Philosophy and
Phenomenological Research

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Is Intuition Based On Understanding?[1]

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According to the most popular non-skeptical views about intuition, intuitions justify beliefs because they are based on understanding. More precisely: if intuiting that p justifies you in believing that p it does so because your intuition is based on your understanding of the proposition that p. The aim of this paper is to raise some challenges for accounts of intuitive justification along these lines. I pursue this project from a non-skeptical perspective. I argue that there are cases in which intuiting that p justifies you in believing that p, but such that there is no compelling reason to think this is because your intuition is based on your understanding of the proposition that p.

A common idea about intuition is that it is based on understanding.[2] According to one way of developing the idea, in intuiting that p you simply draw on your understanding of the proposition that p. Kirk Ludwig develops the idea this way:

> I will use "intuition" to mean an occurrent judgment formed solely on the basis of competence in the concepts involved in response to a question about a scenario, or simply an occurrent judgment formed solely on the

[1] I thank Jody Azzouni, George Bealer, John Biro, Otávio Bueno, Robert D'Amico, Greg Ray, Harvey Siegel, Ernie Sosa, and Gene Witmer for helpful discussions about the ideas pursued in this paper. My thoughts about intuition and understanding have benefited from Kirk Ludwig's patient tutelage, friendly encouragement, and critical engagement, and I am very happy to express my gratitude to him for being a teacher, critic, and friend of many years.

[2] See, for example, (Bealer 1998), (Boghossian 1996), (Giaquinto 2007), (Goldman and Joel, 1998), (Goldman 2007), (Grundmann 2007), (Jackson 1998), (Ludwig 2007), (Peacocke 2000, 2004, 2008), and (Sosa 2007). Some authors frame their views as accounts of basic *a priori* belief, not intuition.

> basis of competence in the concepts involved in it (in response, we might say, to the null scenario).[3]

One might use "intuition" more liberally. One might allow that some intuitions derive from sources other than understanding, for example heuristics and biases. So there is an alternative way of developing the idea that intuition is based on understanding. According to it, *if* you intuit that p in a way that justifies you in believing that p, *then* what you are doing is simply drawing on your understanding of the proposition that p. This is close to what Ernest Sosa writes:

> S *rationally* intuits that p if and only if S's intuitive attraction to assent to <p> is explained by a competence (an epistemic ability or virtue) on the part of S to discriminate, among the contents that he understands well enough, the true from the false, in some subfield of the modally strong (the necessarily true or necessarily false), with no reliance on introspection, perception, memory, testimony, or inference (no further reliance, anyhow, than any required for so much as understanding the given proposition).[4]

Sosa uses "rational intuition" for what Ludwig calls "intuition." Here I will adopt Sosa's more liberal usage of "intuition" on which it is not just a matter of definition that intuitions are based on understanding.

The common epistemic view behind these terminological differences is the view that if intuiting that p justifies you in believing that p it does so because your intuition is based on your understanding of the proposition that p. Suppose that your understanding of p consists in your grasp of some appropriate range of concepts.[5] Then gaining intuitive justification for believing that p just depends on grasping concepts. You do not need to gather any additional information about whether p is true or false in order to gain intuitive justification for believing that p. I will call views of this sort understanding-based views of intuition.

I am skeptical about understanding-based views of intuition, and the aim of this paper is to raise some challenges for them. These challenges do not constitute a knockdown refutation, but they do seem to me to motivate exploring alternative views about the epistemology of intuition.[6]

Here is the plan. In section 1, I flesh out my target. I put together a set of theses that I provisionally assume define understanding-based views of

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[3] (Ludwig 2007), pg 135.

[4] (Sosa 2007), pg 61.

[5] I am not distinguishing between "understanding of the proposition that p" and "understanding of p." Where grammar permits it, I prefer the contraction.

[6] See (Horwich 2000), (Schechter and Enoch 2006), and (Williamson 2003, 2006, 2007) for additional challenges to understanding-based views of intuition.

intuition, and I explain why views of this sort seem attractive. In section 2, I discuss some problem cases for understanding-based views of intuition. In section 3, I consider some ways proponents of understanding-based views of intuition might respond to these problem cases. I argue that all of these responses are unattractive.

1. Understanding-Based Views of Intuition

Understanding-based views of intuition develop the idea that if your intuition that p justifies you in believing that p it does so because your intuition is based on your understanding of the proposition that p. The aim of this section is to set out some provisional assumptions about how understanding-based views of intuition should flesh out this core idea.

> (1) Your understanding of p consists in your fully grasping some appropriate range of concepts and their mode of combination in p.[7]

Take the proposition that all dogs have tails. In what does your understanding of this proposition consist? According to one familiar notion of understanding, it consists in your fully grasping the concepts expressed by "all," "dogs," "have," and "tails," and how these concepts are combined in the proposition. I am assuming that this is the notion of understanding relevant to understanding-based views of intuition.[8]

There are alternatives. For example, your understanding of p might consist in something less than your fully grasping some appropriate range of concepts and their mode of combination in p. Maybe it consists in your partly grasping some appropriate range of concepts and their mode of combination in p. We might want to use "understand" so that if S has a propositional attitude with the content that p then S counts as understanding the proposition that p and we might follow Burge in thinking that S can have propositional attitudes with contents about which S makes what seem best described as conceptual errors.[9]

On the other hand, your understanding of p might consist in something more than your fully grasping some appropriate range of concepts and their mode of combination in p. As Nagel observes, fundamental scientific dis-

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[7] I thank an anonymous reviewer for pointing out the need for grasping the mode of combination. An example that make it obvious is: understanding the proposition that $1 > 2$ is different from understanding the proposition that $2 > 1$.

[8] There are plenty of usages of "understanding" and cognates for other notions. The notion of understanding I have highlighted picks out a capacity, for example, though we often use "understand" as in "Alice understands what Bob says" to pick out a performance, an event we might later refer to as "Alice's understanding of what Bob said."

[9] See (Burge 1979).

coveries are often "given out as propositions to which one must subscribe without really understanding them."[10] For example: I tell a child that matter is energy; the child might fully grasp the concepts expressed by "matter," "is," "energy," and how they are combined in the proposition but still fail in an important sense to understand the proposition that matter is energy.

I am provisionally assuming that these alternative notions of understanding are not relevant to understanding-based views of intuition.

