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Utilizing Complex Conceptual Spaces Modeling for Space Situational Awareness

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The overall goal of the approach developed in this paper is to estimate the likelihood of a given kinetic kill scenario between hostile space-based adversaries using the mathematical framework of Complex Conceptual Spaces – Single Observation. Conceptual spaces are a cognitive model that provide a method for systematically and automatically mimicking human decision making. For accurate decisions to be made, the fusion of both hard and soft data into a single decision framework is required. This presents several challenges to this data fusion framework. The first is the challenge involved in handling multiple complex terminologies, which is addressed by drawing on a set of Space Domain Ontologies. Another challenge is the complex combinatorics involved when considering all possible feature combinations. This can be mitigated by using integer linear programming optimization that is outlined by the Complex Conceptual Spaces – Single Observation mathematical model framework. A third challenge is the complicated physics that is involved in a spacecraft collision that must be addressed to obtain a better understanding of threat assessment. Overcoming these various challenges allows for a quantitative ranking for the potential of a kinetic kill collision across multiple spacecraft pairs. In addition to overcoming these challenges this paper will break down threat assessment into four domains and identify a ranking of threat both for each individual domain and for the four domains combined. Simulation results are shown to verify the developed concepts.

I. Introduction

This paper is written to utilize the conceptual spaces (CS) framework, developed in [1], for the fusion of different classifications of data to support an automated anticipatory decision support framework to be used by space operators. The overall objective for this sort of fusion of space event data is to increase Space Situational Awareness (SSA). Although space is currently not considered a hostile zone, readiness for the threat of an orchestrated intentional spacecraft collision caused by hostile parties should be established. As the number of threats in space increases it is becoming progressively more difficult to identify threats in realtime. Throughout this paper spacecraft assumed to pose a threat will be referred to as *Red spacecraft*, and the assumed targets that need to be protected will be referred to as *Blue spacecraft*. The methods described herein measure and rank the threat of a kinetic kill space event, which in this context means an intentional act of a physical collision of a pair of spacecraft that is carried out with the intention of damaging or destroying the target spacecraft.

This paper expands upon the work presented in [2], with the addition of exploring separate rankings for each individual domain for spacecraft pairs. These rankings give the user additional insight into the threat presented by an observed spacecraft pair. This paper utilizes CS to mimic human decision making to assess which Red spacecraft are

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most capable of a kinetic kill to a pre-specified Blue spacecraft. When humans make decisions, they use an abundant amount of potentially helpful information; including measurable data, contextual data, their own human intuition, etc. For space events this information includes observation data that is derived from sensors, such as radar and telescopes, that are used to determine a spacecraft's position and photometry information. Measurable information of these types are examples of *hard data*. Other information about spacecraft that is derived from human perception, judgment and analysis, instead of directly from sensor measurements, are commonly referred to as *soft data*. This type of data is often in textual form. Some examples of soft data include, but are not limited to, social political aspects of the relationship between Red and Blue spacecraft countries of ownership as well as a spacecraft component systems and thruster type. Human derived information is not easily quantified, and its uncertainty is also difficult to quantify. This paper presents a method of utilizing CS to illustrate that both types of data can be exploited to establish the relative threat posed by a given Red to a given Blue spacecraft.

There are a number of data fusion challenges addressed in this paper. One challenge is the task of organizing complex data and data terminology used in the characterization of space events and space threats, especially when that data is from multiple sources and is heterogeneous in nature. Another challenge is the complex physics of a spacecraft-pair collision. The physics that describe the likelihood of a collision is described by *Lambert's Problem*, which is the problem of determining the orbit between endpoints when the two position vectors and the time of flight between each spacecraft's position are known [3]. Another is addressing the underlying mathematics of CS. Conceptual spaces by themselves were originally created as a cognitive model for information fusion, but do not detail the underlying mathematics. For CS to be utilized as a tool, a mathematical framework needs to be implemented. These challenges will be addressed in detail throughout this paper.

II. Complex Terminology

The strategy used to address the challenge of complex terminologies is to utilize an ontology. For present purposes, an Ontology is defined as a controlled vocabulary of terms that are organized hierarchically and curated by experts. Ontologies make information more accessible and discoverable by tagging, and thereby semantically enhancing, items of data in large information systems. Their use increases interoperability between people, organizations, systems and machines, thereby making stored data more findable and analyzable by both humans and computers. Ontologies provide the semantics required to represent both soft and hard data in a structured linked knowledge base, enabling fusion of data from multiple sources and in multiple formats.

For this work data is tagged using the Space Domain Ontologies (SDO), which are a set of modules that are extensions of the Common Core Ontology (CCO) [4, 5]. The SDO have been developed over the course of 5 years by CUBRC Inc. with input from researches at the University at Buffalo [6]. The SDO use international standard ISO/IEC 21838-2 Basic Formal Ontology (BFO) [7, 8] as their top-level ontology. An illustration of the structure of CCO and how the SDO fits into this structure can be seen in Figure 1.

The SDO currently contain more than 800 classes that represent entities of a wide range of different types. Space object data is aligned with classes and relations in the SDO and stored in a dynamically updated Resource Description Framework (RDF) triple store. The triple store can be queried to support SSA and the needs of the spacecraft, ground operators and other users. For each type of entity in the domain, the ontology provides a definition, a classification, and relational links to other entities. The ontology represents the reality and complexity of the entities in the space domain.

