

THE BAD COMPANY OBJECTION AND THE EXTENSIONALITY OF FREGE'S LOGIC

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ABSTRACT

According to the Bad Company objection, the fact that Frege's infamous Basic Law V instantiates the general definitional pattern of higher-order abstraction principles is a good reason to doubt the soundness of this sort of definitions. In this paper I argue against this objection by showing that the definitional pattern of abstraction principles – as extrapolated from §64 of Frege's *Grundlagen*– includes an additional requirement (which I call *the specificity condition*) that is not satisfied by the Basic Law V while is satisfied by other higher-order abstractions such as Hume's Principle. I also show that the failure of this additional requirement in the case of Basic Law V is engendered by an essential feature of Frege's conception of logic and thus that Frege himself should not have regarded the Basic Law V as a definition by abstraction.

Keywords: Abstraction Principles. Bad Company. Content Recarving. Frege's Logic.

RESUMO

Segundo a objeção da Má Companhia, o fato de que a infame Lei Básica V de Frege proporciona o padrão de definição geral dos princípios de abstração superior é uma boa razão para duvidar da validade deste tipo de definições. Neste artigo, eu argumento contra esta objeção, mostrando que o padrão de definição dos princípios de abstração — como extrapolados a partir do §64 do *Grundlagen* de Frege — inclui um requisito adicional (que denomino como *a condição de especificidade*) que não é satisfeito pela Lei Básica V, embora seja satisfeito por outras abstrações de ordem superior, tal como o Princípio de Hume. Mostro também que a falha deste requisito adicional no caso da Lei Básica V é engendrada por uma característica essencial da concepção de Frege da lógica, e que, assim, o próprio Frege não deveria ter tomado a Lei Básica V como uma definição por abstração.

Palavras-chave: Princípios de Abstração. Má Companhia. Reentalhamento do Conteúdo. Lógica de Frege.

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1. Introduction

Frege's early attempt to provide logical foundations to arithmetic is based on a special sort of definitions which have been named *abstraction principles*. Although Frege neither used this terminology nor explicitly formulated a rigorous condition on what it takes for a definition to be an abstraction principle, it is commonly agreed that an abstraction principle is an implicit definition of instances of a certain concept, where conditions of identity between them are framed in terms of an equivalence relation. One of Frege's most famous examples of an abstraction principle is the implicit definition of the concept of 'direction of a straight line': the direction of a line a is identical to the direction of a line b iff line a is parallel to line b . In general, the definitional pattern of a first-order abstraction principle is formally represented as follows:

$$\forall x \forall y [R(x, y) \leftrightarrow \sigma(x) = \sigma(y)] \quad (\mathbf{AP})$$

where R is a first-order equivalence relation and σ is the so-called abstraction operator. It has to be said that abstraction principles which play an effective role in Frege's logicist project are second-order, i.e. the equivalence relation appearing on the left limb of the bi-conditional is second-order. For instance, *Hume's Principle* – i.e. the implicit definition of cardinal numbers – is framed in terms of the relation of *equinumerosity*: the concept F is equinumerous to the concept G iff the cardinal number of F is identical to the cardinal number of G . We schematically represent a second-order abstraction principle as follows:

$$\forall X \forall Y [\mathbf{R}_x(Xx, Yx) \leftrightarrow \sigma_x(Xx) = \sigma_x(Yx)] \quad (\mathbf{AP2})$$

where X, Y are second-order variables, \mathbf{R} is a second-order equivalence relation, and σ is a second-order abstraction operator (an operator that when combined with a monadic conceptual expression forms a singular term). Another famous (and notorious) example of higher-order abstraction is Frege's Basic

Law V which says that whenever two concepts are co-extensional they have the same *value-range*:

$$\forall X \forall Y [\forall x (Xx \leftrightarrow Yx) \leftrightarrow \varepsilon_x(Xx) = \varepsilon_x(Yx)] \quad (\mathbf{BLV})$$

where ε is the value-range operator. It is well-known that the Basic Law V leads to Russell's paradox and thus is inconsistent. This fact has induced a general objection against the soundness of definitions by abstraction which has been traditionally called *the Bad Company objection*. The objection may be summarized as follows: if the definitional pattern of abstraction principles only requires that the relation appearing in the left limb is an equivalence relation, then this definitional pattern cannot be the base for formulating correct definitions, for the relation of co-extensionality appearing in the left limb of the Basic Law V is an equivalence relation, yet the correspondent abstraction principle is inconsistent. Thus abstraction principles that are reasonably consistent – such as Hume's Principle – are in bad company, for they share the same definitional pattern with the Basic Law V.