> (2) In fully grasping a concept you have some information—which we can call the content of that concept.

There are a variety of theories of concepts in the literature. This assumption is meant to be compatible with all of them. To see this, we must note two important qualifications.

First, (2) does not include any assumption about the amount of information you have in grasping a concept. Fodor is a conceptual atomist.[11] He thinks that most lexical concepts do not have any constitutive structure. Fully grasping the concept expressed by "dog," then, is compatible with lacking all sort of commonsense information about dogs, such as that they are animals. On Fodor's view the contents of most lexical concepts are minimal, perhaps empty.[12] Other philosophers and psychologists defend the theory theory of concepts.[13] According to them concepts are constitutively related to other concepts in theories, and fully grasping a concept involves having a mental representation of the appropriate theory. On this view fully grasping the concept expressed by "dog" likely involves having the information that dogs are animals. (2) is meant to be compatible with both theories—as well as with others, such as the classical theory of concepts, prototype theories, etc.[14]

Second, (2) does not include any assumption about the format in which the content of a concept is stored. Suppose the classical theory of concepts

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10 (Nagel 1979), pg 177.

11 See (Fodor 1998).

12 (2) is compatible with Fodor's view only if we allow that no information is an amount of information. I see this as a terminological choice, like allowing that no money is an amount of money—that amount you might have at some point when falling into debt. If one resists this allowance and insists that an amount of information must include some quantity of information even if it is very small, then (2) will not be compatible with Fodor's view. This will not make a difference to the following discussion, however, since Fodor's view of concepts is not conducive to understanding-based views of intuition anyway.

13 See, for example, (Gopnik and Meltzoff 1997).

14 For a helpful overview of different theories of concepts see the introduction to (Margolis and Laurence 1999).

is correct so that fully grasping the concept C consists in mentally representing its definition, i.e. a set of necessary and sufficient conditions for its application.[15] There are different formats this mental representation can take. You might mentally represent the definition explicitly in a sentence in your language of thought. You might mentally represent the definition implicitly because it is implied by other information that is explicitly represented by sentences in your language of thought. Or you might mentally represent the definition tacitly by having inferential dispositions such that you are disposed to apply C to things you take to meet its sufficient conditions and treat things to which you apply C as if they meet its necessary conditions.[16] These are simply illustrations. A proponent of Conceptual Role Semantics, say, might think that the contents of concepts are tacitly represented in the roles those concepts play in cognition, but endorse a view about what the contents of concepts are that differs from the classical theory of concepts.[17]

Here is a worry.[18] Having a competence or ability does not consist in being in a mental state that is evaluable as correct or incorrect. But having some information does consist in being in a mental state that is evaluable as correct or incorrect. So (2) is not compatible with views according to which grasping a concept consists in having a competence or ability. One might challenge the claim that having a competence or ability does not consist in being in a mental state that is evaluable as correct or incorrect.[19] I prefer a more concessive reply. As Dummett has noted in discussing knowledge of a language, though the knowledge is "practical knowledge", "this is no objection to its representation as propositional knowledge; mastery of a procedure, of a conventional practice, can always be so represented, and whenever the practice is complex, such a representation often provides the only convenient mode of analysis of it."[20] A competence or ability, let us say, *embodies* the information presented in such a representation. In this paper I will use "have information" in an extended sense so that one might have some information by having a competence or ability that embodies it.

These qualifications in place, (2) should not seem like such a bold assumption.

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15 I am allowing that for some concepts C, C's definition is just: For all x, x is C iff x is C. If we rule out such trivial definitions, then we must complicate the formulation of the classical theory of concepts.

16 I am construing the notions of explicit, implicit, and tacit mental representation in accordance with (Dennett 1982).

17 Cf. (Greenberg and Harman 2005).

18 I thank an anonymous reviewer for pressing me to address this worry.

19 Cf. (Stanley 2011).

20 See (Dummett 1976), pgs 36 ff. in (Dummett 1993).

(3) What it is for your intuition that p to be based on your understanding of the proposition that p is for it to be the result of a psychological process that draws solely on the content and mode of combination of the concepts full grasp of which constitutes your understanding of the proposition that p.

This seems to me to be the most natural articulation of what it is for your intuition that p to be based on your understanding of the proposition that p. The assumption leaves open the exact nature of the psychological process. It can be conscious, unconscious, or partly one and partly the other. It can involve computing logical implications, generating analogies, manipulating mental imagery, or whatever.[21]

The assumption fits what proponents of understanding-based views of intuition have said. According to Ludwig, for example, to fully grasp a concept we must have "implicit knowledge" of its application conditions that is "expressed in a skill we have in deploying the concept appropriately."[22] What happens when we have a thought experiment intuition is that we "draw out the implicit knowledge we have of the applications conditions of our concepts as it is embodied in our dispositions to deploy words expressing them."[23,24] This is a psychological process that draws solely on the content and mode of combination of the concepts full grasp of which constitutes your understanding of the judgment you make. Peacocke thinks that it is possible to appreciate that the axiom schema A → (A or B) is valid on the basis of your understanding of it.[25] He describes how this might work. What you do is evaluate the schema under various suppositions about the truth-values of A and B, where your evaluations are derived from an implicit representation with the content "that any Thought (content) of the form A or B is true if and only if either A is true or B is true." What makes this a case in which your appreciation is based on your understanding is that this implicit representation partly constitutes your fully grasping the concept expressed by "or."[26] Peacocke doesn't spell out the full story.

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21 (Peacocke 1986) helpfully discusses the general notion of a psychological process drawing on a body of information.

22 (Ludwig 2007), pg 131. Note that Ludwig's usage of "implicit" corresponds to the usage of "tacit" that I have adopted.

23 (Ludwig 2007), pg 133.

24 Giaquinto develops a similar view about visual intuitions in mathematics; see (Giaquinto 2007).

25 Peacocke focuses on your understanding of the schema, A → (A or B). It seems to me that if what you appreciate is that the schema is valid, then your appreciation should be based on your understanding of the proposition that says this, i.e. that the schema, A → (A or B), is valid. There is no need to work out the exact details of Peacocke's view here, however.

26 See (Peacocke 2008), pgs 114–117.

For example, he does not say anything about the role of your implicit representation of a rule governing the evaluation of conditionals. But it is clear that once spelled out Peacocke's account will yield a process that draws solely on what he takes to be the content and mode of combination of the concepts full grasp of which constitutes your understanding of the proposition that A → (A or B) is a valid schema.[27]

> (4) The psychological process in virtue of which your intuition that p is based on your understanding of the proposition that p reliably generates intuitions with true contents.