Once ontologically structured, data is available to be utilized for any number of applications. The fundamental idea is that for data to be unambiguous across all applications, it must be properly represented in the ontology. For this paper, the primary application of the ontologies is to support the CS model that is being created for space event characterization. The SDO is used to extend the CS framework in two ways. First, by extending the CS framework to entities outside the sphere of the human psychology of 'concepts,' and second, by ontologically structuring the data used in the CS modeling process.

The goal of ontologies is to reduce the effort required to turn collected data into information accessible to, and supportive of, effective and timely decision making. Of concern here are the data relevant to threat rank assessments for Red and Blue spacecraft pairs in the space operating environment. The aim is to allow insofar as possible the inclusion of any available sorts of data usable by analysts to establish the presence, likelihood and severity of a threat. The application of the CS model to this varied yet ontologically structured data is intended to produce an automated form of threat analysis and ranking. This automated ranking system is designed to be another time critical aide to analysts and decision makers responsible for SSA.

One of the challenges analysts face is the amount of data available and the heterogeneous nature of the data collected.



Fig. 1 Space Domain Ontologies Structure

Data provided to analysts often will come from different kinds of sources, and pertain to different and sometimes seemingly unrelated entities, processes, and features. An ontology – analogous to a map – places each of these data elements into a unique location. It defines each type of entity, structures those types into a hierarchy of classes and subclasses to which they belong, and provides relations that link instances of types together. The ontology contains definitions in both a human readable and a machine readable format.

Multi-sourced data poses challenges and terminological difficulties. Consider the term 'vessel.' In different contexts the term 'vessel' may indicate a spacecraft, in another a kind of watercraft, and in a biological setting an organic structure used to transport materials within an organism, for example, a blood vessel. Another common difficulty is maintaining the distinction between a piece of information and what that information is about. Consider the term 'mission' as in 'spacecraft mission.' 'Mission' can indicate an objective to be achieved (for example, to monitor weather), but it can also indicate a process that a spacecraft is engaged in over a period of time, for example, the process of monitoring the weather. The former is achieved by the latter, and while humans easily adjust their understanding between these two cases, machines cannot. An ontology structures the terms and definitions in a machine readable way that allows machines to unambiguously processes information about mission plans and objectives without conflating them with processes prescribed by those mission plans.

There are several benefits to ontologically structured data. First, when working with ontologically tagged data both analysts and machines are assured to be talking about the same things. Second, the ontology provides a way to automate inferences and reason over large quantities of data. Simple inferences, such as from the fact that a spacecraft has a thruster of type T it can be concluded that it uses a propellant of type P, are automatically derived from running reasoners over the ontology. Consequently, sophisticated queries can be formulated to call and utilize data as desired, including looking for correlations and interesting connections among the data. This provides analysts reasoning tools beyond single human capabilities. Third, data of heterogeneous types, for example a quantity of fuel, a velocity of spacecraft at a certain time, the type and number of thrusters a spacecraft possesses, the owner of the spacecraft, and their history of station-keeping maneuvers, are all linked in a logical and searchable way within a single framework.

Conceptual spaces, originally presented by Gärdenfors's [1], are meant to combine aspects of symbolic (rule based) [9–11] and associationist (feature based) [12, 13] concept representation. A CS model is meant to be a geometric representation of how humans understand concepts. Originally CS were based on a theory of human concept possession,

however their geometrical approach to similarity and difference is applicable to any and all dimensions of reality, including in this case threats to spacecraft. One important difference in this work is that CS are not being utilized to represent concepts in an individual's minds but instead of certain types of objects, events and qualities in the world. Specifically, this paper is interested in *Spacecraft, Kinetic Kill* events and *Threat* values, and a CS model is applied to these types of entities.



Fig. 2 Characteristics lying within a single domain of a CS model

The most basic building blocks of CS representations are called *quality dimensions*; for example hue, saturation and brightness in the color domain or pitch, timbre and loudness in the domain of musical tones, as depicted in Figure 2a. A dimensions is essentially an aspect or feature that has a measurable extent of some kind. The fundamental role of dimensions is to build up the domains that are needed to represent what the CS model calls a 'concept' but for this paper is understood to be a type or kind. A *domain* can be defined as a set of integral dimensions that are separable from all other dimensions. The dimensions of the color domain, also referred to as the color space, as mentioned previously, are hue, saturation, and brightness as seen in Figure 2b. For example, if someone is attempting to define a color they would not know what color it is unless the values for its hue, saturation and brightness are known. However, to identify a color they wouldn't need to know anything about its pitch, timbre or loudness, as these are dimensions of the sounds domain and do not help us identify the color. The fundamental reason for decomposing a concept into domains is the assumption that the concept can be assigned certain properties within the domain that are independent of other properties.

Breaking down single domains into smaller sections, one can begin to define *properties*, which are represented in the CS model as a convex region of a domain space. If for example someone is looking for the color blue, it would be represented by a certain set of values from each dimension within the color domain. It can be seen in Figure 3 the colors on the left side represent blue. The color blue would be bounded by constraints on the hue, brightness and saturation. One can relax or constrict the constraints on the property to make it more or less specific. One may say that true blue is only represented by a single point in the color domain while others would allow for a range of lighter to darker blues to all be classified as blue.