As remarked by Linnebo (2009) “The bad company problem shows that a deeper understanding is needed of the conditions under which abstraction is permissible”. In other words, the fact that a relation R is an equivalence relation is not enough to ensure that the correspondent bi-conditional (**AP2**) should count as an abstraction principle, i.e. as a valid definition. A possible reply to the bad company objection is based on a deeper analysis of the definitional pattern that characterizes abstraction principles resulting in new requirements on the relation appearing in the left limb such that the Basic Law V may be shown to fail to satisfy these requirements that other reasonably consistent abstraction principles meet. This is the strategy that I pursue in this paper. In section 2 I will analyze the definitional pattern of abstraction principles by considering the passage in which Frege introduces this sort of definitions; I will argue that in order to formulate a permissible abstraction principle, the equivalence relation R must satisfy what I call *the specificity condition*. In section 3 I will show that differently from Hume's Principle, the Basic Law V do-

es not satisfy the specificity condition. In section 4 I will try to spell out what feature of the relation of co-extensionality in Frege's system of logic prevents the Basic Law V to be a permissible abstraction principle.

2. The definitional pattern of abstraction principles

The notion of definition by abstraction is introduced by Frege in §64 of his *Grundlagen*, where he introduces the famous example of the implicit definition of the concept of 'direction of a straight line':

The judgment '*line a is parallel to line b*', [...] can be taken as an identity. If we do this, we obtain the concept of direction and say: '*The direction of line a is identical with the direction of line b*'. Thus we replace the symbol \parallel by the more generic symbol $=$, through removing what is specific in the content of the former and dividing it between *a* and *b*. We carve up the content in a way different from the original way, and this yields us a new concept. (FREGE, 1950), pp. 74-75

The main difficulty concerning this passage consists in the understanding of the crucial notion of 'content' (*Inhalt*) and the way this notion is handled. For instance, it is not clear how to understand the fact that a certain part of the content of the relation of parallelism is "removed" and successively "divided" between *a* and *b*. What seems to be clear from the passage is that the sentence 'the direction of *a* is identical to the direction of *b*' may be obtained by performing certain content preserving operations on the sentence 'line *a* is parallel to line *b*'; as a result, the two sentences express the same content. To my knowledge, the relevant literature has focused mainly on attempts to define such a relation of identity of content between sentences ((HALE, 1997), (POTTER; SMILEY, 2001), (YABLO, 2008)) rather than on spelling out the operation of content removal and division described in the passage. Albeit this paper does not aim at entirely explaining away Frege's metaphorical language, some hints on the definitional pattern Frege is trying to present may be extrapolated.

I understand the passage as saying that the same content may admit different internal organizations that result in different syntactic structures. For instance, being *C* of the sentence '*a* \parallel *b*', we may obtain a different internal or-

ganization of C by performing the procedure described in the passage on C as already organized according to the syntactic structure of ' $a \parallel b$ '. In particular, we may remove part of the content of the relation of parallelism and attach it – in an unspecified way – to the content of ' a ' and ' b '. As a result, the “remainder” of the content of the relation of parallelism after removing its “specific content” is the content of the relation of identity; and the outcome of attaching this content to ' a ' and ' b ' is the content of new singular terms, i.e. ‘the direction of a ' and ‘the direction of b '. Thus the two limbs of an abstraction principle have the same content in virtue of the fact that what is removed from one constituent of the content is attached to others.

As mentioned in the previous section, there is a general agreement on the fact that the relation R appearing in the left limb of an abstraction principle must be an equivalence relation. Yet Frege seems to suggest that something more is required, something that at first glance may appear almost trivial, but as we will see is a fundamental requirement to rule out some abstraction principles as unsound. Frege explicitly appeals to a specific part of the content of an equivalence relation; moreover, when he says that we replace a relational symbol “by the more generic symbol” of identity, he seems to suggest that what is specific in the content of an equivalence relation R is what makes it more specific than identity, i.e. what characterizes the difference in content between R and $=$. In what sense does an equivalence relation R is more specific than an identity (i.e. has a content including a specific part with respect to the relation of identity)? Frege does not give any hint regarding such a comparison between equivalence relations. Thus a hypothetical answer to the previous question must be formulated outside of the context of Frege's texts.

