Simplicity: A unifying principle in cognitive science?

Nick Chater
Institute for Applied Cognitive Science
Department of Psychology
University of Warwick
Coventry, CV4 7AL
UK

Paul Vitányi,
Centrum voor Wiskunde en Informatica
Kruislaan 413
1098 SJ Amsterdam
The Netherlands

Summary

Much of perception, learning and high-level cognition involves finding patterns in data. But there are always infinitely many patterns compatible with any finite amount of data. How does the cognitive system choose 'sensible' patterns? A long tradition in epistemology, philosophy of science, and mathematical and computational theories of learning argues that patterns 'should' be chosen according to how simply they explain the data. This article reviews research exploring the idea that simplicity does, indeed, drive a wide range of cognitive processes. We outline mathematical theory, computational results, and empirical data underpinning this viewpoint.

Key words: simplicity, Kolmogorov complexity, codes, learning, induction, Bayesian inference

*30-word summary:*This article outlines the proposal that many aspects of cognition, from perception, to language acquisition, to high-level cognition involve finding patterns that provide the simplest explanation of available data.

The cognitive system finds patterns in the data that it receives. Perception involves finding patterns in the external world, from sensory input. Language acquisition involves finding patterns in linguistic input, to determine the structure of the language. High-level cognition involves finding patterns in information, to form categories, and to infer causal relations.

Simplicity and the problem of induction

A fundamental puzzle is what we term the problem of induction: infinitely many patterns are compatible with any finite set of data (see Box 1). So, for example, an infinity of curves pass through any finite set of points (Box 1a); an infinity of symbol sequences are compatible with any subsequence of symbols (Box 1b); infinitely many grammars are compatible with any finite set of observed sentences (Box 1c); and infinitely many perceptual organizations can fit any specific visual input (Box 1d). What principle allows the cognitive system to solve the problem of induction, and choose appropriately from these infinite sets of possibilities?

Any such principle must meet two criteria: (i) it must solve the problem of induction successfully; (ii) it must explain empirical data in cognition. We argue that the best approach to (i) is to choose patterns that provide the simplest explanation of the data; and that this approach provides a powerful approach to (ii), in line with a long tradition of psychological research.

The physicist and philosopher Mach[1] proposed the following radical idea: that the cognitive system *should* (criterion i), and *does* (criterion ii), prefer patterns that provide simple descriptions of the data. Here, a description must allow the data to be reconstructed; and the simplicity of a description is measured by its length.

Mach’s proposal traces back to Ockham’s razor, that, in explanation, entities should not be multiplied beyond necessity; and to Newton’s statement in the *Principia* that we “admit no more causes of natural things than are both true and sufficient to explain the appearances.” But to make Mach’s proposal precise requires a theory of description complexity, which awaited further mathematical developments.

Quantifying simplicity

These mathematical developments came in two steps. First, Shannon’s information theory justified $\log_2(1/p)$ as a code length for items with probability p. This is helpful for providing code lengths of highly repetitive data patterns, which can be assigned probabilities, such as low level perceptual properties, phonemes, words and so on[2]. Second, the critical generalization to algorithmic information theory by Kolmogorov, Solomonoff and Chaitin[3] defined the complexity $K(x)$ of any object, x, by the length of the shortest program for x in any standard (universal) computer programming language. Surprisingly, it turns out that the choice of programming language does not matter, up to a constant additive factor; and moreover, algorithmic information theory turns out to agree closely with standard information theory, where the latter theory applies at all. Crucially, the algorithmic definition of simplicity applies to individual objects, whereas Shannon's definition depends on associating *probabilities*with objects.

Intuitively, then, we can see the cognitive system's goal as compressing data: coding it in such a form that it can be recovered by some computable process (the mathematics allow that compression may be 'lossy'---i.e., information may be thrown away by the cognition system, but we do not consider this here.) Choices between patterns are determined by the compression they provide---compression thus provides a measure of the strength of evidence for a pattern. This viewpoint forges potential connections between compression and pattern-finding as computational projects. Note that the shortest code for data also provides its least *redundant* representation; elimination of redundancy has been viewed as central to pattern recognition both in human[4,5] and machine[6].

