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Denoting Concepts and Ontology in Russell's *Principles of Mathematics*

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Bertrand Russell's *Principles of Mathematics* (1903) gives rise to several interpretational challenges, especially concerning the theory of denoting concepts. Only relatively recently, for instance, has it been properly realised that Russell accepted denoting concepts that do not denote anything. Such empty denoting concepts are sometimes thought to enable Russell, whether he was aware of it or not, to avoid commitment to some of the problematic non-existent entities he seems to accept, such as the Homeric gods and chimeras. In this paper, I argue first that the theory of denoting concepts in *Principles of Mathematics* has been generally misunderstood. According to the interpretation I defend, if a denoting concept shifts what a proposition is about, then the aggregate of the denoted terms will also be a constituent of the proposition. I then show that Russell therefore could not have avoided commitment to the Homeric gods and chimeras by appealing to empty denoting concepts. Finally, I develop what I think is the best understanding of the ontology of *Principles of Mathematics* by interpreting some difficult passages.

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1. Introduction

Many of Bertrand Russell's philosophical views changed and developed rapidly in the years 1900–1905. *Principles of Mathematics* (hereafter: *PoM*), published in 1903, provides an interesting insight into that period, but its interpretation has proven difficult. The book went through several drafts over the course of more than three years, with some elements retained throughout and some changed drastically. Russell even added and substantially changed passages in the final proofs that came back to him from Cambridge University Press.[1] Hence it is not surprising that *PoM* gives rise to several interpretational challenges.

My first aim in this paper is to develop and defend a novel interpretation of the theory of denoting concepts in *PoM*. In Section 2, I introduce the basic terminology and framework of *PoM*, with special attention to its mereology and the theory of denoting concepts. In Section 3, I discuss what I will call *the standard interpretation*, according to which the function of a denoting concept is to shift what a proposition is about to something that is not itself in any form a constituent of the proposition.[2] I argue that the standard interpretation has very little textual support and that, in fact, *PoM* strongly suggests that Russell's view is more complicated. In Section 4, I develop what I take to be a more faithful interpretation of *PoM*'s theory of denoting concepts, which requires an aggregate of relevant terms to be a constituent of any proposition in which a denoting concept shifts aboutness.[3] In Section 5, I then discuss Russell's developments of the theory of denoting concepts in the years after the publication of *PoM*, which provide further evidence for my interpretation.

My second aim is to connect my interpretation of the theory of denoting concepts in *PoM* with the ontology of that book. Traditionally, *PoM* is interpreted as defending a radically inclusive ontology, according to which there are not only tables and chairs, classes and propositions, but also such unusual entities as chimeras and Homeric gods. Whereas tables and chairs both have being and exist, chimeras and Homeric gods do not exist and only have being. This abundant ontology is thought to be entailed by Russell's theory of propositions in *PoM*, according to which (i) every logically significant phrase (one indicating a term) in a meaningful sentence must correspond to some entity in the proposition expressed and (ii) there are meaningful sentences about chimeras and Homeric gods.[4] This interpretation is supported quite directly by several passages in the book, for instance when Russell writes that 'numbers, Homeric gods, relations, chimeras and four-dimensional spaces all have being, for if they were not entities of a kind, we could make no propositions about them' (*PoM* §427). In the last few decades, however, commentators have argued that this traditional interpretation of the ontology of *PoM* is mistaken and that Russell's theory of propositions and denoting concepts does not by itself commit him to there being Homeric gods and chimeras.[5] This newer

[1] For detailed commentary on the history of *PoM*, see for instance Grattan-Guinness (1996); Blackwell (1984); Byrd (1987, 1994, 1996, 1999a,b).

[2] At least if what the denoting concept denotes is not already a constituent of the proposition for another reason, for instance because it is elsewhere directly indicated by a name.

[3] See Oliver and Smiley's *Plural Logic* (2016, 26) for an earlier interpretation along similar lines.

[4] This interpretation has been defended in, for instance, Quine (1966, 658), Pears (1967, 13–14), Ayer (1971, 28), Jager (1972, 55–56) and Griffin (1980, 119–21).

[5] See, for instance, Hylton (1990; 2003, 211–21), Griffin (1996), Candlish (2007, 112) and Stevens (2011).

interpretation seems to have replaced the traditional interpretation almost entirely, so much so that some have claimed that the traditional interpretation 'has been firmly discredited' (Stevens 2011, 57). Others have even gone so far as to argue that Russell, at the time, did *not* think that chimeras and Homeric gods have being at all, contradicting the quote above (Griffin 1996).

