In particular, it fulfills the requirement according to which complete falsities are also completely uninformative. According to VT3, the degree of strongly semantic information of A is construed as the information about the truth yielded by A – an idea which appears to be the most convincing of the three. Indeed, we suggest that the notion of partial truth captures all the essential intuitions underlying Floridi's theory, and that cont_{PT} should be favored over Floridi's cont_S as a measure of strongly semantic information. This suggestion is further developed in Cevolani (2012).

Another conclusion to be drawn from our comparison is that the post-Popperian theories of verisimilitude offer to philosophers of information a full arsenal of both conceptual and formal tools. Frické (1997) and D'Alfonso (2011) argue hat these tools can be fruitfully employed in developing philosophical theories of information, and our paper supports this claim. More particularly, the notions of verisimilitude and partial truth can illuminate the debate about the veridicality thesis. In this connection, our analysis does not support any definite conclusion about the soundness of this thesis, and does not allow, *per se*, to choose between VT and the opposite thesis of the "alethic neutraliy" of information (Floridi 2011a). However, in the author's

[16] Obviously, one could try to defend VT1 by saying that a *literal* (but not more complex statements) is informative if and only if it is true. However, this would devoid VT1 of most of its interest.

[17] We are here excluding from consideration the notion of "depth information" developed by Hintikka.

opinion, it does suggest, as a methodological recommendation, that truth and information are better analysed as independent concepts, and that other notions, like for instance strongly semantic information and verisimilitude, can then be defined on their basis.

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