Virtual Mathematics

the logic of difference

Edited by
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Interlacing the singularity, the diagram and the metaphor

Gilles Châtelet *(edited by Charles Alunni*)*

Introduction by Charles Alunni

> you on whom the future counted so much, you didn't fear to put fire to your life
> We will wander for a long time around your example.
> It is necessary to return . All will have to be started again.
> René Char, *Dans l'atelier du poète*.[1]

The paper presented here for the first time is derived from a lecture that Gilles gave at the École Normale Supérieure in Paris, in a seminar series whose general title was *Possible Worlds*.[2] Not personally having been able to be present, and Gilles having prepared his talk on a series of scattered notes, I asked him, during April, to write his notes up as a paper for publication. The reverberations that I had heard from this lecture, in particular through the report of one of our common students, encouraged me to be pressing on this point. As always, Gilles had captivated a public which, at the beginning, and for reasons of differences in philosophical position, was far from accepting of him a priori. His speculative power, doubled by his 'heroic fury', had shaken the listeners of the rue d'Ulm. Several months passed, without us ever finding ourselves in a situation to resume the point of this project, and without me ever knowing whether Gilles had the least intention to carry it out. Less than two weeks before his suicide, I prepared one of the lectures of my own seminar at the table of a cafe where we were in the habit, for approximately five years, of periodically meeting.[3] Alexis de Saint-Ours, our common student, then came to find me, accompanied by Gilles whom he told me he had 'convinced to come as far as here'. Rather a rare thing, Gilles carried a briefcase. After having settled opposite me, he opened it and produced a small bundle of documents, which he showed me without allowing me to consult them, and all the while saying: 'You see, I listened to you: I am about to finish my paper'

A few days later, in the early morning of June 15, 1999, the announcement of the appalling news was communicated to me by our old mutual friend Bernard Besnier, 'Director of Studies' (*caïman*) at the ENS in Fontenay-Saint Cloud. When, a few days later again, we found ourselves with his sister and her closest friends in his apartment on boulevard Rochechouart, we found his work table entirely occupied by bundles of documents and opened books, which showed that Gilles was working there at the time when he killed himself; these were the elements of the paper in question. In agreement with our friends, I decided to collect them to try to reconstitute the final text of the paper that was to become his last fundamental contribution. The task proved to be long and difficult. He who admired William Burroughs so much (and whom he had met during his stay in the United States) proceeded to compose his manuscripts by an operation of textual *cut- up*.[4] Refusing any use of the computer, he developed a handwritten manuscript on which was then glued other pieces of printed text (quotations photocopied then cut out and stuck, mixed with other pieces of his own texts that he was in the habit of typing). The difficulty was made even worse both by a systematic absence of any numbering of the documents, and by the use of a 'secret' code, marked on the top of a page and on separate paper fragments (of the type $\Phi 1$, $\Phi 2\alpha$, etc. .). Lastly, as with the photocopies of whole books which he made use of and bound, he excluded the title page from the reproduction (making it sometimes difficult to identify the author and the work), in the same way, in his manuscripts Gilles practically never indicated the source of his quotations. In the text which interests us, this was particularly the case for his references to the 'knot theorist', Louis H. Kauffman. Although working on the same sources as Gilles, it has taken approximately two months of work to rebuild this kind of textual and theoretical *puzzle* in the form of a paper.

The manuscript, as with all Gilles' manuscripts, is deposited with the *Gilles Châtelet Archives* at the École Normale Supérieure in Paris, which is, by convention, under the responsibility of myself and Alain Prochiantz. These *Archives* were able to be repatriated to rue d'Ulm thanks to the *École Polytechnique* and to the mediation of the former director of his department of mathematics, François Laudenbach. This was made possible following the donation which was made to the École Normale Supérieure by Doctor Edwige Bourstyn-Châtelet, sister of Gilles, and to whom the oeuvre was bequeathed. For which I'd like to take the opportunity here to express my sincere gratitude.

