HISTORY AND PHILOSOPHY OF LOGIC, 2022
Vol. 43, No. 2, 143–157, https://doi.org/10.1080/01445340.2021.1924570

A Pragmatic-Semiotic Defence of Bivalence

MARC CHAMPAGNE
Department of Philosophy, Kwantlen Polytechnic University, Surrey, BC, Canada
marc.champagne@kpu.ca

Received 7 March 2020 Accepted 28 April 2021

Since Peirce defined the first operators for three-valued logic, it is usually assumed that he rejected the principle of bivalence. However, I argue that, because bivalence is a principle, the strategy used by Peirce to defend logical principles can be used to defend bivalence. Construing logic as the study of substitutions of equivalent representations, Peirce showed that some patterns of substitution get realized in the very act of questioning them. While I recognize that we can devise non-classical notations, I argue that, when we make claims about those notations, we inevitably get saddled with bivalent commitments. I present several simple inferences to show this. The argument that results from those examples is 'pragmatic', because the inevitability of the principle is revealed in *use* (not mention); and it is 'semiotic', because this revelation happens in the use of *signs*.

1. Introduction

Combined with ongoing developments in historical scholarship,[1] the recent release of Charles Sanders Peirce's writings on Existential Graphs[2] will likely enlarge his list of contributions to logic. Those contributions are already significant. According to Alonzo Church, '[t]he explicit use of two truth-values appears for the first time in a [1885] paper by C. S. Peirce'.[3] Not only did Peirce originate the truth tables for two-valued logic,[4] he was also the first to define the operators for *three*-valued logic.[5] Can we proceed from these historical facts to the conclusion that, philosophically, Peirce knew but rejected the principle of bivalence? Robert Lane writes that 'Peirce was concerned to accommodate within formal logic propositions which are neither true nor false; and this means that he believed that some propositions are, indeed, neither true nor false. He thus rejected the Principle of Bivalence'.[6] This seems to be the received view.[7] However, at the risk of complicating this picture, I want to argue that the strategy used by Peirce to defend logical principles can be used to defend the principle of bivalence.

We can invent and manipulate any notation we want; and those inventions and manipulations can prove useful or not. But, when we make higher-order claims about those notations, we inevitably get saddled with bivalent commitments. Such inevitability gives bivalence a special status. This, at any rate, is what I propose to show.

My source of inspiration will be historical, but philosophical debates about bivalence are alive and well. The fact that the past colors the present is even used as a premise in some arguments. Graham Priest, a major critic of bivalence and the excluded-middle, writes that '[s]ince a logical theory [that allows contradictions] is so easy to construct, it is worth asking *why*' such a theory 'should be so outrageous to the sensibility of modern

[1] *Brady 2000*.
[2] *Peirce 2019*; *Peirce 2020*; and more volumes forthcoming.
[3] *Church 1956*, 25fn67; Peirce 1931-58, vol. 3, para. 365, hereafter cited as CP 3.365.
[4] *Anellis 2012*.
[5] *Fisch and Turquette 1966*, 73–5.
[6] *Lane 1998*, 1–2.
[7] *Quine 1986*, 84.

philosophers'.[8] The culprit, in his estimate, is nothing more than 'Aristotle's magisterial authority in the Middle Ages'.[9] There is no denying that bivalent systems have been historically prevalent—they are termed 'classical', after all. Likewise, the fact that workable non-classical systems can be devised is not in dispute. Yet, surely one can combine a recognition that classical assumptions are revisable[10] with a reluctance[11] or refusal[12] to do so. My goal will be to show that, far from being dogmatic, such a reluctance or refusal can be principled.

There are considerable motivations for keeping the principle of bivalence—to avoid the triviality generated by the principle of explosion, for example.[13] However, I want to use Peirce's ideas to tell a story that stands on its own, apart from such benefits.

2. The Pattern of Deductive Inference

I will be making my point by stringing together signs, so a good place to start would be to ask how signs relate to logic. In one of his earliest articles, Peirce asked '[w]hether we can think without signs'.[14] Answering that question, Peirce argued that, since 'every thought must address itself to [or determine] some other' thought, and since this incessant referral *from* something *to* something 'is the essence of a sign', we can conclude that 'all thought is in signs'.[15] Far from clashing with Peirce's rigorous work in logic, this semiotic account sheds light on logical reasoning. Premises are signs (usually symbols) that represent a (real or hypothetical) state of affairs in the world. A conclusion represents another aspect of that state of affairs. 'In an argument, therefore, a symbol is substituted for another that has the same object'.[16] Seen in this light, logic becomes the study of 'certain substitutions of equivalent representations'.[17]

To make this tangible, consider what happens in a logic classroom. We give students signs (in the form of exams, say) and ask them to produce still more signs in return. Of course, a student is free to write whatever she wishes on an examination sheet. Jotting down random gibberish may scar one's transcript, but it does not violate the laws of physics. However, we credit students with mastering an inference rule only when they can produce the right kind of signs in the right circumstances. Not only must these sign substitutions conform to certain patterns, they must also be habitual.[18] To grasp a valid deductive form like a constructive dilemma, for example, it is not enough to generate the correct conclusion in a single instance. Rather, we expect a mind that has grasped this pattern to spot and complete it in an open-ended series of cases. Good logic teachers vary the exercises precisely to ensure and verify that general patterns are being discerned, irrespective of how (in natural language, say) those patterns are realized.

Yet, as the possibility of producing random gibberish attests, one can always substitute signs in an unconstrained or random manner. However, to count as truth-preserving, a substitution must direct interpretation to the same object that was expressed in the starting signs. This Peircean account of logic thus holds fast to the tenet that signs always beget more signs—without lapsing into any kind of relativism.

[8] *Priest 1995b*, 5.
[9] *Priest 1995b*.
[10] *Van Benthem et al. 2006*.
[11] *Haack 1996*.
[12] *Suszko 1975*.
[13] *Priest 2007*.
[14] *CP* 5.250.
[15] *CP* 5.253; see *Champagne 2009*.
[16] *Bellucci 2017*, 16.
[17] *Bellucci 2017*.
[18] *Pietarinen and Bellucci 2016*.

