Quantifier Comprehension

A Comment on the Existing Study
Proposal of a New Experiment

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Abstract

- McMillan et al. (2005) measured brain activity.
- Subjects were judging the truth-value of sentences.
- They compared FO and non-FO quantifiers.
- They claim that computational semantics is plausible.
- I challenge this statement.
- They classification does not capture quantifiers complexity.
- I suggest other studies on quantifier comprehension.
- They can throw light on the role of working memory.

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1. Monadic Quantifiers and Automata
   - Definition and examples
   - Quantifiers and computation
2. Neuroimaging Data (McMillan et al. 2005)
   - Methods
   - Results
   - Discussion
3. Proposal of Improved Experiment
   - FO and Divisibility Quantifiers
   - Aristotelean and Cardinal Quantifiers
   - Quantifiers and Ordering
4. Conclusion

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Instead of introduction

- Every poet has low self-esteem.
- Some dean danced nude on the table.
- At least 3 grad students prepared presentations.
- An even number of the students saw a ghost.
- Most of the students think they are smart.
- Less than half of the students received good marks.
- An equal number of logicians, philosophers, and linguists climbed Elbrus.

Lindström definition

Definition

A monadic generalized quantifier of type $\underbrace{(1,\ldots,1)}_{n}$ is a class Q of structures of the form $M = (U, A_1, \ldots, A_n)$, where A_i is a subset of U. Additionally, Q is closed under isomorphism.

Few examples to make it clear

- $K_{\exists} = \{(U, A) : A \subseteq U \wedge A \neq \emptyset\}$.

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- $K_{D_n} = \{(U, A) : A \subseteq U \wedge card(A) = k \times n\}$.
- $K_{Most} = \{(U, A_1, A_2) : card(A_1 \cap A_2) > card(A_1 - A_2)\}$.

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- $K_{Most} = \{(U, A_1, A_2) : card(A_1 \cap A_2) > card(A_1 - A_2)\}$.
- $K_{Equal} = \{(U, A_1, \ldots, A_n) : card(A_1) = \ldots = card(A_n)\}$.

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How do we encode models?

- We restrict ourselves to finite model $M = (U, A, B)$.
- We list all elements of the model: $c_1, \ldots, c_5$.
- We label every element with one of the letters: $a_{\bar{A}\bar{B}}, a_{A\bar{B}}, a_{\bar{A}B}, a_{AB}$, according to constituents it belongs to.
- We get the word $\alpha_M = a_{\bar{A}\bar{B}} a_{A\bar{B}} a_{AB} a_{\bar{A}B} a_{\bar{A}B}$.
- α_M describes the model in which: $c_1 \in \bar{A}\bar{B}, c_2 \in A\bar{B}, c_3 \in AB, c_4 \in \bar{A}B, c_5 \in \bar{A}B$.
- The class K_Q is represented by set of words describing all models from the class.

ILLUSTRATION

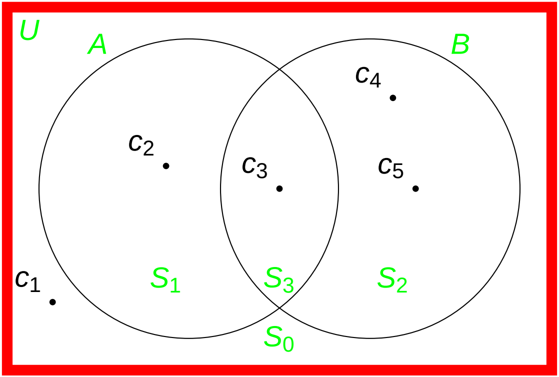


FIGURE: This model is uniquely described by $\alpha_M = a_{\bar{A}\bar{B}} a_{A\bar{B}} a_{AB} a_{\bar{A}B} a_{\bar{A}B}$.

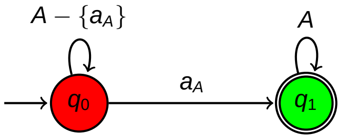
Constituents – general definition

The class K_{Q} of finite models of the form $(M, A_1, \ldots, A_n)$ can be represented by the set of nonempty words L_{Q} over the alphabet $A = \{a_1, \ldots, a_{2^n}\}$ such that: $\alpha \in L_{\mathrm{Q}}$ if and only if there are $(U, A_1, \ldots, A_n) \in K_{\mathrm{Q}}$ and linear ordering $U = \{c_1, \ldots, c_k\}$, such that $length(\alpha) = k$ and i-th character of α is a_j exactly when $c_i \in S_1 \cap \ldots \cap S_n$, where:

$$S_l = \begin{cases} A_l & \text{if integer part of } \frac{j}{2^l} \text{ is odd} \\ U - A_l & \text{otherwise.} \end{cases}$$

Languages corresponding to quantifiers

- $L_{\exists} = \{\alpha \in A^* : n_{a_A}(\alpha) > 0\}$.