When building a CS model different domains will be examined. In CS there is a distinction between properties and concepts. Properties are convex spaces in a single domain while concepts may be based on multiple domains. The unique characteristic of concepts compared to properties is that concepts span several domains. This is closer to how a human would identify objects, events and qualities in the world, as knowing only one property such as color generally isn't enough knowledge to know what is being described. The distinction between properties and concepts is not drawn on in traditional symbolic or associationist representation models. This distinction provides an added benefit to the CS approach. Concepts can have associations between multiple domains; such links between domains is known as



Fig. 3 Properties within the Color Domain



Fig. 4 Cross-Domain Property Associations

cross-domain property associations and are useful for building concepts. For example, see Figure 4, with an apple the color and texture domains are usually associated. If an apple is brown it will generally be wrinkly, however, if an apple is red or green it is usually smooth. The apple that is being observed is an individual object and is represented by a point in the CS model. An *object* that is being observed will hold a single value in each of the dimensions across all of the domains. There may several objects, such as an apple that is red and smooth or an apple that is brown and wrinkled. Each apple is a different object, and each is represented by a different point in the CS. If there is a CS model that is built with domains and properties to represent the type 'apple,' then it should be able to identify both of these objects.

A CS model can be constructed to identify any sort of entity, including material objects such as apples and spacecraft, but also events, dispositions, qualities and relations. For example, an event, such as a station-keeping maneuver of a particular spacecraft could be represented using dimensions such as duration, location, orbital path, angular velocity, etc., and have properties and cross-domain properties that indicates a station-keeping maneuver. If the same CS model has properties and cross-domain properties to identify and orbital maneuver it would be able to uniquely distinguish a station-keeping maneuver from another type of orbital maneuver.

Following this line of thinking employed for various association schemes in the fusion community, similarity between two objects or events increases as a function of the number of features they have in common and in the proximity of measurable values of those features. In our framework, dimension values can be treated as independent, and the distance between values of an observed quality and a specified ideal or canonical value is sufficient to capture how similar an observed object or event is to that ideal. Therefore, similarity or associability should be a function of shared dimensions and interdomain correlations. The next step in working with CS models is to quantify the above representation so that mathematical analysis can be exploited to provide the quantitative data needed to rank likelihoods of an event occurring The measurement value of a given CS quality dimension is tagged with the CCO term Measurement Information Content entity (M-ICE). M-ICE is defined in the CCO as Descriptive Information Content Entity that describes the extent, dimensions, quantity or quality of an entity relative to some standard. Our CS model uses ontologically structured classes of measurable entities for its quality dimensions and structured measurement data in the form of M-ICEs to tag the results of its calculations. This allows the ontology to represent various CS qualities of space objects and events, as well as their connections, and thereby disambiguates quality dimensions from one another.

III. Complex Physics

The second challenge that will be discussed, which arises in this data fusion problem, involves the complex physics of a collision between two spacecraft. In order for a kinetic kill event to occur, a thrust maneuver must be performed so that the targeting spacecraft is on a collision course with the spacecraft that is targeted. The physics of the spacecraft-pair collision likelihood is described by Lambert's Problem, which is the problem of determining the orbit between endpoints when the two position vectors and the time-of-flight between each spacecraft's position are known. Lambert's problem is the two-point boundary value problem for the differential equation associated with restricted two body dynamics [14]:

$$\ddot{r} = -\mu \frac{\dot{r}}{r^2} \tag{1}$$

In essence this is the problem of finding an orbit around a primary body, in this case Earth, that passes from one point to another point in a specified amount of time. For our situation, the first point will be the original starting point of the Red spacecraft and the ending point will be the location of the Blue spacecraft propagated forward to account for the time-of-flight required between the two spacecraft.



Fig. 5 Orbit Transfer

Viewing Figure 5, the solution to Lambert's problem determines the amount of thrust, which is related to the change in velocity Δv_1 , required between an original orbit represented by position vector one r_1 , and an ending orbit represented by position vector two r_2 . The resulting impact velocity, Δv_2 , will determine the force of the impact. Lambert's problem has no closed-form solution and must be solved by an iterative method. There are many solutions to Lambert's problem including Gauss's solution[3], Battin's method [3, 15] and universal variables [3, 16].

Lambert's problem was named after Johann Heinrich Lambert because he was the first person to pose and then solve the problem; the original solution was formulated in 1761 [3]. Gauss's original solution was developed to rediscover the pseudo-planet of Ceres. His method was derived in 1801 and didn't require any range information. Gauss was able to solve the orbit problem, however Bate, Mueller and White indicated that the derivation of Gauss's solution is long [16]. Gauss' solution is also limited by the type of orbit transfer and the spread between the vectors that is allowed [3]. Battin's method was later developed by Richard Battin in 1987. His technique is more robust and doesn't suffer from the difficulties of a 180° orbit transfer that other solutions have difficulty with. The universal variable algorithm presented by Bates, Muller, and White [16] uses a iterative scheme to find the universal variables.

