A Logical Reconstruction of Medieval Terminist Logic in Conceptual Realism

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Abstract

The framework of conceptual realism provides a logically ideal language within which to reconstruct the medieval terminist logic of the 14th century. The terminist notion of a concept, which shifted from Ockham's early view of a concept as an intentional object (the fictum theory) to his later view of a concept as a mental act (the *intellectio* theory), is reconstructed in this framework in terms of the idea of concepts as unsaturated cognitive structures. Intentional objects (ficta) are not rejected but are reconstructed as the objectified intensional contents of concepts. Their reconstruction as intensional objects is an essential part of the theory of predication of conceptual realism. It is by means of this theory that we are able to explain how the identity theory of the copula, which was basic to teminist logic, applies to categorical propositions. Reference in conceptual realism is not the same as supposition in terminist logic. Nevertheless, the various "modes" of personal supposition of terminist logic can be explained and justified in terms of this conceptualist theory of reference.

1 Introduction

Medieval logic was a philosophical logic, by which we mean a logic that is based on a philosophical theory of predication, which in turn is based on a theory of universals. The three major types of medieval theories of universals were nominalism, conceptualism, and realism. Today, each of these types of theories can be reconstructed and represented by distinctive formal theories of predication; in particular, each can be logically reconstructed as a second-order logic with nominalized predicates (and propositional forms) as abstract

singular terms.¹

The traditional view of logic as based on a theory of predication, which in turn is based on a theory of universals, is sometimes described as the view of logic as language, a view that was defended by the founders of logicism, Gottlob Frege and Bertrand Russell. The logistic systems of Frege and Russell have not fared well in 20th century logic, however, which is not generally viewed as a philosophical logic. The predominant modern view of logic is that of a formal calculus subject to varying set-theoretical interpretations, a view sometimes described as logic as calculus.² We do not dispute the importance of such a view, and in fact we find it useful as a mathematical tool. Nevertheless, the traditional view is what is important for philosophical purposes, and, when formalized using the tools of modern mathematical logic, it can be used to represent not only a theory of universals but a general formal ontology, a system of knowledge representation, and a variety of different positions in the philosophies of mind and language. One such framework is what elsewhere we have called *conceptual realism*, a logistic system that is based on a conceptual theory of predication, but which also contains a natural realism amounting to a modern form of Aristotelian essentialism and an intensional realism amounting to a naturalized form of platonism.³ This logistic system is also a form of second-order logic with nominalized predicates as abstract singular terms.

Reference in conceptual realism is not restricted to so-called singular terms, e.g. proper names and definite descriptions, but involves quantifier phrases containing common nouns or proper names, where the former can be complex as well as simple. This uniform account of both singular and general reference is similar in a number of respects to the medieval *suppositio* theory of the 14th century; and, in fact, we maintain that with minor modifications the latter can be reconstructed within the framework of conceptual realism.

¹See Cocchiarella [1986] for a formal exposition of such logical reconstructions. Nominalism, for example, can be represented by standard predicative second-order logic under a substitutional interpretation for predicate quantifiers, extended to allow for nominalized predicates as abstract singular terms—but where the first-order part of the logic is free of existential presuppositions, and where all abstract singular terms are posited as nondenoting, thereby representing the nominalist thesis that there are no universals as values of either the bound individual or the bound predicate variables.

²See van Heijenoort [1967] for a characterization of these two views of logic.

³See Cocchiarella [1996] for a description of conceptual realism as a formal ontology and Cocchiarella [1995] for a description of conceptual realism as a system for knowledge representation in artificial intelligence.

Indeed, conceptual realism provides a general philosophical framework within which we can reconstruct and explicate not just the medieval *suppositio* theory as a theory of reference, but a number of other issues that were central to medieval logic as well, such as the identity theory of the copula (for categorical propositions) and the difference between real and nominal definitions. One of the benefits of such a reconstruction is that we can defend medieval logic against the kinds of arguments that Peter Geach has given against a uniform account of general and singular reference in his book *Reference and Generality*.⁴

2 Terminist Logic and Mental Language

The medieval logic we will be concerned with here is what came to be called "terminist logic" because it was primarily concerned with the semantics or "properties of terms" (proprietates terminorum), i.e. of adjectives and proper and common count nouns, which hereafter we will speak of as common names. This approach to logic began in the 13th century with such logicians as Peter of Spain, Roger Bacon, Lambert of Auxerre, and William of Sherwood. Around 1270, however, terminist logic "went into a kind of hibernation" and was replaced by an alternative movement known as "speculative grammar". The hibernation ended in the early 14th century when "terminist-style semantic theory woke up again". The major terminist logicians of this later period are William of Ockham, John Buridan, Walter Burley, and Gregory of Rimini. Ockham and Buridan are generally described as nominalists, but really were conceptualists on our account. Burley is said to be a realist, but his realism was really a form of conceptual natural realism (which we will not go into here).

The conceptualism of the terminist logicians is clearly seen in their assumption that underlying spoken and written language there exists a mental language made up of both categorematic and syncategorematic concepts and mental propositions, or what in conceptual realism we prefer to call

⁴See T.K. Scott [1966b] for criticism of Geach's account of Ockham's *suppositio* theory. Also, see Cocchiarella [1998] for a refutation of Geach's general arguments against of the kind of uniform account of reference given in conceptual realism.

⁵Spade [1996], p.43. Spade gives a useful account of the history of this period. We will be relying on this text throughout this essay.

 $^{^6}$ Ibid.

judgments—or thoughts when they are only entertained and not asserted or judged.⁷ This "language of thought", which is generally referred to today as Mental, was assumed to be common to all humans. Unlike spoken and written languages, which were said to be "conventional" languages, Mental was said to be a "natural" language, by which was meant a language somehow established by nature. Apparently, what makes Mental natural is that its categorematic concepts (mental terms) "get their signification by nature and not by convention." Signification is the basic semantical relation of Mental, but it applies only to categorematic concepts, syncategorematic concepts being said not to signify at all. The signification of a categorematic concept is not an intensional object, but rather the things that fall under the concept, by which was meant, in a narrow sense, the things that now fall under the concept, but which, in a wider sense, included the things that could fall under the concept, i.e. the things that can fall under the concept, and therefore the things that did, do, or will fall under the concept as well. This distinction between narrow and wide signification was possible because our thoughts (mental propositions) occur in time and, by means of tense and modal modifications, can be oriented toward the past or the future, as well as the present, and even toward what is merely possible. Mental is a tensed and modal language, in other words, containing among its syncategorematic concepts certain operators that correspond to the tenses and modal modifications of verbs, or what the medieval logicians called ampliation. Moreover, because mental terms can signify in a wider sense, Mental is ontologically committed to a form of possibilism, though the possible objects signified (in the wide sense) by a concept seemed to be only those that are possible in nature.¹⁰

That mental terms get their signification by nature is based on the idea that there is a "natural likeness" between concepts and the things they sig-

⁷In conceptual realism, a proposition is the intensional content of a speech or mental act, and not, as in terminist logic, the assertion or judgment made. We will allow some laxity in the use of 'proposition' in our account of terminist logic, however.

⁸Geach in [1957], p.102, is one of the first to use 'Mental' this way. See Trentman [1970], Normore [1990], and Spade [1996], chapter 4, for an account of Mental. Normore [1985], p.189, explicitly refers to it as as "a language of thought."

⁹Spade [1996], p.93.

¹⁰See Normore [1985], p.191. It is not clear Normore would agree that all possibilia in Ockham's and Buridan's ontology are possible in nature, i.e. that the modality in question is a natural possibility (as opposed, e.g., to a logical or metaphysical possibility). A natural possibility and necessity seems to be what Burley had in mind, however.

nify, a natural likeness that is caused, apparently, by the things signified.¹¹ This suggests that concepts are something like images, which is much too restrictive a view in that it excludes those concepts of things that cannot be "imaged," i.e. things other than the visible objects of the macrophysical world. Scientific concepts of things, processes and events in the microphysical world can be mathematically modelled, and in that sense "imagined," but, because they are smaller than the wavelength of light, they cannot be "imaged" or pictured literally, and therefore there can be no "natural likeness" between our concepts and such things. Nor, of course, can there be a "natural likeness" (or a causal relation) between concepts and mathematical objects, be they numbers, sets, fields, or whatever. 12 This idea of a "natural likeness" also suggests that the concepts corresponding to common names must all be sortal concepts, i.e. concepts that have identity criteria associated with them. But the concepts corresponding to the common names 'thing', 'object', 'individual', and even 'physical object', and 'abstract object', do not have identity criteria associated with them, the way, e.g., the common names 'man', 'dog', 'carrot', 'chair', etc., do. Even the common name 'furniture', unlike different sorts of furniture (such as tables and chairs) and the common name 'event', unlike sorts of events (such as a running, kicking, or kissing) do not have identity criteria associated with them. In what sense is there a "natural likeness" between things in general and the common-name concept thing, or between physical objects and the complex common-name concept physical object? We can form an image of a chair or table, i.e., of a particular sort of furniture, but we cannot form an image of furniture in general, nor can we form an image of events in general, though we can form an image of a kissing (between, say, Abelard and Heloise, or Bill and Monica). The notion that all categorematic concepts have a "natural similarity" with the things they signify is much too restrictive and is one of the semantic features of Mental that we do not assume applies to the kind of conceptualism we

¹¹According to Spade, "Ockham says the things a concept signifies are all such that:
(a) The concept is like every one of them. (b) It is not like any one of them any more than it is like any other one; it is equally like all of them. (c) It is like any one of them more than it like anything else—anything that is not signified by the concept." ([1996], pp.153f)

¹²It seems that mathematical objects were not accounted for at all in terminist logic. The reconstruction we give below in terms of conceptual realism provides a way of giving an account along logicist lines, but without the latter's commitment to logical realism (as a form of metaphysical platonism).

have in mind here. For this reason, we will not refer hereafter to Mental as a "natural" language, but will instead follow contemporary practice and speak of historically real languages such as Latin, English, French, etc., as natural languages. We prefer the contemporary usage partly because it is contemporary, but also because we want to distinguish artificial languages (such as HST^*_{λ} described below), which are also "conventional," from the historically developed natural languages spoken by a linguistic community.

In regard to the relation between Mental and spoken (and written) languages, Ockham's view differs somewhat from Buridan's. According to Ockham, for example, a linguistic term used in an assertion signifies the same things that are signified by the concept corresponding to the term. Buridan, on the other hand, says that the spoken linguistic term signifies the concept corresponding to the term, and only indirectly, through the concept, signifies the things that it signifies. ¹³ In conceptualism, we take a position similar to Ockham's in that, e.g., a referential expression of English that occurs as the noun phrase of an assertion in English refers to the same things that are referred to by the referential concept that the expression stands for—and, in fact, the linguistic act is just the mental act expressed overtly in English.¹⁴ On the other hand, we do want to say in conceptualism that a referential expression of a spoken language stands for (stat pro) a referential concept, and similarly that a predicable expression stands for a predicable concept, that might be exercised in a given speech act, which might seem in some respects similar to what Buridan says. But then, Ockham does have a notion of subordination, which he says holds between a linguistic term and the corresponding concept, which seems to be essentially what is meant in saying that the term stands for the concept. It is Ockham's position, in other words, that is closer to conceptualism as we understand it here.

The relation between a conventional language such as Latin or English and a language of thought such as Mental is sometimes said to be analogical, especially by writers sympathetic to nominalism. Peter Geach, for example, held the "general thesis" that "language about thoughts is an analogical development of language about language," and, in particular, that "the concept *judging* is ... an analogical extension of the concept *saying*." Sim-

¹³Cf. Normore [1985], p.190, and Spade [1996], chapter 3.