Ulm, Paris, March 2005

Notes

1 [*Trans.* toi sur qui l'avenir comptait tant, tu n'as pas craint de mettre le feu à ta vie ./ Nous errerons longtemps autour de ton exemple./ Il faut revenir .Tout est à recommencer'.]
2 Meeting of April 15, 1999. [*Trans.* – at the ENS in rue d'Ulm, Paris.]
3 For the meetings of the 'Pensée des sciences' seminar, held twice a month at the École Normale Supérieure (Wednesday evenings from 8 to 10.30pm), we had founded a kind of ritual which consisted in (and consists in still today) continuing the debates over late victuals. The debates were so animated that the owner called us 'the folk group' (*le groupe folklorique*). This bistro, located opposite rue d'Ulm, used to be known as *Le Normal Bar*. In the sixties, it was already a meeting place: that of Jacques Lacan and his group.
4 Remember that it was between 1958 and 1960, at the time of his Parisian stay at the famous Beat Hotel of rue Gît-le-Cœur, that Burroughs became impassioned with the results of this technique of the cut-up developed by the painter and poet Brion Gysin.
5 Call number Ined.01.

Gilles Châtelet

If the allusive stratagems can claim to define a new type of systematicity, it is because they give access to a *space where the singularity of the diagram and the metaphor may interlace*, to penetrate further into the physico-mathematic intuition and the discipline of the gestures which precede and accompany 'formalisation' This interlacing is an operation where each component backs up the others: without the diagram, the metaphor would only be a short-lived fulguration because it would be unable to operate: without the metaphor, the diagram would only be a frozen icon, unable to jump over its bold features which represent the images of an already acquired knowledge; without the subversion of the functional by the singular, nothing would come to oppose the force of habit.

We would thus like to undertake research which would give priority to certain key axes, which would analyse the increasingly crucial role played by the allusive stratagems in the articulation of the intuitive practices of two different domains or disciplines: Physics and Mathematics, Geometry and Algebra, an articulation *that does not embody a relation of instrumentality of one practice over the other.*

A) *Analysis of the relationship between philosophical metaphors and scientific metaphors.*

Aristotle already noted that the metaphor could be understood as a 'syllogism to complete': it is precisely this *invitation to complete* that permits that which is not actually presented there to be shown. The metaphor allows one to think between the lines and thus is not only a linguistic impertinence necessarily devoted to precariousness and quickly absorbed by convention.

In philosophy, metaphors are not content to play a subsidiary role, which one could, if absolutely necessary, do without, but often appear as centrepieces of what Jean Ladrière calls the 'support of the line of argument': by creating the effect of *veracity*, this support establishes itself as complementary to studied deduction by means of logic in the narrow sense that ensures the *transfer of the supposed truth.*

These metaphors of a particular type reign over a whole context and globally command a whole system of more traditional metaphors devoted to the local illustration of propositions. Without what might be referred to as '*orchestrating metaphors*', the propositions would appear isolated, even if they respected the habitual protocols of sequences.

It is precisely this veracity and this allusive capacity that nourishes the argumentative support found at the heart of the intuitive practices of the most formalised sciences.

Whether they are scientific or philosophical, metaphors organise the key points of reactivation and acceleration, the fulcra, the 'Archimedes levers' able to retain a whole context and propel a whole set of concepts to a higher speed, allowing for example in physics or mathematics the almost instantaneous transit of deductive chains of considerable length.

This great proximity between the 'scientific' metaphor and the 'philosophical' metaphor gives rise to the thought that each of these fields expresses two different, yet capable of being articulated, modes of intervention of allusive stratagems. Thus, one can observe that the philosophical metaphor is in a relation of *rivalry-complicity* with conceptual grasp – with the reciprocal *threat of overflowing*: the incontinent proliferation of the metaphor, the domestication of metaphorical impertinence by conceptual grasp.

This relation of rivalry-complicity brings the philosophical metaphor closer to the scientific metaphor, if one recalls that *the latter is a part of the implicit text which accompanies any demonstrative development.* This implicit text – whose importance we emphasised in

Figuring Space (Châtelet 2000)[6] – allows an overall view of this development, entering into a relationship of alliance and rivalry with the official text presenting the *procès verbal* of the demonstrations. If it seems difficult to speak about the *procès verbal* of the demonstrative in philosophy, one can nevertheless point out that the philosophical metaphor is organised in *regulated sequences* ensuring an effect of convergence, of allusions and large unsteady oscillations (*mises en bascule*), an effect intended to force conviction, just as the implicit text allows anticipation of the already acquired stages of a proof.