This view of logic nevertheless invites the following concern. If '[a]ll reasoning consists for Peirce in the substitution of representations with other representations of the same object',[19] must logic abandon the aspiration for stability and permanence that has driven its study since at least Plato? Peirce could be said to advance a 'process philosophy'[20] and, by extension, a 'process philosophy of logic'.[21] The latter can be hard for many to envision. However, one of the virtues of Peirce's account is that it identifies reliable moments of stability amid the incessant flow of sign substitutions. Consider the following example:[22]

Rule: All men are mortal.
Case: Napoleon III is a man.
Therefore,
Result: Napoleon III is mortal.

This rule/case/result sequence fits the canonical format of a deductive inference.[23] Now, suppose that one questions the validity of this argument. Surely a philosopher committed to fallibilism[24] must be open to testing any claim, including the claim that the conclusion of a valid argument follows. What, then, licenses the logician's confidence that, given the truth of the rule and the truth of the case, the result must be true as well?

According to Peirce, confidence in the truth of the result pivots on the acceptance of an unstated claim, which we might state as follows: 'If a rule and a case falling under that rule are both true, then their result is true'. Grafting this new claim to the previous argument yields a new, enlarged, argument:[25]

Rule: If a rule and a case falling under that rule are both true, then their result is true.
Case: The rule ('All men are mortal') and case ('Napoleon III is a man') of the previous argument are both true.
Therefore,
Result: The result of the previous argument ('Napoleon III is mortal') is true.

We started by saying that a joint endorsement of the rule and the case commits one to endorsing the result. But, when we were pressed to justify why this holds, we invoked another rule and thereby turned the starting argument about Napoleon's mortality into a case falling under that rule. Clearly, one is entitled to ask why *this* new inference should be binding. The attempt to justify an argument thus requires that one argue again. Naturally, this engenders a regress. One can, if one wishes, continue invoking more rules to justify other rules. However, Peirce's insight is that, by the time one can formulate a second round of concern, one has effectively confirmed that the iterated pattern enjoys a special status. He summarizes the situation as follows:

> [L]et the premisses of any argument be denoted by *P*, the conclusion by *C*, and the leading principle by *L*. Then, if the whole of the leading principle be expressed as a premiss, the argument will become

[19] *Bellucci 2017*, 42.
[20] *Hausman 2002*.
[21] *Champagne and Pietarinen 2020*.
[22] *Bellucci 2016*, 255; *Bellucci 2017*, 25.
[23] *CP* 2.623.
[24] *Margolis 1998*.
[25] Like Peirce, I will express rules as conditionals and cases as antecedents of such conditionals.

L and *P*

∴ *C*

> But this new argument must also have its leading principle, which may be denoted by *L*'. Now, as *L* and *P* (supposing them to be true) contain all that is requisite to determine the probable or necessary truth of *C*, they contain *L*'. Thus *L*' must be contained in the leading principle, whether expressed in the premiss or not. Hence every argument has, as portion of its leading principle, a certain principle which cannot be eliminated from its leading principle. Such a principle may be termed a *logical principle*.[26]

Thus, I am free to ask why, given my acceptance of a rule and a case falling under it, I must accept the result. But, in so asking, I am essentially asking why, given my acceptance of a rule and a case falling under it, I must accept the result. These two questions are identical, which is the ground of the solution: a 'pure' rule is one that adds nothing. Indeed, 'Peirce's point is that the very setting in motion of the regress is the symptom that we have reached the end of logical analysis'.[27] Since the highest grade of mastery in the handling of signs is a habit, grasping a principle consists in realizing that the token rules generated by a regress are tokens of a type.

This Peircean account leaves interpreters free to substitute representations with other representations. However, when those substitutions conform to or instantiate a principle, the signs end up saying the same thing. Recursivity, not self-evidence, becomes the hallmark of logical certainty.

At one point in Lewis Carroll's dialogue between the Tortoise and Achilles, the sceptical character says 'force me, logically, to accept Z as true'.[28] The protagonist foolishly takes on this errand. Alas, logic is not the kind of topic that can be 'forced' unto anyone. This does not mean, however, that anything goes or that what underpins logic is mere social agreement.[29] Although the patterns catalogued by logicians differ from the patterns tracked by natural and social scientists, logicians must still back up their claims with some form of evidence. When one labels an argument as valid or invalid, there must be *something* one can point to in order to justify what one says.[30] The problem is that such a dispute 'cannot be remedied by any further explanatory signs [...]. The fact that further conditional statements are demonstrably useless if the Tortoise does not already understand entailment shows that what is seen by Achilles and not by the Tortoise must be *already present*, *internal* to the sign-act pair'.[31] The regress is ultimately halted by a recognition that further use of the logical principle as a premise produces only redundancy.

As Peirce puts it, '*logic* supposes that reasonings are *criticised*; and as soon as the reasoner asks himself what *warrant* he has for concluding from S is M that S is P, he is driven to *formulate* his leading principle'.[32] This formulation is yet another sign.

> But you might ask, why not express this *new* leading principle as a premise, and so obtain a *third argument* having a leading principle *still more* abstract? If, however, you try the experiment, you will find that the third argument so obtained has no

[26] *CP* 2.466.
[27] *Bellucci 2017*, 26.
[28] *Carroll 1895*, 279; see *Zhang 2017*, 2.
[29] *Dutilh Novaes 2015*.
[30] *Champagne 2016*.
[31] *Legg 2012*, 13; emphasis in original.
[32] *Peirce 1976*, 4, 175; emphasis in original.

> more abstract a leading principle than the second argument has. [. . .] This leading principle has therefore attained a *maximum degree* of abstractness [. . .].[33]

In this passage, Peirce invites us to 'try the experiment'. This reflects both his pragmatism and his view of logic as diagrammatic reasoning: 'By diagrammatic reasoning, I mean reasoning which constructs a diagram according to a precept expressed in general terms, performs experiments upon this diagram, notes their results, assures itself that similar experiments performed upon any diagram constructed according to the same precept would have the same results, and expresses this in general terms'.[34] Staying true to the question-and-answer format[35] of Peirce's argument and continuing his line of reasoning so that a diagrammatic pattern may emerge, we obtain a pyramidal structure with an ever-widening base:

L and P

$\therefore C$

Why? Because . . .