Once Lambert's problem is solved, information about the required thrust and the impact velocity will be obtained. This information is useful in the understanding of a spacecraft collision. In order to maximize the amount of damage that is obtained from a collision the operator of the Red spacecraft will want to maximize their impact velocity.

IV. Mathematical Framework for Conceptual Spaces with Multiple Observations

Conceptual spaces gives a geometric representation of how humans understand concepts. In CS representation there are only a few mathematical notions, such as convexity of geometries, distances between points and similarities based on these distances. For the most part Gäredenfors leaves the creation of a mathematically representative model to others. Some work that has been done on modeling CS mathematically can be seen in [17–22]. References [21, 22] use a Conceptual Spaces – Mathematical Programming Hybrid model in order to describe the underlying mathematics that represents CS. Mathematical programming, also known as *optimization*, is defined as the action of making the best or most effective use of a situation or resource.

In the optimization community, optimization is better stated as a means by which one minimizes or maximizes a function over a set of constraints. A set of constraints, also known as a constraint set, represents attributes of the problem that are either feasible or not feasible. Take for example the vehicle routing problem in which the problem asks "What is the optimal set of routes for a fleet of vehicles to traverse in order to deliver to a given set of customers?" Some constraints that would need to be considered is the range of the vehicle, the drivers work day, that the driver needs periodic breaks, along with other constraints. All of the constraints, known as the *constraint set*, for the problem combine together to generate a region that satisfies the problem's constraints, known as a *feasible set*, wherein all the possible solutions to the problem exist. The functions to be minimized or maximized are known as the *objective functions*, and can then be optimized over the feasible set to find the best solution.

There are many types of optimization problems that can be used to solve different problems. Some of the most common type of optimizations problems are known as linear programming (LP) problems [23]. The general form of linear programming is to minimize a linear objective function subject to linear constraints. Stated in standard form, it is given by

$$\min c^T x$$

s. t. $Ax = b$ (2)
 $x \in S$

where x is a vector of unknown variables, also known as the decision variable, c is known as the cost vector, b is a vector, A is the constraint matrix, and S represents the set that the decision variable belongs to. In general the constraint matrix does not need to be square, meaning the problem is underdetermined, and thus the problem is not easily solved. A more special form of linear programming is integer linear programming (ILP), in which all of the variables are restricted to take on integer values. If only some of the variables are restricted to take on integer values then this problem becomes a mixed integer linear programming (MILP) problem [24].

In [21, 22] two different types of CS with different properties that must be considered for mathematical modeling are used. The first is single domain CS, in which each concept contains only one domain and one property within the domain. This is used as a stepping stone to generate more practical mathematical models. The second is complex CS, which have more then one domain and one or more property within each domain. Complex CS are more common than single domain CS. References [21, 22] also divide the mathematical models into two different subsections for single and multiple observations. Its important to note that the wording of these models is misleading. Both models are capable of handling multiple observations, however the single observation model is only capable of handling a single concept at a given time. In theory the multiple observation model would be more computationally efficient because it is capable of handling multiple concepts at a given time. However, with the use of parallelized computing the single observation model run in parallel for each concept can be much more efficient depending on the operating environment.

The model used for this paper was the mathematical Holender's model titled complex CS with a single observation. Multiple concepts can be ran in sequence or in parallel in order to examine more than one concept. When dealing with complex conceptual spaces model there are two questions that must be considered: do the observed properties of the object being examined exist within the concept and do cross-domain properties associate with one another exist in the observation the same way they do within the concept?

In order to set up the optimization problem for CS one must first define a set of property combinations, for domain *i* and properties *j*, that can be observed together to represent a specific concepts:

$$F = \left[\left(i^{1}, j^{1} \right), \left(i^{2}, j^{2} \right), ..., \left(i^{n}, j^{n} \right) \right]$$
(3)

The optimization problem is to now find the best F based on a cost function, or to find the similarities between an observed object and each property defined in the concept. Stated mathematically, it is given as follows

$$\max \sum_{(i,j)\in F} s_{ij} \tag{4}$$

where s_{ij} is the similarity of property *j* from domain *i*. Finally, the fact that not every decision variable, x_{ij} , needs to be taken into account leads to

$$x_{ij} = \begin{cases} 1, \text{ if property } j \text{ from domain } i \text{ is considered} \\ 0, \text{ otherwise} \end{cases}$$
(5)

Next, one can set up the following objective function that represents the observed object and the following constraint set:

$$\max\sum_{i}\sum_{j}s_{ij}x_{ij} \tag{6a}$$

s.t.
$$\sum_{i=1}^{n_i} x_{ij} = 1 \forall i$$
 (6b)

$$x_{ij} + x_{i'j'} \le 1 \forall \{(i,j), (i',j')\} \in F$$
(6c)

$$x_{ij} = 0 \text{ or } 1 \forall i, j \tag{6d}$$

In this model presented the objective function represents the observed object, and is maximized such that the results model the CS appropriately. It is important to note that most concepts can have multiple properties that are possible within the same domain. For example, with an apple it can either have the property of being red, green or brown within the color domain, however a green apple is no less an apple than a red apple. The constraint set consisting of equality and inequality constraints are what form the representation of the feasible region of the concept. The equality constraints represent only one property from each domain. The inequality constraint set represents the association between cross-domain properties. The solution gives an optimal value that can be normalized by the number of domains in the concept representation to obtain the similarity between the concept and the observed object.