The crucial strategic character of the metaphor – whether in science or philosophy – lets us suspect that it would help in better determining the proximities and differences of these two fields of rationality; this is why we propose to analyse it in several precise cases.

B) *Analysis of the role of allusive stratagems in contemporary physico-mathematics: a new conception of notation.*

Recent spectacular developments in *Knot Theory* (*Théorie des Nœufs* (sic!)), the work of Vaughan Jones marking the turning point, renders manifest a profound articulation between Geometry, Algebra, Topology and Physics.[7]

These developments should not only be appreciated as distinguishing themselves by 'varied applications' – as one terms the extensive character of a transfer of technology from one discipline to another – but as falling under a tradition of implementation of a *graphic reason* in the exact sciences.

We have already noted that the 'orchestrating metaphors' were able to exceed [to dissolve] the duality of the deductive and the argumentative by establishing a new relation between illustrating and the illustrated. This is also the case for some contemporary research which completely renews *the very notion of indexation.* No explicit intuition accompanies the 'classical' behaviour of calculations: in formulae of the type $\sum_i =$, the set of indices is neutral and the indexation remains completely external to the development of these calculations, behaving like a 'notation' which is completely indifferent to that which it notes. These formulae remain captive to a linear successiveness, x_1 then x_2 then x_3 etc., an artificial sequence a little analogous to the chain of verbs *veni, vidi, vici* where the temporal order of the processes of enunciation (*énonciation*) replicate exactly the order of the processes of the statement (*énoncé*).

The contemporary point of view makes the notation *concrete* by identifying it with a diagram already used in an *a priori* foreign domain (knot theory [*théorie des nœufs* (sic)[8]]). This domain thus 'evokes' gestures which are classical for it, but completely new in the domain where it is imported as 'notation' Thus certain *a priori* not very suggestive, complicated formulae of tensor calculus, can be condensed in a fulgurating way and launch new calculations.[9]

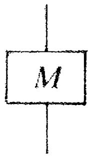
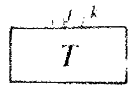
This upsets the very notion of indexation which becomes *bi-dimensional* in freeing itself from the successive: it is very much a victory of the hand that comments on itself, the indexation no longer being delivered by an external 'set', but by a process of deformation and modification of diagrams.[10] This confronts us with a remarkable situation: theorems of mathematics make it possible to support the notation for this same mathematics (See Kauffman 1991, 15).

We propose to analyse in detail this *revenge* of the hand which is no longer content to drone out x_1 then x_2 then x_3 etc., as prescribed by linear successivity, but can play on all the routes permitted by the (interlacing) tracery. The notation *contaminates* to some extent the calculations, in order to create a new context like literary metaphor.

Let us recall again that *Figuring Space* (Châtelet 2000) concludes by stressing that the knots (*nœufs* [sic]) and the (interlacing) tracery are reduced neither to an ornament, nor to a particular chapter of the topology of ordinary Space, but introduce a new mode of intervention of the geometrical figure, just as they had introduced a new manner of making the image penetrate the text in order to avoid too linear a reading of it, and thus expressing *an ability to rupture* which is a reminder of the type of intervention of metaphors already described; this seems to be associated with an aptitude of the 'tied' to interweave an 'over-under' with a cursory reading which *passes* simply from right to left or from left to right. The 'tied' puts in question the traditional opposition between habitual, totally undifferentiated, geometric space and. . strongly differentiated (high and low, right and left. .) 'psychological' space (Ernst Mach) which induces *evaluations and orientations.*

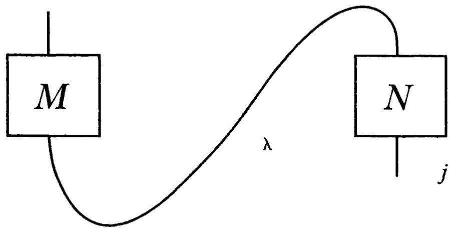
Matrix algebra already used the high-low opposition, and Einstein's convention of tensor products $\sum T_i^{\lambda\mu} T_{\lambda\mu}^j$ clearly showed the subsidiary role of the silent index: it diverted attention to the *intrication* of this *opposition* and of the *successivity* of the summation.