The role that (4) plays in an understanding-based view of intuition depends on the background epistemology within which the understanding-based view is developed. It is worth distinguishing three background epistemologies.

The first two accept the following conditional: intuitions justify beliefs only if they are reliable. The conditional is neutral with respect to explanatory priority. According to one background epistemology, intuitions justify beliefs because they are reliable.[28] If this is your background epistemology, then (4) articulates what it is about the psychological process in virtue of which your intuition that p is based on your understanding of the proposition that p that enables it to explain why your intuition justifies you in believing that p. According to an alternative background epistemology, intuitions justify beliefs only if they are reliable, but not because they are reliable.[29] If this is your background epistemology, then (4) articulates a condition that the psychological process in virtue of which your intuition that p is based on your understanding of the proposition that p must meet if it is to explain why your intuition justifies you in believing that p. On either of these background epistemologies, (4) must be true.

A third background epistemology rejects the claim that intuitions justify beliefs only if they are reliable. According to some philosophers, intuitions

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[27] Some proponents of understanding-based views of intuition are noncommittal about the psychological processes that generate intuitions. I do not take this as evidence that they reject (3), just that they believe the exact natures of the psychological processes do not bear on the epistemology of intuition. For Sosa what is important is that the processes are reliable in a virtue-theoretic way, not what the exact details are; see (Sosa 2007). Similarly, for Bealer what is important is that the processes are "modally reliable," not what the exact details are; see (Bealer 1998). Even if Sosa and Bealer remain noncommittal about the psychological processes that generate intuitions, it seems to me that they are committed to the existence of processes conforming to (3).

[28] Some philosophers invoke properties stronger than reliability. Cf. (Peacocke 2004), 148 ff. especially pg 173; (Sosa 2007), ch 3; (Bealer 1998).

[29] See, for example, (Boghossian 1996, 2008), (Hale and Wright 2000), and (Ludwig 2007).

justify beliefs in virtue of their phenomenology.[30] If this is so, then even if intuitions are unreliable, so long as they have the right phenomenology, they might nonetheless justify beliefs. On the face of it, anyone attracted to such a phenomenology-based view of intuition should reject understanding-based views of intuition. Consider, however, the following alternative conditional: intuitions are a ground of knowledge only if they are reliable. Justification might not require reliability, but it is difficult to deny that knowledge does. Someone attracted to a phenomenology-based view of intuitive *justification* might also be attracted to an understanding-based view of intuitive *knowledge*.[31] So even if you do not think that justification requires reliability, you still might be interested in developing a view of intuition along the lines I have been describing, but you would take it to be a view of intuitive knowledge, not intuitive justification. And any such view would incorporate a claim like (4).[32]

Invoking the reliability of processes raises two additional issues. First, there is the Generality Problem.[33] Here we can pose it this way. Suppose your intuition that that p is based on your understanding of the proposition that p. There are many process types that are instantiated by the token causal chain from your understanding to your intuition. Which one is relevant to making an assessment of reliability? Second, there is what we might call the Benchmark Problem.[34] Suppose we have picked out a process that connects your intuition that p to your understanding of the proposition that p. What is it for it to be reliable? For our purposes we do not need to defend exact answers to these questions, or even the view that they have exact answers. All we need to do is make sure that the challenges to be developed in section 2 do not hinge on assuming implausible answers to them.

For the Generality Problem I will assume that whatever the process is it should be natural and explanatory from the point of view of psychology.[35] So it should be a psychological kind or naturally specifiable in terms of psychological kinds. And it should be the sort of process that can be invoked in giving psychological explanations. For the Benchmark Problem I will assume that actual performance is not all that matters. What also matters is how the psychological process would perform in counterfactual

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30 See, for example, (Huemer 2001, 2005). I defend this view as well in (Chudnoff 2011).

31 See, for example, (Huemer 2005), especially pages 122–127.

32 Understanding-based views of intuitive justification are still the main target of this paper. I explain why in footnote 62.

33 Cf. (Conee and Feldman 1998), (Alston 2005) especially chapter 6.

34 Conee and Feldman also mention this issue, but do not discuss it in detail.

35 This is similar to Alston's view. Conee and Feldman argue that it will not determine a unique process. This is not a problem in the present context since my worry will be that there are not enough reliable psychological processes.

circumstances. One useful notion is that of safety. For our purposes we can say: a psychological process is safe just in case were it to generate an intuition that p from your understanding of the proposition that p, it would be true that p.[36] I will assume that safety is a good benchmark for reliability.

> (5) An appropriately developed understanding-based view of intuition will account for all intuitive justification.

I add this assumption for completeness.

These are the assumptions I will make about understanding-based views of intuition. The attraction of understanding-based views of intuition can best be appreciated by comparing them with their ancestors—linguistic theories of the *a priori*. One sort of linguistic theory of the *a priori* is the version of logicism about mathematics the logical positivists defended.[37] Benacerraf gives a nice summary of what made this view so attractive:

> But, in reply to Kant, logicists claimed that these [mathematical] propositions are a priori because they are analytic—because they are true (or false) merely "in virtue of" the meanings of the terms in which they are cast. Thus, to know their meanings is to know all that is required for a knowledge of their truth. No empirical investigation is needed. The philosophical point of advancing the view was nakedly epistemological: logicism, if it could be established, would show that our knowledge of mathematics could be accounted for by whatever would account for our knowledge of language.[38]

Logicism as developed by the logical positivists, and linguistic theories of the *a priori* in general, are attractive because, if they could be established, they would show that the cognitive resources required for knowing a language are also sufficient to account for knowledge of mathematics, or knowledge of *a priori* truths in general. There is no need for Kantian intuition.

Recent understanding-based views of intuition differ from linguistic theories of the *a priori* in several ways. For example, recent proponents of understanding-based views of intuition do not invoke a metaphysical notion of analyticity—i.e. truth in virtue of meaning.[39] They focus on grasping

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[36] Cf. (Sosa 1999) and (Sosa 2007), especially pg 26 where he introduces the notion of basis-relative safety.