Denoting concepts that do not denote anything are central to this new way of understanding the relation between the ontology and theory of denoting in *PoM*. If my interpretation of the theory of denoting in *PoM* is correct, however, denoting concepts that do not denote anything *cannot* be used to avoid the ontological commitment to chimeras and Homeric gods in the way recent interpreters have suggested. I make this point in the last two sections of the paper. In Section 6, I discuss the difficult §73 of *PoM*, in which denoting concepts which do not denote anything are introduced. In Section 7, I then explain how I think one should resolve the conflict between §73, in which Russell seems to claim that there are no chimeras, and §427, in which Russell claims that chimeras have being.

2. The Basics

A central notion in *PoM* is that of a proposition. The new approach to propositions developed by Moore and Russell in the late 1890s and early 1900s played a crucial part in their breaking away from the then dominant idealist tradition (Hylton 1984; Moore 1898). They took propositions to be mind-independent and language-independent combinations of entities: propositions are true or false objectively and not linguistic or mental.[6] Sentences are only derivatively true or false insofar as they express a true or false proposition; a sentence is meaningful if it expresses a proposition.

[6]'Entities' is Russell's terminology (he also uses 'term', see below). What Russell means by these corresponds most closely to what Moore (1898) calls 'concepts'.

According to *PoM*, words have meaning 'in the sense that they are symbols which stand for something other than themselves' (*PoM* §51). Russell calls this relation between words and their meanings *indication*.[7] So, for example, the name 'Boris Johnson' indicates the person Boris Johnson and 'humanity' indicates the concept *humanity*. A meaningful sentence expresses a proposition that contains all the indications of the logically significant phrases of the sentence. So the sentence 'Boris Johnson is a politician' expresses a proposition of which Boris Johnson, the actual physical person, is a constituent.[8] In short, there are meaningful sentences, which consist of linguistic items, and these express propositions, which contain the non-linguistic items indicated by the logically significant parts of the sentence.[9]

It will sometimes be useful to represent propositions with angle brackets, so that 'Boris Johnson is a politician' expresses the proposition ⟨Boris Johnson, *politician*⟩.[10] For Russell, however, a proposition is a *unity* and not just a combination of its constituents. Not only can two different propositions have exactly the same constituents, as in ⟨Boris Johnson, *seeing*, Theresa May⟩ and ⟨Theresa May, *seeing*, Boris Johnson⟩, but Russell also thinks that propositions have a certain assertoric force—due to the way that verbs unify propositions—that a mere combination of terms lacks (*PoM* §§54–55, §80; Rauti 2004). So we should not think of this representational device as capturing propositions properly, as though they were simply ordered n-tuples of constituents.

[7]Although Russell is not consistent in this terminology (Griffin 1996, 38).

[8]Russell is happy to accept propositions about deceased people. The sentence 'Margaret Thatcher was a politician', for example, will express a proposition of which Margaret Thatcher literally is a constituent. Russell does not seem to see any problems with this in *PoM*: he implicitly endorses a kind of eternalism.

[9]Of course if the sentence is about linguistic items, then the proposition will have linguistic items as constituents (*PoM* §51).

[10]This is on the 'Boris Johnson is-a politician' interpretation, as opposed to the 'Boris Johnson is a-politician' one (*PoM* §57, second footnote).

For Russell, *term* is the broadest philosophical category.[11] 'Whatever may be an object of thought, or may occur in any true or false proposition, or can be counted as *one*', is called a term (*PoM* §47). He uses several synonyms, among which is *entity*, which I will also use. Boris Johnson is a term, as is the concept indicated by the verb 'to walk' and the relation *taller than*. The notion of a term is also intimately connected with the notion of a proposition: every term occurs in many propositions—for instance in the proposition that it is self-identical—and every constituent of a proposition must be a term. All terms *are*, or *have being*, but not all of them exist.