¹⁴This is not to say that there are no "inner episodes" of thinking, i.e. of referring and predicating, that are nonlinguistic.

¹⁵Geach [1957], p.98.

¹⁶Ibid., p. 75

ilarly, Wilfrid Sellars claimed that our view of thoughts as "inner episodes" is a theoretical construction modelled upon our view of meaningful linguistic behavior, and in particular that "concepts pertaining to the intentionality of thoughts," such as that of reference, are "derivative from concepts pertaining to meaningful speech." The thesis does seem to be one that Ockham followed in that some of the features of Latin are assumed to carry over into Mental. Indeed, according to Geach, "the grammar of Mental turns out to be remarkably like Latin grammar. There are nouns and verbs in Mental; nouns have cases and numbers, and verbs have voice, mood, tense, number, and person." ¹⁸

Despite his general thesis, Geach warns us not to carry the analogy too far. Ockham, in particular, according to Geach, "merely transfers features of Latin grammar to Mental, and then regards this as explaining why such features occur in Latin." Except for mood and tense, Geach maintains, "the grammatical properties ascribed by Ockham to Mental words may all be easily dismissed." The exception is noteworthy because tense "does enter into the content of our thoughts," and, according to Geach, "there are modal differences between thoughts—though the moods of a natural language like Latin are a very inadequate indication of this, being cluttered with a lot of logically insignificant idiomatic uses." ²¹

Geach's claim that Ockham carried the analogy of thought to language too far has been attacked on several fronts.²² Nevertheless, his critics agree that the proper comparison is not of Mental with Latin but of Mental with the kind of "ideal languages" that logicians and philosophers have constructed in the twentieth century, i.e. with a logistic system from the point of view of logic as language. Thus, according to J. Trentman, "Ockham's real criterion ... for admitting grammatical distinctions into Mental amounts to asking whether the distinctions in question would be necessary in an ideal language—ideal for a complete, true description of the world."²³ Similarly,

¹⁷Sellars [1981], p.326. Also, see "Empiricism and the Philsophy of Mind" in Sellars [1963].

¹⁸Geach [1957], p.102. See Spade [1996], chapter 4, for a more detailed list of the "common" and "proper accidents" of nouns and verbs of Latin, but where only the "common accidents" are part of the grammar of Mental.

¹⁹Ibid.

 $^{^{20}}$ Ibid., p.103.

²¹Ibid., pp.103f.

²²See Trentman [1970] and Spade [1996], chapter 4.

²³[1970], p.589.

according to Paul Spade, "mental language is to be a kind of ideal language, which has only those features it needs to enable it to discern the true from the false, to describe the world adequately and accurately."²⁴ Of course, an ideal language should account not only for an adequate and accurate description of the world, but also for valid reasoning about the world. In other words, Mental should be constructed as a logically ideal language, relative to which analyses of natural language sentences can be given, thereby resulting in logically perspicuous representations of the truth conditions of those sentences. The logical forms representing these truth conditions would then determine which sentences follow validly from other sentences as premises, i.e. they would then determine the conditions for valid reasoning in terms of the recursive operations of logical syntax.²⁵

The kind of analyses of natural language sentences intended here are what the terminist logicians called "expounding" (exponere) or "exposing" (exposito). Thus, according to Calvin Normore, "exposito is a natural result of the recognition that the surface grammar of a sentence is not always a reliable guide either to its truth conditions or its inferential connections to other sentences," which suggests that the analysis (or exposition) of a natural language sentence (of Latin, English, etc.) should not result in another sentence of that language but in a logical form of an ideal language based on logical grammar. That is why the proper province of logic for Buridan, according to Normore, is "the articulation of truth conditions for grammatically complex sentences," i.e. the process of making "the logical form of sentences explicit". 27

What is needed in terminist logic, but up until now has not be given, is a representation of Mental as a logistic system based on the view of logic as language, and in particular on a conceptualist theory of predication. Such a system should provide a perspicuous representation of the truth conditions of our speech and mental acts, and thereby, in terms of the recursive operations of logical syntax, the validity of our arguments as well. Moreover, as a

²⁴[1996], p.110.

²⁵The recursive operations of logical syntax will generate some logical forms that do not represent propositions of Mental, however, but which are needed for the deductive machinery of the ideal language by which to prove the validity of arguments—and for the generation of those forms that do represent the propositions of Mental as well. (See §7 for more on this point.)

²⁶Normore [1985], p.192.

 $^{^{27}}$ Ibid.

conceptualist theory of our speech and mental acts, the system should also provide logical forms that perspicuously represent the cognitive structure of those acts, including in particular the referential and predicable concepts underlying them. It is these concepts that in one way or another correspond to what the terminist logicians called supposition. The system we have described for conceptual realism, we believe, can be used in just this way, even if some features of the system may seem to be in conflict with certain terminist theses.

3 Conceptual Intensional realism

There are two kinds of realism in conceptual realism: an intensional realism in which some, though not all, nominalized predicates and propositional forms are posited as having denotata, and a natural realism in which, on the basis of empirical data, natural kinds and natural properties and relations are posited as corresponding to certain of our sortal and predicable concepts. Our concern here will be only with the system of conceptual intensional realism, the core of which is an extension of standard second-order predicate logic in which predicate and propositional forms can be nominalized and allowed to occur as abstract singular terms on a par with individual variables—but with the one modification that the first-order part of the logic is free of existential presuppositions for singular terms.²⁸ Complex predicates are formed by means of λ -abstraction, so that where φ is a formula and $y_1, ..., y_n$ are distinct individual variables, $[\lambda y_1...y_n\varphi]$ is an *n*-place predicate expression. A predicate expression is always accompanied by a pair of parentheses (and commas if it is relational) when it occurs in its functional role as a predicate, as in $F(x_1,...,x_n)$ and $[\lambda y_1...y_n\varphi](x_1,...,x_n)$, where F is an n-place predicate letter. (We drop the parentheses and commas when referring to predicate expressions simpliciter.) We nominalize a predicate by dropping the parentheses (and commas) and allow the result to occur as an abstract singular term—as in G(F), $G([\lambda y_1...y_n\varphi])$, R(x,F), and even G(G), and R(x,R),

²⁸Strictly speaking, this core system is constructed on the basis of a conceptually prior system of constructive conceptualism, i.e. one in which only "predicative" and no "impredicative" concepts (such as the concept of a limit in mathematics) can be formed. This "predicative" logic is part of the core system, but we will ignore it here because the distinctions in concept-formation it was design to represent were not considered in terminist logic. We assume that tense and modal operators are also part of the system, but we will generally ignore their use here.

where G is a 1-place and R a 2-place predicate. (We use capital letters for predicate variables and constants, lower-case letters for individual variables, and Greek letters for formulas.) Nominalized formulas are represented as 0-place predicates, as in $[\lambda \varphi]$, read as 'that φ ', which we will write more simply as $[\varphi]$.

The comprehension principle of our core theory, which we call HST^*_{λ} , can be stated in the following simple form (where F is not free in φ),

$$(\operatorname{CP}_{\lambda}^*) \qquad (\exists F)([\lambda x_1...x_n \varphi] = F),$$

from which the more usual (but weaker) form,

$$(\mathsf{CP}^*) \qquad (\exists F)(\forall x_1)...(\forall x_n)[F(x_1,...,x_n) \leftrightarrow \varphi],$$

follows. Russell's paradox, as represented through the Russell predicate $[\lambda x(\exists G)(x = G \land \neg G(x))]$, is not derivable in HST^*_{λ} . By $(\mathrm{CP}^*_{\lambda})$, the Russell predicate stands for a concept, i.e.,

$$(\exists F)([\lambda x(\exists G)(x = G \land \neg G(x))] = F),$$

and therefore

$$(\exists F)(\forall x)[F(x) \leftrightarrow (\exists G)(x = G \land \neg G(x))],$$

are provable in HST^*_{λ} ; but all that follows by Russell's argument is that when nominalized, the abstract singular term corresponding to the Russell predicate fails to denote an object (as a value of the bound individual variables), i.e.,

$$\neg(\exists y)\left(\left[\lambda x(\exists G)(x=G\land\neg G(x))\right]=y\right)$$

is provable.²⁹ This shows that not every concept can be nominalized in conceptual realism, where, by the nominalization of a concept we mean a projection (or hypostatization) of the concept's intensional content into the domain of objects.

Predicable concepts, in other words, are not the intensional abstract objects (if any) that are denoted by nominalized predicates as abstract singular terms. For example, the predicable concepts that we exercise in saying

 $^{^{29}\}mathrm{HST}^*_{\lambda}$ is in fact consistent relative to weak Zermelo set theory, and it is equipollent to the theory of simple types.

of someone that she is wise, kind, and beautiful, or of a black cube that it is cubical and black, are distinguished in this way from the intensional "properties" of being wise, being kind, being beautiful, being cubical and being black—i.e. the "properties" wisdom, kindness, beauty, cubicity, and blackness—which conceptually we grasp through the concepts by means of the process of nominalization.³⁰ In conceptualism, as we understand it here, predicable concepts are intersubjectively realizable—and in that sense objective—cognitive capacities, or cognitive structures based upon such capacities, to characterize and relate objects in various ways. Some, but not all, predicable concepts may be initially based on experience, and some might even precede the initial acquisition of language. But, as expressed in natural language, regardless of how they are connected with experience, all predicable concepts are the cognitive capacities that underlie our rule-following abilities for the correct use of predicate expressions. In other words, predicable concepts are those features of thought and communication that determine the truth conditions that predicate expressions have in different possible contexts of use. It is through the exercise of these capacities that our mental acts, and therefore our speech acts as well, are informed with a predicable nature.

As cognitive capacities that can be exercised by different people at the same time, as well as by the same person at different times, concepts in general have an unsaturated nature, which is similar to but not the same as Frege's notion of unsaturatedness, and which is the basis of our understanding of predicable concepts as "universals." The joint exercise of a referential and a predicable concept, as in the expression of a categorical proposition, does not result in a truth value (or an intensional propositional object), but in a judgment, i.e. a mental act (event), and a speech act if expressed overtly. The concepts are not themselves constituents of the judgment or speech act, i.e. they are not components (objects) of that act (as an event), but rather are the determinants that inform the act with a referential and a predicable nature. In this regard, concepts are not objects (values of the bound individual variables), though the exercise of a referential and predicable concept results in an object, namely, a speech or mental act.

A nominalized predicate, accordingly, cannot denote the concept the pred-

³⁰We speak of the intensional content of a predicable concept as a "property" here in conformity with traditional use. These "properties," or intensions, it should be noted, are not the same as the properties of conceptual natural realism, which for us are unsaturated causal structures that are realized as the nexuses of possible states of affairs. See Cocchiarella [1996], §5, for more on this distinction.

icate stands for in its role as a predicate, if it denotes anything at all. That a nominalized predicate does denote, and, in particular, that it denotes the intension or intensional content of the concept the predicate stands for, is an assumption that leads from conceptualism to conceptual intensional realism.³¹ Here, by the intension or intensional content of a concept we understand the truth conditions determined by the concept in its different possible contexts of use; and that the intensions of some concepts, such as that expressed by the Russell predicate, cannot be *object*ified, i.e. reified as objects that are values of the bound individual variables, means that those truth conditions would lead to a contradiction if such intensions were to "exist" as objects (i.e. as values of the bound individual variables). Such a result does not affect the status of those concepts as concepts, nor of their intensional content, i.e. the truth conditions they determine; rather, it only means that those truth conditions cannot be reified as objects. The intensions of most concepts, or at least those used in ordinary thought and communication, can be objectified, however, without leading a contradiction.³²

Traditionally, these reified intensional contents have been called platonic forms, because it was Plato who first argued for their "existence," but in the sense of being part of a timeless realm that is independent of the natural world, and therefore independent of the evolution of consciousness. It is the notion of such a timeless realm, and of platonic forms as part of such a realm, that is rejected by nominalists—though their rejection of abstract entities applies even to moderate realism, which assumes that universals "exist" only in things, and hence only in time. There is an alternative, however, which amounts to a naturalized platonism, which we call conceptual intensional realism. On this account, our cognitive access and subsequent knowledge of all abstract objects is possible only by means of our concepts. That is, it is only by starting out from concepts as cognitive capacities that we are able to conceptually grasp and understand the contents of those concepts as abstract objects; and the primary mechanism by which we are able to do this

³¹Denotation in conceptual realism is not the same as reference, it should be noted. In particular, whereas reference (described in section 6 below) is a pragmatic notion that applies to speech and mental acts, denotation is a semantic notion that is involved in the evaluation of truth conditions. That a singular term a fails to denote (in a given context) means that $\neg(\exists y)(a=y)$ is true (in that context), where y is an individual variable not occurring free in a.