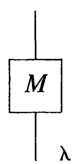
By proposing for a matrix

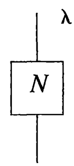
$M = (m^i_j) =$  $\delta =$  ou $T = T^{ijk} =$ 

one accentuates the operation of disappearance of the silent indices in favour of *incidental and emergent features,* and of a *compact reading of products.*

Thus, a product of matrices becomes:

$\sum M^i_\lambda N^\lambda_j \longleftrightarrow$ 

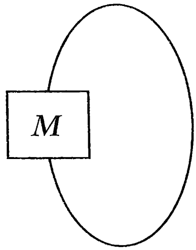
To sum is no longer to drone out but to connect 

with 

the product becoming 

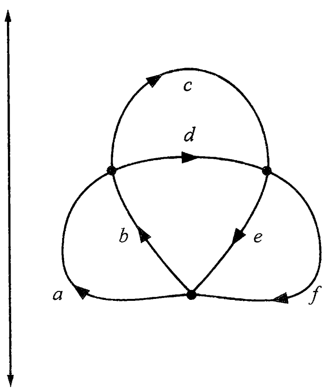
The formula becomes very spectacular for the *trace*:

$$\sum M_i^i \longleftrightarrow$$



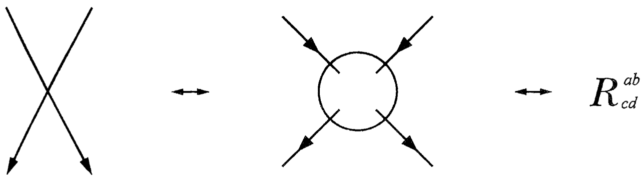
Current mathematical physics succeeded in uniting operations already very powerful by themselves: all the 'imagery' of Feynman diagrams and the diagrams of homological algebra and algebraic topology which give priority to the point of view of arrows and above all *blocks of arrows* (exact series, sequences. . cf. C) at the expense of the classical point of view of 'alpha and omega sets' (*ensembles de départ et d'arrivée*). They induce a new grasp of the relation between the image and the calculation: to think from the start at the level of blocks, is to capture the operativity to a greater degree – which is not without recalling the 'global effects' of the orchestrating metaphor described in paragraph A.

One is thus led to associate each knot (*nœuf* [sic]) – more precisely, its projection on a plane – with a tensor expression [thus benefiting from all of the autospatiality]:

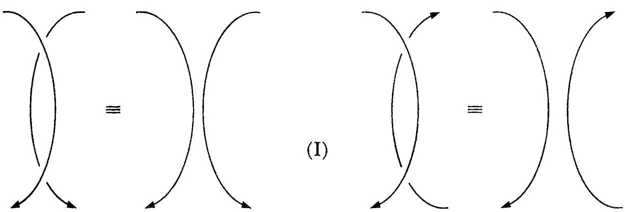


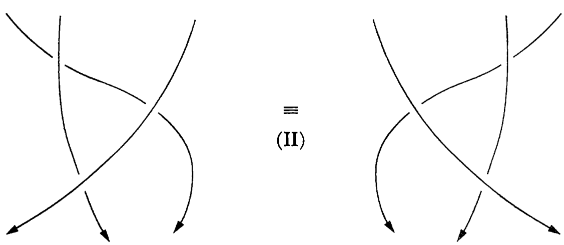
$$\longleftrightarrow \quad T(k) = \sum_{a,b,c,d,e,f} R^{ba}_{dc} R^{dc}_{ef} R^{ef}_{ba}$$

But there is more [to this link between geometry and algebra]: a third component comes [to be connected and] to complete the new notation – the intersections of the projection of the knot (*nœuf* [sic]) can also be seen as collision diagrams of particles:

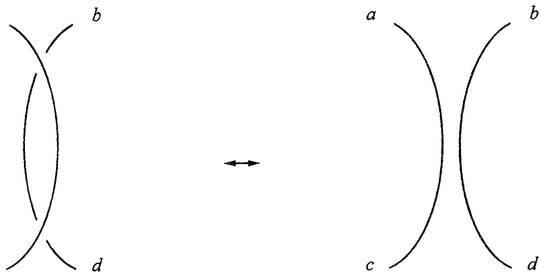


One can show that deformations of the graph do not affect the type of knot (*nœuf*); they can be classified as follows:





These deformations resulting directly from the concrete intuition of the sliding of knots (*nœuf*) materialised by bits of string, induce classical tensor relations concerning the constraints of Quantum Field Theory (and reciprocally). It is thus necessary to take all of this terminology of categories of 'braids', of 'ribbons' which irrigate algebra with geometrical allusions seriously. All these effects are multiplied ten-fold by association with Quantum Field Theory; one would be tempted to say that the two operations (algebraic and quantum) reinforce one another, mathematical arrows becoming physical, and *reciprocally.* Thus, the deformation:



is *equivalent* to the unitarity of matrices associated by convention (I). (II) is *equivalent* to a classical relation of statistical Mechanics.

We see to what point we are far from the classical figuration and illustration which always lead to a kind of dissymmetrical predation of the concrete by the abstract. We already knew that the knot (*nœuf*) is not captured by an intuition of volume or of a given container: it does not occupy a 'place' in our space – there is no *outside* and *inside* of a knot (*nœuf*). The knot (*nœuf*) is not a figure: it is, if you like, an experiment in autospatiality. This is why it so upsets the indexation of physical formulae which, to it, seemed *a priori* foreign. Indexation is no longer reduced to the external evaluation of a collection, but becomes the protagonist of an experiment which secretes its own overflow.

It reveals the grasp of Feynman diagrams as convenient conventions which associate integral calculus with a reproductive imagery of real particle collisions as definitively null and void, leaving the capacity of these diagrams for auto-procreation in suspense.

To index diagrams by knots (*nœuf*) is not to associate two imageries operating by resemblance, but to grasp in a single act the two dynamics of allusion (the collisions not 'resembling' knots (*nœuf*)). The conditions of the *knot* (*nœuf*) *diagram* become here *physical conditions* by the transfer of operations (and reciprocally).

The success of this synthesis of indexation by knots (*nœuf*) would certainly have been appreciated by C.S. Peirce, who liked to say that 'algebra is nothing other than a kind of diagram', stressing that it had the privilege of articulating three functions: that of an *icon* (similarity in reality between its signifier and its signified – *resemblance*); that of an *index* (contiguity in reality between signifier and signified – *auto-overflowing*) and that of a symbol (*instituted*, learned, contiguity between signifier and signified – *convention*), the most perfect being the one where these functions 'are in as equal proportion as possible' [Cf. Châtelet 2000, 188 n. 40].

This indexation by knots (*nœuf*) incarnates this ideal of equilibrium between image, allusion and calculation.

One can now appreciate the great subversive proximity of the unsteadily oscillating relation, illustrating-illustrated, of the orchestrating metaphor, and this driven by allusive stratagems, particularly in the case of the indexation-knot (*nœuf*).

One could detect here an invaluable pivot point: that which would refuse the quartering denounced by Heidegger; that of an informative language aiming at the most massive and most rapid production of messages, and sanctioned by 'yes-no' decisions, aiming to force nature to appear in a calculable objectivity, to debit compact and irreducible units of signification, and of a language of plastic tradition, able to stammer, and which lets things appear.

C) *The revolution of Grothendieck as the articulation between concrete geometrical and concrete algebraic.*

Let us recall that, in the '60s, Alexandre Grothendieck wanted to undertake a vast *programme of* reciprocal *'translation'* between Algebra and Geometry, implying a rupture with traditional intuition and the intervention of new techniques which seem 'abstract', but which however are revealed to be the most adapted to this 'translation', as his introduction to the language of diagrams (*schémas*) emphasises: 'as in many of the parts of modern mathematics, the first intuition moves further and further away, in appearance, from the language appropriate to express it in all the desired precision and generality' (Grothendieck and Dieudonné 1960).