Terms not only divide into existent and non-existent, but also into *things* and *concepts*:

> Among terms it is possible to distinguish two kinds, which I shall call respectively *things* and *concepts*. The former are the terms indicated by proper names, the latter those indicated by all other words. Here proper names are to be understood in a somewhat wider sense than usual. (*PoM* §48)

Although Russell introduces the distinction initially via the kinds of words that indicate things and concepts, it corresponds to a distinction between the possible roles a term can play in a proposition: things, whenever they occur in a proposition, have to play the role of subject, whereas concepts sometimes play the role of predicate, sometimes the role of subject (*PoM* §§48–49; Hylton 1990, 174). In the proposition expressed by 'Socrates is human', 'Socrates' indicates a thing, which plays the subject role, and 'human' indicates a concept, which plays a predicative role; in the sentence 'humanity is a characteristic of Socrates', 'humanity' indicates the same concept as 'human' indicated before, but this concept is now a subject of the proposition. This shift in the role the concept plays is not available to Socrates: he can never play a predicative role. Hence what 'humanity' and 'human' indicate is a concept and what 'Socrates' indicates is a thing.

Russell calls a concept C a *class-concept* when 'x is a C' stands for a propositional function (*PoM* §58, §73). Examples of class-concepts—what we would nowadays call (modifications of) count nouns—are *cat*, *football* and *British politician*. With every class-concept is associated a class-as-many, which consists of the terms that, when substituted for x in 'x is a C', make a true proposition. With the class-concept *football*, for instance, is associated a class-of-many of which all and only the footballs are members (these are the terms that make a true proposition when substituted for x in 'x is a football'). A class-as-many is not a single entity but essentially plural, and of a higher type than the terms of the class (*PoM* §104; Klement 2014). Besides classes-as-many, there are also classes-as-one: these are not plural but single terms and of the same type as their members. The class-as-one associated with the class-concept *football* is all the footballs taken together as a single entity, whereas the class-as-many of footballs takes the footballs plurally.

The theory of classes is related to the mereology of *PoM*. Whatever is not a class-as-many, Russell calls a *unit* (*PoM* §133). Some units are simple, some units are complex. A complex unit is a *whole*. Some wholes are *unities*: these are not 'completely specified when all its simple constituents are specified' (*PoM* §136). In fact, unities just are propositions: a proposition is not specified by its parts because, as we saw, two different propositions can have exactly the same constituents. The other wholes are *aggregates*: these are completely determined by their parts (*PoM* §135, §136). An aggregate is 'a single term' (*PoM* §141); in fact, generally, 'a whole is a new single term, distinct from each of its parts and from all of them: it is one, not many' (*PoM* §137). Russell is thus very clear that in general, aggregates are single terms, unlike classes-as-many. When there is only one term which makes

[11] Although in a footnote, Russell claims that *object* is even broader, which he says is connected with 'grave' problems (*PoM* §58). *Object* is meant to cover both the singular (e.g., simples and aggregates) and the plural (e.g., classes-as-many) as well as the ambiguous (e.g., variable disjunctions).

'x is a C' is true, there is no corresponding aggregate.[12] Russell identifies the classes-as-one with the aggregates, as long as the class-as-one contains more than one term (*PoM* §135, §139). So the class-as-one of all the footballs just is the aggregate of all the footballs: they together form a single aggregate term that is not many, like the class-as-many, but one. In the degenerate case where the class-as-one contains just one term, there is no corresponding aggregate.

Among the linguistic items, *denoting phrases* are of special importance. These consist of 'all', 'every', 'any', 'a(n)', 'some', 'the', or 'some synonym of them' (*PoM* §58) followed by a class-concept.[13] These phrases indicate *denoting concepts*, which I will represent using / / following Griffin (1996). So, for example, the denoting phrase 'all footballs' indicates the denoting concept /all footballs/ and 'the present queen of England' indicates /the present queen of England/. These denoting concepts in turn *denote* certain objects, for instance the footballs or Queen Elizabeth II. If C is a class-concept, I will call the various denoting concepts that can be formed with C the *denoting concepts of* C. So the denoting concepts of *football* are /all footballs/, /every football/, /any football/ and so on.

When a denoting concept is a constituent of a proposition, the proposition is, as a rule, not about the concept, but about what it denotes (*PoM* §56). So the sentence 'every football is a sphere' is about footballs even though 'every football' does not indicate footballs but the denoting concept /every football/. The denoting concept shifts what the proposition is about to what it denotes. Although he does not acknowledge any exceptions to this principle when he introduces denoting concepts for the first time, Russell does in fact think that there are propositions that are about denoting concepts themselves (e.g., in §65).