[37] There are differences in the exact ways that different logical positivists developed this view. See, for example, the selections by Carnap (both selections), Ayer and Hempel collected in (Putnam and Benacerraf 1983). The differences do not bear on anything I say here.

[38] (Benacerraf 1981), pgs 42–43 in (Demopoulos 1995).

[39] See (Boghossian 1996, 2008) for the distinction between metaphysical analyticity and epistemic analyticity.

concepts rather than knowing meanings. And they pose their theories as ways of legitimizing reliance on intuition (though not intuition as Kant thought of it), not as setting up an alternative to reliance on intuition. Still, we can explain the attraction of understanding-based views of intuition along lines similar to those traced by Benacerraf. Understanding-based views of intuition are attractive because, if they could be established, they would show that the cognitive resources required for grasping concepts and their modes of combination in propositions are also sufficient to account for intuitive justification. They provide an alternative to Platonist views of intuition according to which intuition is a form of intellectual perception, so that the capacity for intuition includes a capacity for non-sensory awareness of abstract subject matter.[40] Writers in the Platonic tradition have generally thought that we have capacities—such as the capacity for non-sensory awareness of abstract subject matter—over and above those that are plausibly required for concept possession. But these additional capacities have struck many as mysterious. If intuition is based on understanding, then there is no need to think we have such extra capacities, and this seems to me to be the main motivation for pursuing understanding-based theories of intuition.[41]

2. Problem Cases

As characterized in the previous section, understanding-based views of intuition imply the following conditional:

> The Conditional: If your intuition that p justifies you in believing that p, then your intuition that p is the result of a reliable psychological process that draws solely on the content and mode of combination of the concepts full grasp of which constitutes your understanding of the proposition that p.

One might challenge The Conditional on the basis of an opposing epistemological theory. As mentioned above, some philosophers think intuitions justify beliefs in virtue of their phenomenology. I also pointed out, however, that these same philosophers might be interested in the prospects

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[40] Proponents of understanding-based views might agree that intuitions bear some *phenomenological* similarities to perceptions. Thinkers in the Platonic tradition—e.g. Augustine, Descartes, Gödel, Husserl—think that intuitions are also *metaphysically structured* like perceptions in that, in at least some cases, they include standing in an awareness relation to their subject matter. My own sympathies lie with the Platonists. For more on intuitions so conceived see (Chudnoff forthcoming-a, forthcoming-b).

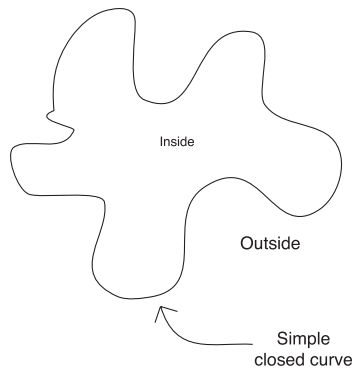
[41] Goldman and Peacocke are very clear on this point. See (Goldman and Pust 1998), (Goldman 2007), and (Peacocke 2000, 2004).

of pursuing an understanding-based theory of intuitive *knowledge*, which would imply an analogous conditional. Further, there are obvious dialectical reasons to avoid relying on controversial, alternative epistemological theories. So what I will do in this section is develop two counter-examples to The Conditional that are motivated independently of any opposing epistemological theory.

Another feature of these counter-examples is that they do not depend on substantive background assumptions about the natures of our concepts. This will allow us to be concessive with respect to the background assumptions proponents of understanding-based views of intuition do make about the natures of our concepts.[42] It was the aim of developing counter-examples with this feature that motivated me to explore the particular cases I do, which cases are, admittedly, not the first that might pop into mind when you think about intuition.

Consider the following famous theorem of topology:

> The Jordan Curve Theorem (JCT): A simple (i.e. non-self-intersecting) closed curve in a plane separates the plane into two disconnected regions —an inside and an outside.



Most people find this theorem intuitively compelling; that is, most people, after only modest reflection, have an intuition that JCT is true. And on the face of it, when a typical person has an intuition that JCT is true this intuition justifies him or her in believing JCT.

What I will argue now is that a typical person's intuition that JCT is true is not the result of a reliable psychological process that draws solely on the contents and mode of combination of the concepts full grasp of which constitutes that person's understanding of JCT. Imagine a typical person and call him Albert. Albert intuits that JCT is true.

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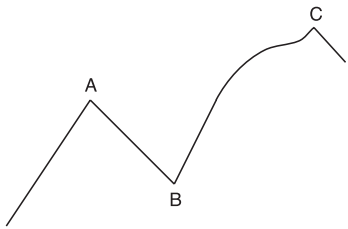
[42] This distinguishes the present challenge from the ones pressed in (Horwich 2000) and (Williamson 2003, 2006, 2007).

Here is a plausible albeit sketchy story about Albert.[43]

The first part of the story is about some of his concepts. Albert's concepts of simple closed curve and disconnectedness are visual recognitional concepts. So their contents include information about how simple closed curves and disconnected regions visually appear. Simple closed curves look like circles and deformations of circles. Disconnected regions look like there is no way of tracing a path from one to the other without passing a point that belongs to neither. Albert's concept of universal quantification is a logical concept. Its content includes information about valid inference rules. Albert's grasp of it is partly constituted by his disposition to infer in accordance with the rule of universal generalization. So from the premise that an arbitrary F is a G Albert is disposed to infer that all Fs are Gs.

The second part of the story is about a psychological process going on in Albert that draws on the contents of these concepts in generating his intuition that JCT is true. Albert consciously initiates the process by imagining what he takes to be an arbitrary simple closed curve. He visually recognizes that this figure separates the plane into two disconnected regions. Then he exercises his inferential disposition to infer that all Fs are Gs from the premise that an arbitrary F is a G, thereby generating the intuition that JCT is true.

This story provides all the elements for an understanding-based account of Albert's intuition that JCT is true except one: reliability. The psychological process described is unreliable. In general, it is unsafe to make generalizations about all curves on the basis of what holds true for curves that can be visualized. Curves that can be visualized are not really arbitrary. An example will illustrate the point. Reflection on curves we can visualize suggests that a continuous curve must have a well-defined tangent at most of its points. For even if there are kinks in the curve—see points A, B, and C below—it would seem that there must be points *between* the kinks at which there are well-defined tangents.



Lacks tangents at A, B, and C, but has tangents at other points

[43] It is similar to the accounts of visual thinking Giaquinto develops in (Giaquinto 2007). Chapter 9 of that book suggests he shares my skepticism about the epistemic significance of this sort of account in the case of JCT.