An interesting way of understanding the higher specificity of the relation of parallelism with respect to identity may come from some fundamental algebraic properties of equivalence relations. The idea is the following: an equivalence relation is a congruence with respect to a restricted range of properties – commonly referred to as *invariant properties* – while identity is a congruence with respect to all properties. In other words, two parallel lines are

indistinguishable with respect to their *orientational properties*, whereas two identical lines are indistinguishable with respect to all properties. As a consequence, an equivalence relation R may be understood as a relation of *partial or restricted indistinguishability* while an identity is understood as a relation of total or general indistinguishability. Given an equivalence relation R , the more restricted is the range of R -invariant properties, the more specific R is with respect to the relation of identity. This higher specificity of an equivalence relation R may be formalized in terms of a restricted quantification over properties. Being I_R the higher-order predicate ‘ X is an R -invariant property’, the following holds for every two individuals a and b :

$$Rab \rightarrow \forall X [I_R(X) \rightarrow (Xa \leftrightarrow Xb)] \quad (*)$$

$$a = b \rightarrow \forall X [Xa \leftrightarrow Xb] \quad (**)$$

In other words, the relation R is concerned with a specific range of properties I_R , whereas the relation of identity is concerned with all possible properties that individuals may have. The formulas (*) and (**) may be used to formulate a crucial requirement that a relation R must satisfy in order to run Frege’s principle of content carving. According to the quoted passage of §64, the content of an abstraction operator is formed using the specific part of the content of an equivalence relation R and when we remove this part from the content of R we obtain the content of the relation of identity. Whatever expressions like “removing” and “dividing” a content might mean, it is clear that if there is no “specific part” of the content of R , there is no available content to be “divided” to define the content of the abstraction operator. Therefore an equivalence relation R must satisfy the following condition: *R must express a content that includes “a specific part”*. According to the previous remark, a relation R expresses a content including a specific part only if R is more specific than the relation of identity, which in turn may be understood as the fact that R admits a range of invariant properties that does not coincide with the totality of properties. In other words, to determine whether R is more specific than identity we have to check whether there is a property φ that is

not an R -invariant property. How may this requirement on properties be expressed? According to (*) if a property φ is such that given two R -related individuals a and b , either a has φ and b lacks φ , or a lacks and b has φ . In other words, if Rab is true and a property φ may be used to distinguish between a and b , then φ is not an R -invariant property, i.e. R is more specific than identity. We may summarize this reasoning by stating the following requirement:

Specificity condition (first-order)

A first-order equivalence relation R expresses a content including a specific part iff there are two individuals a , b and a property φ such that:

$$Rab \wedge [(\varphi(a) \wedge \neg\varphi(b)) \vee (\varphi(b) \wedge \neg\varphi(a))] \quad (\mathbf{SC})$$

The specificity condition may appear as a trivial requirement: it suffices that an equivalence relation R is not the relation of identity for R to satisfy (SC). It is immediately evident that the relation of parallelism satisfies this requirement, for surely there are two distinct straight lines a and b that are parallel. As we will see, the specificity requirement becomes interesting when we consider higher-order equivalence relations:

Specificity condition (higher-order)

A second-order equivalence relation \mathbf{R} expresses a content including a specific part iff there are two concepts F , G and a second-order property Φ such that:

$$\mathbf{R}_x(Fx, Gx) \wedge [(\Phi_x(Fx) \wedge \neg\Phi_x(Gx)) \vee (\Phi_x(Gx) \wedge \neg\Phi_x(Fx))] \quad (\mathbf{SCHO})$$

