Fundamentally vague laws

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Are laws of nature exact? Perhaps not, says philosopher **Eddy Keming Chen** – meaning we may need to look beyond mathematics for a complete description of the world

WHAT links a heap of sand, the edge of a cloud and actor Patrick Stewart's baldness? If you're only vaguely grasping what I'm getting at, you're on the right track: they are all examples of imprecision in our description of the world. How many grains of sand can you take away from the heap and still call it a heap? Where exactly does the cloud end and the sky begin? How many hairs is Patrick Stewart allowed to have, and of what length, before he is classed as not bald? It's hard – perhaps impossible – to tell.

Such vague concepts, with their messy boundaries and borderline cases, are all around us. Until now, we've tended to assume they represent imperfections in our state of knowledge, in our ways of communication, or in our modes of description of the world. At some level, we think, the world must be precisely defined. Underpinning it all, in the end, are the laws of physics, which are expressed using cast-iron mathematical equations that admit no vagueness.

I'm not so sure that's the case. I think I've uncovered an instance in which a fundamental physical law is itself vague. If so, and we must admit vagueness into our physical laws, that casts doubt on the ability of conventional mathematics to provide us with a full description of the world – and perhaps gives us entirely new avenues to find even better physical theories.

Philosophers like me spend a lot of time thinking about vague terms such as "bald" and "heap". We've been doing it for a long time, too. The heap question is known as the sorites paradox, and it was noted as early as the 4th century BC. If a million grains make a heap, then one million minus one grains also make a heap, as do one million minus two grains and so on. Follow that logic, and eventually a single grain also makes a heap. That is absurd. So should we accept that there is a sharp boundary, some number of grains, below which grains do not make a heap? That is hard to swallow, too.

Vagueness is pervasive in natural language, and yet it resists logical analysis. The principle of bivalence that is central to classical logic – every statement is either true or false – seems to fail for vague terms. Imagine Tom, who is a borderline case of "bald". It is not true that Tom is bald. It is not false that Tom is bald. Classical bivalent logic is at risk.

Philosophical reflections have identified three types of vagueness. First, there is "semantic" vagueness. This is down to genuine indeterminacy in our language. Perhaps the terms we actually use really are so vague in such a way that leave some statements without being clearly true or clearly false; it is just a feature of how we communicate.

Second, it may be due to our ignorance of some facts. Even though it may not be clear how to draw a sharp boundary between bald and non-bald, or between heap and non-heap, there may be an objective cut-off that we’re not aware of. This we call “epistemic” vagueness, and it neatly preserves bivalent logic, because there’s a true or false answer to any statements, even if we don’t know which it is.

Third, vagueness may be due to some genuine indeterminacy in the world. This is called “ontic” vagueness. There are some objects we define using natural language, such a cloud, or Mount Everest, that simply do not have determinate physical boundaries in space or time.

While we can continue to debate what type of vagueness is at play in any one situation, one thing that unites most philosophers is that all of this has little to do with fundamental laws of nature. Vagueness may also appear in some high-level sciences, such as biology, where terms such as “cell”, “organism” and “life” are imprecisely defined – a virus of the sort that’s exercising us right now seems to be a classic borderline case of a living organism, as a bundle of genetic material that can only replicate inside the cells of another organism. But that vagueness should disappear when we drill down to more fundamental levels of explanation.

Fundamental laws of nature are supposed to be written in the exact, non-messy language of mathematics. Mathematics as we currently conceive is built around set theory, and a mathematical set seems to be the very definition of being not vague. Something is either a member of a set – the set of all odd numbers, say, or all numbers divisible by 11 – or it isn’t. Sets are rigidly defined via a notion of equality: if two sets have the same members, they are the same set. Similarly, any mathematical function, topological space or geometrical shape built from sets is precisely defined. It is hard to see how the fundamental laws of physics could be completely and faithfully expressed in these terms if they admitted any vagueness.

Think, for example, of Isaac Newton’s universal law of gravitation, or his second law of motion, force = mass x acceleration, $F = ma$. Physical laws such as these are arbitrary, in the sense that they are set by nature. From all different ways the world could be, they pick out a small subset of physically possible worlds that are compatible with those equations. They admit no borderline cases: the behaviour of objects within this world will adhere to these equations exactly, no ifs, no buts.

These laws also have a quality that will become important later on, “traceability”. Our world is sensitive to changes in the laws. Any shift in the gravitational constant, G, for example, will be felt by the massive objects and will change, however slightly or significantly, the motion of planets around stars, the formation of galaxies, the distribution of matter in the cosmos or how a falling vase shatters when it hits the ground. G’s exact value leaves a trace in what there is and how things move in the world. Similarly, if we change $F=ma$ to $F=ma_{1.001}$, it can produce observable physical changes.

The same arguments hold with all other fundamental equations of physics, for example Schrödinger’s equation that defines the evolution of a quantum system, or Einstein’s field equations of general relativity that determine the evolution of the universe at large.

Did I say all? Not quite. There seems to be one essential element of fundamental physics that has every right to be considered a law, but doesn't fit into this pattern.

Its origin lies in the puzzling observation that the fundamental equations of physics are all time-symmetric – they work equally well backwards as forwards – yet the world around us is distinctly time-asymmetric and irreversible. An arrow of time exists, a fact often encapsulated in the second law of thermodynamics, which puts limits on the sort of processes that can occur in reality. Ice cubes melt when placed in a drink to cool it, for example, but don't spontaneously form in it.

Explaining why leads to an influential proposal known as the Past Hypothesis. It says that the universe had a very special initial condition: it was initially in a state of low entropy, one with a high degree of order, and has been evolving away from that ever since. This is about as fundamental a law about how the universe works as we have – and yet it is screamingly vague. In its weakest version, it simply says that the initial state of the universe has low entropy. How low is low?