³²The axiom for positing such intensional objects in HST^*_{λ} is $(\exists/HSCP^*_{\lambda})$, as described in Cocchiarella [1986].

is the conceptual analogue of nominalization. Of course, once nominalization becomes institutionalized in natural language and the intensions of our concepts are objectified by this mechanism, we may go on to use abstract singular terms other than nominalized predicates—such as, e.g., numerals and other mathematical expressions in our theoretical account of mathematical entities³³—though, in the end, all abstract objects are based in one way or another on our ability to form concepts as cognitive capacities.

But abstract objects, as we understand them here, are not only epistemologically dependent on the concepts by which they are "grasped"; rather, they also have their being in the concepts whose intensions they are, or those intensions they are ultimately based upon. All abstract objects, on this account, which include the propositional contents of our myths and scientific theories as well as the intensions of the concepts exercised in our speech and mental acts, are products of the evolution of consciousness, language and culture, including in particular the evolution and development of the forms of nominalization in language and thought. Abstract objects do not "pre-exist" the evolution of consciousness, in other words, but depend essentially upon it, even though they subsequently come to have a certain amount of autonomy and come to play a role in the further evolution of culture.³⁴

4 Ockham's Early theory of *Ficta* as Intensional Objects

The ontology of intensional objects, or naturalized platonism, we have described above would seem to be in conflict with the nominalism commonly associated with Ockham and the terminist logicians. Ockham, for example, clearly rejected the platonist interpretation of nominalized predicates; but that is because he associated it with a platonist or realist theory of predication. On this theory, a person is said to be wise, for example, because he

³³See Cocchiarella [1989], §4, for an explanation of how the natural numbers are initially constructed through a correlation first of numerical quantifier phrases with predicable concepts and then by a nominalization of the latter.

³⁴The naturalized platonism we have in mind here is similar to Karl Popper's theory of what he calls the third world, or the world of objective contents of thought, which he distinguishes from the first world of physical objects and physical states and a second world of mental states, including dispositions to act. See Popper [1967], p.106, and P2 of Popper and Eccles [1983].

exemplifies the quality or property denoted by 'wisdom'. That is, a predication of the form 'x is wise' is explained on the basis of a supposedly more basic sentence of the form 'x exemplifies wisdom', which means that signification in the sense in which 'wise' signifies wise individuals is not a basic semantic notion after all.³⁵

This is not how intensional objects are understood in conceptual realism, however, where predication (as described below) is explained in terms of the mutual saturation of a referential and predicable concept in a speech or mental act. Intensional objects, as we have said, are products of language and culture that do not pre-exist the evolution of consciousness, and, as such, can in no sense be the basis of a realist theory of predication. Something like a (modal moderate) realist theory of predication is part of conceptual natural realism, but the natural kinds and properties that are part of that theory are not objects, and therefore they are not the intensional objects denoted by nominalized predicates. Natural kinds, for example, are unsaturated causal structures that are the basis of causal laws; and they become saturated only as the nexuses of states of affairs. A terminist logician, such as Ockham (reconstructed somewhat as described below), would have rejected the (empirical) posits of conceptual natural realism—although another terminist logician, such as Burley, might well have accepted them—but Ockham could have accepted the intensional objects of conceptual intensional realism, just as he once accepted ficta as intentional objects.³⁶

Ockham, in other words, did accept something like our account of intensional objects in his early view of concepts as ficta.³⁷ On this view, which is sometimes called the fictum theory, concepts are the intentional objects of acts of intellection (e.g. judgments). Ficta were not regarded as independently "real" entities, but were said to have only an "intentional being" (esse objectivum), according to which "their being is their being cognized." ³⁸

³⁵See Loux [1974], p.6.

³⁶For a description how the empirical posits of conceptual natural realism are formulated, see Cocchiarella [1996], §§5-6. The same logistic formulation of natural realism would apply to Ockham as to Burley, incidentally, except that Burley would have accepted the thesis that some concepts have a natural kind, or a natural property, corresponding to them, whereas Ockham would have denied (negated) that thesis.

³⁷Spade [1996], p.154, and Normore [1990], p.59, for a description of Ockham's *fictum* theory.

³⁸Spade [1996], p.156. Marilyn McCord Adams prefers to speak of Ockham's early view of concepts as the "objective existence theory." She objects to calling such objects *ficta*, apparently because sometimes the intentional objects thought about are real and

Ficta included not only "universals", such as humanity and triangularity, but also logical objects, such as propositions (in the modern sense), and fictitious objects, such as chimeras and goatstags, and also impossible objects, such as the round square.³⁹ These are just the sort of objects that are accounted for in conceptual realism as intensional objects, which suggests that the latter might not really be so alien to Ockham's ontology after all, even if he later changed his mind about ficta.⁴⁰

Ockham described his earlier view of concepts as intentional objects in his *Reportatio*, where he made a distinction between two kinds of mental language. One was Mental as we have described it so far, and "the other was a language whose terms were the mental representations of the spoken expressions of natural language itself," i.e. a mental language that is "posterior to spoken language," and therefore based on spoken language. ⁴¹ This view of *ficta* is not unlike our view of intensional objects in conceptual realism, where conceptual nominalization and the *object*ification, or reification, of these objects is "posterior" to linguistic nominalization both in the historical development of language and in the conceptual development of individuals in their acquisition and use of language.

Of course, Ockham gave up his early theory of concepts as ficta in favor of his later theory of concepts as mental acts (intellectiones). But that does not mean that the role ficta played in explaining how we can think about "unreal" objects can now be explained by concepts as mental acts; and, in fact, we maintain they cannot, and that something like ficta—namely, the intensional objects of conceptual realism—are needed to fulfill this role. Ontologically, ficta and the intensional objects of conceptual realism are similar in the way they depend upon concepts; for just as ficta, as intentional objects, had their being in being cognized in the mental acts that Ockham later identified with concepts, so too intensional objects similarly have their being in the concepts

not fictitious things ([1977], pp.151f). Our concern here, however, is not with intentional objects as such, whether real or fictitious, but with Ockham's early theory of *ficta* as a way of seeing why the intensional objects of conceptual realism can be accommodated in a reconstructed version of terminist logic.

³⁹See Adams [1977], p.147. T.K. Scott, [1966a], p.16, notes that Gregory of Rimini argued for propositions (in the modern sense), or what he called enunciables (*enuntiabile*), which were denoted by infinitive or gerundive expressions that amounted in effect to nominalized sentences. These enunciables, like the abstract objects of conceptual realism, were real, but not existent, objects.

⁴⁰For an analysis of the round square as a fictum, see Cocchiarella [1996], §7.

⁴¹Normore [1990], p.59.

whose intensions they are. Ockham was right in his rejection of concepts as ficta, but wrong in then rejecting ficta altogether. They have a role to play in mathematics and the semantics of fiction, and stories and theories in general; and, perhaps even more importantly, as we will see, in the semantics of those concepts that intensional verbs (such as 'seek', 'promise', 'owe', etc.) stand for, as well as in our conceptualist theory of predication for concepts based on relations in general, including the copula. Our proposal here is to take the intensional objects of conceptual realism as a "logical reconstruction" of Ockham's early theory of ficta as intentional objects.

5 Ockham's Later Theory of Concepts as Mental Acts

Ockham's later theory of concepts, which is sometimes called the *intellectio* or mental-act theory, does not identify concepts with intentional objects, but with mental acts themselves, i.e. with actual mental occurrences.⁴² The common-name concept man, for example, "is the very act of thinking of men"⁴³; that is, as a mental occurrence, that very act signifies all men. This theory is similar to contemporary nominalism where, e.g., an actual spoken linguistic token of the word 'man' is said be *true of* all men, but as a matter of convention only, and not because there is a "natural likeness" between the word and men.

One problem with the mental-act theory is how the same concept, e.g., man, can be common to all humans as a term of Mental. That is, if a person's concept man is just a mental act (event) of that person, and one person's mental act is never the same as another's (or of the same person at another time), then how can the same concept man be common to different people (or to the same person at different times)? The answer that was given, apparently, is that although one person's concept man is "numerically different" from another person's concept man—i.e. their mental acts of thinking of men are different mental events—nevertheless, the two concepts are "exact duplicates of one another." But in what sense can the mental acts of two or more people, or of the same person at different times, be exact duplicates

⁴²Spade, [1996], p.155, and Adams [1977], p.145.

⁴³Spade [1996], p.155.

⁴⁴Ibid., p.93.

of one another? Is it because there is an assumed "natural likeness" between concepts and the things they signify, i.e. that one person's concept man will then have a "natural likeness" with another person's concept man (or with the same person's concept man at a different time)? If so, then, for reasons already given against the supposed "natural likeness" between concepts and the things they signify, this is an answer not acceptable today, or at least not for conceptual realism as we understand it here.

Ockham does suggest an alternative—namely, that concepts as mental acts are qualities of the mind, and in particular qualities that "exist" only when a mind is exercising the mental act in question, as in moderate realism (but restricted to qualities that inhere only in minds⁴⁵). Different mental acts of thinking of men are then just different instances of a mind's having the same quality. 46 This version of the mental act theory is sometimes called the quality theory of concepts, according to which "the concept is a real quality inhering in the mind just like any other real property."47 This theory might explain how the same concept can be exercised in two actual mental acts—namely, by being the same mental quality inhering in the mind or minds whose acts they are—but it doesn't account for concepts that are in fact never exercised and that we nevertheless "tacitly know" or have in our conceptual repertoire—e.g., concepts of very large numbers. Also, it is not clear how a concept as a quality inhering in a mind only when it is exercised can explain how the mind can exercise that concept, nor how it might inform the act with a referential or predicable nature.

Our proposal is to "reconstruct," or replace, Ockham's theory of concepts as mental qualities with the theory of concepts as cognitive capacities that we have described above for conceptual realism. Concepts in this sense do not have an "existence" independently of the more general capacity that humans have for language and thought, and yet, as capacities that are intersubjectively realizable, they are objective in at least as strong a sense as Ockham's notion of a "natural likeness" between concepts and what they signify—but without the problems the latter notion raises. Also, as the rule-following capacities underlying our use of predicate and referential expressions in natural languages, concepts have by their very nature the function of informing

⁴⁵Why a moderate realism restricted to mental qualities should be acceptable, but not a moderate realism that applied to the "external" world as well, is an issue we leave to others to explain—if it can be explained at all.

⁴⁶Spade [1996], p.155, and Adams [1977], p.145.