V. Building of Conceptual Spaces Model

This section will focus on building a CS model that can be utilized for data fusion in order to increase SSA with respect to threat assessment. Building a CS model requires the identification of the type of entity to be modeled, as well as the identification and definition of the domains and dimensions belonging to that concept model. The CS model presented here aims to model and rank spacecraft threats, specifically the threat a given Red spacecraft poses to a given Blue spacecraft with respect to a potential kinetic kill attack. Following Little E. & Rogova L. (2006) and Steinberg (2009) an adversarial threat consists of three elements: Intent, Capability and Opportunity [25, 26]. Threat levels can be calculated in terms of these three elements combined with target vulnerability in such a way as to determine relative rankings of the obtaining threats for a given configuration of Red and Blue assets. The resulting four elements constitute the four domains of the CS model presented here.

Some of the dimensions that can be identified within these domains will now be discussed in detail. These dimensions are thought to be an initial starting point for the CS model and should be expanded upon as expert knowledge is gained and as the ontology expands. To assess the relevant capabilities, intentions, opportunities and vulnerabilities of each spacecraft, data tagged with the SDO will be used. These include data about the space operational environment, including data about objects, events, qualities, relationships, dispositions and agents.

A. Intent

Intent is a disposition that includes the beliefs, desires, attitudes and intentions of an agent. The intent of concern here are those dispositions that constitute the propensity of an adversary to initiate a hostile action. In BFO a disposition is a realizable entity, by which is meant an entity that can (but need not) be manifested in some process – as the disposition of fragility is manifested in the process of breaking through fragmentation. A disposition is borne by a given material entity in virtue of the latter's physical make up. Its realization occurs when and because its bearer is in some special triggering circumstance. It is assumed that intent is borne by individual people primarily and borne by groups of people, such as nation states, space agencies and military organizations derivatively. The focus for this paper will be on the geopolitical intent, an important aspect of the geopolitical climate that holds between the respective owners of Red and Blue spacecraft pairs. Geopolitical intent is a set of dispositions of a geopolitical entity that are directed towards other geopolitical entities and are realized in interactions with those entities. By finding values for these constitutive dispositional elements a model of the intent of an organization that wields authority and control over a spacecraft can be designed. Intent is part of the psychological component of a threat, and is difficult to measure directly. However, there are measurable indicators that can be used.

The first dimension of intent that will be discussed is political tension. If the tension is high between the owners, respectively, of a Red and a Blue spacecraft then they will have a greater disposition to hostile action, and thus pose a higher threat to each other. There are several indices that can be used to measure political tension including, but not limited to, the Geopolitical Risk Index, the World Economic Forum annual global risk report, the Heidelberg Institute for International Conflict Research conflict barometer, and the EU Global Conflict Risk Index [27–30]. These professional analyses along with other resources can be combined in order to generate a similarity value for the political tension that obtains between countries. How data can be combined into a single value, essentially how the weights in front of the variables are derived was never addressed in [21, 22]. One method that can be explored to combine these different sources of data is a statistical analysis, such as Bayesian networks.

Figure 6 illustrates a value for the measure of geopolitical tension as represented by the SDO. Governments and governmental organizations with authority over spacecraft operations consists of a groups of people that bear dispositions, and among those dispositions are the political beliefs, desires and intentions that are directed towards other countries and geopolitical entities. Geopolitical tension is a function of, and bears as a part, those dispositions that would be realized in hostile acts towards another geopolitical entity. The nodes and edges colored orange indicate instances, while those in blue and black above the type/instance line indicate types. The ontology allows data to be linked from a particular data source represented here by the GPRI, and a particular data value of 88.7 to spacecraft and the countries that own and operate them. The measurable values derived from sources like the GPRI are, following the CCO, are classified as Measurement Information Content Entities.

Another dimension of intent is the rate of change of political tension between owners. Political tension levels can change for a variety of reasons, including an escalation of conflicts, changes in national goals or priorities, violations of treaties, and many others. A sudden heightening of tension between two adversaries can lead to hostile actions even before political tension passes a predefined threshold. This is usually due to the fear that if tension is rising rapidly then hostile action will be inevitable.

A third dimension for the domain of intent are the kind and quality of the plans, policies and official statements of one organization towards another. The Union of Concerned Scientist publishes, for example, a catalog of known spacecraft that includes a description of each spacecraft's primary and secondary missions [31]. Missions that are known to be benign indicate a low threat level while those known to be hostile indicate a greater threat. The quantity, frequency, sources and content of the policies and assertions of an organization can be, and often are, used to evaluate the strength of the beliefs, desires and intentions of the group of people that bear those dispositions, and thereby are predictive of behavior [27, 32]. Evaluations such as these provide another measurable dimension for intent.

