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The Lottery Paradox Generalized?

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ABSTRACT

In a recent article, Douven and Williamson offer both (i) a rebuttal of various recent suggested sufficient conditions for rational acceptability and (ii) an alleged 'generalization' of this rebuttal, which, they claim, tells against a much broader class of potential suggestions. However, not only is the result mentioned in (ii) not a generalization of the findings referred to in (i), but in contrast to the latter, it fails to have the probative force advertised. Their paper does however, if unwittingly, bring us a step closer to a precise characterization of an important class of rationally unacceptable propositions—the class of lottery propositions for equiprobable lotteries. This helps pave the way to the construction of a genuinely lottery-paradox-proof alternative to the suggestions criticized in (i).

1 Probability and Acceptance

The idea that rational acceptability supervenes on probability[1] in some way or other is an attractive one.[2] Its truth would entail the existence of a second-order function f mapping each and every probability function Pr on to some corresponding 'acceptance' function A with the same domain. The latter would indicate the rational acceptability or otherwise, with respect to the class of probability functions mapped on to it, of the various propositions in its domain. Let us take the range of acceptance functions to be $\{0,1\}$, with

[1] In line with current orthodoxy, probability functions will be taken to be functions from a field of propositions $\mathcal{F}$ defined over a set of possible worlds Ω to the interval [0,1] of the reals, axiomatized in the standard Kolmogorovian manner. A probability model $\mathcal{M}$ will be defined as a triple $\langle \Omega, \mathcal{F}, \mathrm{Pr} \rangle$.

[2] This is not to say that this view is entirely uncontroversial. For instance, those who hold that the acceptability of a proposition hinges partly on the practical consequences of true/false negatives/positives will reject this thesis (see Rudner [1953], for example).

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$A(\varphi) = f(\mathrm{Pr})(\varphi)$ taking on the value 1 in the event that φ is rationally acceptable with respect to Pr and 0 in the event that it is not.

If the task of *fully* specifying f is far from straightforward, there are a number of intuitive ground-rules that can be established from the outset. Amongst these, the following three will figure in our subsequent discussion:

NON-UNANIMITY: For some probability model $\mathcal{M} = \langle \Omega, \mathcal{F}, \mathrm{Pr} \rangle$ and some proposition $\varphi \in \mathcal{F}$, $\mathrm{Pr}(\varphi) < 1$ and $f(\mathrm{Pr})(\varphi) = 1$.

AGGREGATIVITY: For any probability model $\mathcal{M} = \langle \Omega, \mathcal{F}, \mathrm{Pr} \rangle$ and propositions $\varphi, \psi \in \mathcal{F}$, if $f(\mathrm{Pr})(\varphi) = 1$ and $f(\mathrm{Pr})(\psi) = 1$, then $f(\mathrm{Pr})(\varphi \cap \psi) = 1$.

ZERO-NORMALIZATION: For any probability model $\mathcal{M} = \langle \Omega, \mathcal{F}, \mathrm{Pr} \rangle$, $f(\mathrm{Pr})(\varnothing) = 0$.

Further noteworthy intuitive desiderata on f, induced by natural constraints on rational acceptance functions, include

UNIT-NORMALIZATION: For any probability model $\mathcal{M} = \langle \Omega, \mathcal{F}, \mathrm{Pr} \rangle$, $f(\mathrm{Pr})(\Omega) = 1$.

DEDUCTIVE CLOSURE: For any probability model $\mathcal{M} = \langle \Omega, \mathcal{F}, \mathrm{Pr} \rangle$ and propositions $\varphi, \psi \in \mathcal{F}$, if $\varphi \subseteq \psi$ then $f(\mathrm{Pr})(\varphi) \leq f(\mathrm{Pr})(\psi)$.

FINITE SUPERADDITIVITY: For any probability model $\mathcal{M} = \langle \Omega, \mathcal{F}, \mathrm{Pr} \rangle$ and propositions $\varphi, \psi \in \mathcal{F}$, $f(\mathrm{Pr})(\varphi \cup \psi) \geq f(\mathrm{Pr})(\varphi) + f(\mathrm{Pr})(\psi) - f(\mathrm{Pr})(\varphi \cap \psi)$.[3]