⁴⁷Spade [1996], p.155.

a speech or mental act with a predicable or referential nature; and, of course, they are the very same capacities that are exercised in the production of those speech and mental acts. Finally, the unsaturated nature of a concept explains its non-occurrent, or quasi-dispositional status—that is, its status as a capacity that could, but need not, be exercised in an appropriate context, or, that might in fact never be exercised at all.

6 Reference in Conceptual Realism

Reference in conceptual realism is a pragmatic notion that applies only when referential concepts are exercised in speech or mental acts. Reference in terminist logic is also a pragmatic notion, but applies to the way categorematic terms are said to *supposit* for the things they signify when used in a speech or mental act. Both systems distinguish reference to concepts from reference to things, but only conceptual realism is explicit in distinguishing reference to concepts in terms of predicate quantifiers. Reference to concepts in terminist logic, which is called *simple supposition*, does not explicitly involve predicate quantifiers, but this might be a matter only of surface grammar.⁴⁸ In any case, our concern here will be with reference to things, which in terminist logic is called *personal supposition*.

Personal supposition in terminist logic is not the same as reference to things in conceptual realism, because (as we explain in §9 below) categore-matic terms can have personal supposition either as subjects or predicates of categorical propositions, whereas referential concepts in conceptual realism can never function as predicable concepts, nor can predicable concepts function as referential concepts. Nevertheless, except for the so-called merely confused personal supposition of predicates containing an intensional verb or modal operator, the personal supposition of terms in categorical propositions does coincide with a combined notion of activated and deactivated reference in conceptual realism, where deactivated reference is involved in the truth conditions determined by a predicable concept. Both systems, moreover, give a uniform account of general and singular reference to things.

Like predicable concepts, referential concepts in conceptual realism are unsaturated cognitive structures; but the structures are not the same. Rather,

⁴⁸There is also another type of supposition, *material supposition*, in which a term stands for itself or other spoken or written signs. We will not deal with this type of supposition here.

like the way that quantifier phrases have a structure that is complementary to predicate expressions, or the way that noun phrases are complementary to verb phrases, referential concepts and predicable concepts are cognitive structures that are complementary to one another. This complementarity is such that when they are exercised together in a speech or mental act each saturates the other; and just as the predicable concept is what informs that act with a predicable nature, so too the referential concept is what informs the act with a referential nature. Thus, for example, an affirmative assertion that is analyzable in terms of a noun phrase and a verb phrase (regardless of the complexity of either) is semantically analyzable in terms of an overt application of a referential concept with a predicable concept; and the assertion itself, as a speech act, is the result of the mutual saturation of their complementary structures in that act. It is just this sort of mutual saturation of complementary cognitive structures that constitutes the nexus of predication in conceptualism. It is also what accounts for the unity of a speech or mental act, i.e. of an assertion or judgment, a problem that Ockham, who anticipated F.H. Bradley's infinite regress argument, was unable to resolve.⁴⁹ Ockham, for example, assumed that a judgment that every man is an animal was literally made up of a universal quantifier, the concept man, the mental copula is, and the concept animal.⁵⁰ But then what unifies these mental terms into a single unified mental act? A fifth mental term that "tied" these items together would need a sixth to "tie" it with the others, which in turn would need a seventh, and so on ad infinitum. That is not how a judgment or assertion is understood in conceptual realism, where concepts, as unsaturated cognitive structures, are not objects, and therefore cannot be actual constituents of a mental act (event).⁵¹

Referential concepts, as we have indicated, are what the quantifier phrases of our logistic system stand for when the latter are affixed to the symbolic counterparts of names, where both proper and common names are under-

⁴⁹See Spade [1996], chapter 4, §3.

⁵⁰Ibid., p.123. Spade points out that not all terminists agreed with Ockham, and Buridan as well, on this view of judgments or mental propositions as complexes of syncategorematic and categorematic mental terms. Gregory of Rimini and Peter of Ailly, in particular, criticized the view, and argued instead that judgments, or mental propositions, unlike the assertions of spoken language, were "structureless mental acts" that occur, as it were, all at once. This view is similar to the notion of a judgment or assertion in conceptual realism, and might well be "reconstructed" in terms of the latter.

⁵¹On our account of predication as the mutual saturation of a referential and predicable concept, there cannot be even a first step toward Bradley's infinite regress.

stood to have such counterparts, just as they do in Mental, the language of thought of terminist logic. For convenience, we refer to the symbols as proper or common names as well. A proper name is distinguished in the system from common names by a meaning postulate to the effect that at most one thing can be referred to by that name, and that the name refers to the same thing in every possible world in which it refers to anything at all.⁵² A proper name may be used with or without existential presupposition, a topic we will return to later. Common names are the counterparts of common nouns, which include sortal common nouns (such as 'man', 'tree', 'chair', etc.) whose use in thought and communication is associated with certain specific identity criteria, as well as such non-sortal common nouns as 'thing', 'object', etc. whose use is not associated with any specific identity criteria. Both kinds of common nouns stand for common-name concepts. Names, whether proper or common are different from predicate expressions, as Geach has pointed out, because they can be used in "simple acts of naming" outside the context of a sentence.⁵³ Naming is not the same as referring, it should be emphasized, because the latter is an act that does not occur outside the (implicit if not explicit) context of a sentence used in a speech act, i.e. independently of an associated act of predicating.

When used to refer in an assertion, a name occurs, as we have said, as part of a quantifier phrase. For example, when the common name 'raven' of English is used in an assertion to refer to every raven, or to some raven, then in the logical form representing the assertion in question the symbolic counterpart of 'raven'—which, for convenience, we will take to be 'Raven'—will be affixed to a universal, or an existential, quantifier (indexed by a individual variable), respectively. Thus, an assertion that every raven is black will be symbolically represented by $(\forall x Raven)Black(x)$, or, using a λ -abstract, by $(\forall x Raven)[\lambda x Black(x)](x)$, where, again for convenience, we use 'Black' as the symbolic counterpart of the English predicate adjective 'black'. The sym-

$$PN(S) =_{df} \Box(\forall xS)(\Box(\forall yS)(y=x) \land \Box[E!(x) \to (\exists yS)(x=y)]),$$

where E! stands for the concept of *concrete existence*. Proper names of concrete objects can be assumed to be vacuous in any world in which those objects do not exist. The second conjunct is vacuously true in the case of abstract objects, because, on conceptual grounds, no abstract object exists concretely.

 $^{^{52}}$ Using the notation described below, a symbolic name, S, can be stipulated to be a proper name—a stipulation we abbreviate as PN(S)—as follows:

⁵³[1980], p.52.

bolic counterpart of 'Some raven is not black', with the negation understood to be internal to the predicate, is represented as $(\exists x Raven)[\lambda x \neg Black(x)](x)$. In each case, the quantifier phrase stands for the referential concept that is being exercised in the speech act in question, and the predicate expression stands for the predicable concept of being-black, or being-not-black. The notation is logically perspicuous in the way it represents the cognitive structure of each assertion as the mutual saturation of the referential and predicable concepts in question, analogous to the way that the corresponding quantifier and predicate expressions mutually saturate each other syntactically.

Denials, or negative assertions, are equivalent to affirmative assertions in which the negation is internal to the predicate; but they do not have the same cognitive structure. Thus, for example, an assertion of 'No raven is white' is really a denial that some raven is white, which, using an external negation, we symbolize as $\neg(\exists x Raven)White(x)$.⁵⁴ The equivalent affirmative assertion, namely that every raven is such that it is not white is symbolized as $(\forall x Raven)[\lambda x \neg White(x)](x)$. Despite their logical equivalence, and therefore their having the same truth conditions, the assertions have different cognitive structures, which it is important to distinguish in a perspicuous representation of our speech and mental acts. In the denial or negative assertion, for example, the referential concept that the quantifier phrase ' $(\exists x Raven)$ ' stands for has been "deactivated", by which we mean in part that no reference to a raven is involved in such an assertion, whereas the quantifier phrase in the equivalent affirmative assertion stands for a referential concept that has been "activated", i.e. exercised in the speech act in question.⁵⁵

The objectual quantifier phrases $(\forall x)$ and $(\exists x)$ (and their rewrite variants), which are already present in our system, are assumed to be abbreviated versions of $(\forall xObject)$ and $(\exists xObject)$, where 'object' is assumed to be synonymous with 'thing', and hence where these phrases may also be read as

$$Not([(\exists x Raven) White(x)]),$$

where 'Not' is a one-place predicate read as 'is not the case'. We use the standard notation instead for convenience.

⁵⁴'No raven is white' is obtained through a transformation of 'There is no raven that is white', which in turn is a transformation of 'It is not the case that there is a raven that is white', or, equivalently, of '[That there is a raven that is white] is not the case'.

⁵⁵The deactivation is more explicit in '[That some raven is white] $_{NP}$ [is not the case] $_{VP}$ ', where the sentence 'some raven is white' is nominalized. We could symbolize this as

'everything' and 'something', respectively. The fact that assertions of the form 'Every S is F' and 'Some S is F' have the same truth conditions as assertions of the form 'Everything is such that if it is an S, then it is F' and 'Something is an S and it is also F' is captured in the following meaning postulates:

(MP1)
$$(\forall xS)F(x) \leftrightarrow (\forall x)[\lambda x((\exists yS)(x=y) \rightarrow F(x))](x),$$

(MP2)
$$(\exists xS)F(x) \leftrightarrow (\exists x)[\lambda x((\exists yS)(x=y) \land F(x))](x).$$

Despite their logical equivalence, however, the assertions in question do not have the same cognitive structure, a fact that is perspicuously represented in our notation.⁵⁶

Complex common names are generated in English from more basic common names by attaching a (defining) relative clause to the latter. We adopt an analogous procedure in our logic by introducing an operator '/', which generates a (complex) name by taking a name and a formula as arguments. Thus, where S is a name and φ is a formula, then ' S/φ ' is a (complex) name, read as 'S (who, which) that is (are) φ '. For example, a symbolic representation of an assertion of 'Every citizen (who is) over 21 is eligible to vote', which has the complex common name 'citizen (who is) over 21' as its grammatical subject, can now be given perspicuously as

$$(\forall x Citizen/Over-21(x))Eligible-to-Vote(x).$$

The reference in this assertion is to every citizen who is over 21, and not just to citizens, as it would have been had one asserted the different, but equivalent sentence, 'Every citizen is such that if he is over 21, then he is eligible to vote', which is symbolized as

$$(\forall x Citizen)[\lambda x (Over-21(x) \rightarrow Eligible-to-Vote(x))](x).$$

To account for the logical equivalence of these sentences, and others like them, we assume the following as meaning postulates (where S is a variable for names, F is a one-place predicate variable, and φ a formula):

(MP3)
$$(\forall x S/\varphi)F(x) \leftrightarrow (\forall x S)[\varphi \to F(x)],$$

(MP4)
$$(\exists x S/\varphi) F(x) \leftrightarrow (\exists x S) [\varphi \wedge F(x)].$$

⁵⁶Strictly speaking, the λ -abstracts in (MP1) and (MP2) cannot represent the exercise of a predicable concept, because the quantifier phrase $(\exists yS)$ in each has not been "deactivated". Deactivation in these cases involves the copula, Is, discussed in §8 below.