More crucially, formalizing simplicity provides a candidate solution to the problem of induction, described above. The infinity of patterns, compatible with any set of data, are not all equal: the cognitive system should prefer that pattern that gives the shortest code for the data.

Regarding criterion (i) above, there are two beautiful and important mathematical results[7] that justify this choice as a solution to the problem of induction. One result is that, under quite general conditions, the shortest code for the data is also the most probable (according to a Bayesian analysis, using the so-called "universal prior."). A second result is that the shortest code can be used for prediction, with a high probability of 'convergence' on largely correct predictions. A third powerful line of justification for simplicity as an effective method of induction is its widespread use in machine learning[8,9] and statistics[10].

Simplicity as a cognitive principle

So simplicity appears to go some way towards meeting criterion (i): justifying why patterns should be chosen according to simplicity. What about criterion (ii)? Does simplicity explain empirical data in cognitive science? Table 1 describes a range of models of cognitive phenomena, from low and high level visual perception, language processing, memory, similarity judgements, and mental processes in explicit scientific inference. The breadth of domains in which simplicity has proved to be a powerful organizing principle in cognitive modelling is encouraging.

But how does the simplicity principle stand up to direct empirical testing? This question is difficult to answer, for two reasons. (1) *The representation problem*: Although, in the asympotote, and assuming the brain has universal Turing machine power, Kolmogorov complexity is language invariant, for many specific, non-asymptotic empirical predictions from simplicity depend on assumptions about mental representation, which will affect what regularities can be detected. But the mental representation of perceptual and linguistic stimuli is highly contentious in cognitive science. (2) *The search problem*: The cognitive system may prefer the simplest interpretation that it can find, but be unable to find a simple pattern of interest. Thus, without creating a full-scale cognitive model, involving assumptions about representation and perhaps also search, precise predictions from the simplicity viewpoint cannot be obtained[11].

There are, however, a number of lines of evidence that appear consonant with the simplicity viewpoint.

- A vast range of phenomena in perceptual organization, including the Gestalt laws of closure, good continuation, common fate, and so on, have been widely interpreted as revealing a preference for simplicity. Box 2 discusses some complex cases. The main theoretical alternative, the Bayesian approach to visual perception[12] is mathematically closely related to the simplicity principle[13]
- Items with simple descriptions are typically easier to detect in noise and easier to detect[2,11].
- The simplicity of a code for a stimulus quantifies the amount of structure uncovered in that stimulus. The more structure people can find in a stimulus, the easier they find it to process and remember[14] and the less random it appears[15].
- The speed of learning for Boolean concepts (e.g., A or B or C; A and (B or C) etc) is well predicted by the shortest code length for those concepts[16].
- Similarity can be viewed as a function of the simplicity of the *distortion* required to turn one representation into the other. This viewpoint makes empirical predictions which are not captured by existing spatial or feature-based theories of similarity, but which have been confirmed[17].
- Shepard’s Universal Law of generalization[18], which implies that items have a probability of confusion that is a negative exponential function of the distance between them in an internal ‘space,’ can be derived from the assumption that the psychological similarity between two objects is a function of the complexity of the simplest transformation between them, and minimal additional assumptions[19].
- The physiology of early vision, including receptive field shapes, and phenomena such as lateral inhibition, seems adapted to maximize information compression in vision[20]. On the other hand, both theoretical and empirical arguments suggest that,

> the brain also uses highly redundant ‘sparse’ neural codes for perceptual input[21, 22].

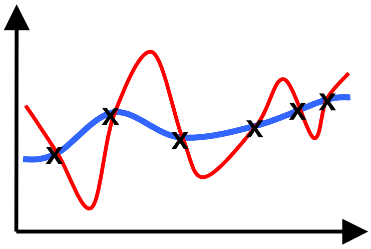
To summarize, since Mach, a range of theorists have proposed the sweeping idea that much of cognition concerns compression[23], or the elimination of redundancy[24], and the simplicity principle has been developed into a mathematically rigorous method for finding patterns in data[3]; served as the foundation for a broad range of cognitive models; and is consistent with a range of empirical data. We suggest that simplicity is worth pursuing as a potentially important unifying principle across many areas of cognitive science.