In this case it makes no sense to say that \mathbf{R} meets the specificity condition whenever is distinct from identity, for there is no defined identity relation between concepts: identity is characteristic of objecthood. However, even though there is no relation of identity between concepts, it is possible to define a relation of distinguishability which is expressed by the fact that there is a se-

cond-property property that one concept has and the other lacks. It is evident that the relation of equinumerosity satisfies the specificity condition; let F be the concept defined by the open formula $x = a$ and G defined by $x = b$. Clearly, F and G are equinumerous (both have only one instance). The second-order property ‘ X has a as instance’ is a property that F has and G lacks, thus the property ‘ X has a as instance’ may be used to distinguish between two equinumerous concepts. The main result of this section is that the definitional pattern of an abstraction principle is not limited to the fact that the relation appearing in the right-limb must be an equivalence relation, for the specificity condition must also be met in order to run the procedure of content recarving and provide a content to the abstraction operator. In the next section we will see that the relation of co-extensionality – which appears in the right limb of the Basic Law V – does not meet this requirement. This is a crucial point to argue that the Basic Law V is not a bad company to Hume’s Principle, for – in virtue of the fact that it does not instantiate the same definitional pattern – it is not even a company.

3. Basic Law V

As anticipated at the end of the previous section, in this section I will show that the relation of co-extensionality does not satisfy the specificity condition in both Frege’s system of logic and standard higher-order logic, and thus cannot be conceived as an instance of the definitional pattern of abstraction principles. The main consequence of this fact is that the Basic Law V cannot be considered as a “bad company” for Hume’s Principle. As a preliminary historical remark, notice that the view that according to Frege the Basic Law V should be obtained by recarving the content of the relation of co-extensionality is not uncontroversial. Frege never justifies the truth of this axiom by invoking the procedure of §64; he rather seems to consider the Basic Law V as a primitive logical law². The fact that in the *Grundgesetze* he considers the two limbs of the Basic Law V as being “gleichbedeutend” (i.e. identical in meaning) is no evidence for the thesis that they should be identical in content: for

² (FREGE, 2013, p.14)

at the time the *Grundgesetze* was written, the word “gleichbedeutend” is undoubtedly translated as “identical in reference”, thus identical in truth value. As a consequence, nothing in Frege’s mature writings suggests that he ascribed a relation between the two limbs of the Basic Law V stronger than logical equivalence. However, in the essay *Function and Concept* (FREGE, 1952) Frege seems to suggest that a sentence saying that two concepts are co-extensional expresses the same sense as a sentence saying that their extensions coincide (as highlighted in (BURGE, 1984)); hence, if we assume that identity of sense implies identity of content – i.e. that the notion of content is coarser in grain than the notion of sense – then Frege’s example suggests that the two limbs of the Basic Law V are identical in content. And this fact represents a good reason to suppose that an identity of extensions is obtained by a content recarving procedure from the content of the relation of co-extensionality. The main purpose of this paper is to show that the Basic Law V cannot be considered as a bad company of Hume’s Principle due to the fact that it does not satisfy the definitional pattern described in §64 of the *Foundations*. Thus it is not crucial for our purpose to show that Frege should have considered the Basic Law V and Hume’s Principle as different instances of the same definitional pattern.

In the remaining part of this section I will show that the Basic Law V cannot be considered as an instance of the definitional pattern described in *Grundlagen* §64 in virtue of the fact that it fails to satisfy the specificity condition. We will start by formulating this condition for the relation of co-extensionality, i.e. the equivalence relation that appears in the left limb of the Basic Law V:

Specificity Condition (BLV)

The relation of co-extensionality expresses a content having a specific part iff there are two concepts F , G and a second-order concept Φ such that:

$$\forall x(Fx \leftrightarrow Gx) \wedge [\Phi_x(Fx) \wedge \neg\Phi_x(Gx)] \vee [\Phi_x(Gx) \wedge \neg\Phi_x(Fx)]$$

(SCBLV)

The fact that the relation of co-extensionality does not satisfy (SCBLV) in the formal system of the *Grundgesetze* may be proven by showing that the following principle – which I call the *Principle of Extensional Equivalence* – holds: for every higher-order open formula $\Phi_x(Xx)$ where the variable X occurs free,

$$\forall x(Fx \leftrightarrow Gx) \rightarrow (\Phi_x(Fx) \rightarrow \Phi_x(Gx)) \quad \text{(PEE)}$$

The Principle of Extensional Equivalence says that two co-extensional concepts are interchangeable *salva veritate* in all contexts. In other words, there is no higher-order formula that may distinguish between two co-extensional concepts. The proof of (PEE) in Frege’s system will be presented through an adaptation of Frege’s notation to make it understandable even to the reader who is not familiar with the concept script. Let ι be an operator that when applied to a value range v returns the value c if c is the only element of v , otherwise the false. Following §34 of (FREGE, 2013) we define the membership operator as follows:

$$(a \in b) = \iota \varepsilon_z[\exists g (b = \varepsilon_u g(u) \wedge g(a) = z)] \quad \text{(DEF)}$$

which says that the function $a \in b$ of the arguments a, b has the same value as the function g of the argument a if there is a function g such that the value range of g is b , otherwise the value of $a \in b$ is the False.