7 Active Versus Deactivated Reference

We noted above that in denials, or negative assertions, the referential concept that the quantifier phrase stands for has been deactivated, and that in fact no reference is really being made in such assertions. The notion of a deactivated referential concept, or deactivated reference, is fundamental to the theory of reference in conceptual realism.⁵⁷ A basic thesis of this theory, for example, is that a referential concept is never part of what informs a speech or mental act with a predicable nature, but functions only as what informs such an act with a referential nature, i.e., as what accounts for that act's intentionality or aboutness. Every basic assertion, as expressed by a noun phrase and a verb phrase, is the result of applying just one referential concept and one predicable concept. This means that a quantifier phrase that occurs as part of a complex predicate of a sentence that is used in an assertion does not stand for the referential concept that it stands for when it is used as a grammatical subject. In such a context, the referential concept that such a quantifier phrase stands for has been "deactivated". What the quantifier phrase stands for in such a context, according to another basic thesis of our theory, is the intensional content of the referential concept it stands for when used as a grammatical subject. Thus, an applied predicable concept that is represented in natural language by a complex predicate in which a referential (quantifier) expression occurs is not formed on the basis of the referential concept that that referential expression stands for, but on the basis of the intensional content of that referential concept.

By the intensional content of a referential concept we mean the intensional object that is specified through a two-step process in which the referential concept is first "transformed" into (i.e. correlated with) a predicable concept, which is then transformed, by nominalization, into the intensional content of the latter. This "double transformation" (correlation), or "nominalization", of a referential concept, represented by a quantifier phrase (QxS), into its intensional content, represented by [QxS], is defined as follows⁵⁸:

$$[QxS] =_{df} [\lambda x(\exists F)(x = F \land (QxS)F(x))].$$

⁵⁷A deactivated referential concept, it should be noted, is not a referential concept (just as a dead man is not a man); rather, as we explain below, it is the intensional content of the referential concept in question.

⁵⁸Strictly speaking, the definition given is of the predicate determined by the quantifier phrase (QxS); and it is this predicate that can then be nominalized.

In using this notation, we assume that any quantifier phrase that occurs within an abstract singular term, i.e. within a nominalized complex predicate expression (which is represented by a λ -abstract in our system) has been deactivated and does not represent an active use of the referential concept the phrase otherwise stands for.

To take a well-known example from Richard Montague, consider a context in which the sentence 'John seeks a unicorn' is asserted (and in which the name 'John' is used with existential presupposition).⁵⁹ The speaker in this context purports to refer only to John, and not to a unicorn, which means that the referential concept that the quantifier phrase 'a unicorn' stands for has been deactivated. The logical form representing this assertion in our system is:

$$(\exists x John)[\lambda x Seek(x, [\exists y Unicorn])](x).$$

The same analysis applies to an assertion of 'John finds a unicorn', the logical form of which is:

$$(\exists x John)[\lambda x Find(x, [\exists y Unicorn])](x).$$

Despite their having the same logical form, there is a difference in what follows from these two assertions. This is because the transitive verb 'find' is extensional in its range, as well as in its domain, whereas the transitive verb 'seek' is extensional only its domain. This means that the following meaning postulate is implicit in the use of 'find'⁶⁰:

$$[\lambda x \operatorname{Find}(x,[Qy \, S])] = [\lambda x \, (Qy) \operatorname{Find}(x,y)].$$

It is because of this implicit meaning postulate that one can argue validly as follows:

John finds a unicorn; therefore, a unicorn is found by John.

⁵⁹See Montague [1974], pp.266f. Our analysis of quantifier phrases that occur as parts of predicates (verb phrases) is similar to Montague's, except that Montague's intensional logic is a sense-denotation theory of (simple) types based on a logical realist ontology.

 $^{^{60}}$ We use 'Q' as a schema letter for determiners in general, which in addition to \forall and \exists , include the symbolic counterparts of 'most', 'few', 'the', etc., and counterparts of the demonstrative pronouns 'this' and 'that'.

And it is because no such meaning postulate is implicit in the use of 'seek' that the following argument is not valid:

John seeks a unicorn; therefore, a unicorn is sought by John.

The two arguments do not differ in the logical form, but in the kind of relational concept involved in the formation of their respective predicable concepts.

We should perhaps emphasize here that although

$$(\exists x John)[\lambda x Find(x, [\exists y Unicorn])](x) \leftrightarrow (\exists x John)(\exists y Unicorn) Find(x, y)$$

is provable in our system (by λ -conversion and the above meaning postulate), i.e. although the two sides of this biconditional represent the same truth conditions, the formula on the right-hand side does not represent the form of a speech or mental act. Only one referential concept can be activated in a basic assertion, which means that the second quantifier phrase on the right-hand side must be deactivated, i.e. "nominalized", before the formula can be taken to represent the cognitive structure of a speech or mental act.⁶¹ This means that not all of the logical forms of our theory can be taken as representing a proposition of Mental. That is as it should be, we believe, because Mental was intended to generate only mental propositions (as based perhaps on an ars combinatoria for the analysis of concepts), and not also all of the deductive logical machinery of a calculus ratiocinator. 62 In other words, Mental is not so much a logically perspicuous medium for the representation of truth conditions as it is a logically perspicuous medium for the representation of the cognitive structure of our speech and mental acts.⁶³ That logical forms that do not represent the cognitive structure of our speech and mental acts

$$(\exists x John \land \exists y Unicorn) Finds(x, y).$$

But we would still be using one referential concept in such a basic kind of assertion.

⁶¹It is possible to use a "conjunctive referential concept", as in 'John and a unicorn are such that he finds it', which might be symbolized as:

 $^{^{62}}$ Of course, our present system for conceptual realism goes well beyond the logical resouces of terminist logic in a number of other ways as well. Our interest here is to reconstruct terminist logic *within* conceptual realism, not to claim that the two are equivalent systems.

⁶³This is all the more obvious if truth conditions are cashed out in terms of a logic of events and states of affairs, as is suggested in Cocchiarella [1997], §4.

might be needed to account for the deductive relations between propositions seems not to have been an issue ever considered by Ockham and the other terminist logicians.

Another point that should be emphasized here is that the predicable concepts that 'seek a unicorn' and 'find a unicorn', and their symbolic counterparts, $[\lambda x Seek(x, [\exists y Unicorn])]]$ and $[\lambda x Find(x, [\exists y Unicorn])]$, stand for are cognitive structures underlying our rule-following abilities in the use of these verb phrases. These concepts are not a complex of a real relation and an intensional object, and the assertions in question should not be interpreted as asserting that a real relation holds between John and an intensional object. That the predicable concepts are formed on the basis of the relational concepts that 'seek' and 'find' stand for, together with the intensional content of the referential concept that 'a unicorn' stands for, means only that the truth conditions determined by these predicable concepts are based on these entities. And that the predicable concept that 'find a unicorn' stands for is based on an extensional relation means that the contribution to those truth conditions of the intensional content of 'a unicorn' can be described in an equivalent extensional way, i.e. as involving an actual unicorn.

8 The Identity Theory of the Copula

All predicable concepts formed on the basis of a relational concept are to be analyzed as above when they are to be exercised in a speech or mental act. This applies no less to the copula in its use to express an identity than it does to transitive verbs such as 'seek' and 'find'. But, because quantifier phrases occurring within a complex predicate do not stand for the referential concepts they stand for when used as grammatical subjects, we need to distinguish predicable concepts based upon the copula from those based on strict identity. We introduce a new symbol, 'Is', that we will use for this purpose, and note that, like the transitive verb 'find', the copula Is is extensional in its range as well as in its domain, except that in this case the copula becomes a strict identity. The schematic meaning postulate for Is then is as follows:

$$[\lambda x Is(x, [QyS])] = [\lambda x (Qy)(x = y)].$$

With \exists as a special case of the schematic determiner Q, we have

$$[\lambda x Is(x, [\exists y S])] = [\lambda x (\exists y)(x = y)]$$

as a particular meaning postulate or conceptual truth regarding the copula Is. By means of this notation, we can perspicuously represent the cognitive structure of an assertion of, e.g., 'Socrates is a man' as follows (assuming 'Socrates' is being used with existential presupposition):

$$(\exists x Socrates)[\lambda x Is(x, [\exists y Man])](x),$$

which, by λ -conversion and the above meaning postulate, is equivalent to:

$$(\exists x Socrates)(\exists y Man)(x = y).$$

This last formula, however, unlike the one above, does not represent the structure of a speech or mental act, although it does represent the same truth conditions.

Something like this kind of analysis was involved in the so-called the twoname theory, or the identity theory of the copula in terminist logic. Apparently, Ockham and other terminists thought that every affirmative categorical proposition amounted to asserting an identity between the personal suppositions of the subject and the predicate terms of the proposition, as, e.g., the suppositions of the names 'Socrates' and 'man' in an assertion of 'Socrates is a man'. Negative judgments, on the other hand, amounted to a denial of such an identity.⁶⁴ As a result, the identity theory of the copula came to be developed as a theory of the truth conditions of categorical propositions, a theory that is now referred to as the doctrine of supposition proper.⁶⁵

The personal suppositions of a proper or common name are the thing(s) signified by that name, but, unlike signification, supposition is understood to be relativized to the propositional context of a speech or mental act where a quantifier might occur with the name. That is why the theory of supposition of terminist logic is really a theory of the truth conditions of categorical propositions as linguistic or mental acts. These truth conditions, as we have said, are determined by the identity theory of the copula together with the quantifiers that occur with the names. Thus, for example, an assertion of

⁶⁴See Spade [1996], p.133, for a discussion of this "old and venerable theory" of the intellect's "composing and dividing" of concepts in making affirmative or negative judgments, where dividing is "composing negatively."

⁶⁵See Scott [1966], p.30, and Spade [1996], chapter 8, §iii.

⁶⁶Tense or modal modifications of the copula will "ampliate" the personal supposition of the terms, and in that way modify the truth conditions of the speech or mental act in question. (See, e.g., Scott [1966], p.33, and Spade [1966], chapter 10.) For simplicity of presentation, we restrict ourselves here to present tense uses of the copula.

'Some man is a thief', which on our analysis has the form

$$(\exists x Man)[\lambda x Is(x, [\exists y Thie f])](x),$$

is equivalent, by λ -conversion and the above meaning postulate, to

$$(\exists x Man)(\exists y Thief)(x = y),$$

which indicates that the truth conditions of this assertion amount to the identity of some supposition of the term 'man' with a supposition of the term 'thief', where each supposition, as a result of the quantifiers attached to the terms, amounts to a restriction on what on what the terms signify.⁶⁷ Similarly, an assertion of 'Every man is an animal', which on our analysis has the form,

$$(\forall x Man)[\lambda x Is(x, [\exists y Animal])](x),$$

is equivalent, by λ -conversion and the above meaning postulate, to

$$(\forall x Man)(\exists y Animal)(x = y),$$

which indicates that the truth conditions of the assertion involve an identity between each supposition of the categorical term 'man' and some supposition of the categorical term 'animal', where, again each supposition is a restriction, as determined by the quantifiers attached to each term, of what they signify.⁶⁸

The same kind of analysis also applies to categorical propositions expressed by means of a predicate adjective with the 'is' of predication instead

$$(\exists x Man \land \exists y Thief)[\lambda xy(x=y)](x,y),$$

i.e., where a conjunctive referential concept is used involving both 'man' and 'thief'.

$$(\forall x Man \land \exists y Animal)[\lambda xy(x=y)](x,y),$$

which is equivalent to our original assertion.