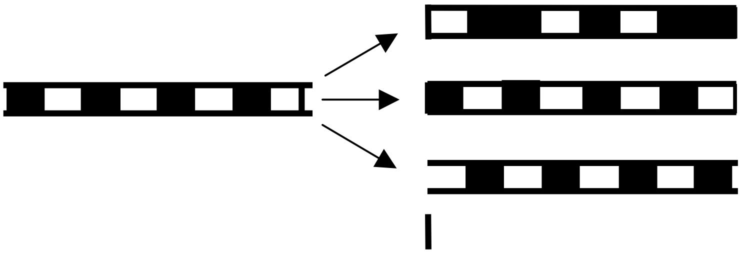
Table 1: Pattern-finding by simplicity: A sample of research

Cognitive process	Data	Codes	Computer science/mathematical approaches	Cognitive science applications
Low-level perception	Sensory input/artificially captured images	Filters in early vision	• Image compression[25]	Early vision as compression[23,22]
High-level perception	Sensory input/output of early perceptual processing	Representations of higher level structure	• Pattern theory[26]	• 'principle of economy[1] • perceptual organization[27,14]
Language acquisition	Linguistic input	Representations of language structure	• Text compression[28]	• Phonological29] and morpholgical analysis[30] segmentation[31, 24] and grammar induction[32,33]
High-level cognition	High-level representations of knowledge	similarity, causal relations	• Information distance[34] • Gencompress[35]	• Similarity as representational distortion[18] • Categorization by compression[36]
Scientific inference	Scientific data	Theoretical knowledge	• Machine induction systems[9] • Foundations of Statistics[10]	• Ockham, Newton • Mach's principle of economy[1] • Formal measures of simplicity[37,38]

Table 1: Many pattern-finding problems have been successfully approached by mathematicians and computer scientists using a simplicity principle. In many of these areas, the simplicity principle has also been used as a starting point for modelling cognition.

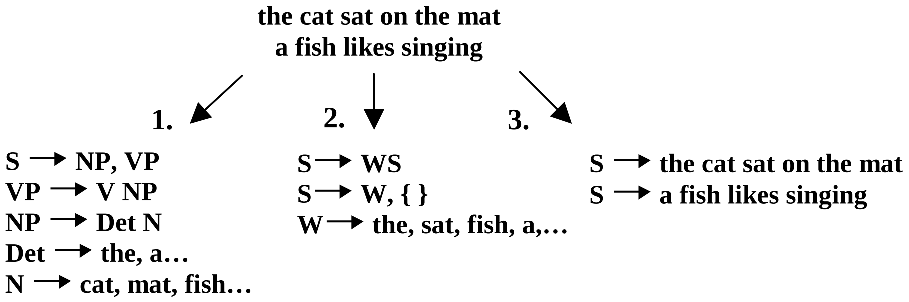
Box 1: Finding patterns and the problem of induction





1a. The abstract problem of induction: the continuous case.

1b. The abstract problem of induction: the discrete case.



the cat sat on the mat
a fish likes singing

1.

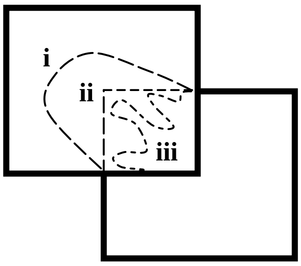
S → NP, VP
VP → V NP
NP → Det N
Det → the, a…
N → cat, mat, fish…

2.

S → WS
S → W, { }
W → the, sat, fish, a,…

3.