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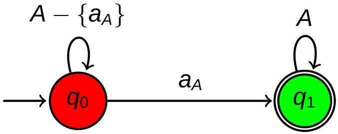
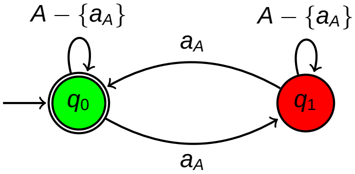


- $L_{D_2} = \{\alpha \in A^* : n_{a_A}(\alpha) \equiv 0 \ (mod 2)\}$.



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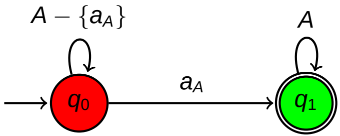

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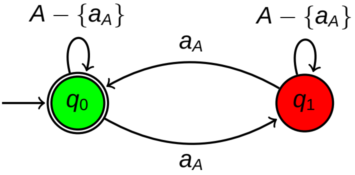

- $L_{MOST} = \{\alpha \in A^* : n_{a_{AB}}(\alpha) > n_{a_{A\bar{B}}}(\alpha)\}$.

What does it mean that class of monadic quantifiers is recognized by class of devices?

Definition

Let $\mathcal{D}$ be a class of recognizing devices,
Ω a class of monadic quantifiers.
We say that $\mathcal{D}$ accepts Ω if and only if
for every monadic quantifier Q:

$$\mathrm{Q} \in \Omega \iff \text{there is device } A \in \mathcal{D}(A \textit{ accepts } L_{\mathrm{Q}}).$$

Relevant results: acyclic FA and FA

Theorem (J. van Benthem)

Quantifier Q is first–order definable iff L_Q is accepted by acyclic finite automaton.

Theorem (M. Mostowski)

Monadic quantifier Q is definable in the divisibility logic iff L_Q is accepted by finite automaton.

FA do not use any kind of working memory device.

Odds of "even"

- "Even" and "odd" are non-FO.
- They can be however recognized by FA.
- But opposite to FO quantifiers you need FA with cycle.
- Difference between FA and acyclic FA.

Relevant results

Theorem (J. van Benthem)

Quantifier Q of type (1) is semilinear iff L_Q is accepted by push–down automaton.

PDA use stack which is simple working memory device.

Observation

There are many natural language quantifiers which lie outside the context–free languages.

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Subjects and technique

- 12 healthy right-handed native English-speaking adults (8 males, 4 females).
- Mean age 24.4 years.
- Mean education 16.4 years.
- BOLD fMRI.

MATERIALS

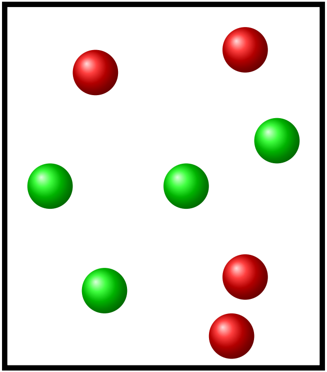
- 120 grammatically simple propositions.
- 6 different quantifiers probing color:
  - First-order: “all”, “some”, “at least 3”.
  - Higher-order: “less than half of”, “an even number of”, “an odd number of”.
- Half of each type of item was true.
- 2 consecutive 10s events:
  1. Presentation of the sentence.
  2. Presentation of the sentence with addition to an array.
- 8 randomly distributed familiar objects.
- Does the proposition accurately describe stimulus array?

Example of the task

Every ball is green.

Example of the task

Every ball is green.



Example of the task

Even number of balls are green.

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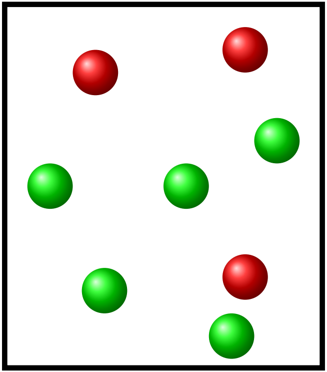
Even number of balls are green.



Example of the task

Most of the balls are green.