Grothendieck, in his writing, sometimes refers to Galois' theory, as an example still to be reconsidered of what one could call *a pure algebraic concretism*, as much to the work on classical theories such as those of Lie and Sylow groups, as on the most recent theories. One can understand Galois' theory as *training*: that of the progressive discernment of roots, the formal conditions of such a discernment paying attention to the *sequences of reduction,* while the explicit formulae of resolution become subsidiary. To bring to bear all the effort of research on the sequences of groups, fibres, 'bundles', etc., to be able to grasp in flight *the very gesture of learning. . .,* such would be, according to Grothendieck, Galois' unforgettable lesson.

This algebraic concretism can then espouse another concretism, that of Geometry, which is that of *learning the gestures for grasping Space.* We would like to analyse in detail some examples of this 'geometrico-algebraic' concretism in the work of Grothendieck. His conception of the point is particularly enlightening: the 'classical' point of Geometry is simply the trace in space-time of the act of designation of *this point here*, while the point conceived by Grothendieck (and now by all contemporary geometers-algebraists) is an operation, an infinite panoply of virtualities, whose designation would be the most trivial, a *monadic point* able 'to concentrate in one point' all that previously claimed to hold separately the attention of the mathematician. The modern point is re-knotted with this 'evidence' that was always anticipated, but never grasped before Grothendieck as mathematical evidence, making available a new operative power (*puissance*) and above all a formidable allusive power: the 'concentration in one point' is the complete opposite of a subsidence in one point but appears, on the contrary, as an *operation of liberation and amplification of geometrical virtualities.*

Grothendieck saw clearly that mathematics never succumbs to an abstraction deprived of all the richness of determination: the 'generalisations' of mathematics are never confused with inoffensive generalities: there is definitely an audacity specific to mathematicians, certainly associated with a strict discipline of verification, but above all permitting access to a field where yet unclarified virtual determinations emerge. Installation in such a field possesses all the character of a *diagnosis* that operates in a decisive way, well before any exhaustive analysis: the most flagrant example is that of the attack on such and such a conjecture by a stronger – and thus *a priori* more difficult to deduce – conjecture which completely displaces a problem and reveals the old conjecture to be a poorly posed problem. To confuse mathematics with simple deductive chains is to be unaware of the crucial character of the sense of the 'good conjecture' – of that which we have called the *diagnostic of a mathematician*: this is why Grothendieck's 'evidence' is not related to the proximity of two terms in a deductive chain, but to the 'natural' effect related to the abolition of the space between the symbol which captures and the gesture which is captured.

Statements of the type 'Let us consider such a point. . ., such a subset. . . Let us extend this segment of line. . .' are frequent during a demonstration.[11] They are certainly inserted in the deductive chain but have to some extent a *strategic character* appreciated by experts, and they surreptitiously introduce another rhythm. 'It is here that something happens. It was necessary to have the idea to consider this or that. These elements, these statements or these constructions were quite available, but asleep and seem to become animated abruptly by virtue of this 'let us consider', which puts all the attention on what becomes a *pivotal element,* expressing a type of concretism which is much more intense than the 'concrete' allegedly encountered at street-corners by the naïve empiricist.

We are at the antipodes of the 'abstraction' which always results from the violent deduction of a part, and thus of a mutilation, whereas while the 'lever' does not subtract anything and acts like certain fragments of a *puzzle* which, from the outset, emerge and impose or dictate the solution: to be absolutely concrete is to persevere to some extent in *a kind of tangential approach of thought which grasps its own movement.*

Grothendieck's undertaking – like any translation – is not content to define *a simple bank of reciprocal references* between 'purely algebraic' and 'purely geometrical' concepts that are left intact. The theory almost forcefully dislodges the attention of the mathematician from 'points' and fixed sets towards arrows (these morphisms) and makes it possible to understand algebraically geometrical syntheses.[12] One almost wants to say that, thanks to the introduction of topology, the structures of commutative algebra themselves fabricate an 'environment' without remaining under the supervision of co-ordinates. We would like to show that one may understand the programme of translation as crowning a *tradition of discovery and development of analogies* between Number Theory and Geometry initiated by Kronecker and developed by Weil (the analogy between bodies of algebraic numbers and 'paths' of coverings of an algebraic curve, etc.).