S → the cat sat on the mat
S → a fish likes singing

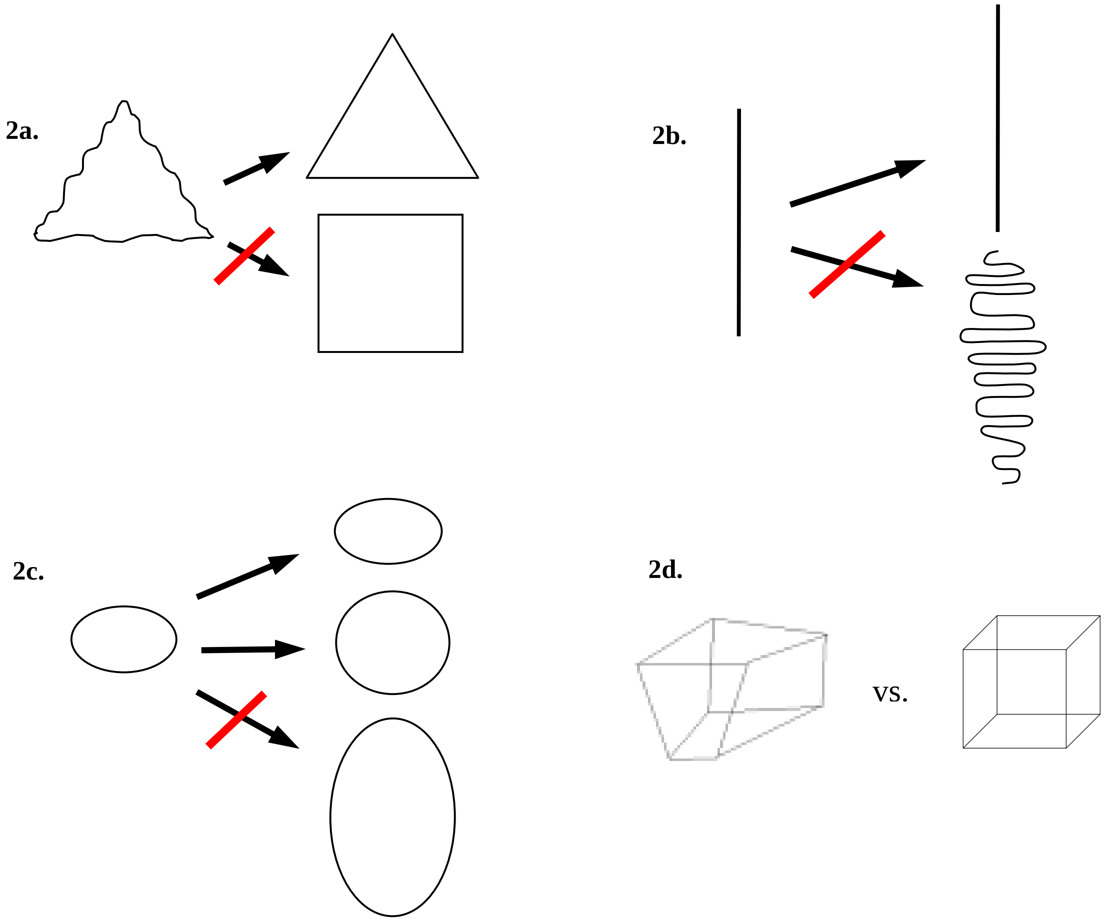


1c. Grammar Learning

1d. Perception: Figural Completion

There are always infinitely many patterns compatible with any finite body of data. This raises the critical question of how the cognitive system makes appropriate choices from among this infinite range of options. The general problem is illustrated in 1a, where there are clearly any number of continuous functions that can be made to pass through a set of data points. The same issue arises for discrete data. The alternating black/white squares on the left hand of 1b illustrate a sequence of binary data. But, as the right hand of 1b indicates, the overall pattern of which this data is a part could continue in any way. The ‘middle’ continuation is more cognitively natural. But why? And are cognitively natural continuations reliable in prediction? 1c extends the point to grammar induction from a tiny ‘corpus’ of language data. Grammar 1 provides a linguistically reasonable analysis; Grammar 2 can produce any word sequence whatever and is clearly wildly overgeneral; Grammar 3 produces just the sentences in the corpus and nothing more. Human learners favour reasonable analyses; but why? Finally, 1d illustrates the limitless possible hypotheses for elaborating partial perceptual input---only ii. is seriously entertained, though i. and iii. are also compatible with the data. These illustrations are quite abstract; but, importantly, the same issue arises even if the input is arbitrarily rich: although some specific patterns will be eliminated but such enrichment, an infinite number of incompatible patterns will always remain.