L' and (L and P)

$\therefore C$

Why? Because . . .

L'' and (L' and (L and P))

$\therefore C$

Why? Because . . .

L''' and (L'' and (L' and (L and P))

$\therefore C$

Why? Because . . .

L'''' and (L''' and (L'' and (L' and (L and P)))

Importantly, despite the (potentially endless) generation of new tokens, the propositional content of the logical principle L is the same each time. Some patterns are realized in the very act of questioning them. This speaks to a pattern's reliability (or at any rate inevitability), so Peirce reserved the label 'principle' for propositions that have this feature.

How much redundancy must take place before redundant manifestations of a principle are recognized *as* redundant? This is not for the philosopher of logic to say. However, the fact that such recognition is an individual accomplishment does not shroud it in privacy. On the contrary, as the pyramidal structure shows, requests to justify justification display a common pattern available for public scrutiny. If the sequence was rendered in acoustic logic,[36] it would almost sound like a tune on repeat play. The same goes for the dialogue

[33] *Peirce 1976*, 4, 175–6; emphasis in original.
[34] *Peirce 1976*, 4, 47–8.
[35] *Pietarinen 2006*, 77–102; *Hintikka 2007*.
[36] *Champagne 2015c*.

between Achilles and the Tortoise. Hence, not only does questioning give rise to a pattern, that pattern can be observed—in a suitably enlarged sense of 'observation'.[37]

A 'final' justification of a principle would only generate another token inference whose merit could be called into question, so Peirce wisely never 'justifies [his] assertion that our reasoning proceeds in accordance with such implicit principles', preferring instead to let those principles become 'evident to all those who reflect on their own reasoning'.[38] Hence, those turning to Peirce in search of explicit axioms will find something quite different.

Which view of logic should one adopt? If the foregoing considerations are correct, justifications of views on this central issue must ultimately culminate in a distinctive form of practical wisdom that 'operates in specific circumstances which cannot be anticipated by universal rules. No matter how clear the rule, we always apply it to particulars, which is a matter of seeing the right thing to do relative to the particular situation and actors'.[39] An appeal to what is present before one is an attempt to *cease* discourse, not extend it. As a result, '[s]eeing what to do in a situation does not stand in need of further justification, nor could it receive any—at least none that would persuade one who is lacking in *phronêsis*'.[40] So, when an argument fails to persuade, it is not always because of some fault in the argument.

In a comment that connects with Peirce's claim that 'all knowledge without exception comes from observation',[41] Thomson likens the failure to recognize a logical principle to 'a failure to appreciate the transitivity of the relation *sameness of length*'[42]—also a diagrammatic pattern that can be redundantly exhibited. Far from being contentious, such recognition is a keystone of staple systems like first-order predicate logic. Consider, for example, what justifies the universal generalization inference rule or UG. If we enumerate all that a universally-quantified claim like ∀xFx picks out, we obtain a long conjunction: Fa *and* Fb *and* Fc *and* Fd and so on (where the particulars named vastly outnumber the letters in the alphabet). Suppose, then, that we have ∀x(Gx → Fx) and ∀xGx as premises and wish to prove that the predicate '… is F' is true of all particulars. Schematically, an enumerative proof would resemble Table 1.

It becomes—or should become—apparent by line 11 or 15 that these sub-proofs repeat themselves. The derivation, however, will continue until we exhaust all the particulars that the universal conclusion ranges over. So, even though a derivation of each sub-conclusion is easy, we will never be able to show line 1 and box all the lines beneath it. Like Zeno's paradox of motion, we will never reach the finish line.

UG authorizes us to skip such an exhaustive demonstration, provided that a pattern of inference holds and the conclusion reached in that pattern holds for any arbitrarily-selected individual. Both conditions are met: not only is a pattern apparent, there is nothing special about the individual 'a'. So, instead of engaging in a rigmarole, we can cut to the chase and generalize the result obtained at line 4—thereby completing the derivation (Table 2).

There are philosophical problems with the inference rule deployed at line 8 of Table 2.[43] Still, the demonstrative strategy of UG has been part of natural deduction since its inception.[44] Without this rule (or something similar, such as universal derivation), predicate logic as we know it would not be feasible. The example of UG is telling because a

[37] *Hull and Atkins 2017*; *Legg and Franklin 2017*.
[38] *Hitchcock 2003*, 70–1.
[39] *Braver 2012*, 169.
[40] *Braver 2012*.
[41] *Peirce 1976*, 4, 48.
[42] *Thomson 2011*, 24.
[43] *Cellucci 2009*.
[44] *Gentzen 1969*, 78.

Table 1. Incomplete derivation of universal claim by enumeration.

1.	Show	∀x	Fx		Assertion (not yet derived)
2.	∀x	(Gx	→	Fx)	Premise
3.	∀x	Gx			Premise
4.	~~Show~~	Fa			Assertion (directly derived)
5.	Ga	→	Fa		2, UI (x to a)
6.	Ga				3, UI (x to a)
7.	Fa				5, 6, MP
8.	~~Show~~	Fb			Assertion (directly derived)
9.	Gb	→	Fb		2, UI (x to b)
10.	Gb				3, UI (x to b)
11.	Fb				9, 10, MP
12.	~~Show~~	Fc			Assertion (directly derived)
13.	Gc	→	Fc		2, UI (x to c)
14.	Gc				3, UI (x to c)
15.	Fc				13, 14, MP
16.	Show	Fd			Assertion
…	…				…

Table 2. Completed derivation of universal claim by generalization.