The latter is somewhat laborious to unpack in intuitive terms. It essentially tells us the following. With respect to any probability function and propositions φ, ψ in its domain, it is only rationally permissible to: (i) accept $\varphi \cup \psi$, accept φ, accept ψ and accept $\varphi \cap \psi$, or (ii) accept $\varphi \cup \psi$, accept φ (ψ), not accept ψ (φ) and not accept $\varphi \cap \psi$, or (iii) not accept $\varphi \cup \psi$, not accept φ, not accept ψ and not accept $\varphi \cap \psi$. Adding FINITE SUPERADDITIVITY to UNIT- and ZERO- NORMALIZATION still allows, in line with our intuitions, for propositions to be such that neither they, nor their negation, are rationally acceptable. In other words, the following comes out false, as it should do:

OPINIONATION: For any probability model $\mathcal{M} = \langle \Omega, \mathcal{F}, \mathrm{Pr} \rangle$ and proposition $\varphi \in \mathcal{F}$, either $f(\mathrm{Pr})(\varphi) = 1$ or $f(Pr)(\overline{\varphi}) = 1$.

This constraint could be obtained, of course, by strengthening FINITE SUPERADDITIVITY to

FINITE ADDITIVITY: For any probability model $\mathcal{M} = \langle \Omega, \mathcal{F}, \mathrm{Pr} \rangle$ and propositions $\varphi, \psi \in \mathcal{F}$, $f(\mathrm{Pr})(\varphi \cup \psi) = f(\mathrm{Pr})(\varphi) + f(\mathrm{Pr})(\psi) - f(\mathrm{Pr})(\varphi \cap \psi)$.

[3] Note that we recover both DEDUCTIVE CLOSURE and AGGREGATIVITY from the conjunction of UNIT-NORMALIZATION, ZERO-NORMALIZATION and FINITE SUPERADDITIVITY.

Note that, formally speaking, this strengthening would make $f(\mathrm{Pr})$ a probability function, for all Pr. None of these further desiderata on f will play a major role in the discussion that follows.

2 Three Proposals and a Counterargument

In a recent article entitled 'The Lottery Paradox Generalized', Douven and Williamson ([2006]) offer a result undermining a trio of recent proposals offering a partial characterization of f, in the sense of offering a merely sufficient condition for the acceptability of any proposition φ with respect to any given probability function in the domain of which φ is included. All three proposals appear to be guided by a concern to satisfy the requirements of NON-UNANIMITY, AGGREGATIVITY and ZERO-NORMALIZATION.

The starting point for the proposals is an initial suggestion that $\mathrm{Pr}(\varphi) \geq t$, for some sufficiently high $t \in (0.5,1)$, is sufficient for rational acceptability, presumably to do justice to NON-UNANIMITY. This however famously leads to the members of an inconsistent set of propositions being individually acceptable. Consider the set of so-called lottery propositions, of the form 'Ticket i will lose' for a fair n-ticket lottery ($1 - 1/n \geq t$) with a guaranteed winner. Each proposition in that set receives a probability greater than t, and hence, by the above proposal, would qualify as rationally acceptable. By AGGREGATIVITY, however, we then obtain a violation of ZERO-NORMALIZATION, as the intersection of these propositions is the empty set. So the idea is then to render the would-be sufficient condition more stringent by imposing a relevant additional requirement that prevents lottery propositions from being individually acceptable. In other words, all three proposals propose to flesh out the following general template:

TEMPLATE: For any probability model $\mathcal{M} = \langle \Omega, \mathcal{F}, \mathrm{Pr} \rangle$ and proposition $\varphi \in \mathcal{F}$, if $\mathrm{Pr}(\varphi) \geq t$ (for some appropriate $t < 1$) and defeater D does not hold (for some appropriate D), then $f(\mathrm{Pr})(\varphi) = 1$.

The proposals regarding the defeater D—loosely attributed to Pollock ([1995]), Ryan ([1996]) and Douven ([2002]), respectively—are as follows:

DEFEATER 1: $D = \varphi$ is a member of a minimally inconsistent set of propositions Γ, such that for all $\psi \in \Gamma$, $\mathrm{Pr}(\psi) \geq t$ (where Γ is minimally inconsistent iff Γ is inconsistent and there is no $\Gamma^* \subset \Gamma$ such that Γ^* is inconsistent).

DEFEATER 2: $D = \varphi$ is a member of a set of propositions Γ, such that for all $\psi \in \Gamma$, $\mathrm{Pr}(\psi) \geq t$ but $\mathrm{Pr}(\bigcap \Gamma) < t$.

DEFEATER 3: $D = \varphi$ is a member of a probabilistically self-undermining set of propositions Γ (where Γ is probabilistically self-undermining iff, for all $\psi \in \Gamma$, $\mathrm{Pr}(\psi) \geq t$ but $\mathrm{Pr}(\psi | \bigcap(\Gamma - \psi)) < t$).