Translated by Simon Duffy

Notes

* The notes between square brackets are mine [CA]; the others are Gilles' The parentheses within the text are also mine [CA]

6 See in particular, ch. 3.4, 'Indifference centres and knots of ambiguity, fulcra of the balances of Being', p. 88 ff; and above all, ch. 5, 'Electrogeometric Space', § 5, 'The electrogeometric experiment as square root', C, 'The screw as bold metaphor', p. 176 ff.

7 The last paragraph of Châtelet 2000, ch. 5.6, entitled: 'Towards the knot as secularisation of the invisible' (p. 183–6). The following new material is an extension of this work.

8 In an astonishing way, Gilles Châtelet no longer speaks here of 'kno (*nœuds*) (cf. Châtelet 2000, 183 ff), but of '*nœufs*'(sic), without any expla tion. The mathematical context in which the author intercedes refers to Kau man, *Knots and Physics* (1991). In this fundamental work, Gilles Châte notes, by hand, above the first two diagrams on the 'Trifoil' and the Yan Baxter equation (p. 108): '*nœuf*' and 'inverted *nœuf*' This syntagm is th not a typographical error, and the author obviously had an idea in mind. I therefore reduced by this to propose a 'conjecture' about a possible li between the mathematical concept of knot and (perhaps) the number ni (*neuf*), the idea of a reference to the egg (*œuf*) appearing to me at the ve least inconsistent. The syntagm *nœuf* (for *nœud* or knot) could then be relat to the fact that, in the use of the knot diagram, considered as an 'Abstra Tensor Diagram' for the Yang-Baxter 'nodal' solution, Kauffman establish a link to a list of 9–tuples from which this equation can be read (Kauffma 1991, 318).

9 One of the best examples is that of Yang-Baxter's formula which connects relation of matrix commutation to a knot (*nœuf*) diagram deformation.

10 We can appreciate here the whole path traversed since the work of Yukaw Heisenberg still anxious to illustrate, endeavouring to fix diagrammaticall the concept of 'particle of exchange' But this still remained 'to the side' calculations and too captive to a relation of *illustration* and similitude wi chemical imagery. Contemporary diagrams do not draw their force fro similitude but from the capacity of their new indexations to ensure *a co-pe etration of the image and the calculation.*

11 The example of the demonstrations and constructions of so-called elementa Geometry is very enlightening: it is enough to think of the proof of the poin wise convergent character of the sides of a triangle, transformed by astu extensions into the perpendicular bisector of another triangle. There is a 'effect of synthesis' caused by certain points or remarkable constructions a by no means given by a simple representation on a figure: *the figure becom diagram* because it suggests a dotted line.

12 Two central intuitions traverse Grothendieck's work:

a) the substitution of the point of view of the arrow for the point of view excessively fixed sets: the arrow deposes sources and targets;

b) the grasp of the point as capable at the same time of *condensation* (th most sophisticated structures could become 'points'): this is the case fo vectorial fibre classes on X seen as points of K(X), and also of multipli cation of geometrical virtualities (this is the case with singular points Let us take some very simple examples:

1) that of the notion of the ideal; one should not consider it as a simple 'ge eralisation' of the multiples of arithmetic, but as an autonomous entity, *point* which has its place and which holds at the same time to the set (and to the element – this is the point of view of the *spectrum* (*spectre*).

2) that of an *A-modulus M* of finite type: it seems more complicated to define it by the existence of exact sequences of the type $A^p \to M \to O$, than in the usual way. It is however this type of definition which encourages *operating on blocks* – exact sequences which appear just as condensed as a geometrical point, and which are found at the core of the development of Bundle Theory.