1.	~~Show~~	∀x	Fx		Assertion (directly derived)
2.	∀x	(Gx	→	Fx)	Premise
3.	∀x	Gx			Premise
4.	~~Show~~	Fa			Assertion (directly derived)
5.	Ga	→	Fa		2, UI (x to a)
6.	Ga				3, UI (x to a)
7.	Fa				5, 6, MP
8.	∀x	Fx			4, UG (a to x)

similar pattern-based shortcut is at the core of the Peircean account: pick any argument you wish, question it, and a pattern will manifest itself. Rephrasing Peirce's reasoning in information-theoretic terms, we might say that if a pattern can be identified, then it can be *compressed*; and if it can be compressed, then it is *real*.[45] Coupled with such realism, a commitment to rationality does not commit one to making endless replies. Since reality is mind-independent, one can at some point walk away from a justification to let the world do the rest.

The ontology and grasp of a logical principle must square with what other sciences besides logic are telling us, so it cannot hinge on some supernatural domain or special intuitive faculty.[46] Since Peirce never relinquished his early contention that 'all thought is in signs',[47] his account never pretends to come into contact with some extra-semiotic realm where the truths of logic are somehow 'intuited' or 'given'. In the end, the distinctive finality of logical principles does not stem from their ability to stop the production of further signs, but from their ability to stop the production of further *informative* signs. We can literally *see* when this transition occurs.

The question of how many iterations are needed before someone realizes that token appeals to rules instantiate a type is a matter of personal psychology. Even so, one virtue of Peirce's account is that it specifies the defining attribute of a principle independently

[45] *Dennett 1991*.
[46] *Benacerraf 1973*.
[47] *CP* 5.253.

of any psychological recognition. Fallibilism is not scepticism, so the demand to justify logic can be met. Amid the incessant interpretive turmoil of sign-action, this discernment of recurring patterns satisfies (in a naturalist way) the craving for permanence and stability that has driven interest in logic since Plato.

3. The Inevitable Pattern of Bivalence

I want to argue that the account of logical principles just sketched allows for a defence of bivalence and, by extension, the excluded-middle.[48] Bivalence is the view that any well-formed claim admits of two and only two possible truth-values, truth and falsity. It is usually called *the principle of* bivalence. I think bivalence deserves to be ranked as a principle for precisely the reasons that Peirce brought to light, since it must be relied on in the very act of questioning it. To see this, consider the following argument:

Rule: If something is a premise in an argument, then it is true or false.
Case: 'Napoleon III is a man' is a premise in an argument.
Therefore,
Result: 'Napoleon III is a man' is true or false.

The truth of this argument's second premise can be verified by looking at the previous section. What about the first premise? Is it true that 'The premises in an argument are either true or false'? Suppose that one takes this claim to be true. Grafting this new claim to the previous argument yields a new, enlarged, argument:

Rule: If something is a premise in an argument, then it is true or false.
Case: 'If something is a premise in an argument, then it is true or false' is a premise in the previous argument.
Therefore,
Result: 'If something is a premise in an argument, then it is true or false' is true or false.

I submit that the defence of bivalence just sketched is just as tenable as the previous defence of logical entailment.

Some critics of bivalence may flirt with relativism, but Peirce was crystal clear about what led him to consider a limit case between truth and falsity:

> [A]lthough it is true that 'Any proposition you please, *once you have determined its identity*, is either true or false;' yet *so long as it remains indeterminate and so without identity*, it need neither be true that any proposition you please is true, nor that any proposition you please is false. So likewise, while it is false that 'A proposition *whose identity I have determined* is both true and false', yet until it is determinate, it may be true that a proposition is true and that a proposition is false.[49]

This strikes me as perfectly sensible. While on a rapid reading Peirce seems to endorse truth-value 'gaps' and 'gluts',[50] nothing in his remarks requires one to abandon the principle of bivalence or excluded-middle. If I believe in truth and falsity, but I also believe that I sometimes don't know which truth-value to assign, does this acknowledgment of my own ignorance mean that I am somehow committed to some third truth-value? Surely it would be illicit to saddle me with this extra commitment.

[48] *Sayward 1989*.
[49] *CP* 5.448; emphasis in original.
[50] *Priest 1995a*.

Lane agrees that 'Peirce's logico-semantics for object-indeterminate propositions is neither mysterious nor absurd, nor a threat to [the principle of bivalence]'.[51] As Peirce writes, '[t]here are two conceivable certainties with reference to any hypothesis, the certainty of its truth and the certainty of its falsity', such that any 'values intermediate between them indicate, as we may vaguely say, the degrees in which the evidence leans toward one or the other'.[52] When we encounter a propositional sign[53] that does not allow us to ascertain its truth or falsity, we retain 'the privilege of carrying its determination further',[54] either by reformulating that proposition, engaging in further inquiry, and/or waiting (in the case of future contingents). Yet, even when we do none of those things, a proposition *about* an indeterminate proposition will be fully determinate. For example, when a slot machine wheel is whirling, one cannot (yet) tell which icon it will land on, so one's inability to assign a truth-value to 'The wheel lands on cherries' is an artifact of one's epistemic position. However, the proposition 'One cannot tell whether the slot machine wheel lands on cherries' is true and the proposition 'One can tell whether the slot machine wheel lands on cherries' is false—with no third truth-value in between.

Just as the very act of questioning a valid deduction compels one to rely on a valid deduction, questioning whether a proposition must be true or false compels one to employ a proposition that must be true or false. This can seem like a circle. But, when adjudicating between methods of adjudicating inferences, 'it is certainly difficult to see how a proof of the soundness of one's preferred rules could be other than rule-circular'.[55] Patterns that are inescapable have a good claim of being foundations, if anything does. That is what we seek in a 'foundation', practically speaking: something we can always count on. This, at any rate, is how Peirce sees the situation. I agree with him. I merely think his account of principles also vindicates bivalence against contemporary attacks (that Peirce could never have foreseen).