By way of objection to these suggestions, Douven and Williamson prove the following theorem, where D can be interpreted in any of the three above ways and $\mathbb{M}$ denotes the set of probability models $\mathcal{M} = \langle \Omega, \mathcal{F}, \Pr \rangle$ such that $\mathcal{F}$ includes as a subset some partition $\mathcal{P}$ of Ω such that $1\text{–}1/|\mathcal{P}| > t$, and $\Pr(\psi) = \Pr(\psi^*)$ for all $\psi, \psi^* \in \mathcal{P}$:

Theorem 2.1 *For any probability model* $\mathcal{M} = \langle \Omega, \mathcal{F}, \Pr \rangle \in \mathbb{M}$ *and any* $\varphi \in \mathcal{F}$, *if* $\Pr(\varphi) < 1$ *then D holds of* φ.[4]

In other words, the accounts on offer fail to license as acceptable an extremely large class of propositions that intuitively are so. In particular, the accounts do not establish of *any* sub-unit probability proposition belonging to the field of a model with a fine enough equiprobable partition of the outcome space that it is rationally acceptable. Indeed, with respect to $\mathbb{M}$, we wind up with something equivalent to the claim that $f(\Pr)(\varphi) = 1$ if $\Pr(\varphi) = 1$ (Douven and Williamson say that the proposal 'trivializes' in this domain). So at best, we obtain the result that the proposals really do not take us very far at all towards a complete characterization of f.

But matters are presumably worse than this. Consider any further condition C that the authors cited may wish to offer as being sufficient for rational acceptability. If any proposition φ were to meet this condition, it had better not be the case, on pains of paradox, that φ is a lottery proposition. But then, presumably, Pollock, Ryan and Douven would have to resort to equipping C with the proviso that defeater D does not hold of the proposition under consideration. The upshot of this would then be that violation of D is necessary for rational acceptability and hence, by Theorem 2.1, that, although NON-UNANIMITY holds in general, it fails in $\mathbb{M}$. But this would be an unhappy result: the acceptability of an arbitrary less-than-certain proposition—e.g. the proposition that it will not snow in Toulouse next Christmas Day—clearly needn't *ipso facto* be defeated by the mere fact that there exists some extremely fine equiprobable partition of the outcome space—e.g. the partition of propositions of the form 'Ticket n will win' induced by the existence of a 10^6-ticket fair lottery with guaranteed winner.

3 Generalizing the Counterargument?

Noting that the proposals may be subject to various modifications so as to avoid the undesirable results just reported, Douven and Williamson ponder

[4] The proof is given by noting that for any φ such that $1 > \Pr(\varphi) \geq t$, the following set of propositions Γ meets the requirements of DEFEATER 1–3: $\Gamma = \{\varphi\} \cup \{\overline{\varphi} \cup \psi | \psi \in P\}$; see (*op. cit.*, p. 760).

over whether a more general baseline result could be offered that would tell against a much broader class of suggestions:

> […] there remains a nagging doubt that there might be some presently overlooked 'trivialization argument' similar to the one propounded previously. As it turns out, such a doubt would be justified, for the [previous argument] generalizes: it can be proved that a large class of proposals similar to the ones considered above fail for what is at root the same reason for which those were seen to fail. (*op. cit.*, p. 761)

The 'large class of proposals' that Douven and Williamson wish to consider consists of those proposals 'that are *formal* in the sense that they define the defeater in terms that are probabilistic or broadly logical' (*op. cit.*, p. 758, emphasis in original). It turns out that they have a precise—and plausible—characterization of what qualifies as a formal property, in the relevant sense. In their view, such a property is a property whose extension in any probability model is invariant under probability- and set-theoretic-operation-preserving permutations of the propositions in the field of that model. To unpack things:

Definition 3.1 An *automorphism* π of a probability model $\mathcal{M} = \langle \Omega, \mathcal{F}, \Pr \rangle$ is a 1:1 function $\mathcal{F} \mapsto \mathcal{F}$, such that, for all $\varphi, \psi \in \mathcal{F}$:

(1) $\pi(\varphi \cap \psi) = \pi(\varphi) \cap \pi(\psi)$
(2) $\pi(\overline{\varphi}) = \overline{\pi(\varphi)}$
(3) $\Pr(\varphi) = \Pr(\pi(\varphi))$

Definition 3.2 A property is *structural* with respect to probability model $\mathcal{M} = \langle \Omega, \mathcal{F}, \Pr \rangle$ iff for any proposition $\varphi \in \mathcal{F}$ and any automorphism π of $\mathcal{M}$, if φ has that property, then so too does $\pi(\varphi)$.