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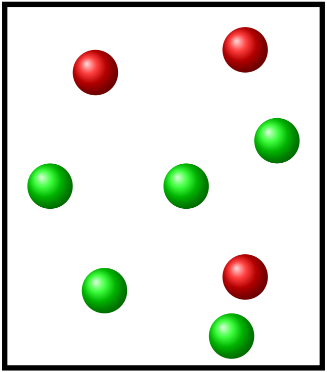
Most of the balls are green.



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Results

- FO judgments: 92,3% , non-FO: 84,5%.
- FO and non-FO recruit right inferior parietal cortex – the region of brain associated with number knowledge.
- Only non-FO recruit right dorsolateral prefrontal cortex – the part of brain associated with working memory.

Additional support

- Corticobasal degeneration (CBD) – number knowledge.
- Alzheimer (AD) and frontotemporal dementia (FTD) – working memory limitations.

Additional support

- Corticobasal degeneration (CBD) – number knowledge.
- Alzheimer (AD) and frontotemporal dementia (FTD) – working memory limitations.
- CBD impairs comprehension more than AD and FTD.
- FTD and AD patients have greater difficulty in non-FO.

Main Claim

Claim

Our computational model explains differences in processing. Especially it predicts the use of working memory.

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Reminder

definability	example	recognized by
FO	exactly 6	acyclic FA
$FO(D_n)$	even	FA
semilinear (1)	most	PDA

TABLE: Quantifiers and complexity of corresponding algorithms.

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- The explanation is based on the wrong assumption.

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- The explanation is based on the wrong assumption.
- Overlooked computational differences between quantifiers.
- The experimental design may be improved.

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Use Complexity Distinctions

Compare 3 classes of quantifiers:

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1. recognizable by acyclic FA,

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Use Complexity Distinctions

Compare 3 classes of quantifiers:

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2. recognizable by FA,
3. recognizable by PDA.

Predictions based on computational model

1. Comprehension of divisibility quantifiers – but not FO – depends on the executive resources (FA vs. acyclic FA).

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1. Comprehension of divisibility quantifiers – but not FO – depends on the executive resources (FA vs. acyclic FA).
2. Only quantifiers not definable in divisibility logic will activate working memory.

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Aristotelean vs. Cardinal Quantifiers

- Aristotelean: “all”, “every”, “some”, “no”, “not all”.
- Cardinal, like: “at least 3”, “at most 7”, “between 8 and 11”.

Aristotelean vs. Cardinal Quantifiers

- Aristotelean: “all”, “every”, “some”, “no”, “not all”.
- Cardinal, like: “at least 3”, “at most 7”, “between 8 and 11”.
- FO representation of cardinal is psychologically ill-suited.
- Consider the translation of “at least 3 balls” into FO:

$$\exists x \exists y \exists z (x \neq y \wedge y \neq z \wedge x \neq z \wedge ball(x) \wedge ball(y) \wedge ball(z)).$$

The rank of cardinal quantifiers

- The complexity of FO-translation is proportional to the quantifier rank.
- Processing of cardinal quantifiers is more similar to non-FO quantifiers than to Aristotelean?
- Use cardinal quantifiers of higher rank, e. g: “at least 7”.
- Subitizing opposed to counting?

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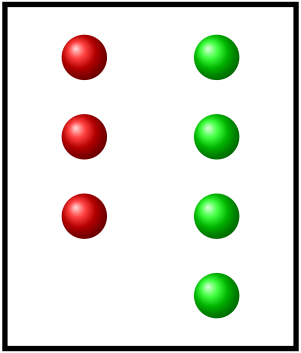
Verify the role of working memory

- Ordering of elements as new independent variable.
- Quantifier processing in ordered vs. random universes.
- Over ordered universe the working memory is not needed.
- In this case non-FO quantifier can be recognized by FA.

Majority over ordered universe

Most of the balls are green.



Majority over randomized universe

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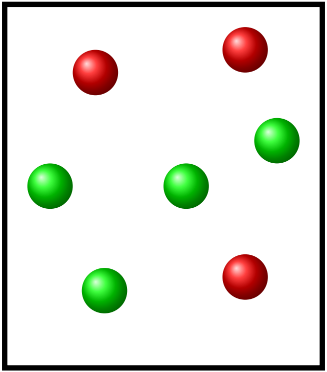

Prediction

- “Most” over ordered universes will not activate working memory.
- Ordering will not influence FO and divisibility processing.

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Conclusion

- Logical distinction on FO and non-FO quantifiers is not sufficient for investigating the role of working memory in quantifier comprehension.
- It is high time for conducting improved experiments starting with reaction time studies!

References

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For Further Reading

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