I deliberately have been somewhat vague about what denoting concepts denote. Call denoting concepts of the form /the C/ *definite* and all others—/a C/, /some C/, etc.—*indefinite*. There is a relatively well-known interpretational challenge concerning the objects denoted by the indefinite denoting concepts.[14] Russell sometimes suggests that the different indefinite denoting concepts that can be formed with, say, the concept indicated by the word 'football' each denote different combinations of the footballs. On this theory, /any football/ denotes a *variable conjunction* of all the footballs, while /some football/ denotes a *constant disjunction* of all the footballs, and so on (*PoM* §§59–65). At other places, he suggests that all these denoting concepts simply denote the footballs taken together, but do so in different ways, so that there are different denoting relations but only one combination of footballs that is denoted each time. To complicate matters further, he also sometimes talks about a kind of ambiguous denotation of a single term of the relevant class:

> The fact is that the concept 'any number' does denote one number, but not a particular one. This is just the distinctive point about *any*, that it denotes a term of a class, but in an impartial distributive manner, with no preference for one term over another. (*PoM* §88)

I cannot untangle this complicated aspect of *PoM* here, partly because it would require a paper of its own and partly because it would distract from the main contentions of this paper. The

[12] In §148, where Russell summarises the chapters of the second part of the book, he writes: 'we also saw that, by extending the notion of aggregates to single terms and to the null-class, we could regard the whole of the traditional calculus of Symbolic Logic as an algebra specially applicable to the relations of wholes and parts in the definable sense'. As far as I can tell, Russell does not in fact discuss this extension anywhere in the book and I will assume, for the purposes of this paper, that he does not accept aggregates consisting of a single term.

[13] Although I distinguish between the linguistic items and their worldly counterparts more rigorously than Russell does in *PoM*, I follow him in using 'class-concept' both for the relevant words and the concepts themselves: 'football' and *football* are both class-concepts in this usage. This should not lead to any confusion.

[14] See, for instance, Dau (1986) and Bostock (2009).

question I am interested in is not what denoting concepts denote, but rather how Russell conceived of propositions of which denoting concepts are constituents.

3. The Standard Interpretation

According to what I will call *the standard interpretation*, Russell thinks that the sentence 'every even number is even', to take a simple example, will express a proposition that contains just the denoting concept /every even number/ and the concept *even*, so that it may be represented as ⟨/every even number/, *even*⟩. This proposition is about every even number, even though it does not have any number as a constituent, because /every even number/ is a denoting concept: it shifts what the proposition is about to the even numbers. In general, the denoting concepts of *C* can change what a proposition is about to the *C*s without the *C*s being a constituent of the proposition in some way.

The standard interpretation has been accepted almost universally, but surprisingly it is not directly supported by the text. Note first of all that in Russell's initial introduction of denoting concepts, *he simply does not say* that what a denoting concept denotes, or the aggregate of the relevant terms, will typically not be part of the propositions that contain it. Given that many interpreters take this to be the *raison d'être* of denoting concepts, this is already somewhat disconcerting. There is only one place in *PoM* where the standard interpretation is suggested, namely in Russell's discussion of 'the' and identity:

> If we say 'Edward VII is the King', we assert an identity; the reason why this assertion is worth making is, that in the one case the actual term occurs, while in the other a denoting concept takes its place. . . Often two denoting concepts occur, and the term itself is not mentioned, as in the proposition 'the present Pope is the last survivor of his generation'. (*PoM* §64)

Russell's phrasing in the first sentence seems to suggest that the proposition expressed by 'Edward VII is the King' is not ⟨Edward VII, *identity*, Edward VII⟩, but ⟨Edward VII, *identity*, /the King/⟩, the denoting concept *taking the place of* the second occurrence of Edward VII. If this reading is right, then this would be an example where Russell seems to confirm the standard interpretation. The phrase 'the term itself is not mentioned' in the second sentence also seems to suggest that the present Pope is not a constituent of the proposition expressed by 'the present Pope is the last survivor of his generation'. But this reading can easily be contested, because another way of understanding 'the term itself is not mentioned' is that *the sentence* 'the present Pope is the last survivor of his generation' does not contain *a name* that indicates the present Pope directly.[15]

Given that Russell does *not* explicitly endorse the standard interpretation and that there is only one passage where it is suggested, the textual evidence directly supporting the standard interpretation is incredibly slim. Someone might argue that Russell is perhaps not explicit, but that the standard interpretation is nonetheless obviously implicit throughout his discussion of denoting: denoting concepts shift what a proposition is about to something that is *not* a constituent of the proposition—*that* is the point of denoting concepts.