Definition 3.3 A property is *formal* iff it is structural with respect to every probability model.

It will also be useful to mention the following potential constraint on *f*:

Structurality: For any probability model $\mathcal{M} = \langle \Omega, \mathcal{F}, \Pr \rangle$, any automorphism π of $\mathcal{M}$ and any proposition $\varphi \in \mathcal{F}$, $f(\Pr)(\varphi) = f(\Pr)(\pi(\varphi))$.

As Douven and Williamson point out, the class of formal properties includes, as they put it, 'what can be plausibly regarded as the primitive predicates from (meta-)logic, set theory and probability theory' (*op. cit.*, p. 766). *Does* rational acceptability supervene on formal properties, so-defined? Although it seems at least prima facie plausible that it does, a considered assessment of this claim will have to await another occasion. In the meantime, it will be interesting to follow Douven and Williamson in investigating what consequences this 'formalistic' constraint would have on a theory of rational acceptance.

In order to present their findings, one last piece of terminology must be introduced:

Definition 3.4 A property of a proposition is *aggregative* iff, for all propositions φ and ψ, if φ has P and ψ has P, then their intersection $\varphi \cap \psi$ does so too.

The main results of the paper are then as follows, where $\mathbb{M}'$ denotes the set of probability models $\mathcal{M} = \langle \Omega, \mathcal{F}, \Pr \rangle$ such that $\Omega = \{w_1, \ldots, w_n\}$ is finite, $\mathcal{F} = \wp(\Omega)$, and for all $w, w^* \in \Omega$, $\Pr(\{w\}) = \Pr(\{w^*\})$:

Theorem 3.1 *For any probability model $\mathcal{M} = \langle \Omega, \mathcal{F}, \Pr \rangle \in \mathbb{M}'$, if property P is structural with respect to $\mathcal{M}$, Q aggregative and P sufficient for Q, then if there exists a proposition $\varphi \in \mathcal{F}$ such that* $\Pr(\varphi) < 1$ *and φ has P, then $\varnothing$ has Q.*[5]

This, in turn, has the following immediate corollary:

Theorem 3.2 *For any probability model $\mathcal{M} = \langle \Omega, \mathcal{F}, \Pr \rangle \in \mathbb{M}'$, if property Q is both aggregative and structural with respect to $\mathcal{M}$, then if there exists a proposition $\varphi \in \mathcal{F}$ such that* $\Pr(\varphi) < 1$ *and φ has Q, then $\varnothing$ has Q.*

The pertinent interpretation of Q is obviously the following: φ has Q if $f(\Pr)(\varphi) = 1$. Theorem 3.1 pertains to attempts to provide a sufficient condition for acceptability, in the same vein as the proposals considered in the previous section. Theorem 3.2, on the other hand, pertains to attempts to specify a condition that is both sufficient and necessary. Of course, it is plausible that those who require of some *particular* property sufficient for rational acceptability that it be structural will make the same requirement of *any* property sufficient therefore, i.e. endorse STRUCTURALITY. Theorem 3.2 is therefore clearly the more interesting of the two.

After presenting these results, Williamson and Douven then offer the following informal gloss on Theorem 3.1:

> [...] any proposal properly called a *solution* to the lottery paradox—which cannot allow the inconsistent proposition to be rationally acceptable—is, if structural, trivial, just as [the conjunctions of TEMPLATE with DEFEATER 1–3] were seen to be. (*op. cit.*, p. 763, emphasis in original)

5 See Appendix for proof. Note that the result generalizes to any model, the strongest consistent propositions in the field of which are equiprobable; these propositions needn't consist in the $\{w_i\}$.

In other words, according to them, just like the proposals discussed in Section 2, any structural sufficient conditional for rational acceptability will be unable to license as acceptable many sub-unit probability propositions that are intuitively so. The gloss on Theorem 3.2 is the following:

> [...] if propositions with imperfect probability can be rationally acceptable while the inconsistent proposition is not, then rational acceptability is not a structural property. (*op. cit.*, p.763)

Now the first thing to note here is that Theorem 3.2 in no way provides a generalization of the results discussed at the end of the previous section. Indeed, the domain $\mathbb{M}'$ with respect to which we would obtain a violation of NON-UNANIMITY is not a proper superset of $\mathbb{M}$.