Peirce was led by his general interest in continuity to experiment with a 'limit' between truth and falsity. According to what he called 'synechism', every sharp distinction that we make is in fact an oversimplification of a continuous reality. As Peirce writes, '[n]o sane man can dream that the ratio of the circumference to the diameter could be exactly ascertained by measurement'.[56] So, on the hypothesis that the world is inherently continuous[57] or analog, any discontinuous or digital representation necessarily does injustice to what(ever) it represents. This might generalize to logic. Hence, as Peirce explains,

> I have long felt that it is a serious defect in existing logic that it takes no heed of the *limit* between two realms. I do not say that the Principle of Excluded Middle is downright *false*; but I *do* say that in every field of thought whatsoever there is an intermediate ground between *positive assertion* and *positive negation* which is just as Real as they.[58]

Continuity is not the only reason to consider a third truth-value between truth and falsity. Although such 'limit-propositions' are not themselves modal propositions,[59] 'Peirce's discovery of triadic logic was motivated by his consideration of triadic modality'[60] of

[51] *Lane 1998*, 51.
[52] *CP* 2.647.
[53] *Stjernfelt 2014*.
[54] *CP* 5.447.
[55] *Rumfitt 2015*, 3.
[56] *CP* 1.172.
[57] *Vargas and Moore 2021*.
[58] *Peirce 1976*, 3, 851; emphasis in original. Quoted in *Lane 1999*, 290.
[59] *Lane 1998*, 208.
[60] *Fisch and Turquette 1966*, 79.

possibility, actuality, and necessity. For instance, in the Gamma portion of his Existential Graphs,[61] Peirce accommodates modality by adding the 'broken cut' to the notational mix. Yet, Peirce's interpretation of the broken cut was always epistemic, not alethic.[62] This is important, because a sign expressing possible-that-P does not have as its object a *tertium quid* between the true and the false. Rather, when a proposition like 'It rains' is surrounded by a broken cut, then the resulting graph 'only asserts that the alpha and beta rules do not compel me to admit that it rains, or what comes to the same thing, a person altogether ignorant, except that he was well versed in logic [. . .] *would not know* that it rained'.[63]

A system of logic can certainly be rich enough to express both knowledge and ignorance. In fact, many logics capable of expressing a lack of knowledge[64] take their inspiration from Peirce.[65] But, Peirce rightly warns that 'we should fall into inextricable confusion in dealing with the broken cut if we did not attach to it a sign to distinguish the particular state of information to which it refers'.[66] Were one fully informed, a given proposition would fall squarely into the class of true propositions or the class of false propositions.[67]

This account thus dovetails with Peirce's 'convergence' theory of truth,[68] according to which truth is the asymptotic limit that a community of inquirers *would* reach if and when all defeaters have been eliminated. The kinship can be made visible by asking whether the claim 'Logical principles can be demarcated from non-logical ones' is true. Peirce clearly intends it to be. Like any other truth, that truth can (a) be called into question yet (b) should get confirmed in the long run. Not only does bivalence satisfy (a) and (b), it is essentially baked into a self-correcting convergence towards truth. For, if there are unsettled truth-values at the end of inquiry, we are not at the end yet. Or, to put the same moral in terms of Peirce's diagrammatic logic, a sheet of assertion representing all claims should contain no unenclosed dotted line.

It would of course be baseless confidence to assert that all currently unsolved problems admit of solutions,[69] such that true propositions about those problems or solutions always await. Bivalence is neither a cure-all nor a substitute for inquiry. Even so, when glossed as a regulative assumption[70] and/or implicit hope,[71] bivalence prompts inquirers never to rest content with unclear verdicts and thus to keep searching. So, if we rule out unwarranted doubts, as Peirce recommends we do,[72] it is not senseless to think that every well-formed claim is squarely true or false. Certainly the truth of a claim about a claim (e.g. 'I used the word "squarely" in the previous sentence') can be verified without much effort. Since this is all that is required in order to exhibit the principle of bivalence, a demonstration is always within easy reach.

Humans nevertheless have a capacity to call into question anything. We can thus question the reliability of what we rely on. However, the fact that one has effectively answered one's own question is something that one must realize for oneself. Although one would

[61] *Roberts 1973*, 64–86.
[62] *Ma and Pietarinen 2018*, 3642.
[63] *Peirce 2020, 448*; emphasis added.
[64] *Hintikka 1962*.
[65] *Pietarinen 2019*.
[66] *Peirce 2020*, 450.
[67] *Dummett 1959*, 149–50. Note that semantic paradoxes do not have to pose the level of worry often attributed to them. See, for instance, the Peirce-inspired argument developed by *Prior 1976*, 141.
[68] *Legg 2014*.
[69] *Raatikainen 2004*, 133.
[70] *Howat 2013*, 453–5.
[71] *Misak 2004*, 157; *Lane 2018*, 181.
[72] *CP* 5.265.

hope that such a realization will take less time than more, Peirce's account lets each inquirer test logic and glean the results of those tests, at their own pace.

It may be that, historically, bivalence has tended to coast on the authority of Aristotle. But, if my interpretation is correct, this widely-held principle finally gets a vindication with the ideas of 'the American Aristotle'.[73]

4. Some Possible Objections

One could object that Peirce's arguments for logical principles do not apply to bivalence. The root of such a worry would be that, whereas *modus ponens* is a syntactic structure, bivalence has 'semantic' implications that require a different treatment. Consider, for instance, the rule to the effect that 'If something is a premise in an argument, then it is true or false'. It does seem that, when this rule figures as a premise in an argument for bivalence, the argument begs the question. Yet, in response to this concern, one could say the same for the original argument by Peirce, which also seems to beg the question in favor of *modus ponens*. No red flags were raised there.

While Peirce illustrated his original argument with the *modus ponens* inference rule, the philosophical issue he addressed is so fundamental that his solution yields dividends in other debates.

> What makes an instance of reasoning not only sound but *evident* to reason? These are central questions in the philosophy of logic still today. Peirce addresses and cleverly refutes the so-called paradox of the justification of deduction, according to which reasoning that rests upon logical principles must not itself be used to justify those principles. Peirce's solution is that the *statement* of the principle of reasoning, which he had termed leading or guiding principle, and *reasoning according to* that principle are not the same thing: the former is an abstract statement of the principle in another form than the form in which the principle according to which we reason is being used.[74]

By analogy: just as the activity of walking, not the verb 'to walk', is what gets one from one point to another, the activity of reasoning proceeds just fine irrespective of whether it is rendered in language. We may name an activity and display what a verb like 'reasoning' or 'walking' refers to, but ultimately this will involve showing, not telling.