This claim is false. In the 19th century, mathematicians discovered pathological curves that are everywhere continuous and nowhere differentiable—i.e. they lack tangents at all their points.[44] Such pathological curves are defined as limits of infinite sequences of other curves and are too complicated to visualize. I have given just one example of the process going wrong. But there are many others.[45] So it is reasonable to conclude that the psychological process under consideration is unsafe: it is not the case that if it were to generate an intuition that p from your understanding of the proposition that p, it would be true that p. One might insist that the relevant process is generalizing from a curve that is arbitrary rather than generalizing from a curve that is, sometimes mistakenly, taken to be arbitrary. But if the process is one that invokes visual recognitional concepts, then this is impossible since, as pathological curves show, curves that can be visualized form a special class. One might try circumscribing the process so that it only occurs in non-pathological cases. But there is no way to do this naturally in psychological terms. One might try circumscribing the counterfactual scenarios relevant to making a judgment about safety so that the process does come out looking safe. But this seems blatantly *ad hoc*.

One response to the existence of pathological curves is skepticism about intuition.[46] You might argue that Albert's intuition fails to justify him in believing JCT. This would be rash. As Descartes pointed out in the *Meditations*, we might not be able to visualize a 1000-sided figure, but this does not prevent us from appreciating various truths about it using our intellect.[47] The reach of intuition is not limited by our capacity to visualize.[48] There are simple closed pathological curves. Albert cannot visualize these curves. But he can intellectually appreciate that even though they are pathological, and so lack tangents everywhere, they still separate the plane into two disconnected regions. JCT still holds of them despite their pathology and that JCT still holds of them is intuitively clear even if not visually evident.

The question now is: is there an understanding-based account of *this* intuition? If so, then Albert's concepts of simple closed curve and disconnectedness must not be *merely* visual recognitional concepts. And plausibly

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44 Bolzano first discovered a pathological curve. Weierstrass first published a paper proving the existence of a pathological curve—a different one from the one Bolzano discovered. (Strichartz 2000) proves that Bolzano's curve is pathological. (Dunham 2005) gives an account of Weierstrass' proof.

45 (Hahn 1956) describes some others.

46 See (Hahn 1956). (Feferman 2000) defends a more moderate response.

47 See (Descartes 1984), pg 50.

48 Kantians might disagree. In this paper, I am not assuming a Kantian conception of intuition, according to which the scope of intuition is fixed in part by the nature of our sensory faculties. See (Parsons 1983, 2007) for a well worked out Kantian view of intuition.

they are not. It is not obvious how to establish what their contents are. One supposition that is concessive to proponents of understanding-based views of intuition is that Albert mentally represents rigorous definitions of simple closed curve, disconnectedness, and other notions deployed in JCT.[49] There are different ways of defining these notions, and the details will not matter here. But to get an idea of what such definitions are like, here is a standard way of defining a simple closed curve in a plane: a simple closed curve in a plane is a one-one bi-continuous mapping from the unit circle to points in the plane. This rigorous definition agrees with and extends the visual recognitional concept of a figure that looks like a circle or a deformation of a circle. The supposition that Albert mentally represents definitions of this sort for all the notions deployed in JCT is concessive to proponents of understanding-based views of intuition because together such definitions logically imply JCT. Given the supposition, then, in one reasonable sense of implicit JCT is implicit in the contents of the concepts full grasp of which constitutes Albert's understanding of JCT.

Even conceding this much, however, it seems to me that we lack what is required for an understanding-based account of Albert's intuition that JCT is true. The reason why is that even if the contents of the concepts full grasp of which partly constitutes Albert's understanding of JCT logically imply JCT, it does not follow that Albert's intuition that JCT is true is the result of a psychological process that is sensitive to this implication. One might argue that surely there are psychological mechanisms in Albert that compute logical implications. No doubt. The problem is this. First, on any psychologically plausible view these mechanisms compute logical implications by resolving them into simple steps. Second, resolving the logical implication in this case into simple steps yields a deduction that is incredibly long. Of course, there is no way to give an exact number of steps without specifying what premises can be taken as given and what counts as a simple step. But there is reason to think that any reasonable specification of these parameters will yield a deduction that it is implausible to think Albert unconsciously performs in intuiting that JCT is true. The mathematician Thomas Hales carried out the project of formalizing a proof of JCT and arrived at a deduction containing "approximately 20 million primitive logical inferences."[50] Further, let us make the fantastical assumption that Albert's psychology includes not only a mechanism for computing logical implications by resolving them into simple steps, but also a creative mathematician who teases out implications from the contents of Albert's concepts by drawing on sophisticated background mathematics such as algebraic

[49] Cf. Peacocke's claim that Newton implicitly represented the rigorous definition of a limit; see (Peacocke 2008), pgs 119 ff.

[50] (Hales 2007), pg 883.

topology and analysis. This mathematician cuts down the length of the proof by millions of steps—but still it would take several pages of dense symbolism to write down.[51]

In reply, a proponent of understanding-based views might suggest we consider savants that can solve certain mathematical problems as fast as a calculator.[52] Either they quickly perform an enormous number of simple steps or they do not. If they do quickly perform an enormous number of simple steps, then perhaps we do too when intuiting JCT. If they do not, then whatever it is they are doing, perhaps we are doing that as well when intuiting JCT. One point in response to this argument is that there is some reason to doubt that savants are quickly performing an enormous number of simple steps. There is evidence that savant abilities depend, at least in part, on the disposition to represent problems in a novel way that makes them more readily solvable.[53] This is a unique disposition, not one widely shared. Further, the second horn of the dilemma is mistaken. There is no *a priori* reason to think that whatever process it is that goes on in savants must be the sort of process that draws solely on the contents and modes of combination of concepts. But if it does not, then it is not the sort of process that can be invoked in an understanding-based view of intuition. A reason to think there is *some* reliable mechanism that underlies savant abilities, or our intuition of JCT, does not necessarily constitute a reason to think the mechanism is one that can be invoked in an understanding-based view of intuition. Perhaps, for example, the mechanism works more like perception in that it provides us with information not already in the contents of our concepts.