Secondly, and more importantly, if taken at face value, the second quote clearly inflates the logical strength of Theorem 3.2. The statement should be revised so as to make clear the somewhat important fact that the result has *only* been established for the very specific case of equiprobable distributions over the strongest consistent propositions in the powerset of a finite set of possible worlds. It would indeed be very 'damaging [...] to the project of finding a formal solution to the lottery paradox' (*op. cit.*, p. 763) if the result held *generally*, for all probability models, as the quote clearly suggests. This would amount to NON-UNANIMITY, ZERO-NORMALIZATION, STRUCTURALITY and AGGREGATIVITY being jointly inconsistent, and that would indeed be unwelcome news to many.[6]

Theorem 3.2 can of course be used to show that these properties are jointly inconsistent, albeit in the presence of a *further* constraint on *f*. The truth, for instance, of the following condition would force NON-UNANIMITY to obtain in $\mathbb{M}'$ and hence, by Theorem 3.1, force a violation of ZERO-NORMALIZATION:

> **MONOTONICITY:** For any pair of probability models $\mathcal{M} = \langle \Omega, \mathcal{F}, \mathrm{Pr} \rangle$ and $\mathcal{M}^* = \langle \Omega, \mathcal{F}^*, \mathrm{Pr}^* \rangle$ and propositions $\varphi \in \mathcal{F}$ and $\psi \in \mathcal{F}^*$, if $\mathrm{Pr}(\varphi) \leq \mathrm{Pr}^*(\psi)$ then $f(\mathrm{Pr})(\varphi) \leq f(\mathrm{Pr}^*)(\psi)$.

To see why adding this leads to a violation of ZERO-NORMALIZATION, assume NON-UNANIMITY and consider a specific model $\mathcal{M} = \langle \Omega, \mathcal{F}, Pr \rangle \notin \mathbb{M}'$ and proposition $\varphi \in \mathcal{F}$ such that $\mathrm{Pr}(\varphi) < 1$ but $f(\mathrm{Pr})(\varphi) = 1$. It is easy to establish that there will exist a model $\mathcal{M}^* = \langle \Omega, \mathcal{F}^*, \mathrm{Pr}^* \rangle \in \mathbb{M}'$ and $\psi \in \mathcal{F}^*$ such that $1 > \mathrm{Pr}^*(\psi) \geq \mathrm{Pr}(\varphi)$. By MONOTONICITY, $f(\mathrm{Pr}^*)(\psi) = f(\mathrm{Pr})(\varphi) = 1$

6 Including to many of those who do not subscribe to TEMPLATE; the result would be very general indeed.

and hence NON-UNANIMITY will hold in $\mathbb{M}'$. So if MONOTONICITY is true, then if NON-UNANIMITY is true in general, it is also true in $\mathbb{M}'$, hence, assuming that STRUCTURALITY and AGGREGATIVITY are both true, by Theorem 3.2, ZERO-NORMALIZATION is false. So we can offer the following:

Theorem 3.3 *If f satisfies STRUCTURALITY, AGGREGATIVITY and MONOTONICITY, then either it does not satisfy ZERO-NORMALIZATION or it does not satisfy NON-UNANIMITY.*

However, not only is MONOTONICITY not clearly intuitive in itself but, to the best of my knowledge, there is, as of yet, no compelling argument in support of it either.[7,8] Pending the provision of such an argument, let us therefore focus on the question of how damaging the incompatibility actually demonstrated in Theorem 3.2 is to the prospects of a formal solution to the lottery paradox.

In spite of the importance of the issue to the claims made, Douven and Williamson offer no considerations to believe that it is damaging at all. In fact, upon a modicum of reflection, it seems fairly clear that it is *not*. Whilst failure of NON-UNANIMITY over the universal domain is clearly undesirable, as is failure of NON-UNANIMITY over the slightly narrower domain $\mathbb{M}$ considered in Theorem 2.1, failure of NON-UNANIMITY over the particular domain $\mathbb{M}'$ considered in Theorem 3.1 actually seems *intuitively correct*.