This way of thinking would have been justified if there were not several places where Russell directly contradicts it. Consider, for instance, §68, where Russell writes that '["Socrates is one among men"] explicitly contains the class as a constituent'. In this sentence, 'men' indicates the denoting concept /all men/ which in turn denotes the class-as-many of men.[16] Russell leaves

[15] In fact, as will become clear in the next section, I am forced to defend this reading, since the alternative undermines my interpretation of the *PoM* theory of denoting concepts. In support, note that Russell uses the verb 'to mention' several times in *PoM* to talk about what happens at the sentence-level rather than the propositional-level. More specifically, he uses 'a term being mentioned' and similar phrases when the relevant sentence contains a name directly indicating the term.

[16] Russell treats 'men' and 'all men' as synonymous, and in general takes the plural '*C*s' of a class-concept *C* to be synonymous with 'all *C*s' (*PoM* §67).

no doubt here: the class of men is a constituent of the proposition expressed by 'Socrates is one among men'.[17] So the relevant proposition is not just ⟨Socrates, *is one among*, /all men/⟩, as the standard interpretation would have it. And again: of the proposition expressed by 'Socrates is a man', Russell writes that it 'contains a term, a relation and what I shall call a disjunction (a term to be explained shortly)' (*PoM* §57). The disjunction is what he took the denoting concept /a man/ to denote, hence he thinks that what /a man/ denotes is a constituent of the proposition.[18] There is definitely a mistake in this latter passage, though. For Russell does not mention that the proposition has the denoting concept /a man/ itself as a constituent, which it definitely should have given his theory of denoting concepts. So perhaps this passage can be explained as an instance of Russell being sloppy and talking about what /a man/ denotes when he means to be talking about /a man/ itself.

The plainest example where Russell contradicts the standard interpretation is to be found in his discussion of infinity. One of the motivations for introducing denoting concepts is that they help explain how we can grasp propositions about the infinite. A naive version of Russell's theory, one according to which 'every number' does not indicate a denoting concept but immediately indicates every number, entails that the proposition expressed by 'every number is divisible by 1' has infinitely many constituents. 'But', says Russell, 'this at least is clear, that all the propositions known to us (and, it would seem, all propositions that we *can* know) are of finite complexity' (*PoM* §141). The problem, then, is that we are able to grasp the proposition expressed by 'every number is divisible by 1', which seems to be about every number, but since we are able to grasp it, it cannot actually have every number as a constituent.

[17] He does not say, in §68, whether it is the class-as-one or the class-as-many which is a constituent of the proposition. I briefly return to this point in footnote 22.

[18] A footnote to §57 makes clear that the interpretation of this sentence is as 'Socrates is a-man', so that it involves a denoting phrase.

Russell uses denoting concepts to solve this problem. Most interpreters, under the influence of the standard interpretation, claim that his solution works as follows: the proposition expressed by 'every number is divisible by 1' will contain only the denoting concept /every number/, the number 1, and the relation *divisible by*, so that it can be represented as ⟨/every number/, *divisible by*, 1⟩. Since there are only finitely many constituents, the proposition itself is finitely complex and hence can be grasped by our finite minds. The proposition is still about every number, however, because the denoting concept /every number/ shifts what the proposition is about to what /every number/ denotes.

But this is not how Russell describes the solution at all, for he says this about 'any number has a successor':

> For example, the proposition 'any number has a successor' is composed of a finite number of constituents: the number of concepts entering into it can be enumerated, and in addition to these there is an infinite aggregate of terms denoted in the way indicated by *any*, which counts as one constituent. (*PoM* §141)

The number of concepts can indeed be enumerated: they are presumably the denoting concept /any number/ and the non-denoting concept *has a successor*. But crucially, the aggregate of all the numbers is also claimed to be a constituent of the proposition.[19] Russell then tells us that 'every' and 'all' similarly enable propositions of finite complexity to be about infinite classes (*PoM* §141). There can be no doubt that Russell is here asserting that the proposition expressed by 'any number has a successor' contains the aggregate of all the numbers, that is, the class-as-one

[19] An anonymous reviewer suggests that the 'which' in 'which counts as one constituent' could be read as referring to what '*any*' indicates, in which case it would refer to a denoting concept after all. This seems to me a mistake: Russell starts the sentence with 'the number of concepts entering into [the proposition] can be enumerated, *and in addition to these*. . . ': hence what he will mention next as a constituent is not going to be another concept. Besides, '*any*' itself is not a denoting concept, so it requires an implausible interpretative leap to take Russell here to be referring to a denoting concept.

of all the numbers, in addition to /any number/, which goes directly against the standard interpretation.