JCT poses a challenge to understanding-based views of intuition. We all find JCT intuitively compelling, and, on the face of it, when we do so we all gain justification for believing JCT. But it is difficult to see how our intuition that JCT is true might be the result of a reliable psychological process that draws solely on the contents and mode of combination of the concepts full grasp of which constitutes our understanding of JCT. I have not surveyed every possibility. But there are general grounds for skepticism. The two sorts of account I have considered bring to light conflicting demands on understanding-based views of our intuitions about curves. First, the view must identify a psychological process that is sensitive enough to the subtleties in the notion of a curve to be reliable. Purely visual reasoning fails here. Second, the view must identify a psychological process that is not so computationally demanding that it is implausible that it is actually causally efficacious in generating our intuitions. Unconscious logical reasoning from rigorous definitions fails here. It is difficult to see how the propo-

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51 See, for example, the proof in (Munkres 1975).

52 I thank an anonymous reviewer for suggesting I address this reply.

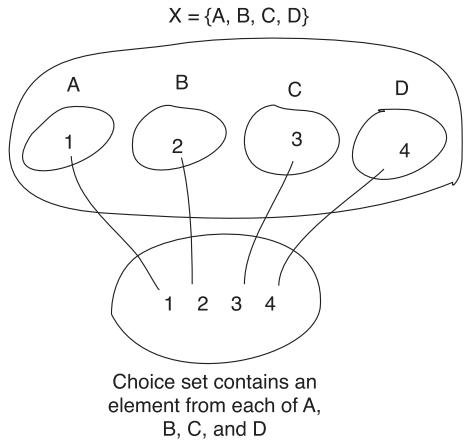
53 For a discussion "from the inside" see (Tammet 2007).

nent of understanding-based views of intuition might negotiate these conflicting demands. One final point: JCT is the most famous example of a theorem that is obvious but difficult to prove, but it is not an isolated case; there are plenty of other less celebrated claims about curves that are also obvious but difficult to prove.

The second problem case I will discuss comes from set theory. Let us imagine that Albert is an introductory set theory student. Here is a story about him at a particular time.

Prior to this time Albert's education in set theory has proceeded thus: he learned about naïve set theory and Russell's paradox showing that it is inconsistent; he was introduced to the iterative conception of sets; he learned the axioms of Zermelo-Fraenkel (ZF) set theory, as motivated by the iterative conception of sets. Now Albert's instructor writes the following on the board and invites him to consider whether it is true:

> Axiom of Choice (AC): For any set X of non-empty, disjoint sets there is a set that contains exactly one member of each of the members of X.



Albert reflects on AC, and has an intuition that it is true.

That is the story. Here are two claims about it:

> (1) Albert's intuition that AC is true justifies him in believing that AC is true.
> (2) Albert's intuition that AC is true is not the result of a reliable psychological process that draws solely on the contents and mode of combination of the concepts full grasp of which constitutes Albert's understanding of AC.

If these claims are true, then the story presents us with a counterexample to The Conditional, and so another problem case for understanding-based views of intuition.

Claim (1) seems plausible to me. Albert's intuition that AC is true doesn't look very different from other mathematical intuitions. If some mathematical intuitions justify some mathematical beliefs, then why shouldn't Albert's intuition justify him in believing AC? One might worry that AC has been the subject of controversy in the mathematical community. But Albert is ignorant of the controversy. Perhaps if he were to learn about it, his justification for believing AC would be defeated—at least until he sees his way through the controversies. But this does not mean that he doesn't have some justification for believing AC to begin with. One might worry that AC has counter-intuitive implications, such as the Banach-Tarski theorem that a unit sphere can be decomposed into a finite number of pieces that can be recombined into two unit spheres. Perhaps if Albert were to learn about this, his justification for believing AC would be defeated—at least until he sees his way to accepting the Banach-Tarski theorem and recognizing that AC is indispensible for standard mathematics. But, once again, this does not mean that he doesn't have some justification for believing AC to begin with. One might worry about the non-constructive nature of AC. While some choice sets can be constructed using a rule for picking an element from each set out a collection of sets, AC implies that there are choice sets even when there is no such rule. This is the source of its power and controversy. While some philosophers of mathematics have taken the non-constructive nature of AC to count against it, their reasons for doing so derive from their background views about the metaphysics and epistemology of mathematics. Albert isn't so burdened. Once again, while it is possible for Albert to lose his justification for believing AC by becoming a constructivist and worrying about AC on constructivist grounds, this does not imply that he fails to have some justification to begin with.[54]

Let's consider claim (2). What is the content of the concepts full grasp of which partly constitutes Albert's understanding of AC?

At one point in Albert's life as a student of set theory we might have best articulated this content using the axioms of naïve set theory—i.e. the extensionality axiom and the naïve comprehension axiom. AC is an easy consequence of naïve set theory. So is everything else, however, since naïve set theory is inconsistent. A psychological process that generates intuitions about sets by computing consequences of naïve set theory is not a reliable psychological process. But, as I told the story, Albert has already progressed beyond naïve set theory. So let us consider what other psychological processes might be in play.

We can immediately set aside psychological processes that generate intuitions about the infinite by analogy with the finite. First, it isn't clear how such processes are anchored in Albert's understanding of AC. Sec-

[54] For a historical discussion of AC see (Moore 1982).

ond, even though finite examples do analogically suggest AC, drawing conclusions about the infinite by analogy with the finite is notoriously unreliable. For example: no finite set can be put into one-one correspondence with a subset of itself, but every infinite set can be put into one-one correspondence with a subset of itself. We will do better to consider psychological processes that draw on the content of Albert's matured, non-naïve understanding of AC.

A proposal that is concessive to understanding-based views of intuition is that the content of the concepts full grasp of which partly constitutes Albert's matured, non-naïve understanding of AC is just the content of the iterative conception of sets.[55] We need not explore the content of this conception here in detail.[56] Just two observations about it will matter. First, there is broad agreement that the content of the iterative conception of sets is captured by the Zermelo-Fraenkel axioms.[57] That is, just as the axioms of naïve set theory capture the content of the naïve conception of sets, so the axioms of ZF capture the content of the iterative conception of sets.[58] Given the story, then, it is plausible that at the time we are considering Albert the content of the concepts full grasp of which constitutes his understanding of AC is no richer than ZF in this sense: this content logically implies p only if ZF logically implies p. The second observation that matters here is that AC is logically independent of ZF. That is, ZF neither logically implies AC nor logically implies the negation of AC.[59] It follows that the content of the concepts full grasp of which partly constitutes Albert's understanding of AC does not logically imply AC. So even if Albert's psychology includes a mechanism that generates intuitions about sets by computing logical consequences of the contents of his set theoretical concepts, this psychological mechanism will not account for his intuition that AC is true.