And indeed, agents with maximally entropic distributions of the sort specified above are after all in the worst possible state of indecision with respect to the contingent makeup of the world: if there ever was a situation in which abstaining from taking a stance on any contingent matter of fact is epistemically recommendable, this is surely it![9]

To unpack this more explicitly: (i) for any field of propositions, there is at least one probability function defined over that field with respect to which no contingent proposition is rationally acceptable; (ii) for any field of propositions and associated probability function, if no contingent proposition is rationally acceptable given that function, then the same applies to any asso-

[7] It may be interesting to note that, in a recent co-authored paper, Douven has explicitly expressed doubts regarding the analogue of a condition somewhat weaker than MONOTONICITY in the structurally similar context of judgment aggregation. This leads me to suspect that he would be fairly unmoved by the result reported in Theorem 3.3. See (Douven and Romeijn [2007]) for both their concerns regarding this weaker condition in the context of judgment aggregation and their insightful discussion of the structural parallels between aggregation and acceptance.

[8] Note that if MONOTONICITY were independently intuitive, the proposals discussed in Section 2 could be dismissed very quickly indeed: MONOTONICITY is violated by all three suggestions.

[9] Suspension of judgment could arguably be prudentially irrational, for whatever reason, but it is epistemic rather than prudential rationality that is the focus of the present discussion.

ciated probability function that has greater entropy; therefore (iii) for any field of propositions, no contingent proposition is rationally acceptable given a maximally entropic probability function.[10]

It is worth noting here that there is what could be seen as a precedent for this view in the belief revision literature. In Grove's well-known framework (Grove [1988], p. 160), for instance, we find a function mapping total preorders $\succeq$ in Γ, the set of strongest consistent propositions in $\mathcal{F}$, on to a subset K of $\mathcal{F}$.[11] According to Grove, relation $\succeq$ 'may be interpreted as a measure of how '[in]compatible' alternative [members of Γ] are with our current beliefs [K]', with $\gamma \succeq \gamma^*$ iff γ is at least as incompatible as γ^*. It is not too far-fetched to interpret the inverse of this relation as a simple relation of comparative probability (although, quite remarkably, this interpretation has not, to the best of my knowledge, been offered yet). Modeling the agent's doxastic structure in terms of this ordered set of propositions is then equivalent to a representation in terms of a set of probability functions, namely the set of all probability functions Pr with domain $\mathcal{F}$ such that, for all γ, $\gamma^* \in \Gamma$, $\Pr(\gamma) \leq \Pr(\gamma^*)$ iff $\gamma \succeq \gamma$. So we would have in effect a special case of a more general framework than the one offered here: a special case of a mapping from possibly *non-singleton sets* of probability functions to sets of accepted propositions (or equivalently, to acceptance functions).

The set of propositions returned by the function in question is the set of all propositions that are supersets of all the members of the minimal set of Γ (i.e. $\varphi \in K$ iff for all $\gamma \in \{\gamma \in \Gamma | \forall \gamma^* \in \Gamma, \gamma^* \succeq \gamma\}$, we have $\gamma \subseteq \varphi$).[12] So if, for all γ, $\gamma^* \in \Gamma$, $\gamma \succeq \gamma^*$ and $\gamma^* \succeq \gamma$, then $K = \{\Omega\}$. But of course, in such circumstances, the equivalent representation in terms of sets of probability functions is just a singleton set, containing the unique probability function that assigns equal probabilities to the elements of Γ. The upshot of this then is that, according to a natural interpretation of a highly orthodox view in the belief revision literature, if all members of Γ are considered equally probable, then no contingent proposition is rationally acceptable.

Of course, this framework has nothing to say regarding the acceptability of propositions relative to some singleton set of probability functions, whose member does *not* assign equal probabilities to the elements of Γ. This level

[10] Note, of course, that all that is needed here is for failure of NON-UNANIMITY in $\mathbb{M}'$ to be more plausible than the failure of either STRUCTURALITY or AGGREGATIVITY. To the extent that the latter is fairly implausible in my view, that failure of the first is highly plausible strikes me as being more than needs to be established.

[11] K is assumed to be closed under the subset relation, so that if $\varphi \in K$ and $\varphi \subseteq \psi$, then $\psi \in K$. It needn't however be the case that for all $\varphi \in \mathcal{F}$, either $\varphi \in K$ or $\overline{\varphi} \in K$.

[12] I ignore, both here and in what follows, the somewhat exotic case in which $\min(\Gamma) = \varnothing$. To deal with this, we should rather say that $\varphi \in K$ iff there exists a $\gamma \in \Gamma$ such that, for all $\gamma^* \in \Gamma$ such that $\gamma \succeq \gamma^*$, $\gamma^* \subseteq \varphi$.

of opinionation is simply not modeled. One could however be tempted to make the following suggestion:

PROPOSAL: For any probability model $\mathcal{M} = \langle \Omega, \mathcal{F}, \Pr \rangle$ and proposition $\varphi \in \mathcal{F}$, $f(\Pr)(\varphi) = 1$ iff for all $\gamma \in \max(\Gamma) := \{\gamma \in \Gamma | \forall \gamma^* \in \Gamma, \Pr(\gamma) \geq \Pr(\gamma^*)\}$, it is the case that $\gamma \subseteq \varphi$.