There is yet another passage in *PoM* that contradicts the standard interpretation, but I will postpone my discussion of it until Sections 7 and 8 because it is difficult to interpret and connected to several other issues that need to be introduced first. It should in any case be clear that a close reading of *PoM* reveals that Russell does *not* claim that denoting concepts can shift what a proposition is about to what is *not* also in one form or another a constituent of the proposition. This observation immediately casts doubt on the standard interpretation. When we then consider some of his examples and how he handles them, it becomes clear that Russell, in several cases, claims that if a denoting concept of *C* occurs in a proposition and shifts what the proposition is about to whatever it denotes, then the *C*s will also be a constituent of the proposition in one form or another (as a class, as a disjunction, as an aggregate).

Besides textual evidence, there is also a theoretical reason for rejecting the standard interpretation. As I mentioned above, Russell thinks that there are propositions that are about denoting concepts:

> it is possible to consider and make propositions about the concepts themselves, but these are not the natural propositions to make in employing the concepts. 'Any number is odd or even' is a perfectly natural proposition, whereas '*any number* is a variable conjunction' is a proposition only to be made in a logical discussion.[20] (*PoM* §65)

Similarly, when he first introduces denoting concepts, he says 'of the *concept* "any number", almost all the propositions that contain the *phrase* "any number" are false. If we wish to speak of the concept, we have to indicate the fact by italics or inverted commas' (*PoM* §56). The usage of italics or inverted commas is here recommended as a way of disambiguating what is meant. But how can we differentiate the *propositions* expressed by 'every number is prime' and '*every number* is prime'? Since these sentences clearly differ in meaning (even though they are both false), they must express different propositions.

On the standard interpretation, there seem to be two possibilities. First, the sentences express the same proposition, namely ⟨/every number/, *prime*⟩. This would leave the difference in meaning mysterious, so this option can't be right. The alternative is to take the italics to effect a change of indication: where 'every number' indicates the denoting concept /every number/, the phrase '*every number*' indicates the denoting concept //every number//, which denotes /every number/. But this second option is not as easily read into *PoM* as it might initially seem. First of all, nowhere in *PoM* does Russell talk about denoting concepts that denote denoting concepts. In particular, when he mentions that we can speak of denoting concepts themselves, he does not mention that this requires a further denoting concept. If he did have this theory in mind, he almost certainly would have mentioned it here.

Perhaps more importantly, //every number// simply cannot be a denoting concept in the framework of *PoM*: a denoting concept, after all, consists of *all*, *any*, *a*, *some*, *every* or *the* together with a class-concept: 'all denoting concepts, as we saw, are derived from class-concepts; and *a* is a class-concept when "*x* is an *a*" is a propositional function' (*PoM* §73). But what could possibly be the class-concept in //every number//, which we are imagining to be indicated by the phrase '*every number*'? It cannot be *every number*, because '*x* is an *every number*' is not a propositional function. It cannot be *number*, because '*x* is a number' is false when *x* takes /every number/ as value, which cannot be right if //every number// is indeed to denote /every number/. There just is no way of reading the phrase '*every number*' so that it indicates a denoting concept: to introduce //every number//

[20] Note that Russell here makes a mistake: the denoting concept itself is of course not a variable conjunction, but it denotes one. Instead, he could have written '*any number* denotes a variable conjunction' or '*any number* is a denoting concept'.

as its indication is to completely disregard what Russell writes about the possible forms a denoting concept can take.

So much for the standard interpretation and its demerits. Those who accept it clearly face more interpretative difficulties than has been acknowledged. Instead of defending the standard interpretation, I want to develop a rival interpretation, however.

4. Denoting Concepts and Aggregates

Suppose we take the passages quoted above seriously and so reject the standard interpretation. How can we incorporate them in a plausible interpretation of the theory of denoting concepts in *PoM*? A first suggestion might be the following: if, in a proposition, a denoting concept occurs and shifts what the proposition is about, then what it denotes will also be a constituent of the proposition. This would make sense of the first two passages quoted, namely Russell's claim in §68 that the proposition expressed by 'Socrates is one among men' contains the class of men, which is denoted by /all men/, and the claim in §57 that the proposition expressed by 'Socrates is a man' contains the variable disjunction which is denoted by /a man/. On this reading, when he writes that the proposition expressed by 'Socrates is one among men' contains the class as a constituent, he must mean the class-as-many, since this is what is denoted by /all men/.