Box 2: Empirical data



Box 2: Various qualitative aspects of the resolution of perceptual ambiguity can be understood in terms of simplicity. In each of 2a-c, the left hand side schematically represents a visual input, and the right hand figure represents possible interpretations. 2a illustrates that preferred perceptual organizations typically have a relatively good (although not necessarily perfect) fit with the data---here a somewhat irregular triangle interpretation is favoured over a very irregular square interpretation. Patterns with good data fit provide short codes for the data, given the pattern, and are preferred by the simplicity principle. 2b illustrates the complementary preference for simple patterns: the 2D straight line projected image is thus preferred to a highly irregular curve in the plane, even though, viewed from one specific angle, this can project a perfect 2D line. 2c reveals the importance of the *precision*, in visual coding. The figure illustrates a preference for interpreting a small ellipse as that ellipse in the plane perpendicular to the viewer, rather than a larger, but geometrically similar, ellipse at a

highly skewed angle (another possible interpretation is a circle, at a moderately skewed angle). Thus, data fit, and, apparently, complexity of pattern appears identical here. How can the simplicity principle distinguish the two elliptical interpretations. The answer is that the projection is much more stable for the perpendicular ellipse; for the highly skewed ellipse the angle of orientation must be specified more precisely, costing additional code length, to obtain an equally good fit with the data. Finally, 2d illustrates that simpler interpretations are taken to have causal significance. The right hand 2D figure is perceived as a projection of a wire cube, the left hand figure is perceived as an irregular 2d figure. Crucially, the joints of the wire cube are perceived as rigid---whereas the joints of the irregular 3D figure appear potentially flexible. The joints of the cube are perceived as rigid, presumably because otherwise this ‘simple’ arrangement would be merely a remarkable coincidence (analogously, a sequence of 100 heads from a coin would be interpreted as indicating that the coin is biased). Thus causal structure may be inferred on the basis of simplicity.

Qualitative demonstrations of this kind have also been supplemented by formal psychology theories which seek to explain the interpretations of perceptual figures as minimizing code length [a], [b].

References:

[a] Hochberg, J. & McAlister, E. (1953) A quantitative approach to figure "goodness." *Journal of Experimental Psychology* 46, 361-364

[b] Van der Helm, P.A. & Leeuwenberg, P.A. (1996). Goodness of visual regularities: A non-transformational approach. *Psychological Review 103,* 3, 429-456

Acknowledgements: This work was partially supported by the Human Frontiers Science Foundation, EU Training Network HPRN-CT-1999-00065, the Leverhulme Trust, an ESRC/DTI Link Award, and by Oliver, Wyman & Company. We thank Khalilah Khan and Ursula Richards for invaluable help in preparation of figures and text.

References

1 Mach, E. (1959) *The analysis of sensations and the relation of the physical to the psychical*. New York: Dover Publications. (Original work published 1914)

2 Hochberg, J. & McAlister, E. (1953) A quantitative approach to figure "goodness." *Journal of Experimental Psychology* 46, 361-364

3 Li, M. & Vitányi, P. (1997) *An introduction to Kolmogorov complexity and its applications*. New York: Springer-Verlag. (2nd edition)

4 Attneave, F. (1954) Some informational aspects of visual perception. *Psychological Review* 61, 183-193.

5 Barlow, H. B. (1959). Possible principles underlying the transformation of sensory messages. *Sensory Communication*. (Rosenblith, W. ed.) pp. 217-234. MIT Press.

6 Watanabe, S. (1960). Information-theoretical aspects of inductive and deductive inference. *IBM Journal of Research and Development* 4, 208-231.