It is then trivial to show:

Theorem 3.4 PROPOSAL *entails* (i) NON-UNANIMITY, (ii) UNIT-NORMALIZATION, (iii) ZERO-NORMALIZATION, (iv) FINITE SUPERADDITIVITY, *and* (v) STRUCTURALITY.

Theorem 3.5 PROPOSAL *entails the negations of* (i) MONOTONICITY *and* (ii) OPINIONATION.[13]

This is a favorable result, at least to the extent that STRUCTURALITY is a plausible constraint to impose. We do not however, at this point in time, have anywhere near sufficient grounds to endorse this suggestion. After all, there are presumably many alternatives that also meet the aforementioned desiderata, some of which may be alone in meeting a number of further constraints that have yet to be unearthed. Further research would be required here.

For instance, the fact that the account would license acceptance of propositions with arbitrarily small probabilities may be of concern to some (rightly or wrongly). This could however be dealt with by either adding a requirement to the effect that the members of the maximal set receive a probability above some relevant threshold t (i.e. replacing $\max(\Gamma)$ with $\max'(\Gamma) := \{\gamma \in \Gamma | \forall \gamma^* \in \Gamma, \Pr(\gamma) \geq \Pr(\gamma^*) \wedge \Pr(\gamma) \geq t\}$) or simply by taking the set of accepted propositions to consist in the set of those propositions that are supersets of every member of Γ whose probability lies above some suitable sub-unit probability threshold t (i.e. replacing $\max(\Gamma)$ with $\max''(\Gamma) := \{\gamma \in \Gamma | \Pr(\gamma) \geq t\}$).

4 The Lottery Paradox: Towards a 'Formal' Solution

Setting aside PROPOSAL and cognate suggestions for another occasion, one might want to note a somewhat surprising fact about Douven and Williamson's theorems. They very helpfully, if unwittingly, suggest a first step towards a characterization of an important class of rationally unacceptable propositions: the class of lottery propositions for an equiprobable lottery.

13 See Appendix for simple proofs of both theorems.

Indeed, the models in $\mathbb{M}'$ can be seen as providing formal representations of the state of mind of agents whose opinions range exclusively over the closure under union of the members of a finite partition of lottery proposition, of the form 'Ticket i will win' ($1 \leq i \leq n$), with $1-1/n \geq t$ and Pr(Ticket i will win) = Pr(Ticket j will win) (for all j such that $1 \leq j \leq n$).[14] Let us therefore baptize the main point argued for in the previous section as follows:

Weak Lottery-Proofness: For any probability model $\mathcal{M} = \langle \Omega, \mathcal{F}, \Pr \rangle \in \mathbb{M}'$ and proposition $\varphi \in \mathcal{F}$, if $\Pr(\varphi) < 1$, then $f(\Pr)(\varphi) = 0$.

Of course, this constraint remains a very permissive one. It only singles out as unacceptable a very small selection of all lottery propositions for equiprobable lotteries. It leaves out, for instance, those lottery propositions that are *not* the union of a proper subset of the set of strongest consistent propositions in an agent's field. However, it does take us one small step further towards characterizing f and paves the way for the provision of stronger restrictions on rational acceptability, based on possible extensions of Williamson and Douven's results. So far from spelling doom for the enterprise of providing a formal account of rational acceptability, 'Generalizing the Lottery Paradox' turns out to provide a non-negligible contribution to this project.

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Appendix

Proof of Theorem 3.2: Assume $\Pr(\varphi) < 1$. Then there exists some possible world $w^* \in \Omega$ such that $\{w^*\} \nsubseteq \varphi$. Now let π_i denote a transposition of $\{w_i\}$ and $\{w^*\}$—i.e. a permutation of $\wp(\Omega)$ such that $\pi_i(\{w_i\}) = \{w^*\}$, $\pi_i(\{w^*\}) = \{w_i\}$ and $\pi_i(\{w_j\}) = \{w_j\}$, for $j \neq i$. Let g_i denote the automorphism of $\mathcal{F}$: $g_i(\varphi) := \bigcup_{\{w\} \subseteq \varphi} \pi_i(\{w\})$. Assume P to be structural and sufficient for Q. It follows that, for $1 \leq i \leq n$, $g_i(\varphi)$ has P and hence Q. Assume Q to be aggregative. It follows that $\bigcap_{1 \leq i \leq n} g_i(\varphi)$ has Q. Finally, since for $1 \leq i \leq n$ we have $\{w_i\} \nsubseteq g_i(\varphi)$, it follows that $\bigcap_{1 \leq i \leq n} g_i(\varphi) = \varnothing$ and hence that $\varnothing$ has Q.