A purely deductive justification of deduction is thus out of the question. Moreover, I agree with Peirce that, no matter where our inquiries end up, '[w]e must begin with all the prejudices which we actually have when we enter upon the study of philosophy'.[75] Yet, as we have seen, assumptions that continually re-surface with each round of questioning are elevated, in the best case scenario, to the status of principles. Seeing how those assumptions were present from the start, their subsequent vindication will always be open to a circularity complaint.[76] This, however, is to be expected. After all, a ubiquitous pattern cannot be proven without resorting to that pattern, on pain of no longer being ubiquitous.

One might object that circles can vindicate invalid patterns as easily as valid ones. It is unclear, however, whether fanciful counter-examples[77] still work when expressed with a diagrammatic notation. In any event, I do not see why the circularity involved in the defence of *modus ponens* should be regarded as virtuous whereas the circularity involved

[73] *Everett forthcoming*.
[74] *Pietarinen 2020*, 135–6; emphasis in original.
[75] *CP* 5.265.
[76] *Dummett 1978*, 291–2; *Haack 1976*.
[77] Cf. the *modus morons* of *Haack 1996*, 186–7.

in the defence of bivalence should be regarded as vicious. By parity of reasoning, either both circles are acceptable, or neither are.

When a stubborn interlocutor in a Tortoise-and-Achilles-style dialogue finally gets it, her insight is retroactively applied, since the pattern was present all along. It is precisely because the principles catalogued by the logician are operative even in untutored reasoning[78] that, '[i]n everyday business, reasoning' can be 'done as well without the aid of theory as with it'.[79] Schooling can nevertheless make the obvious seem remote. Since there will always be holdouts, must an account of a principle like bivalence convince *everyone* in order to be tenable? Like Peirce, I do not think philosophy is tasked with responding 'to a generalized scepticism [. . .] motivated by the mere *possibility* that our logical laws and principles might be invalid or false'.[80] The adversaries I countenance must therefore make positive commitments.[81] Such commitment or claim-making is crucial, since it provides a way to adjudicate which view is correct. When a regress begins, the principle that the regress instantiates may not be initially apparent. It is in the next step, when one states and employs the logical principle as a premise, that one obtains an argument whose logical principle is the same as the preceding one.[82] Hence, one can initially put forward an argument that denies the very principle(s) that enable(s) it, such as the following:

Rule: If something is a premise in an argument, then it is true, false, or indeterminate.
Case: 'Napoleon III is a man' is a premise in an argument.
Therefore,
Result: 'Napoleon III is a man' is true, false, or indeterminate.

Yet, asking why the result ought to be accepted prompts this next argument:

Rule: If something is a premise in an argument, then it is true, false, or indeterminate.
Case: 'If something is a premise in an argument, then it is true, false, or indeterminate' is a premise in an argument (above).
Therefore,
Result: 'If something is a premise in an argument, then it is true, false, or indeterminate' is true, false, or indeterminate.

One could perhaps be brazen enough to assert that a non-classical account is neither true nor false, or both. Yet, the proponent of a non-classical account clearly intends her account to be *true* (otherwise what is she a *proponent* of?). To secure that truth, however, one must deny not just its falsity—which is sensible enough—but also whatever truth-value allegedly falls outside of the classical pair. For, if the claim 'If something is a premise in an argument, then it is true, false, or indeterminate' is itself false or indeterminate, then the initial argument is no longer sound. The proponent of trivalence must therefore denature or abandon her commitment in the very attempt to defend it. Importantly, such a denaturing or abandonment is not required in an argument for bivalence. That is why Peirce had the wherewithal to end his pioneering notes by stating that 'Triadic Logic is Universally true'.[83]

[78] *CP* 2.204.
[79] *CP* 1.623.
[80] *Howat 2014*, 482; see *Champagne 2015a*.
[81] *Brandom 1994*.
[82] *Bellucci 2017*, 43.
[83] Manuscript *R*339; reproduced in *Fisch and Turquette 1966*, 75.

The pragmatic maxim recommends that we focus on the 'practical bearings' of our ideas.[84] Heeding that recommendation, the more critics of bivalence insist that what they have concluded about that principle is *true*, the less I am inclined to believe them.

5. Conclusion

Currently in the mainstream, '[w]hat is generally credited to Peirce are truth-tables and the discovery of three-valued logic'.[85] Scholarship is progressing, so more innovations are likely on the way. In this vein, I have argued that—surprisingly perhaps—we can use ideas in Peirce's corpus to vindicate bivalence.

The argument I have presented ties together the early and late bookends of Peirce's intellectual career. In a manuscript written five years before his death in 1914, Peirce observed that '[a]n argument is simply a construction of premisses which constitute a *sign* of the truth *of the truth* of its conclusion, no matter what kind of reasoning it uses [. . .]'.[86] There is a growing appreciation that '[t]his is the semiotic conception of inference that Peirce had adopted since 1865'.[87] Staying true to his early contention that 'all thought is in signs',[88] Peirce saw that, amid the open-ended generation of signs, some logical patterns continually get confirmed. He dubbed such patterns principles. We may express a principle with a proposition, but even the clearest proposition cannot pre-empt questioning. Such questioning, however, produces more grist to the mill. Peirce and his interpreters take this inescapability to vindicate logical entailment, but I have shown how it can also vindicate bivalence.

If Peirce is right that '[t]he *final* upshot of thinking is the exercise of volition, and of this thought no longer forms a part',[89] then even a perfect argument would not make voluntarism vanish.[90] Additionally, signs can always beget more signs, so '[f]ar from identifying meaning with a finite list of verification conditions, Peirce's pragmatism entails that there can be no end to their enumeration'.[91] As a result, one remains free to criticize my arguments. Such criticisms, however, supply endless opportunities to verify the principle in question. Indeed, any objection to my arguments cannot be formulated without appealing to premises—notably 'The author defends bivalence'—that are regarded as *true*. I have argued that this inevitable reliance makes bivalence a principle, in the strictest sense.

