Disproof of Bell's Theorem

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We illustrate an explicit counterexample to Bell's theorem by constructing a pair of dichotomic variables that exactly reproduce the EPR-Bohm correlations in a manifestly local-realistic manner.

Central to Bell's theorem [1] is the claim that no local and realistic model can reproduce the correlations observed in the EPR-Bohm experiments. Here we construct such a model. Let Alice and Bob be equipped with the variables

$$A(\mathbf{a},\,\lambda)\,=\,\{-\,a_j\;\boldsymbol{\beta}_j\,\}\,\{\,a_k\;\boldsymbol{\beta}_k(\lambda)\,\}\,=\,\begin{cases}+1 & \text{if} \quad \lambda\,=\,+1\\ -1 & \text{if} \quad \lambda\,=\,-1\end{cases} \tag{1}$$

$$\text{and }\;B(\mathbf{b},\,\lambda)\,=\,\{\,b_j\;\boldsymbol{\beta}_j(\lambda)\,\}\,\{\,b_k\;\boldsymbol{\beta}_k\,\}\,=\,\begin{cases}-1 & \text{if} \quad \lambda\,=\,+1\\ +1 & \text{if} \quad \lambda\,=\,-1\,,\end{cases} \tag{2}$$

where the repeated indices are summed over x, y, and z; the fixed bivector basis $\{\boldsymbol{\beta}_x,\,\boldsymbol{\beta}_y,\,\boldsymbol{\beta}_z\}$ is defined by the algebra

$$\boldsymbol{\beta}_j\,\boldsymbol{\beta}_k\,=\,-\,\delta_{jk}\,-\,\epsilon_{jkl}\,\boldsymbol{\beta}_l\,; \tag{3}$$

and—together with $\boldsymbol{\beta}_j(\lambda)=\lambda\,\boldsymbol{\beta}_j$—the λ-dependent bivector basis $\{\boldsymbol{\beta}_x(\lambda),\,\boldsymbol{\beta}_y(\lambda),\,\boldsymbol{\beta}_z(\lambda)\}$ is defined by the algebra

$$\boldsymbol{\beta}_j\,\boldsymbol{\beta}_k\,=\,-\,\delta_{jk}\,-\,\lambda\,\epsilon_{jkl}\,\boldsymbol{\beta}_l\,,\quad \text{where } \lambda\,=\,\pm 1 \text{ is a fair coin [2]}, \tag{4}$$

δ_{jk} is the Kronecker delta, ϵ_{jkl} is the Levi-Civita symbol, $\mathbf{a}=a_x\,\mathbf{e}_x+a_y\,\mathbf{e}_y+a_z\,\mathbf{e}_z$ and $\mathbf{b}=b_x\,\mathbf{e}_x+b_y\,\mathbf{e}_y+b_z\,\mathbf{e}_z$ are unit vectors, and the indices $j,\,k,\,l=x,\,y,$ or z. The correlation between $A(\mathbf{a},\,\lambda)$ and $B(\mathbf{b},\,\lambda)$ then works out to be

$$\mathcal{E}(\mathbf{a},\,\mathbf{b})\,=\,\frac{\lim_{n\to\infty}\left\{\frac{1}{n}\sum_{i=1}^{n}A(\mathbf{a},\,\lambda^i)\,B(\mathbf{b},\,\lambda^i)\right\}}{\{-\,a_j\;\boldsymbol{\beta}_j\,\}\,\{\,b_k\;\boldsymbol{\beta}_k\,\}}\,=\,\lim_{n\to\infty}\left[\frac{1}{n}\sum_{i=1}^{n}\frac{A(\mathbf{a},\,\lambda^i)\,B(\mathbf{b},\,\lambda^i)}{\{-\,a_j\;\boldsymbol{\beta}_j\,\}\,\{\,b_k\;\boldsymbol{\beta}_k\,\}}\right] \tag{5}$$

$$=\lim_{n\to\infty}\left[\frac{1}{n}\sum_{i=1}^{n}\{\,a_j\;\boldsymbol{\beta}_j\,\}\,\{\,A(\mathbf{a},\,\lambda^i)\,B(\mathbf{b},\,\lambda^i)\}\,\{-\,b_k\;\boldsymbol{\beta}_k\,\}\right]\,=\,\lim_{n\to\infty}\left[\frac{1}{n}\sum_{i=1}^{n}\,\{\,a_j\;\boldsymbol{\beta}_j(\lambda^i)\,\}\;\{\,b_k\;\boldsymbol{\beta}_k(\lambda^i)\,\}\right] \tag{6}$$

$$=\,-\,a_j\,b_j\,-\lim_{n\to\infty}\left[\frac{1}{n}\sum_{i=1}^{n}\,\{\,\lambda^i\,\epsilon_{jkl}\;a_j\,b_k\;\boldsymbol{\beta}_l\,\}\right]\,=\,-\,a_j\,b_j\,+\,0\,=\,-\,\mathbf{a}\cdot\mathbf{b}\,, \tag{7}$$

where the denominators in (5) are standard deviations. The corresponding CHSH string of expectation values gives

$$|\mathcal{E}(\mathbf{a},\,\mathbf{b})\,+\,\mathcal{E}(\mathbf{a},\,\mathbf{b}')\,+\,\mathcal{E}(\mathbf{a}',\,\mathbf{b})\,-\,\mathcal{E}(\mathbf{a}',\,\mathbf{b}')|\,\leq\,2\,\sqrt{1-(\mathbf{a}\times\mathbf{a}')\cdot(\mathbf{b}'\times\mathbf{b})}\,\leq\,2\sqrt{2}\,. \tag{8}$$

Evidently, the variables $A(\mathbf{a},\,\lambda)$ and $B(\mathbf{b},\,\lambda)$ defined above respect both the remote parameter independence and the remote outcome independence (which has been checked rigorously [2][3][4][5][6][7]). This contradicts Bell's theorem.

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References

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