7 Vitányi, P. & Li, M. (2000) Minimum Description Length Induction, Bayesianism and Kolmogorov Complexity. *IEEE Trans. Information Theory* 46, 2, 446-464

8 Quinlan, J. & Rivest, R. (1989) Inferring decision trees using the minimum description length principle. *Information and Computation* 80, 227-248

9 Wallace, C. & Freeman, P. (1987) Estimation and inference by compact coding. *Journal of the Royal Statistical Society, Series B* 49, 240-251

10 Rissanen, J. (1989) Stochastic complexity and statistical inquiry. *World Scientific Series in Computer Science,* 15. Singapore: World Scientific

11 Van der Helm, P.A. & Leeuwenberg, E.L.J. (1996) Goodness of visual regularities: A non-transformational approach. *Psychological Review 103,* 3, 429-456

12 Knill, D. & Richards,W. (eds.) (1996) *Perception as Bayesian Inference*. Cambridge: Cambridge University Press

13 Chater, N. (1996) Reconciling simplicity and likelihood principles in perceptual organization. *Psychological Review* 103, 566-581

14 Garner, W. (1974) *The Processing of Information and Structure*. Potomac, MD: Erlbaum

15 Falk, R. & Konold, C. (1997) Making sense of randomness: Implicit encoding as a bias for judgment. *Psychological Review* 104, 2. 301-318

16 Feldman, J. (2000) Minimization of Boolean complexity in human concept learning. *Nature* 407, 630-633

17 Hahn, U. et al (in press) Similarity as Transformation. *Cognition*.

18 Shepard, R. N. (1987) Toward a universal law of generalization for psychological science. *Science* 237**,** 1317-1323

19 Chater, N. & Vitányi, P. (in press) Generalized law of universal generalization. *Journal of Mathematical Psychology.*

20 Blakemore, C. (ed.) (1990) *Vision: Coding and efficiency*. Cambridge, England. Cambridge University Press

21 Gardner-Medwin, A. R. (2001). The limits of counting accuracy in distributed neural representations. *Neural Computation* 13, 477-504.

22 Olshausen, B. A. & Field, D.J. (1997) Sparse coding with an overcomplete basis set: A strategy employed by V1? *Vision Research* 37, 3311-3325

23 Wolff, J.G. (1982) Language acquisition, data compression and generalization. *Language and Communication* 2. 57-89

24 Barlow, H.B. et al. (1989) Finding minimum entropy codes. *Neural Computation* 1, 412-423

25 Fisher, Y. (ed.), (1995) *Fractal Image Compression: Theory and Application*. New York: Springer Verlag,

26 Mumford, D. (1996) Pattern theory: a unifying perspective. *Perception as Bayesian Inference* (Knill, D. & Richards, W., eds.) pp.25-62. Cambridge University Press

27 Leeuwenberg, E. & Boselie, F. (1988) Against the likelihood principle in visual form perception *Psychological Review* 95, 485-491

28 Bell, T.C., Witten, I.H. and Cleary, J. (1990) *Modelling For Text Compression* Prentice Hall

29 Goldsmith, J. (2002) Probabilistic models of grammar: phonology as information minimization. *Phonological Studies*, 5.

30 Goldsmith, J. (2001) Unsupervised Learning of the Morphology of a Natural Language. *Computational Linguistics* 27, 2, 153-198.

31 Brent, M. R., & Cartwright, T. A. (1996). Distributional regularity and phonotactic constraints are useful for segmentation. *Cognition* 61, 93-125

32 Grünwald, P. (1996) Symbolic, Connectionist and Statistical Approaches to Learning for Natural Language Processing. (Wermter, S., Riloff, E. and Scheler, G. eds.) *Lecture Notes in Artificial Intelligence* 1040, pp. 203-216. Springer Verlag, Berlin, Germany

33 Clark, R. (2001) Information theory, complexity, and linguistic descriptions. In S. Bertolo (Ed.) *Parametric Linguistics and Learnability*, pp. 126-171Cambridge: Cambridge University Press.

34 Gacs, P.,Tromp, J. et al. (2001) Algorithmic Statistics, *IEEE Trans. Information Theory* 47, 6, 2443-2463.

35 Li, M. et al. (in press) An Information Based Sequence Distance and its Application to Whole Mitochondrial Genome Phylogeny. *Bioinformatics.*

36 Pothos, E. & Chater, N. (2002) A simplicity principle in unsupervised human categorization. *Cognitive Science*. 26, 303-343

37 Kemeny, J.G. (1953) The Use of Simplicity in Induction. *The Philosophical Review*, 62, 391-408

38 Sober, E. (1975) Simplicity. Clarendon Press