Let ξ be a metavariable and f a first-order function of one argument, by considering the following instance of (DEF):

$$(\xi \in \varepsilon_u f(u)) = \iota \varepsilon_z[\exists g (\varepsilon_u f(u) = \varepsilon_u g(u) \wedge g(\xi) = z)]$$

and by applying the Basic Law V, it follows that:

$$\xi \in \varepsilon_u f(u) = f(\xi) \quad (\mathbf{T1})$$

In other words, for every function f and every term ξ , the expression ‘ $\xi \in \varepsilon_u f(u)$ ’ is coreferential to the expression ‘ $f(\xi)$ ’. We can now prove the Principle of Extensional Equivalence. Let $A(f(\xi))$ a formula in which the function f occurs applied to a term ξ (ξ may be a Roman, German, or uppercase Greek letter). A sketch of the proof follows:

- | | |
|--|---------------------------------------|
| (7) $A(f(\xi))$ | (Assumption) |
| (8) $\forall x(f(x) = g(x))$ | (Assumption) |
| (9) $A(\xi \in \varepsilon_u f(u))$ | (by (T1), 1, and Leibniz’s Law) |
| (10) $\varepsilon_u f(u) = \varepsilon_u g(u)$ | (by Basic Law V, 2, and Modus Ponens) |
| (11) $A(\xi \in \varepsilon_u g(u))$ | (by Leibniz’s Law on 3 and 4) |
| (12) $A(g(\xi))$ | (by (T1), 5, and Leibniz’s Law) |

Which proves that for every formula A in which the function f occurs, if $A(f(\xi))$ and if f and g are co-extensional, then $A(g(\xi))$. Therefore, in Frege’s system co-extensional functions are always replaceable *salva veritate*. Hence there is no higher-order formula of Frege’s system that may distinguish between co-extensional functions which, in turn, means that the relation of co-extensionality does not satisfy the specificity condition. Considered that – as argued in the previous section – the specificity condition should be included as a requirement that an equivalence relation must satisfy in order to run the content recarving procedure and thus introducing the corresponding abstraction principle, the definitional pattern of abstraction principles extrapolated from §64 of the *Grundlagen* is not applicable to the relation of co-extensionality. As a consequence, the Basic Law V and Hume’s Principle are not instances of the same definitional pattern, i.e. the Basic Law V is not a bad company for Hume’s Principle, for it is not even a company. Notice that the fact that the relation of co-extensionality does not satisfy the specificity condition implies that it expresses no specific content in Frege’s logic, thus there is no available content to form the value range operator.

Contemporary higher-order logic deserves some additional comments. The formula (**PEE**) is valid in both standard and Henkin's semantics for higher-order logic; given the incompleteness of higher-order logic, we cannot take this fact as a proof of the fact that the formal system treats conceptual expressions in a pure extensional way. However, as remarked in (HECK, 2012) (p. 137), the principle of extensional equivalence may be proven by induction in the complexity of higher-order formulas. Therefore, both Frege's logic and standard higher-order logic are *purely extensional*, i.e. they allow for the replacement of co-extensional conceptual expressions in every formula *salva veritate*.

One may object that the proposed argument is circular when applied to Frege's system, for – as previously mentioned – we use the Basic Law V to prove the principle of extensional equivalence. Notice that to show that the relation of parallelism satisfies the specificity condition we did not need to assume the abstraction principle of directions; similarly, to show the same fact regarding the relation of equinumerosity, we did not need to assume Hume's Principle.