In inferences and in reality at large, a pattern does not become more real when it is spotted and named. So, while stating a principle can have pedagogical and rhetorical value, we should not be led 'by feelings of loyalty to the old idea that some arguments have suppressed premises'[92] to state and re-state every principle that we rely on in valid reasoning. Some things should and indeed must go unmentioned. So, I am tempted to say that Peirce provides a 'quietist' grounding of the laws of logic.[93] However, this label is misleading, since quite few arguments may be required before a silent recognition of their shared pattern can occur. This generation of multiple instances suggests that Peirce supplies a 'process-philosophical' account, but here again this label fails to do justice to the

[84] *CP* 5.402.
[85] *Béziau 2012*, 251.
[86] Manuscript *R*637, 30; quoted in *Bellucci 2017*, 354.
[87] *Bellucci 2017*, 354.
[88] *CP* 5.253.
[89] *CP* 5.397; see *Champagne 2014*.
[90] *Champagne 2015b*.
[91] *Short 2007*, 288.
[92] *Thomson 2011*, 23.
[93] *Howat 2014*.

fact that unchanging patterns can be discerned. The account might instead be called 'pragmatic', because principles like bivalence are revealed in *use* (not mention); and 'semiotic', because this revelation happens in the use of *signs*.

I think the signs that I have strung together suffice to make their point. Still, as any teacher knows, logic cannot be 'forced' unto anyone, so the question of how many iterations are needed before someone realizes the inevitability of a principle like bivalence is a matter of personal psychology.

Acknowledgements

I want to thank Emily Beattie, Francesco Bellucci, Jeffrey Brian Downard, David Gilbert, Diana Heney, Travis LaCroix, Robert Lane, Ahti-Veikko Pietarinen, Chris Stephens, audience members at the meetings of the Charles S. Peirce Society and the Canadian Philosophical Association, students in my Advanced Formal Logic course, as well as anonymous reviewers for this journal.

ORCID


Marc Champagne http://orcid.org/0000-0002-9841-7538

References

Anellis, I. H. 2012. 'Peirce's truth-functional analysis and the origin of the truth table', *History and Philosophy of Logic*, **33**, 87–97.
Bellucci, F. 2016. 'Charles S. Peirce and the medieval doctrine of *consequentiae*', *History and Philosophy of Logic*, **37**, 244–68.
Bellucci, F. 2017. *Peirce's Speculative Grammar: Logic as Semiotics*, New York: Routledge.
Benacerraf, P. 1973. 'Mathematical truth', *Journal of Philosophy*, **70**, 661–79.
Béziau, J.-Y. 2012. 'A history of truth-values', in D. M. Gabbay, F. J. Pelletier, and J. Woods (eds.), *Handbook of the History of Logic, Volume 11: A History of its Central Concepts*, Amsterdam: Elsevier, 235–307.
Brady, G. 2000. *From Peirce to Skolem: A Neglected Chapter in the History of Logic*, Amsterdam: North-Holland.
Brandom, R. B. 1994. *Making It Explicit: Reasoning, Representing, and Discursive Commitment*, Cambridge, MA: Harvard University Press.
Braver, L. 2012. *Groundless Grounds: A Study of Wittgenstein and Heidegger*, Cambridge, MA: MIT Press.
Carroll, L. 1895. 'What the Tortoise said to Achilles', *Mind*, **4**, 278–80.
Cellucci, C. 2009. 'The universal generalization problem', *Logique & Analyse*, **52**, 3–20.
Champagne, M. 2009. 'Some semiotic constraints on metarepresentational accounts of consciousness', in J. N. Deely and L. G. Sbrocchi (eds.), *Specialization, Semiosis, Semiotics*, Toronto: Legas Press, 557–64.
Champagne, M. 2014. 'Just do it: Schopenhauer and Peirce on the immediacy of agency', *Symposium*, **18**, 209–32.
Champagne, M. 2015a. 'Disjunctivism and the ethics of disbelief', *Philosophical Papers*, **44**, 139–63.
Champagne, M. 2015b. 'Don't be an ass: rational choice and its limits', *Reason Papers*, **37**, 137–47.
Champagne, M. 2015c. 'Sound reasoning (literally): prospects and challenges of current acoustic logics', *Logica Universalis*, **9**, 331–43.
Champagne, M. 2016. 'Brandom, Peirce, and the overlooked friction of contrapiction', *Synthese*, **193**, 2561–76.
Champagne, M., and A.-V. Pietarinen. 2020. 'Why images cannot be arguments, but moving ones might', *Argumentation*, **34**, 207–36.
Church, A. 1956. *Introduction to Mathematical Logic, 1*, Princeton: Princeton University Press.
Dennett, D. C. 1991. 'Real patterns', *The Journal of Philosophy*, **88**, 27–51.
Dummett, M. A. E. 1959. 'Truth', *Proceedings of the Aristotelian Society*, **59**, 141–62.
Dummett, M. A. E. 1978. *Truth and Other Enigmas*, Cambridge, MA: Harvard University Press.
Dutilh Novaes, C. 2015. 'A dialogical, multi-agent account of the normativity of logic', *Dialectica*, **69**, 587–609.
Everett, D. L. Forthcoming. *The American Aristotle: The Life and Mind of C. S. Peirce*, Oxford: Oxford University Press.
Fisch, M., and A. Turquette. 1966. 'Peirce's triadic logic', *Transactions of the Charles S. Peirce Society*, **2**, 71–85.
Gentzen, G. 1969. *The Collected Papers of Gerhard Gentzen*, M. E. Szabo (ed.), Amsterdam: North-Holland.
Haack, S. 1976. 'The justification of deduction', *Mind*, **85**, 112–9.
Haack, S. 1996. *Deviant Logic, Fuzzy Logic: Beyond the Formalism*, Chicago: University of Chicago Press.
Hausman, C. R. 2002. 'Charles Peirce's evolutionary realism as a process philosophy', *Transactions of the Charles S. Peirce Society*, **38**, 13–27.
Hintikka, J. 1962. *Knowledge and Belief: An Introduction to the Logic of the Two Notions*, Ithaca: Cornell University Press.
Hintikka, J. 2007. *Socratic Epistemology: Explorations of Knowledge-Seeking by Questioning*, Cambridge, UK: Cambridge University Press.