A fourth measurable dimension indicative of intent is a measure of the deviation from normal station-keeping activities. Degradation of an orbit is normal due to natural factors such as atmospheric drag. Periodic thruster maneuvers are needed to restore and hence maintain a desired orbit. Two variations in station-keeping activities warrant attention from a threat perspective: a failure to make a timely station-keeping maneuver and the distance a spacecraft deviates from its normal orbital path. A spacecraft becomes of concern to the degree that these two variables increase, since they are indicative of either a problem with the spacecraft's operations or the owners intent, both of which raise the potential threat posed by the spacecraft. For our assessment, intentional maneuvers that differ from normal station-keeping and move a spacecraft to a new orbit indicate a new intent. Maneuvers that decrease the relative proximity of a Red spacecraft to a Blue spacecraft improve the opportunities available to the Red spacecraft for hostile actions. Thus any movement that falls outside of station-keeping and towards an intercept course reduces total distance, and ultimately



Fig. 6 Measure of Geopolitical Tension

time to impact.

B. Opportunity

An opportunity is a realizable entity that exists due to the physical make up of one or more entities external to some agent and in virtue of which that agent can realize a capability. Relevant types of opportunities in the space environment include spatiotemporal opportunities, geopolitical opportunities and physical affordances. For example, the distance between two spacecraft can present an opportunity to successfully exercise a thruster capability and move to intercept, a legal restriction can limit the country of ownership's political opportunities to initiate a kinetic kill, and a physical barrier or avoidance defense technology may limit the collision capabilities afforded to an attacking spacecraft. The SDO has classes and definitions for spatial regions of 0-3 dimensions, orbital paths, and other qualities of the orbiting processes that Red and Blue spacecraft participate in. These sorts of spatial entities are the physical basis in virtue of which spatiotemporal opportunities for successful kinetic kill action obtain.

One way to address spatiotemporal opportunities is to determine the thrust required for an impact to occur. Looking at Figure 5 the required thrust is Δv_1 and the impact force can be determined by the terminal velocity, Δv_2 . For an opportunity to present itself, a maneuver must occur when the required thrust is less than the maximum thrust available by the Red spacecraft. Different sources of intelligence data and observation data can be utilized in order to determine the thrust capabilities of a spacecraft. If the thrust capabilities are not capable of producing an attack, then there is no reason to evaluate the Red asset and it can be ignored. However, if there is a possibility for an attack, then it is evident that the opportunity is higher when there is a larger gap between minimum thrust required and the Red asset's thrusting capability.

In order to evaluate the resulting impact velocity and terminal velocity, the solution to Lambert's problem needs to

be implemented. As mentioned previously Lambert's problem is the solution for the two-point boundary value problem where the points are the position vectors for the respective Red-Blue spacecraft pairing. Since the time-of-flight between the two position vectors is yet to be determined, there is no closed-form solution, and an iterative method must be used to describe all possible intercept possibilities. This analysis can be used to make plots in a similar fashion to the way "Porkchop Plots" are used to determined interplanetary maneuvers as described in [33, 34]. Porkhop plots can be used to illustrate impact velocity or thrust required as seen in Figures 7a and 7b, respectively.



Fig. 7 Porkchop plots for impact velocity and thrust required

Another dimension of opportunity that should be examined is the impact velocity, in order to determine if the impact force is capable of damaging or destroying all or part of the Blue asset. Assuming that the maximum thrust was utilized, it would be further assumed that the malicious spacecraft would desire to attack at a time when the impact velocity would be maximized. This would create the highest opportunity for complete destruction of the Blue asset.

Other dimensions to be considered are geopolitical opportunities due to legal restrictions, as found in treaties and international agreements, public sentiment, as measured by public support for an action, and allied support, as found in the funding and policies of third party organizations. For example, the consequences of a legal violation may hinder the opportunity of a spacecraft owner to initiate a hostile action. Several sources of information will need to be drawn upon in order to comprise a similarity score for geopolitical opportunities.

C. Capability

A capability is a disposition whose realizations are of specific interest to one person or group of people, for example the capability of a sensor to scan an area of view. A realization is of specific interest when it provides a benefit or is done intentionally. Relevant capabilities include those belonging to the spacecraft itself, the ground systems, the controlling operators, and the organizations they are part of. Capabilities are distinguished from opportunities in that capabilities exist in virtue of the physical makeup of the entity bearing them, whereas opportunities exist in virtue of the physical makeup a corresponding capability.

Spacecraft capabilities will include its functioning parts, as well as its systems and subsystems. Identifying the type of spacecraft, as well as the types of systems and subsystems of that spacecraft can reveal the sorts of capabilities possessed by that spacecraft. Information about a spacecraft's capabilities can be gained from human-derived or semantically-derived sources, as in [35], or from intelligence that is publicly available. Analysis of such information can be used to determine the extent of the spacecraft's capabilities.

An indicator of a spacecraft's capability can be determined by the force produced by that spacecraft's main thruster. Thruster force is a function of specific impulse and mass flow rate, which can be calculated as follows [36]

$$F_{thrust} = g_0 \times I_{sp} \times \dot{m} \tag{7}$$

One dimension that can be used to quantify a spacecraft's capability is a measurement of its specific impulse value, I_{sp} , of its main thruster as well as its capable mass flow rate. Since it is difficult to measure mass flow rate it may be more desirable to evaluate the mass left onboard a spacecraft. Historical data about a spacecraft's original fuel mass, along with previously observed maneuvers, can help derive the remaining available fuel mass. The specific impulse can be

indicated by knowledge of the type of thruster onboard the spacecraft. Different types of thrusters are characterized by the propellants they utilize. Typical I_{sp} values for four different types of propellants can be seen in Table 1 [36, 37].