I call this potential vagueness in a fundamental physical law "nomic vagueness". It seems distinct from the three other types, and may be more basic. But let's drill down a little into what it consists of in the case of the Past Hypothesis.

First, its vagueness can be specified in a more precise way. We can characterise the initial state of the universe in terms of macroscopic variables such as temperature, volume, pressure and entropy, in accordance with astrophysical data. But in classical statistical mechanics, this macrostate corresponds to any number of microstates of individual particles with different positions and velocities. Many different microstates look essentially the same to us as we measure the macrostate, and which microstates correspond to which macrostate is a vague matter. There are always going to be borderline cases where a particular configuration of particles might amount to an initial state of that particular temperature, say – or might not.

But what if we just stipulate the exact boundaries of the macrostate by fiat: say the initial state of the universe corresponds to this set of possible microstates and no others? Let's call this the Strong Past Hypothesis. It means that any vagueness about the universe's initial state is due to our inexact knowledge of its macrostate. This is then similar to the epistemic vagueness we discussed earlier. So, nothing to see here?

The problem is that this Strong Past Hypothesis is arbitrary, and not just in the way other laws or constants are arbitrary. It is untraceably arbitrary: whereas changing the value of the gravitational constant makes a difference to what the world is like, there are infinitely many ways of wiggling the boundary of the initial macrostate that make no difference to what the world is like or even the probabilities of events within it.

This leaves us on the horns of a dilemma: we either embrace nomic vagueness or nomic untraceability. That is to some extent a matter of taste, but I suggest we should rather avoid untraceability. Observations of the world often can uncover the nature of traceable laws;

untraceable laws, by contrast, cannot be pinned down by facts in the world. We can't do science to determine what they are; there is a gap between untraceable laws and the world.

A resolution to this dilemma might still come from within physics, and from a rather surprising quarter – quantum theory. At first glance, this would seem to be the last place to look to banish vagueness from physical laws. Quantum objects such as particles are described by "wave functions" that have no definite locations in space or other exactly defined properties. Besides reality thus apparently becoming riddled with ontic vagueness, the very process of measurement that resolves this vagueness, "collapsing" quantum wave functions into exact states, is itself painfully vaguely defined. In the words of physicist John Stewart Bell, "What exactly qualifies some physical systems to play the role of 'measurer'? Was the wave function of the world waiting to jump for thousands of millions of years until a single-celled living creature appeared? Or did it have to wait a little longer, for some better qualified system…with a Ph.D.?"

Vagueness is indeed a feature of orthodox quantum theory – but other, competing interpretations beyond the orthodox are also available. In the many worlds interpretation, for example, when we probe a quantum system more deeply the universe divides according to the possibilities we might see. There is no vagueness at the fundamental level in this depiction: the fundamental stuff is always exactly defined, and the dynamical laws are exactly specified. In Bohmian mechanics, also known as pilot-wave theory, meanwhile, a single universe evolves deterministically at all times in accordance with exact mathematical equations, and in "spontaneous collapse" theories, wave function collapse is just a random and spontaneous feature of the dynamical laws of the universe, banishing any vague or mystical special role for the measurer.

How might this help with the Past Hypothesis? The details are complex, but it amounts to the fact that, unlike classical mechanics, quantum mechanics allows us to connect the initial microstate and macrostate of the universe in an exact and traceable way. Traditionally, the initial microstate of the universe is described by a wave function, and the Past Hypothesis restricts the possible wave functions to a small subset compatible with a low-entropy macrostate. Work I have done shows that we can specify the initial microstate of the universe as something equivalent to a sum over all these possible wave functions.

In pilot wave theories and many worlds theories, the form of the resulting "initial density matrix" will influence how things evolve subsequently; in spontaneous collapse theories, it will determine how collapses randomly happen. Changes to the initial density matrix will typically change what the world is like, just as is the case with other dynamical constants and laws. Thus, quantum theory helps us preserve both nomic exactness and traceability.

But there's no guarantee that such a solution is possible: we are far away from achieving a quantum description of the beginning of the universe, and it may well be that a final theory of the world may not be fully quantum. If so, and we find ourselves on the horn of our dilemma, forced to admit nomic vagueness, the consequences could be profound, not least for our ability to use mathematics to describe the universe. Any way of capturing a fundamental, yet vague, law such as the Past Hypothesis using traditional mathematics based on set theory will miss out something, or will impose too much sharpness somewhere.

That is perhaps an opportunity to think beyond classical mathematics for describing the universe. Other foundations besides set theory do exist for mathematics. Category theory, for example, focuses not on what mathematical objects are in what set but the abstract relationships between objects; homotopy type theory, meanwhile, relaxes the notion of equality between objects central to set theory and defines objects in terms of paths between points in an abstract space. Either approach might provide a better language for capturing all physical laws, offering more flexibility in dealing with vagueness; equally, it is also possible that no mathematics can completely capture vagueness.

What about future laws of physics? That is a big unknown. But if nomic vagueness is possible, perhaps we don't have to restrict ourselves to formulating laws that can only be stated in precise mathematics. For example, physicists Abhay Ashtekar and Brajesh Gupt have recently done some work on loop quantum gravity, one promising approach to unifying quantum theory with general relativity. Their proposal for an initial condition of the universe also contains a law-like initial condition that could be an instance of nomic vagueness, because of a vague boundary of the 'Planck regime', the earliest epoch of the universe when quantum effects of gravity dominate all other forces. It is one hint that a final theory of physics might not be entirely mathematically expressible.

Mathematics will still remain extremely useful. But if there is nomic vagueness, it may never completely capture the objective order of the world. It may turn out that vagueness runs far deeper than defining the number of grains of sand in a heap or of the hairs on a bald man's head.

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