A problem with reading *PoM* this way is that it cannot make sense of the crucial passage in §141, in which Russell claims that the aggregate of numbers is a constituent of the proposition expressed by 'any number has a successor'. What /any number/ denotes cannot be an aggregate because on the one hand, as we have seen, Russell states very clearly that aggregates are one, not many, and, on the other hand, he tells us that the combinations of terms which are denoted by denoting concepts of the form /any *C*/ are 'something absolutely peculiar, which is neither one nor many' (*PoM* §59). Unless we ascribe to him an inconsistency here, what /any number/ denotes cannot be an aggregate.[21]

The alternative option, which I favour, is to take the crucial passage about infinity in §141 as a starting point, in which case we get the following rule: if, in a proposition, a denoting concept of *C* occurs and shifts what the proposition is about to the *C*s, then the aggregate of the *C*s or the C (in case there is just one) will also be a constituent of the proposition. This is how Russell treats the proposition expressed by 'any number has a successor' in §141. This interpretation also makes sense of §68 as long as we take Russell's claim that the class of men is a constituent of the proposition expressed by 'Socrates is one among men' to be the class-as-one, which is an aggregate.[22] Of course on this interpretation, §57 is puzzling because Russell there says that 'Socrates is a man' contains the variable disjunction denoted by /a man/, which cannot be an aggregate. But, as I suggested above, since he here fails to mention the concept /a man/ as a constituent, this might just be an instance of sloppiness.

An important advantage of this interpretation is that it can make sense of a rather obscure and condensed argument in §148. Russell there argues that aggregates must be of finite complexity *because* there are no infinitely complex propositions (or at least not any that can be grasped by humans):

> Infinite aggregates, we found, must be admitted; and it seems that all infinite wholes which are not unities must be aggregates of terms, though it is by no means necessary that the terms should be simple. (They must, however, **owing to the exclusion of infinite unities**, be assumed to be of *finite* complexity.) (*PoM* §148; italics are Russell's, boldface is mine)

[21] If we take seriously Russell's idea of ambiguous denotation (as opposed to the different-combinations theory), what is denoted again cannot be an aggregate.

[22] Admittedly, this is problematic. Russell almost everywhere in *PoM* claims that /all men/ denotes the class-as-many, so this interpretation of §68 does seem to ascribe to him an inconsistency. There is only one other section where he also could be taken to suggest that /all Cs/ may sometimes denote the class-as-one, namely in §74.

The argument hinted at is, I think, this: (i) infinite unities (these are the infinite propositions) are excluded because, even if there are any, humans cannot grasp them; (ii) but humans can grasp propositions that are about infinitely many terms; (iii) therefore, infinite aggregates must be of finite complexity. This argument would fail if the aggregate of *C*s must not also be a constituent of the propositions in which the denoting concepts of *C* occur and function as aboutness shifters. After all, if Russell's position is that the aggregate of numbers is *not* a constituent of the proposition expressed by 'any number has a successor', as the standard interpretation would maintain, then why should we ever think that the aggregate *must* be of finite complexity *because* infinite propositions are excluded? On the standard interpretation, the problem posed by (i) and (ii) would already be solved, whether aggregates are of finite or infinite complexity. All the constituents of the proposition expressed by 'any number has a successor' are, on this view, of finite complexity regardless of the complexity of the aggregate. So Russell's argument hinted at near the end of §148 does not make sense on the standard interpretation, but can be accommodated straightforwardly on my interpretation.

Note also that my interpretation also solves the theoretical issue the standard interpretation faces that I introduced above: how can there be propositions about denoting concepts themselves? When *C* is a class-concept, let [*C*s] stand for the aggregate of the *C*s if there are two or more *C*s and the *C* if there is only one.[23] The sentences 'every number is prime' and '*every number* is prime' will straightforwardly express different propositions on my interpretation: the first expresses the proposition ⟨/every number/, [numbers], *prime*⟩, whereas the second expresses the proposition ⟨/every number/, *prime*⟩. In the first proposition, /every number/ shifts what the proposition is about and accordingly the aggregate of numbers, namely [numbers], is also a constituent; in the second proposition, aboutness is not shifted and accordingly [numbers] is not a constituent. The italics merely indicate which of these two propositions is intended. This is another advantage of my interpretation.