⁶⁷One way to construe the personal suppositions of 'man' and 'thief' here as a form of reference (as is frequently claimed in the literature) is by noting that the assertion that some man is a thief is equivalent to an assertion that some man and some thief are identical,

⁶⁸To "reconstruct" these suppositions as a form of reference, we can again use a conjunctive referential concept to assert that each man and some animal are such that they are identical:

of the copula. An assertion of 'Every swan is white', for example, which in our framework is symbolized as $(\forall x Swan)White(x)$, is not interpreted by the terminists as an identity between swans and white, or whiteness, or whitenesses (whatever any of these might be taken to be as objects). Rather, the predicate adjective 'white' is interpreted as an attributive adjective, so that to say a thing is white is to say that it is a white thing. ⁶⁹ Predicate adjectives, in other words, were analyzed by the terminists as attributive adjectives applied to the common name 'thing'. ⁷⁰ In conceptual realism, however, the common name 'white thing' is interpreted as the complex common name 'thing that is white', which is symbolized in our system as 'Thing/White(x)' (or as 'Object/White(x)'). Thus, whereas the terminist logician would interpret 'Every swan is white' as 'Every swan is a white thing', we can reconstruct the terminists' analysis as 'Every swan is a thing that is white', which can be symbolized as follows:

$$(\forall x Swan)[\lambda x Is(x, [\exists y Thing/White(y)])](x).$$

This formula, by λ -conversion and the above meaning postulate, has the same truth conditions as

$$(\forall x Swan)(\exists y Thing/White(y))(x = y),$$

which, in terms of supposition theory, is to say that each supposition of the common name 'swan' is identical with a supposition of the (complex) common name 'thing that is white', or, more simply, with the common name 'white thing'.

Negative categorical sentences such as 'No raven is white' are interpreted as denials or negations, as we have already said. That is, to assert that no raven is white is to deny that some raven is white: $\neg(\exists x Raven)White(x)$, which is provably equivalent to denying that some raven is a white thing; i.e.,

$$\neg(\exists x Raven) White(x) \leftrightarrow \neg(\exists x Raven) [\lambda x Is(x, [\exists y Thing/White(y)])](x)$$

⁶⁹See Normore [1985], p.194.

⁷⁰It is not clear if in some cases it is a common name subordinate to 'thing'. E.g. in asserting 'Socrates is wise', are we asserting that Socrates is a wise man (or person), or that Socrates is a wise thing?

is provable in our system. But denying that some raven is a white thing is equivalent, by λ -conversion and the above meaning postulate, to

$$\neg(\exists x Raven)(\exists y Thing/White(y))(x=y),$$

which in terms of the theory of supposition describes the truth conditions as a denial that some supposition of the common name 'raven' is identical with a supposition of the complex common name 'thing that is white', or more simply, with the common name 'white thing'.

Finally, the logical form of a negative particular categorical sentence such as 'Some swan is not white', which is symbolized for us as

$$(\exists x Swan)[\lambda x \neg White(x)](x),$$

is understood in terminist logic as the equivalent statement that some swan is not a white thing', which is symbolized as

$$(\exists x Swan)[\lambda x \neg Is(x, [\exists y Thing/White(x)])](x).$$

This last formula, by λ -conversion and the above meaning postulate, is equivalent to

$$(\exists x Swan)(\forall y Thing/White(x))(x \neq y),$$

the truth conditions for which are that some supposition of the common name 'swan' is not identical with any supposition of the complex common name 'thing that is white'.

9 Ascending and Descending: The Way Up is not Always the same as The Way Down

Supposition theory is not one theory but two. The first, supposition theory proper, is a theory of the truth conditions of categorical propositions as described above. The second, called the doctrine of the modes of supposition, has to do with how many things a categorematic term supposits for in a given speech or mental act.⁷¹ This doctrine, despite the reference to "modes

⁷¹Paul Spade, in [1996], p. 294, was the first to propose this interpretation—though he does not explain or develop it, as we do here, in terms of the principle of descent (described in this section). T.K. Scott, in [1966], was the first to distinguish the two doctrines, but he claims that the second has to do with the elimination of quantifiers (pp. 36f).

of supposition," is not a theory about different "ways of referring." Rather, the "modes" are just different types and subtypes of personal supposition. The two basic types are discrete and common supposition, and the purpose of the theory is to explain, or "reduce", the latter in terms of the former. Common supposition is divided into determinate and confused supposition as subtypes, and the latter is further divided into the sub-subtypes of confused and distributive supposition and merely confused supposition.

A term is said to have discrete supposition in a categorical sentence only if it is either a proper name, a demonstrative pronoun (such as 'this' and 'that') or a common name preceded by a demonstrative pronoun (such as 'this man', 'that horse', etc.). Terms that have discrete supposition are said to be discrete terms, and categorical propositions in which they occur as the grammatical subject are said to be "singular propositions."⁷⁴ The explanation, or "reduction", of the types (and subtypes) of common supposition in terms of discrete supposition is given in terms of a "descent" to, and sometimes an "ascent" from, singular propositions. A proper name such as 'Socrates' will have discrete supposition only when Socrates exists, however, which means that even an assertion of 'Socrates is Socrates' will be false when Socrates does not exist.⁷⁵ The situation is similar but not quite the same in conceptual realism, where we distinguish between using a proper name with existential presupposition and using it without such a presupposition. Thus, if, in asserting 'Socrates is Socrates', we are referentially using the name 'Socrates' with existential presupposition, the symbolic counterpart of this assertion is $(\exists x Socrates)[\lambda x Is(x, [\exists y Socrates])](x)$, in which case the assertion is false if it is asserted at a time when Socrates does not exist, or so we can assume.⁷⁶ But if we are referentially using 'Socrates' without existen-

$$(\forall x Socrates) E!(x),$$

where E! stands for *concrete existence*, as opposed to *being* (the value of an individual variable bound by \exists). Note that the symbolic counterpart of 'Socrates exists', where the name 'Socrates' is used only with existential presupposition, is $(\exists xSocrates)E!(x)$. Denying that Socrates exists, i.e. $\neg(\exists xSocrates)E!(x)$, is then equivalent to $(\forall xSocrates)\neg E!(x)$, from

⁷²Spade [1996], p.277.

⁷³Spade [1996], chapter 9.

⁷⁴Spade [1996], p.276.

⁷⁵Scott [1966], p.41.

⁷⁶That is, a proper name, such as 'Socrates', of a concrete, as opposed to an abstract, object can be stipulated to be "existence-entailing" in the sense that if the name can be used to refer to anything, then that thing exists (as a concrete object):

tial presupposition (which is usually not the case), the symbolic counterpart of our assertion is $(\forall x Socrates)[\lambda x Is(x, [\exists y Socrates])](x)$, which is not false but vacuously true when Socrates does not exist.

This last point, of course, has to do with universal sentences not having "existential import," contrary to the way they were interpreted by medieval logicians. That is, in conceptual realism, and in modern logic in general, universal conditionals—and therefore all sentences beginning with a universal quantifier phrase—are true, and not false as interpreted by the terminists, when their antecedents (subject terms) are vacuous. The utility of this view, both for science and natural language, has been more than justified over the last century; and we assume that its acceptance in our reconstruction of terminist logic is but a minor modification, and improvement, of this system—just as the acceptance of a logic free of existential presuppositions for proper names and definite descriptions is a minor modification, and improvement, of so-called standard first-order logic. We need only stipulate when a name S is assumed to supposit something, i.e. when something is an S, if existential import is needed to validate a descent or ascent, as it will be in some cases.

Latin lacks a definite article, incidentally, which means that the terminists did not consider definite descriptions in their analyses at all. We will not, for that reason, go into the distinction between using definite descriptions with, as opposed to without existential presuppositions.⁷⁸ The existential presuppositions of demonstrative phrases is a matter we cannot avoid, how-

which, together with the above stipulation about the name 'Socrates', it follows that Socrates does not have being when he does not exist, e.g., $\neg(\exists x Socrates)(x=x)$. But, because $(\exists x Socrates)[\lambda x Is(x, [\exists y Socrates])](x)$ is provably equivalent to this last formula without the negation, it then follows that it too is false when Socrates does not exist. (It may be preferable, however, to allow that Socrates can exist, and in that sense have being, even when he does not exist.)

⁷⁷Note that by the meaning postulate (MP1) of §6,

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(\forall x Socrates)[\lambda x Is(x, [\exists y Socrates])](x) \leftrightarrow (\forall x)[(\exists y Socrates)(x=y) \rightarrow Is(x, [\exists y Socrates])],
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is provable. The universal conditional on the right-hand side of this biconditional is vacuously true if $\neg(\exists y Socrates)(x=y)$ is true for any value of x as bound by \forall , which it is when Socrates does not exist, or so we may assume (as already noted).

⁷⁸See Cocchiarella [1989], §6, for a detailed account of this distinction. The truth conditions for definite descriptions used with existential presuppositions are essentially those described by Bertrand Russell.

ever, because such demonstrative phrases are central to the descent to (and, in some cases, ascent from) singular propositions. The term 'man', for example, has determinate supposition in 'Socrates is a man', which means that the descent (at a time when Socrates exists) to a certain disjunction of singulars, 'Socrates is this man or Socrates is that man or ... or Socrates is that man', is valid—and so too is the ascent from the singulars to 'Socrates is a man' (again, at a time when Socrates exists), and therefore from their disjunction.

Because the demonstrative 'that' can be used a number of times in a disjunction, or conjunction, of singular propositions, we must, in our logical language, distinguish each use from the others. For convenience, we will use ' $That_1$ ', ' $That_2$ ',..., ' $That_n$ ', etc. (for each positive integer n), as variable-binding operators that operate on a common name (complex or simple), resulting thereby in a quantifier phrase that can then be applied to a formula. We will also read ' $That_1$ ' as the English 'this'. Thus, where S is a common name (complex or simple), e.g., 'Man/Snubnosed(x)', then ' $That_2xMan/Snubnosed(x)$ ', read as 'that man who is snubnosed' (or as 'that snubnosed man'), is a quantifier phrase of our symbolic language; and, as such, the phrase stands for a referential concept. The symbolic analysis of an assertion of 'That man who is snubnosed is wise', which involves the mutual saturation of a referential and a predicable concept, can now be given a logically perspicuous representation as follows:

$$(That_2xMan/Snubnosed(x))Wise(x).$$

We also interpret the use of 'this' and 'that' that occur without a common name, as in 'This is a dog', 'That is man', etc., similar to the way we interpret the objectual quantifiers $(\forall x)$ and $(\exists x)$, i.e. as implicitly containing the common name 'thing' (or 'object'), as in 'This thing is a dog', 'That thing is a man', etc.

⁷⁹Note that more than one referential concept can be exercised in a disjunction, or conjunction, of singular propositions. The detective who says, while pointing to two different men, 'This man is the killer or that man is the killer' is exercising two different referential concepts, expressed by 'this man' and 'that man', in his speech act (whereas, of course, 'the killer' is deactivated in both disjuncts). Similarly, when a school coach says, while pointing to certain boys in his class, 'That boy is on team A and ... and that boy is on team A', he using the referential expression 'that boy' to refer to a certain number of different boys in his conjunctive statement. Conjunctive and disjunctive statements are not basic statements, needless to say, and hence are not subject to the restriction that only one referential, and one predicable, concept can be exercised in them.