The problem AC poses for understanding-based views of intuition is similar to the problem JCT poses. We all find AC intuitively compelling, and, on the face of it, when we do so we all gain justification for believing AC. But it is difficult to see how our intuition that AC is true might be the result

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55 I say that this proposal is concessive because it assigns a lot more content to Albert's concepts than many theorists about the nature of concepts would find psychologically plausible, and the more content assigned the better for understanding-based views of intuition.

56 Among the many discussions of the iterative conception of sets the following are particularly helpful: (Boolos 1971), (Enderton 1977), and (Schoenfield 1977).

57 (Boolos 1971) is clear on this point.

58 We need not assume that ZF is the *only* axiomatization of set theory that captures the content of the iterative conception of sets.

59 These are the famous consistency and independence results of Gödel and Cohen. Proofs can be found in (Kunen 1980).

of a reliable psychological process that draws solely on the content and mode of combination of the concepts full grasp of which constitutes our matured, non-naïve understanding of AC. With JCT the problem was that computing a derivation of it from the content and mode of combination of the concepts full grasp of which constitutes our understanding of JCT is too complicated to be the likely cause of our intuitions that JCT is true. With AC the problem is that there is simply no such derivation to compute: the content of the concepts full grasp of which partly constitutes our matured, non-naïve understanding of AC fails to logically imply AC. Clearly, reverting to a naïve understanding of AC or foregoing logic in favor of drawing analogies between the finite and the infinite are not plausible alternatives. Once again, I have not surveyed every possible psychological process that might generate our intuition that AC is true by drawing on our understanding of AC. But the *prima facie* difficulty for an understanding-based account of this intuition should be evident.

Proponents of understanding-based views of intuition might reply that the content of the concepts full grasp of which partly constitutes Albert's matured, non-naïve understanding of AC is not captured by ZF, but rather by ZFC—i.e. Zermelo-Fraenkel set theory *plus* the Axiom of Choice.[60] In this case there is no problem. Of course, Albert is a made up character and we are making stipulations about the contents of his concepts. The proponent of understanding-based views of intuition should not commit to the claim that it is *impossible* for the contents of someone's set theoretical concepts to be captured by ZF, rather than ZFC. Rather, the reply on behalf of understanding-based views of intuition is that it is impossible for *both* the contents of someone's set theoretical concepts to be captured by ZF *and* for that person to be able to gain intuitive justification for believing AC. The proponent of understanding-based views of intuitive justification might insist on this, but it is a dialectally weak maneuver. I can imagine Albert as described above. Further, my imaginative exercise is grounded in reflection on a real historical situation: early 20th century mathematicians found themselves in the position of having good reason to accept the ZF axioms deriving from their grasp of the iterative conception of set, confronting the open question of whether to also accept AC, and, for at least some of them, taking themselves to have good reason to accept AC deriving from its intuitive plausibility. Perhaps my understanding of the historical situation is faulty, and perhaps my imagination is misleading me about what is possible. But the proponent of understanding-based views of intuition should provide some reason for thinking this is so that is independent of commitment to understanding-based views of intuition.

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[60] I thank an anonymous reviewer for suggesting this reply on behalf of understanding-based views of intuition.

3. Assessing the Damage

There are two categories of response to the problem cases available to proponents of understanding-based views of intuition. One category of response defends The Conditional identified above by challenging the problem cases. Another category of response abandons The Conditional in light of the problem cases and calls into question the assumptions about understanding-based views of intuition from which I derived it.

I do not have much to say about responses in the first category. They take two forms:

> Pessimistic: deny that our intuitions do justify us in believing JCT or AC.
>
> Optimistic: urge that there are reliable psychological processes that I have failed to consider and that can be appealed to in an understanding-based account of our intuitions that JCT and AC are true.

The pessimistic response seems desperate to me. Take JCT. It is as obvious as any mathematical claim. Proponents of understanding-based views of intuition are not skeptics about intuition in general. And there is no motivation for targeted skepticism about our intuitions that JCT is true independent of the present dialectic. I do not have a decisive refutation of the pessimistic response, but I believe everyone should recognize that it is unattractive and that an alternative would be preferable.

The optimistic response seems like a mere promissory note. If it were backed up with a detailed proposal, that would be something. But I am not aware of any such proposal. Note that it is easy to come up with reliable processes. Just consider the proposal that in the case of JCT the process is *somehow coming to believe JCT* and in the case of AC the process is *somehow coming to believe AC*. Both are reliable. But these proposals do not meet the restrictions we imposed on processes when considering the Generality Problem. They are neither natural nor explanatory from the point of view of psychology. One might try relaxing the restrictions. So there is some theoretical wiggle room. Two points, however. First, the restrictions seem very plausible to me. Second, the optimistic response remains a mere promissory note: now a proponent of it must both defend an alternative set of restrictions on processes and describe a process that meets them. Again, I do not have a decisive refutation to offer. In this case, I wouldn't say we should abandon the project of exploring prospective psychological processes. Maybe one will turn up. But we do have reason to reserve our endorsement of understanding-based views of intuition that imply The Conditional. Now I turn to responses in the second category.

Assumptions (1) and (2), recall, are the following:

(1) Your understanding of p consists in your fully grasping some appropriate range of concepts and their mode of combination in p. (2) In fully grasping a concept you have some information—which we can call the content of that concept.

These work in tandem with assumption (3), which is:

(3) What it is for your intuition that p to be based on your understanding of the proposition that p is for it to be the result of a psychological process that draws solely on the content and mode of combination of the concepts full grasp of which constitutes your understanding of the proposition that p.

One way to block a derivation of The Conditional is to make some revision in this group of assumptions. The most natural revisions are revisions in what information is available to psychological processes that draw solely on your understanding of p. For example, you might revise (1) so that your understanding of p consists in something other than your fully grasping some appropriate range of concepts and their mode of combination. (2), then, should be revised to associate some information with this other sort of understanding. And (3) should be revised so that psychological processes that draw solely on your understanding of p draw on this other body of information.