[14] And as has been remarked a number of times in the literature, notably by both Douven ([2006]) and Williamson ([2000]) themselves (see also Hawthorne [2004]), subjects would typically refrain from endorsing this type of proposition. This observation also lends further support to the above-noted intuition that failure of Non-Unanimity in $\mathbb{M}'$ is the right result.

Proof of Theorem 3.4:

(i) Trivial.

(ii) For any probability model $\mathcal{M} = \langle \Omega, \mathcal{F}, \mathrm{Pr} \rangle$ and proposition $\varphi \in \mathcal{F}$, $\varphi \subseteq \Omega$, so *a fortiori*, for all $\gamma \in \max(\Gamma)$, $\gamma \subseteq \Omega$ and hence $f(\mathrm{Pr})(\Omega) = 1$.

(iii) For any probability model $\mathcal{M} = \langle \Omega, \mathcal{F}, \mathrm{Pr} \rangle$ and proposition $\varphi \in \mathcal{F}$, $\varphi \nsubseteq \varnothing$, so *a fortiori*, for all $\gamma \in \max(\Gamma)$, $\gamma \nsubseteq \varnothing$ and hence $f(\mathrm{Pr})(\varnothing) = 0$.

(iv) Finite Superadditivity is false just in case $f(\mathrm{Pr})(\varphi \cup \psi) < f(\mathrm{Pr})(\varphi) + f(\mathrm{Pr})(\psi) - f(\mathrm{Pr})(\varphi \cap \psi)$. However, each way of making this inequality true contradicts Proposal:

(a) $f(\mathrm{Pr})(\varphi \cup \psi) = 1, f(\mathrm{Pr})(\varphi) = 1, f(\mathrm{Pr})(\psi) = 1$ and $f(\mathrm{Pr})(\varphi \cap \psi) = 0$. Assuming $f(\mathrm{Pr})(\varphi) = 1$ and $f(\mathrm{Pr})(\psi) = 1$, by Proposal, for all $\gamma \in \max(\Gamma)$, $\gamma \subseteq \varphi$ and $\gamma \subseteq \psi$ hence $\gamma \subseteq \varphi \cap \psi$ and therefore $f(\mathrm{Pr})(\varphi \cap \psi) = 1$. Contradiction.

(b) $f(\mathrm{Pr})(\varphi \cup \psi) = 0, f(\mathrm{Pr})(\varphi) = 1, f(\mathrm{Pr})(\psi) = 1$ and $f(\mathrm{Pr})(\varphi \cap \psi) = 1$. Assuming $f(\mathrm{Pr})(\varphi) = 1$, by Proposal, for all $\gamma \in \max(\Gamma)$, $\gamma \subseteq \varphi$ hence $\gamma \subseteq \varphi \cup \psi$ and therefore $f(\mathrm{Pr})(\varphi \cup \psi) = 1$. Contradiction. The proof is analogous for the remaining cases.

(v) Assume that for some automorphism π of $\mathcal{M}$ and some proposition $\varphi \in \mathcal{F}$, $f(\mathrm{Pr})(\varphi) \neq f(\mathrm{Pr})(\pi(\varphi))$. Say, for instance, that $f(\mathrm{Pr})(\varphi) = 1$, but $f(\mathrm{Pr})(\pi(\varphi)) = 0$ (the alternative case is analogous). From Proposal, we have $\gamma \subseteq \varphi$, for all $\gamma \in \max(\Gamma)$. Since for all $\gamma \in \max(\Gamma)$, it is the case that $\pi(\gamma) \in \max(\Gamma)$, it follows that it is also the case that $\pi(\gamma) \subseteq \varphi$, for all $\gamma \in \max(\Gamma)$, and hence that $f(\mathrm{Pr})(\pi(\varphi)) = 1$. Contradiction. ■

Proof of Theorem 3.5:

(i) As seen in Theorem 3.3, Monotonicity is incompatible with the conjunction of Structurality, Aggregativity, Non-Unanimity and Zero-Normalization, all of which have been shown to follow from Proposal.

(ii) Trivial. ■

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