Now our point about the existential presupposition of a demonstrative phrase is that when a speaker says, e.g., 'That man is sitting', he is presupposing that the thing he is indicating is a man, i.e. that 'That man is a man' is true. But if the speaker is pointing to a manikin that he has mistaken for a man, then his purported reference has failed, and both the speaker's assertion and the sentence 'That man is a man' are false in such a context. A speaker's use of a demonstrative phrase, we maintain, is equivalent to, if not synonymous with, a use with existential presupposition of a definite description; in particular, that a use of a demonstrative phrase, e.g., 'that S'. where S is a complex or simple common name, is equivalent to using with existential presupposition the definite description 'the S that I am indicating'. 80 A sentence of the form 'The S is an S', where the definite description is used with existential presupposition—as, e.g., in the implicit premise of Descartes's ontological argument, 'The perfect being is a perfect being'—is not a valid thesis; and, because of the equivalence between demonstrative phrases and definite descriptions used with existential presuppositions, neither are sentences of the form 'That S is an $S^{2,81}$

Determinate supposition, we have said, means a descent to a certain disjunction of singulars. More specifically, a common name S has determinate supposition in a (categorical) proposition P if S occurs in P as part of a quantifier phrase and the descent from P to a disjunction of singular propositions $(Q_1 \vee ... \vee Q_n)$, where each Q_i is obtained from P by replacing the quantifier phrase containing S by 'that S', is valid. To ensure validity, the disjunction must be exhaustive of all the S there are (when P is asserted). Thus, in the determinate supposition of 'man' in 'Socrates is a man' (asserted when Socrates exists), which is symbolized as

$$(\exists x Socrates)[\lambda x Is(x, [\exists y Man])](x),$$

$$(That_i yS)F(y) \leftrightarrow (\exists yS)[(\forall zS)(G(z) \leftrightarrow z = y) \land F(y)].$$

⁸⁰We assume here that there can be no cases of correctly using a demonstrative phrase, such as 'that man', without existential presupposition—the way there can be cases of correctly using a definite description without existential presupposition.

The truth conditions for a sentence of the form 'That_i S is F' are the same as 'There is exactly one S that I am (now) indicating and it is F', which can be symbolized as follows (where 'G(z)' abbreviates 'I am indicating z'):

⁸¹ That S is an S' will be true in any context in which 'Something is that S' is true; i.e. when $(\exists x)(That_iyS)(x=y)$ is true.

the descent, to be valid, cannot be to any disjunction of the form

$$(\exists x Socrates)[\lambda x Is(x, [That_1yMan])](x) \lor ... \lor \lor (\exists x Socrates)[\lambda x Is(x, [That_nyMan])](x),$$

but only to a disjunction that is exhaustive of all of the men there are, i.e. of all the men who exist at the time of the assertion of 'Socrates is a man'. In other words, even after having indicated a number of men by means of a demonstrative phrase, we may still not have indicated the man that in fact is Socrates if we have not exhausted all of the men there are. Implicit in such a descent, accordingly, is the assumption that we are indicating all the men there are; that is, that

A thing is a man if, and only if, either it is that₁ man or ... or it is that_n man.

is true (at the time of assertion). A similar assumption applies to any common name, S, of concrete physical objects; that is, for some natural number n, the terminist logicians assumed that at any given time there are exactly n many things that are S^{82} . As a generalized version of the above thesis for the common name 'man', we will call this assumption, for any common name S (complex or simple) of concrete things, and any natural number n, the principle of descent for n many S, or simply $PD_n(S)$. The principle is symbolized as follows⁸³:

$$(\forall x)[Is(x,[\exists yS]) \leftrightarrow Is(x,[That_1yS]) \lor \dots \lor Is(x,[That_nyS])].$$

This thesis, by the meaning postulate for Is, is provably equivalent to

$$(\forall x)[(\exists yS)(x=y) \leftrightarrow (That_1yS)(x=y) \lor \dots \lor (That_nyS)(x=y)].$$

It is important to note here that the thesis of descent amounts to an explicit answer to the question of how many S there (now) are in terms of the identity

⁸²The restriction must be to concrete physical objects, because the thesis will be false for such abstract objects as the natural numbers, and perhaps also for concrete events (which are not physical objects). Unlike the system of conceptual realism, terminist logic gave no explanation of how such abstract objects as the natural numbers are to be accounted for.

⁸³The principle is taken to apply at any moment of time considered as the present. Related principles for ampliated terms are obtained by applying tense and modal operators to $PD_n(S)$. (We ignore the assumption that $That_iyS \neq That_jyS$, for $i, j \leq n$ where $i \neq j$, incidentally, because it is not needed for the inferences noted here.)

theory of the copula, which as we indicated earlier is what the doctrine of the modes of supposition is really about.⁸⁴

The principle of descent for 'man', i.e. $PD_n(Man)$, also validates the ascent (when Socrates exists) from any one of the singulars 'Socrates is this man', ..., 'Socrates is that_n man' to the original proposition 'Socrates is a man' in which 'man' is said to have determinate supposition. For this kind of proposition, in other words, we can validly ascend when and only when we can validly descend.

Determinate supposition applies to common names occurring as subject terms as well as to common names occurring as predicate terms, as in our example of 'Socrates is a man'. Thus, by $PD_n(Man)$, we can validly descend from 'Some man is running' to 'This man is running or that₂ man is running or ... or that_n man is running'; and of course we can ascend from any one of these disjuncts—and therefore from the disjunction as well—to the sentence 'Some man is running'. Similarly, by $PD_n(Man)$, we can validly descend from 'A man is not running' to 'This man is not running or that₂ man is not running or ... or that_n man is not running'; and, again, we can similarly ascend from any, or all, of these disjuncts to 'A man is not running', or to 'Some man is not running', both of which are symbolized the same way in our system. With determinate supposition, in other words, we can validly descend on the basis of the principle of descent when, and only when we can validly ascend on the basis of that principle.⁸⁵

Confused and distributive supposition is more problematic than determinate supposition, however, because, according to Ockham, a common name will have confused and distributive supposition in a categorical proposition only when one can descend to a *conjunction* of singulars, but cannot ascend from any one singular in the conjunction.⁸⁶ The problem is that such

⁸⁴When n is 0, we take the right-hand side of the biconditional within the scope of $(\forall x)$ to be the formula $(x \neq x)$, from which it follows that nothing is S, which is as it should be when n is 0.

Note, incidentally, that when something is an S, i.e. $(\exists x)Is(x, [\exists yS])$, then, by $PD_n(S)$ and distribution of $(\exists x)$ over \vee , each disjunct will have the form $(\exists x)Is(x, [That_iyS])$, which, in effect, stipulates that the existential presupposition of the demonstrative phrase is fulfilled, i.e. that something is that i S.

⁸⁵The argument for the general claim is based on specific examples, to be sure; but that is because the examples can be easily schematized and shown to hold in general.

⁸⁶There is a problem with this characterization when applied to the predicate of a negative particular proposition, however, because, although the descent is to a conjunction, the conjuncts will not be singular propositions. But then each conjunct can in turn be

a descent is valid only when each conjunct can be truthfully asserted if the original premise is true, and hence only when the existential presupposition of the demonstrative phrase in that conjunct is fulfilled, i.e. only when $(\exists x)Is(x, [That_iyS])$ is true for each i such that $1 \le i \le n$, where S is the common name in question and there are exactly n many S. This, as it turns out, is just the issue of existential import, but as applied to demonstrative phrases in particular. Such presuppositions were implicit in what Ockham and the terminist logicians assumed for this type of supposition.

The common name 'man' in 'Every man is an animal', for example, will have confused and distributive supposition by the principle of descent $PD_n(Man)$, but only if the existential presuppositions in question are fulfilled.⁸⁷ If these presuppositions are fulfilled, then, by $PD_n(Man)$, one can validly descend from 'Every man is an animal' to the conjunction 'This man is an animal and that₂ man is an animal and ... and that_n man is an animal', i.e. to

$$(That_1xMan)[\lambda xIs(x, [\exists yAnimal])](x) \wedge ... \wedge (That_nxMan)[\lambda xIs(x, [\exists yAnimal])](x).$$

One cannot validly ascend, as Ockham says, from any one of these singulars to the universal 'Every man is an animal'; but, clearly, given $PD_n(Man)$, one can validly ascend to the universal from the whole conjunction.

Confused and distributive supposition was assumed by the terminists to apply not only to the subject term of a universal affirmative, but to the subject term of a universal negative categorical proposition as well. But,

$$(\forall x)[Is(x, [That_1yMan]) \to Is(x, [\exists yAnimal])] \land \dots \land (\forall x)[Is(x, [That_nyMan]) \to Is(x, [\exists yAnimal])].$$

Hence, if all universals were assumed to have existential import, then the existential presuppositions of these demonstrative phrases would be fulfilled. That is why we have said that this is just the issue of existential import, but as applied to demonstrative phrases.

[&]quot;reduced" by determinate supposition to a disjunction of singulars. Thus, on our characterization, a common name S has confused and distributive supposition in a proposition P if S occurs in P as part of a quantifier phrase and the descent from P to a conjunction $(Q_1 \wedge ... \wedge Q_n)$ is valid, where each Q_i (whether singular or otherwise) is obtained from P by replacing the quantifier phrase containing S by 'that, S'.

⁸⁷All that follows from $PD_n(Man)$ and $(\forall xMan)Is(x, [\exists yAnimal])$, using (MP1) of §6, is the conjunction that [if anything is that₁ man, then it is an animal] and ... and [if anything is that_n man, then it is an animal], i.e.

again, the descent will be valid only if the existential presuppositions of the demonstrative phrases in the conjunction in question have been fulfilled. Consider, for example, the same common name 'man', only now occurring as the subject of a universal negative categorical, such as 'No man is running'. The confused and distributive supposition of 'man' in this proposition means that, by $PD_n(Man)$, one can descend from this sentence to 'This man is not running and that₂ man is not running and ... and that_n man is not running' (with the negation in each conjunct internal to the predicate). But, as in our previous example, the descent will be valid only when $(\exists x)Is(x, [That_iyS])$ is true for each i such that $1 \le i \le n$.

In our reconstruction, we note first that 'No man is running' is understood as denying that some man is running, which is symbolized as $\neg(\exists xMan)Running(x)$, but which is provably equivalent to $(\forall xMan)\neg Running(x)$. By this last sentence and $PD_n(Man)$, it follows that anything that is that man or ... that man is not running, i.e.

$$(\forall x)[(That_1yMan)(x=y) \lor ... \lor (That_nyMan)(x=y) \to \neg Running(x)],$$

and from this and $(\exists x)Is(x, [That_iyMan])$, for $1 \leq i \leq n$, the desired conjunction,

$$(That_1yMan)[\lambda y \neg Running(x)](x) \wedge ... \wedge (That_nyMan)[\lambda y \neg Running(x)](x)$$

follows.⁸⁸ The same argument can be made in reverse order, moreover, which means that, given $PD_n(Man)$, we can validly ascend from the conjunction to the universal negative sentence. In other words, with the confused and distributive supposition of a common name occurring as the subject term of a universal affirmative or a universal negative proposition, one validly descend to a conjunction when and only when one can also validly ascend from such a conjunction to the universal in question.

There are cases of confused and distributive supposition where the issue of existential import is not relevant, i.e. where the existential presuppositions of demonstrative phrases need not be fulfilled. In particular, a common name occurring as the predicate of any negative categorical proposition will have

⁸⁸Without the assumption that $(\exists x)Is(x, [That_iyS])$ is true, for $1 \leq i \leq n$, all that would follow by $PD_n(Man)$ is the conjunction '[If anything is that man, then it is running] and ... and [if anything is that man, then it is running]'.

confused and distributive supposition, regardless of the issue of existential import. The common name 'runner' in 'No man is a runner', for example, unlike the common name 'man', will have confused and distributive supposition independently of whether or not the existential presupposition of any of the demonstrative phrases in question is fulfilled.⁸⁹ In other words, by $PD_m(Runner)$, one can descend validly to the conjunction 'No man is that₁ runner and ... and no man is that_m runner' without any further assumptions about existential import.⁹⁰ As a denial, 'No man is a runner' is symbolized as $\neg(\exists xMan)[\lambda xIs(x,[\exists yRunner])](x)$, which, by quantifier negation and λ -conversion, is equivalent to $(\forall xMan)\neg Is(x,[\exists yRunner])$. This last formula, by $PD_m(Runner)$ and elementary transformations, implies

$$(\forall x Man) \neg Is(x, [That_1yRunner]) \land ... \land (\forall x Man) \neg Is(x, [That_myRunner]),$$

which, by quantifier negation, is equivalent to the conjunction

$$\neg(\exists x Man)[\lambda x Is(x, [That_1yRunner])](x) \wedge ... \wedge \\ \wedge \neg(\exists x Man)[\lambda x Is(x, [That_myRunner])](x).$$

This argument can also be given in reverse order, so that one can validly ascend, by $PD_m(Runner)$, from the conjunction to the sentence 'No man is a runner'. In other words, with confused and distributive supposition, one can validly descend to a conjunction from a universal negative categorical proposition when and only when one can validly ascend from the conjunction to that proposition by the same principle—regardless whether the common name occurs as the subject or the predicate of the proposition, except that when it is the subject, the existential presuppositions of the demonstrative phrases in the conjunction must be fulfilled.