Such a circularity may induce a new version of the bad company objection: given that the Basic Law V is false (due to Russell's paradox), we cannot conclude that (**PEE**) is valid in Frege's system of logic. As a consequence, the relation of co-extensionality may still satisfy the specificity condition and thus the Basic Law V may still be a company for Hume's Principle; in particular, given the falsity of the Basic Law V, it will be a bad company. The objection may be replied by remarking that the Basic Law V may be dispensable in the proof of (**PEE**); indeed, it is very plausible to assume that we may adapt the proof of (**PEE**) in standard higher-order logic described by Heck (2012) to Frege's system. In the next section we will try to understand in further details which feature of both Frege's logic and Frege's conception of higher-order predication prevents the relation of co-extensionality to satisfy the specificity condition.

4. Frege's logic and higher-order predication

In this section we will try to understand more deeply in virtue of which characteristic of Frege's logical system the relation of co-extensionality fails to satisfy the specificity condition. We have seen that the relation of co-extensionality fails to satisfy the specificity condition due to the fact that the principle of extensional equivalence is provable in Frege's logical system. Thus our question becomes: how the proof of the principle of extensional equivalence in Frege's logical system may be blocked? The proof of the principle of extensional equivalence uses basic features of higher-order logic, such as Leibniz's Law and *Modus Ponens* whose validity in the present context is considered out of question. The only point that is debatable is the theorem labeled (T1), which says that for every first-order function f and argument ξ , the expression ' $f(\xi)$ ' may always be replaceable by ' $\xi \in \varepsilon_u f(u)$ ' *salva veritate*. It is important to notice that (T1) does not say that the incomplete expressions ' $f(\dots)$ ' and ' $\dots \in \varepsilon_u f(u)$ ' are names of the same function, yet names of different co-extensional functions. Nevertheless, if ' $f(\dots)$ ' and ' $\dots \in \varepsilon_u f(u)$ ' denote different functions, it should be in principle possible to assert something true of one and false of the other, thus making the principle of extensional equivalence fail. In informal examples, we use free variables to substantivize functions, as in the expression 'the concept x is a horse'. And this makes possible to construct examples of sentences in which the intensional difference between functions implies an extensional difference between sentences: the sentence 'John grasps the concept x is a horse' may not have the same truth-value as the sentence 'John grasps the concept x is in the value range of the concept ' y is a horse'', perhaps by assuming that John grasps the concept horse while being innocent of the notion of value range. Hence the possibility of expressing contexts in which the principle of extensional equivalence fails relies on the possibility of substantivizing functions.

In Frege's concept script there is no way of substantivizing functions. More precisely, Frege does not use predicate letters that can be used as function names to form higher-order predications directly applied to functions; moreover, there are no symbols of the language denoting empty argument places;

not even free variables, given that even Roman letters used to express generality have a defined (implicit) scope, thus acting as bound variables. When Frege wants to denote a function in his informal comments, he either uses meta-variables such as ‘ξ’ or dots standing for empty argument places. This point may be clarified a bit further. Given a first-order function f , I define a *direct higher-order predication of f* as a formula in which f occurs with no specification of its argument or using a free variable as argument. For instance ‘square root is a continuous function’ or ‘the concept x is a horse is difficult to grasp’ are cases of direct higher order predication. On the other hand, I define an *indirect higher-order predication on f* as a formula in which f occurs with its argument placed filled by a proper name of an object, a bounded variable, or a variable with a restricted scope. In Frege’s logic only indirect higher-order predication is possible, for there are no letters denoting function names which may be used without argument specification and the argument places of first-order functions may be filled either by uppercase Greek letters (proper names of objects) or by German letters (bounded variables) or by Roman letters (variables with restricted scopes)³.

The impossibility of substantivizing functions – and thus of direct higher-order predication – is not something that Frege overlooked or that escaped his attention; it is considered as a desirable feature of his logical system, for it allows for *the reduction of orders*, as he clearly says:

First-order functions can be used instead of second-level functions in what follows. This will now be shown. As was indicated, this is made possible by the fact that the functions appearing as arguments of second-level functions are represented by their value-ranges, although of course not in such a way that they simply concede their places to them, for that is impossible. (FREGE, 2013, p.52)

In spite of the fact that Frege recognizes that we cannot simply replace a function name by the name of its value range, Frege holds that value ranges shall be considered as *representative objects* of concepts in higher-order predi-

³ There is also the case of lowercase Greek letters used to represent schematically higher-order functions, as in the expression $M_{\beta}(f(\beta))$. However, the presence of the index β in M_{β} clearly suggests that higher-order functions act as variable binders, thus allowing the substitution $M_{\beta}(\beta \in \varepsilon_u f(u))$.

cation⁴; as a consequence, a higher-order predication applied to a concept $f(x)$ may always be converted into a first-order one applied to the value range of $f(x)$.