Propellent	Specific Impulse, I_{sp} (seconds)
Cold gas	50
Monopropellant hydrazine	230
Solid propellant	290
Nitric acid/monomethylhydrazine	310
Liquid oxygen/liquid hydrogen	455

Table 1 Typical Specific Impulses

Capabilities are always grounded in the physical make up of their bearer. In this example it would be the physical qualities of the propulsion system, including its thruster type, I_{sp} , and fuel mass. The presence of these qualities in part constitute capabilities, one of which when triggered (or ordered as the case may be) is realized in the maneuvering and collision processes.

Another relevant dimension of capability concerns the reliability of spacecraft systems and sub-systems. There have been many studies on the reliability of different kinds of spacecraft and spacecraft sub-systems [26, 38, 39]. Reliability suffers in response to design flaws but also in response to the effects of the space environment. The measurable value of age is often the best predictor of spacecraft reliability and effectiveness.



Fig. 8 Realizing an Organization Capability in a Kinetic Kill Process

Organizations that control spacecraft also bear capabilities that function more or less effectively. Organizational capabilities depend on protocols, experts, authorities, command structure, discipline, morale, knowledge dissemination, and many others factors that play a role in an organization's ability to act as planned in a timely manner. Organizations exercise their capabilities when the relevant members bearing the appropriate authorized roles realize those roles in prescribed intentional acts. In Figure 8 the relation of these capabilities is illustrated. While a spacecraft thrust maneuver is the realization of the capabilities of a propulsion system, the maneuver process is also at the same time an intentional

act. A spacecraft is an artifact, and spacecraft maneuvers are intentional acts of artifact employment. In the illustrated figure the kinetic kill act is an act of an individual member of an organization who bears the authority role required to initiate certain actions prescribed by that organization. In the figure those prescribed actions are represented by a plan, although they could be represented by a direct order or other act of communication.

The domain of capability includes other dimensions to be explored involving different aspects of the spacecraft's subsystems. If some capabilities are not met, at least in part, then the spacecraft might not be able to produce an attack, and it should be eliminated completely from the analysis. Unfortunately, CS modeling does not currently have the capacity of zeroing out a ranking due to a Red asset being deemed incapable of an attack. The best method for this would be to perform a pre-selective filter to eliminate any zero threat assets, and only analyze assets capable of an attack.

D. Vulnerability

Vulnerability is defined in SDO as a disposition that inheres in a bearer in virtue of it being susceptible to having its otherwise reliable participation in processes of a certain type disrupted by a process of another type. Unlike capabilities, vulnerabilities only depend on the Blue asset that is being examined. A Blue asset's lack of capabilities would lead it to be more vulnerable to an attack.

An aspects that should be considered when looking at a spacecraft's vulnerabilities is its ability to protect itself. Items such as protective shielding or onboard munitions that are capable of blocking an attack need to be considered. Of further note is that if an asset possesses few vulnerabilities, and the operator of the Red asset is aware of this fact, then they will be less likely to pursue hostile actions on this target.



Fig. 9 Some Data Classes used in CS model and their Threat Domains as Represented by the SDO

Figure 9 shows the ontological representation of some of the classes for measurable dimensions discussed in the previous sections. The figure demonstrates the variety of data used in establishing threats, and at the same time shows the real world connection these data bear to each other. The ontology captures these heterogeneous elements, and links them utilizing a finite set of relations and a systematic class hierarchy so that values of instance data falling under these and other potential classes is searchable for future data analytics. In this manner queries can be formulated to pull together data of different desired sorts. For example, queries can be structured to return all values used in capability analyses or return all spacecraft system functions.

As additional dimensions are specified and distinguished, and additional measurement data collected, the various domains and dimensions of the threat ranking can themselves be ranked. Conceptual spaces modeling provides a way to extract and rank according to capabilities, intent, opportunity and vulnerability alone or in some conjunction. This can also be done according to threat type. Our current model has focused of the threat of a kinetic kill, however different sorts of threats, such as cyber-attacks, communication disruptions, or chemical attacks, will have their own corresponding Intent, Capability, Opportunities, and Vulnerability dimensions, which are topics for further work.

VI. Simulated Results

Now that the domains of the CS model have been defined and the various challenges of building a CS model has been addressed. an example problem can be worked out to illustrate that the Conceptual Spaces – Mathematical Programming Hybrid model for data fusion can be an effective tool for ranking of threat. For this paper a hypothetical model with different amounts of dimensions in each of the four domains is made with random variables being used for the similarity values. Five different Red spacecraft and one Blue spacecraft are used to generate five observation pairs. The five observation functions are derived using Eq. (6a) in addition to the similarity values found in Tables 2, 3, 4 and 5.