The way up is not always the same as the way down, however. Consider, for example, the common name 'runner' in the negative particular

⁸⁹Our examples come from Spade [1996], chapter 9. Ockham and other terminists assumed that 'is running' can be construed as 'is a runner'. This construal is dubious; but we will accept it here as part of our reconstruction of terminist logic. A separate, alternative treatment can be given for the present participle in conceptual realism in terms of the logic of events.

⁹⁰The conjuncts in this case are not singular propositions, it might be noted; but, by $PD_n(Man)$, each conjunct can be expanded into a conjunction of singulars of the form 'That_{m+i} man is not that_i runner', with the negation internal to the predicate.

proposition 'Some man is not a runner' (where the negation is internal to the predicate), which is symbolized as $(\exists x Man)[\lambda x \neg Is(x, [\exists y Runner])](x)$. Here, the common name 'runner' has confused and distributive supposition, which means that one can descend, by $PD_m(Runner)$, to the conjunction 'Some man is not this runner and ... and some man is not that_m runner'. ⁹¹ Note that, by $PD_m(Runner)$, $(\exists x Man)[\lambda x \neg Is(x, [\exists y Runner])](x)$ implies

$$(\exists x Man) \neg [Is(x, [That_1yRunner]) \lor ... \lor Is(x, [That_myRunner])],$$

which, by elementary transformations, implies (but is *not* implied by)

$$(\exists x Man) \neg Is(x, [That_1yRunner]) \land ... \land (\exists x Man) \neg Is(x, [That_myRunner]),$$

which validates the descent to the conjunction 'Some man is not this runner and ... and some man is not that_m runner. But the reverse order of this argument is not also valid, because, unlike the inference from $(\exists x Man)(\varphi \land \psi)$ to $(\exists x Man)\varphi \land (\exists x Man)\psi$, the inference from $(\exists x Man)\varphi \land (\exists x Man)\psi$ to $(\exists x Man)(\varphi \land \psi)$ is not valid. Thus, despite Heraclitus, the way up is not always the same as the way down.

10 How Confused is Merely Confused

Merely confused supposition has been the one type of supposition that has been controversial in terminist logic. It is the one type, for example, that does not allow for a valid descent to either a conjunction or disjunction of singular propositions. Ockham's main characterization is that a common name has merely confused supposition in a categorical proposition if one can validly descend to a "disjoint predicate." The common name 'animal', for example, has merely confused supposition in the universal affirmative 'Every man is an animal', which, as already noted, is symbolized as $(\forall x Man)[\lambda x Is(x, [\exists y Animal])](x)$. That is, by the principle of descent, $PD_k(Animal)$, λ -conversion, and elementary transformations, we can validly

⁹¹Once again, the conjuncts are not singular propositions, but, by $PD_n(Man)$, each conjunct can be expanded into a disjunction so that the final result is a conjunction of disjunctions of singular propositions.

⁹²Spade [1996], p.284.

descend from this proposition to 'Every man is [this animal or that₂ animal ... or that_k animal]', which is symbolized as follows:

$$(\forall x Man)[\lambda x(Is(x, [That_1yAnimal]) \lor ... \lor Is(x, [That_kyAnimal]))](x).$$

One cannot, of course, validly distribute the universal quantifier $(\forall x Man)$ over the disjunction $[Is(x, [That_1yAnimal]) \lor ... \lor Is(x, [That_kyAnimal])]$ to get 'Every man is this animal or ... or every man is that_k animal'. So, in this case no further valid "reduction" to singulars is possible.

Some authors have found this "reduction" to a disjunctive predicate "very odd", as though the resulting sentence has problematic truth conditions. ⁹³ But there are many sentences in science and natural language with disjunctive predicates that are clearly unproblematic. The sentence of arithmetic, 'Every integer is odd or even', symbolized as

$$(\forall x Integer)[\lambda x (Odd(x) \lor Even(x))](x),$$

is perfectly clear as to its truth conditions, for example, even though it is not further "reducible" to 'Every integer is odd or every integer is even'. Similarly, 'Every person is either male or female' seems perfectly clear in its truth conditions, even though it is not "reducible" to 'Every person is male or every person is female'.

Ockham also thinks that merely confused supposition applies to such sentences as 'John promises Simon a horse'. 94 But then, merely confused

$$Is(x, [That_1yAnimal \lor ... \lor That_kyAnimal])$$

in mind as the problematic "disjunctive predicate", where the copula has not been distributed. If so, then he has a point, because this expression is not well-formed.

 94 It is not clear that Ockham thinks of an assertion of a sentence like this as a categorical proposition. If it is a categorical, then, apparently, it is to be rephrased with a copula, e.g., as 'John is a man who promises Simon a horse'. But then the disjunction has to do with the different demonstrative phrases, 'this man who promises Simon a horse or ... or that_n man who promises Simon a horse', in which case the descent is by determinate supposition, and therefore unproblematic. Of course, we then still have to explain the supposition of the common name 'horse' in the singulars 'John is that_i man who promises Simon a horse'.

 $^{^{93}}$ This is Paul Spade's view in [1996], p.384. Note, however, that in our analysis we have distributed the copula over 'this animal or ... or this_k animal'. Perhaps Spade has something like

supposition must also apply to 'John gives Simon a horse', because this sentence has the same logical form as 'John promises Simon a horse'. On our analysis, the logical form of these sentences as judgments or speech acts (where 'John' is used with existential presupposition) is given as:

$$(\exists x John)[\lambda x Promise(x, [\exists y Simon], [\exists z Horse])](x),$$

and

$$(\exists x John)[\lambda x Give(x, [\exists y Simon], [\exists z Horse])](x).$$

Now, because 'gives' is an extensional verb with respect to all of its arguments, the following identity is a conceptually valid thesis of our system as a result of the meaning postulate for *Give*:

$$[\lambda x Give(x, [\exists y Simon], [\exists z Horse])] = [\lambda x (\exists z Horse) Give(x, [\exists y Simon], z)].$$

But then, by the principle of descent, $PD_j(Horse)$, we can validly descend to the disjunction, 'John gives Simon this horse or ... or John gives Simon that_j horse', in symbols:⁹⁵

$$(\exists x John)[\lambda x Give(x, [\exists y Simon], [That_1 z Horse])](x) \lor ... \lor \lor (\exists x John)[\lambda x Give(x, [\exists y Simon], [That_j z Horse])](x).$$

This descent, however, is not really by merely confused supposition, which, according to Ockham, does not allow descent to a disjunction. Rather, the

 $[\lambda x(\exists z Horse)Give(x, [\exists y Simon], z)] = [\lambda x(\exists w)(Is(w, [\exists z Horse]) \land Give(x, [\exists y Simon], w))]$ is provable; and from this and $PD_i(Horse)$,

$$[\lambda x(\exists z Horse)Give(x, [\exists y Simon], z)] = [\lambda x(\exists w)([Is(w, [That_1z Horse]) \lor ... \lor \lor Is(w, [That_1z Horse])] \land Give(x, [\exists y Simon], w))]$$

follows. From this last identity and the distribution of a conjunction over a disjunction, the disjunction in question follows.

 $^{^{95}\}mathrm{By}$ the above identity, the meaning postulate for Is, and (MP2) of §6 above,

descent seems to be the same as that already described for determinate supposition.⁹⁶

Nothing like this follows for 'John promises Simon a horse', however; and the reason is that, unlike 'give', 'promise' is not extensional in its third argument position, i.e. 'promise' is an intensional verb with respect to its direct object argument position. Yet Ockham maintained that 'horse' has merely confused supposition in this sentence, and that the descent to a disjunctive predicate, as in 'John promises Simon this horse or ... promises Simon that_j horse', is valid. That is, according to Ockham, the descent from 'John promises Simon a horse', as symbolized above, to

$$(\exists x John)[\lambda x(Promise(x, [\exists y Simon], [That_1zHorse]) \lor ... \lor \lor Promise(x, [\exists y Simon], [That_1zHorse]))](x),$$

is supposed to be valid, when, in fact, it is not valid—as many commentators have repeatedly noted over the years. ⁹⁷ Unlike these other commentators, however, we have a theoretical account in terms of deactivated referential expressions that explains why such a descent fails—and why it succeeds in sentences having the same logical form. Based on this account, our proposal is that a common (or proper) name that is part of a deactivated referential expression that cannot, as it were, be "activated" in a given propositional context is a name for which no "mode" of supposition should be said to apply in the context in question.

$$[\lambda x Promise(x, [That_1 y Horse \lor ... \lor That_j y Horse])]$$

is not well-formed. If this is what Ockham intended, then Spade is right to "think this appeal to disjoint terms is ... a mark of desperation" ([1996], p.284).

 $^{^{96}}$ Essentially the same argument would show that 'horse' has merely confused supposition, and not determinate supposition, however, in 'Every man gives Simon a horse'. That is, the descent to 'Every man [gives Simon this horse or ... or gives Simon that_j horse]' by $PD_j(Horse)$ is valid, whereas a "descent" to 'Every man gives Simon this horse or ... or every man gives Simon that_j horse' is not valid.

⁹⁷Note that, as with the distribution of the copula over 'this animal or ... or that_k animal', we have distributed 'promise' over 'this horse or ... or that_j horse'. Otherwise, the so-called "disjoint predicate", as in

11 Conclusion

The framework of conceptual realism provides a logically ideal language within which to reconstruct the medieval terminist logic of the 14th century. The terminist notion of a concept, which shifted from Ockham's early view of a concept as an intentional object (the *fictum* theory) to his later view of a concept as a mental act (the *intellectio* theory), is reconstructed in this framework in terms of the notion of a concept as an unsaturated cognitive structure. Referential and predicable concepts in particular are unsaturated cognitive structures that mutually saturate each other in mental acts, analogous to the way that quantifier phrases and predicate expressions mutually saturate each other in language. Intentional objects (ficta) are not rejected but are reconstructed as the objectified intensional contents of concepts, i.e., as intensional objects obtained through the process of nominalization—and in that sense as products of the evolution of language and thought. Their reconstruction as intensional objects is an essential part of the theory of predication of conceptual realism. In particular, the truth conditions determined by predicable concepts based on relations—including the relation the copula stands for—are characterized in part in terms of these objectified intensional contents. It is by means of this conceptualist theory of predication that we are able to explain how the identity theory of the copula, which was basic to terminist logic, applies to categorical propositions.

Reference in conceptual realism, based on the exercise and mutual saturation of referential and predicable concepts, is not the same as supposition in terminist logic. Nevertheless, the various "modes" or types of personal supposition are accounted for in a natural and intuitive way in terms of the theory of reference of conceptual realism. Ockham's application of merely confused supposition to common names occurring within the scope of an intensional verb is rejected, as it should be, but its rejection is grounded on the notion of a deactivated referential concept—a deactification that, because of the intensionality of the context in question, cannot be "activated", the way it can be in extensional contexts.

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