This leads to the role of Basic Law V in Frege's logic and to the main point of this paper. As shown by Wright (1983), Frege could have done without the Basic Law V and derive the axioms of arithmetic directly from Hume's Principle plus standard higher-order logic. However, Frege did not regard the possibility of deriving the axioms of arithmetic and of defining numbers as logical objects as the only role of value ranges in his logical system. As remarked by Heck (2012) and as shown by the previous quotation, Frege regarded the Basic Law V as having the virtue of *lowering the orders*, i.e. of converting a higher-order predication into a first-order one by introducing value ranges as representative objects of functions. Hence, the additional and crucial role of the Basic Law V is that of making the logic purely extensional by reducing assertion about concepts to assertions about their value ranges. In other words, Frege regarded the Basic Law V as the principle that makes explicit that co-extensional concepts are replaceable *salva veritate* in all contexts. I have shown that this amounts to say that the Basic Law V does not satisfy the specificity condition which implies that it is not a bad company for Hume's Principle. Therefore, Frege's conception of logic requires that the Basic Law V is not an abstraction principle *on a par* with the others; the special role he ascribes to the Basic Law V implies that the relation of co-extensionality must be a sort of identity between concepts, i.e. an absolutely general higher-order equivalence relation with no specific content. Thus the fact that the Basic Law V does not instantiate the definitional pattern described in §64 of Grundlagen is not an unexpected consequence of Frege's axiomatization, yet is essential to the role that value ranges play in Frege's logic.

5. Conclusion

In this paper I have proposed an argument showing that the bad company objection may not be as harmful as it is commonly thought. More precisely, I have shown that the objection relies on the false assumption that the

⁴ (RUFFINO, 2000), (RUFFINO, 2003)

Basic Law V is an instance of the definitional pattern behind all abstraction principles. My argument is based on the assumption that the definitional pattern of abstraction principles is not characterized just by the requirement that the relation appearing in the left limb is an equivalence relation. By analyzing the passage in §64 of the *Grundlagen*, I have shown that a further condition must be met, i.e. what I have called *the specificity condition*. Notice that this is a particular view on what abstraction principles are: one may consider abstraction principles from a different perspective, i.e. independently of how Frege introduces and justifies them, and thus may not accept a requirement extrapolated from §64 of the *Grundlagen* as a necessary condition for the truth of an abstraction principle. Moreover, I have argued that, according to Frege's conception of logic and to his view on the role of value ranges of functions, it is essential to the possibility of reducing the orders that the relation of co-extensionality does not satisfy the specificity condition and thus that the Basic Law V shall not instantiate the definitional pattern extrapolated from *Grundlagen* §64. It is worth clarifying that this paper is not directly concerned with the problem of the inconsistency of the Basic Law V. I have not tried to argue that the Basic Law V is inconsistent *in virtue of* the fact that in Frege's system of logic the relation of co-extensionality does not satisfy the specificity condition. Another point that is worth highlighting is that the argument I have presented is not aimed at formulating a complete list of requirements that abstraction principle must satisfy in order to be acceptable. In other words, I am not saying that given an equivalence relation R , if R satisfies the specificity condition, then the corresponding abstraction principle is true. There might be further requirements that the proposed analysis fails to capture. As a consequence, this paper has mainly a negative aim: it shows that being the specificity condition a necessary condition, and given that the Basic Law V fails to satisfy this condition, then it cannot be counted as an instance of the same definitional pattern of Hume's Principle. However, I presented no argument to the effect that the specificity condition is also sufficient; hence, there may be other inconsistent abstraction principles that satisfy the specificity condition,

thus showing that a deeper analysis of the definitional pattern of abstraction principles is still needed.

On the other hand, I have presented an interesting alternative approach to the philosophical issues regarding abstraction principles by turning more attention on the way Frege introduces this sort of definitions and the justification for their truth that he seems to suggest based on the procedure of content recarving.

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