Hitchcock, D. 2003. 'Toulmin's warrants', in F. H. V. Eemeren, J. A. Blair, C. A. Willard, and A. F. S. Henkemans (eds.), *Anyone Who Has a View: Theoretical Contributions to the Study of Argumentation*, Dordrecht: Springer, 69–82.
Howat, A. 2013. 'Regulative assumptions, hinge propositions and the Peircean conception of truth', *Erkenntnis*, **78**, 451–68.
Howat, A. 2014. 'Peirce on grounding the laws of logic', *Transactions of the Charles S. Peirce Society*, **50**, 480–500.
Hull, K. A., and R. K. Atkins (eds.). 2017. *Peirce on Perception and Reasoning: From Icons to Logic*, New York: Routledge.
Lane, R. 1998. *Charles Sanders Peirce and the Principle of Bivalence*, PhD diss. University of Miami.
Lane, R. 1999. 'Peirce's triadic logic revisited', *Transactions of the Charles S. Peirce Society*, **35**, 284–311.
Lane, R. 2018. *Peirce on Realism and Idealism*, Cambridge, UK: Cambridge University Press.
Legg, C. 2012. 'The hardness of the iconic must: can Peirce's existential graphs assist modal epistemology?', *Philosophia Mathematica*, **20**, 1–24.
Legg, C. 2014. 'Charles Peirce's limit concept of truth', *Philosophy Compass*, **9**, 204–13.
Legg, C., and J. Franklin. 2017. 'Perceiving necessity', *Pacific Philosophical Quarterly*, **98**, 320–43.
Ma, M., and A.-V. Pietarinen. 2018. 'Gamma graph calculi for modal logics', *Synthese*, **195**, 3621–50.
Margolis, J. 1998. 'Peirce's fallibilism', *Transactions of the Charles S. Peirce Society*, **34**, 535–69.
Misak, C. J. 2004. *Truth and the End of Inquiry: A Peircean Account of Truth*, Oxford: Oxford University Press.
Peirce, C. S. 1931–58. *The Collected Papers of Charles Sanders Peirce*, 8 vols., C. Hartshorne, P. Weiss, and A. W. Burks (eds.), Cambridge, MA: Harvard University Press. Cited as CP, giving volume number and paragraph.
Peirce, C. S. 1976. *The New Elements of Mathematics*, 4 vols., C. Eisele (ed.), The Hague: Mouton.
Peirce, C. S. 2019. *Logic of the Future: Writings on Existential Graphs, 1: History and Applications*, A.-V. Pietarinen (ed.), Berlin: De Gruyter.
Peirce, C. S. 2020. *Logic of the Future: Writings on Existential Graphs, 2: The 1903 Lowell Lectures*, A.-V. Pietarinen (ed.), Berlin: De Gruyter.
Pietarinen, A.-V. 2006. *Signs of Logic: Peircean Themes on the Philosophy of Language, Games, and Communication*, Dordrecht: Springer.
Pietarinen, A.-V. 2019. 'To Peirce Hintikka's thoughts', *Logica Universalis*, **13**, 241–62.
Pietarinen, A.-V. 2020. 'Introduction to volume 2: the 1903 Lowell lectures', in A.-V. Pietarinen (ed.), *Logic of the Future: Writings on Existential Graphs, 2: The 1903 Lowell Lectures*, Berlin: De Gruyter, 67–199.
Pietarinen, A.-V., and F. Bellucci. 2016. 'Habits of reasoning: on the grammar and critics of logical habits', in D. E. West and M. Anderson (eds.), *Consensus on Peirce's Concept of Habit: Before and Beyond Consciousness*, Cham: Springer, 265–82.
Priest, G. 1995a. 'Gaps and gluts', *Canadian Journal of Philosophy*, **25**, 57–66.
Priest, G. 1995b. *Beyond the Limits of Thought*, Oxford: Oxford University Press.
Priest, G. 2007. 'Paraconsistency and dialetheism', in D. M. Gabbay and J. Woods (eds.), *Handbook of the History of Logic, Volume 8: The Many Valued and Nonmonotonic Turn in Logic*, Amsterdam: Elsevier, 129–204.
Prior, A. N. 1976. *Papers in Logic and Ethics*, London: Duckworth.
Quine, W. V. O. 1986. *Philosophy of Logic*, Cambridge, MA: Harvard University Press.
Raatikainen, P. 2004. 'Conceptions of truth in intuitionism', *History and Philosophy of Logic*, **25**, 131–45.
Roberts, D. D. 1973. *The Existential Graphs of Charles S. Peirce*, The Hague: Mouton.
Rumfitt, I. 2015. *The Boundary Stones of Thought: An Essay in the Philosophy of Logic*, Oxford: Oxford University Press.
Sayward, C. 1989. 'Does the law of excluded middle require bivalence?, *Erkenntnis*, **31**, 129–37.
Short, T. L. 2007. *Peirce's Theory of Signs*, Cambridge, UK: Cambridge University Press.
Stjernfelt, F. 2014. *Natural Propositions: The Actuality of Peirce's Doctrine of Dicisigns*, Boston: Docent Press.
Suszko, R. 1975. 'Remarks on Łukasiewicz's three-valued logic', *Bulletin of the Section of Logic*, **4**, 87–90.
Thomson, J. F. 2011. 'What Achilles should have said to the Tortoise', in S. M. Cahn, R. B. Talisse, and S. F. Aikin (eds.), *Thinking about Logic: Classic Essays*, Boulder: Westview Press, 19–33.
Van Benthem, J., G. Heinzmann, M. Rebuschi, and H. Visser (eds.). 2006. *The Age of Alternative Logics: Assessing Philosophy of Logic and Mathematics Today*, Dordrecht: Springer.
Vargas, F., and M. E. Moore. 2021. 'The Peircean continuum', in S. Shapiro and G. T. Hellman (eds.), *The History of Continua: Philosophical and Mathematical Perspectives*, Oxford: Oxford University Press, 328–46.
Zhang, L. 2017. 'On the justification problems: towards a Peircean diagnosis and solution', *History and Philosophy of Logic*, **38**, 222–38.