Any such revision will block a derivation of The Conditional. But it will allow the derivation of an alternative. Let's call the alternative body of information associated with your understanding of p INFO. Then we have:

Alternative Conditional: If your intuition that p justifies you in believing that p, then your intuition that p is the result of a reliable psychological process that draws solely on INFO.

To be helpful the revisions to (1), (2), and (3) must be such that the Alternative Conditional is more defensible than The Conditional.

Clearly, revising (1) so that your understanding of p consists in something less than fully grasping some appropriate range of concepts and their mode of combination in p wouldn't be helpful. What the proponent of understanding-based views of intuition would have to do is revise (1) so that your understanding of p consists in something more than fully grasping some appropriate range of concepts and their mode of combination in p. Consider a simple proposal. Suppose we stipulate the following:

Your understanding of JCT partly consists in your knowing in a reliably accessible way that JCT is true.

> Your understanding of AC partly consists in your knowing in a reliably accessible way that AC is true.

If these are true, the problem cases pose no problem for the Alternative Conditional.

The rub should be evident: if your understanding of JCT and AC conform to the stipulations, then what accounts for how you gain this understanding? Recall the attraction of understanding-based views of intuition is that they explain the potentially mysterious—intuitive justification—by appeal to the mundane—understanding, conceived of as full grasp of concepts and their modes of combination. But if understanding is something more, and especially if it is something about which the proposed stipulations hold, then it is potentially mysterious. So understanding-based views of intuition lose their attraction.

The stipulations were just to help illustrate some of the considerations that need to be taken into account in revising the (1)-(2)-(3) cluster. Consider an actual view. According to Bealer there is a way of understanding a proposition that guarantees the reliability of your intuitions with respect to it.[61] When you understand a proposition in this way Bealer says you understand it determinately. This is the notion of understanding that Bealer invokes in developing his understanding-based view of intuition. According to Bealer the notion of fully grasping a concept should be analyzed in terms of determinate understanding. So one might argue that he isn't really revising the (1)-(2)-(3) cluster, but rather pressing a version of the optimistic response to the problem cases identified above. I'm inclined to think that his notion of determinate understanding is a technical notion with little connection to the ordinary notion of fully grasping a concept, but this does not matter. It does not matter whether we classify Bealer's view as a version of the optimistic response to the problem cases or as an understanding-based view of intuition that revises the (1)-(2)-(3) cluster. Either way it faces the same difficulty.

The difficulty is this. In discussing JCT, I allowed that understanding it might consist in mentally representing rigorous definitions of the notions deployed in it. In discussing AC, I allowed that understanding it might consist in mentally representing ZF. These strike me as extraordinary concessions. But even granting them, we found that there are no reliable, psychologically plausible, understanding-based processes for generating intuitions of JCT and AC. Therefore the informational content of our concepts must be even greater on Bealer's view. If so, then gaining this information is a substantive epistemic achievement. And it if is a substantive epistemic achievement, there must be some explanation of how we do it. But it is

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[61] In discussing Bealer's view I am drawing on (Bealer 1998). The points I focus on are not developed in significantly different ways in his other publications on intuition.

unclear what the explanation might be. One possibility is that we come to determinately understand JCT and AC only after intuiting them. But surely this possibility is no comfort to Bealer, since it bases determinate understanding on intuition rather than intuition on determinate understanding. Without an independent account of how we gain determinate understanding, it remains unclear how Bealer's view might circumvent the problem cases.

The problems in finding a reliable, psychologically plausible, understanding-based process for generating intuitions of JCT and AC might suggest rejecting assumption (4):

> (4) The psychological process in virtue of which your intuition that p is based on your understanding of p reliably generates intuitions with true contents.

Recall, however, that (4) is required by a broad range of background epistemologies. Either intuitions justify beliefs only if they are reliable, or they might justify beliefs even if they are not reliable. If the former is true —and all proponents of understanding-based views of intuitions that I am aware of think it is true—then (4) is inescapable. If the latter is true, then (4) can be abandoned. But philosophers who think intuitions might justify beliefs even if they are not reliable tend not to endorse understanding-based views of intuitive justification anyway. As mentioned above, some philosophers think that intuitions justify beliefs in virtue of their phenomenology. If this is so, then what we have is an *alternative* to understanding-based views of intuitive justification, not a different way of developing an understanding-based view of intuitive justification that avoids commitment to (4). Further, as mentioned above, even if a phenomenology-based view of intuitive justification is correct, there is still the question of how to explain intuitive *knowledge*. And here (4) does seem inescapable, since it is difficult to deny that intuitions are a ground of knowledge only if they are reliable. So proponents of phenomenology-based views of intuitive justification should still take an interest in the arguments in this paper, since they give us reason to doubt the prospects of supplementing such a view with an understanding-based view of intuitive knowledge.[62]

Finally, I turn to (5):

> (5) An appropriately developed understanding-based view of intuition will account for all intuitive justification.

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[62] Even so, understanding-based views of intuitive justification remain my primary target. The reason why is that though I do think we can have intuitive knowledge of AC and JCT, it seems to me that a certain partly skeptical response to my arguments in this paper—namely, denying that we have intuitive knowledge which requires reliability but affirming that we have intuitive justification which just requires the right phenomenology—would require more discussion than I can dedicate to it here to rule out.

Revising (5) is a natural idea. The two examples I gave are mathematical. Many proponents of understanding-based views of intuition focus on non-mathematical intuitions. There are, for example, intuitions in metaphysics, epistemology, ethics, and aesthetics that need accounting for. Perhaps the difference in subject matter corresponds to a difference in epistemology. That is, perhaps an understanding-based view of intuition accounts for the epistemology of philosophical intuitions, but some other view accounts for the epistemology of mathematical intuitions.

I will make two brief observations about this line of thought. First, it gives views that offer a unified account of the epistemology of intuition a comparative advantage. Second, it isn't clear that understanding-based views of intuition will work for philosophical intuitions. In this paper I focused on the specific mathematical cases of JCT and AC because they allowed me to challenge understanding-based views of intuition without relying on substantive claims about the nature of concept possession. But it is possible to develop another sort of challenge to understanding-based views of intuition that focuses on precisely those substantive claims about the natures of our concepts that such views presuppose. Developing this line of thought, however, is an endeavor unto itself, quite different from the one pursued in this paper.[63]

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[63] Cf. (Williamson 2003, 2006, 2007).

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