Case Number	<i>s</i> ₁₁	s ₁₂	<i>s</i> ₁₃	s ₁₄	<i>s</i> ₁₅
1	0.1	0.9	0.7	0.1	0.1
2	0.5	0.7	0.1	0.3	0.5
3	0.5	0.7	0.1	0.3	0.5
4	0.7	0.1	0.9	0.7	0.9
5	0.9	0.3	0.3	0.5	0.7

Table 2Intent

Table 3	Capability
I ubic o	Cupuomity

Case Number	<i>s</i> ₂₁	s ₂₂	<i>s</i> ₂₃	s ₂₄
1	0.1	0.7	0.9	0.3
2	0.3	0.9	0.5	0.7
3	0.7	0.1	0.7	0.5
4	0.5	0.5	0.3	0.1
5	0.9	0.3	0.1	0.9

Case Number	<i>s</i> ₃₁	s ₃₂	\$33	<i>s</i> ₃₄	\$35
1	0.9	0.1	0.1	0.1	0.5
2	0.3	0.7	0.7	0.5	0.3
3	0.1	0.3	0.5	0.9	0.9
4	0.7	0.5	0.3	0.7	0.1
5	0.5	0.9	0.9	0.3	0.7

Another aspect of this paper is that in addition to evaluating the overall threat from each spacecraft pairing, each of the domains will be examined independently. This will allows the operator to look at each domain independently in order

Case Number	<i>s</i> ₄₁	\$42	\$43	<i>S</i> 44	\$45	<i>s</i> ₄₆
1	0.5	0.7	0.5	0.1	0.3	0.1
2	0.9	0.9	0.9	0.3	0.1	0.9
3	0.7	0.5	0.3	0.7	0.5	0.7
4	0.3	0.3	0.7	0.5	0.9	0.5
5	0.1	0.1	0.1	0.9	0.7	0.3

Table 5 Vulnerability

to obtain a better understanding of the threat. This can be accomplished by identifying five different concepts, where the first four concepts will only have constraints in each domain individually. The final concept will have constraints in each of the four domains, as well as cross properties constraints. The constraints for Intent, Opportunity, Capability and Vulnerability can be seen respectively by

$$0 \leq \sum_{i=5}^{i=5} x_{1i} \leq 1$$

$$x_{11} \leq x_{12} \leq x_{12} \leq x_{14} \leq 1$$

$$x_{13} \leq x_{15} \leq 0$$
(8)

$$0 \le \sum_{i=4}^{i=4} x_{2i} \le 1$$

$$x_{21} \le x_{23} \le 1$$

$$x_{22} \le x_{24} \le 0$$
(9)

$$0 \le \sum_{i=5}^{i=5} x_{3i} \le 1$$

$$x_{32} \le x_{33} \le x_{34} \le 1$$

$$x_{31} \le x_{35} \le 0$$
(10)

$$0 \leq \sum_{i=6}^{i=6} x_{4i} \leq 1$$

$$x_{41} \leq x_{43} \leq x_{44} \leq x_{45} \leq 1$$

$$x_{42} \leq x_{46} \leq 0$$
(11)

The final concept can be constructed by the combination of these four constraint lists as well as the cross domain constraints:

$$x_{32} + x_{41} = 2$$

$$x_{11} + x_{12} = 2$$

$$x_{11} + x_{41} = 2$$
(12)

Now that the initial equations and constraints are set up, the linear optimization can be preformed in order to identify a ranking of relative threat in each individual category as well as combined. The ranking that is determined is shown in Table 6.

It can be seen in Table 6 that objects with higher rankings in individual domains tend to hold a higher overall threat, but due to cross domain constrains object three can still tie for the highest total threat despite observation two have higher individual values in all four fields. This ranking can be a crucial tool for threat assessment and the development of determining what assets are at risk of an attack. An operator of a Blue spacecraft might make the decision to reposition an asset if they see a single domain rising in ranks. For example, if the vulnerability of a spacecraft is increased due to inoperative equipment, then the operator may want to reposition that spacecraft to a safer location until they can address the concerns in vulnerability.

Case Number	Total	Intent	Opportunity	Capability	Vulnerability
1	4	5	4	4	5
2	1	1	1	1	1
3	1	2	2	2	4
4	3	4	3	2	3
5	2	3	3	3	2

Table 6 Threat Ranking

VII. Conclusion

This paper was capable of expanding on the utilization of conceptual spaces (CS) for space event characterization. Ontology was utilized to overcome the complex terminologies that arise in the space domain as well as to help define dimensions and domains for the CS model. The complicated physics of the dynamics between two spacecraft was addressed by finding the solution to Lambert's problem. The complex combinatorics of data fusion was addressed by implementing the Conceptual Spaces – Mathematical Programming Hybrid model.

Furthermore, this paper investigated breaking down the concepts in the CS model in order to allow for each domain to be viewed individually. This will give more information to operators and decision makers, so that a more well-rounded assessment of threat can be made. Future work will involve better construction of domains as well as looking into methods for constructing similarity values in domains and reevaluating concept constraints, so that they better match the concepts. As expert knowledge and the ontology grows the model should be adjusted in order to better articulate real-world phenomena.

Conceptual spaces can be a great tool for the fusion of hard and soft data due to the fact that all that is needed is to derive a similarity value between the dimension and the observation. For the space domain there is an abundant amount of tangible intelligence data that can be combined to examine the overall threat that is presented to spacecraft. Using all of this available data with the fusion techniques presented here will allow for an overall increase in space situational awareness.

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