What about Russell's motivations for adopting the theory of denoting concepts in the first place? Do his various uses of denoting concepts conflict with my interpretation?

There are four issues that drive Russell into the direction of the theory of denoting concepts in *PoM*. I have already introduced one, namely that finite beings are able to grasp propositions about infinities. It is worth stressing that his solution to this problem is, if I am correct, radically different from what it is usually thought to be. He does *not* solve it by taking the subject (the numbers, or whatever it may be) away from the proposition, putting a denoting concept in its (or: their) place. Rather, he thinks that the numbers *are* a constituent of the proposition, but specifically as an aggregate. Since aggregates are single terms—they are one and not many—the problem of infinity is solved. This is the intensional solution to the problems of infinity that Russell advocates throughout *PoM* (see especially §330). We can specify aggregates extensionally by enumeration, but this method is practically impossible in the case of infinite aggregates. If this was the only method, we could therefore never grasp any proposition of which an infinite aggregate is a constituent. But, says Russell, we can also specify aggregates intensionally, namely through class-concepts. It is this latter method of specification that allows us to grasp propositions that contain infinite aggregates.

A second motivation for the theory of denoting concepts is that they enable an account of the difference in significance between 'Smith met Brown' and 'Smith met a man'. The former expresses a proposition which can be represented as ⟨Smith, *meeting*, Brown⟩. The latter cannot, of course, express the same proposition, even if Brown is the man Smith happened to meet, for then the sentences would express the same proposition,

[23] This notation will not be used when *C* is an empty class-concept.

which they clearly do not. So the question is, what proposition does 'Smith met a man' express if not ⟨Smith, *meeting*, Brown⟩? The theory of denoting concepts provides an answer. According to the standard interpretation, the proposition expressed by 'Smith met a man' is ⟨Smith, *meeting*, /a man/⟩. According to my interpretation, the sentence expresses a proposition which can be represented as ⟨Smith, *meeting*, /a man/, [men]⟩.

The third motivation concerns significant identity statements, which was also already briefly mentioned in the previous section. The sentence 'the present Pope is the last survivor of his generation' is a significant statement, but without the theory of denoting concepts, Russell would have to maintain that 'the present Pope' and 'the last survivor of his generation' both indicate a particular individual. The sentence would then express the proposition that this individual is identical to themselves, which is 'perfectly futile' (*PoM* §64). With the theory of denoting concepts, Russell is able to explain why the sentence is not perfectly futile: it expresses a proposition which contains the denoting concepts /the present Pope/ and /the last survivor of the present Pope's generation/. Although the proposition is accordingly still *about* one individual, which on my interpretation will also be a constituent of the proposition, the proposition itself is not trivial because it does not merely relate a single individual to itself via identity.

In a similar way, denoting concepts explain the importance of definitions in mathematics (*PoM* §63; Makin 2000, 16–17). At first sight, the fact that definitions are so important seems paradoxical, since

> definitions, theoretically, are nothing but symbolic abbreviations, irrelevant to the reasoning and inserted only for practical convenience, while yet, in the development of a subject, they always require a very large amount of thought, and often embody some of the greatest achievements of analysis. (*PoM* §63)

If definitions in logic and mathematics really just amount to giving another way of indicating the same term, then how can they nonetheless sometimes be deep discoveries? According to Russell, the theory of denoting helps to answer this question:

> This fact seems to be explained by the theory of denoting. An object may be present to the mind, without us knowing any concept of which the said object is *the* instance; and the discovery of such a concept is not a mere improvement in notation. The reason why this appears to be the case is that, as soon as the definition is found, it becomes wholly unnecessary to the reasoning to remember the actual object defined, since only concepts are relevant to our deductions. In the moment of discovery, the definition is seen to be *true*, because the object to be defined was already in our thoughts; but as a part of our reasoning it is not true, but merely symbolic, since what the reasoning requires is not that it should deal with *that* object, but merely that it should deal with the object denoted by the definition. (*PoM* §63)

This passage needs spelling out. Although we may already be familiar with an object, say the number two, it may still be a genuine discovery that this object is 'defined' by a certain concept, say *the even prime*. The realisation that the number two is defined by this concept may enable us to infer things about this number that we would not have been able to infer otherwise. So the proposition expressed by '2 is the even prime' must amount to more than ⟨2, *identity*, 2⟩, since grasping this latter proposition can never be a genuine discovery. Again, the solution is that 'the even prime' indicates a denoting concept. The proposition expressed is therefore better represented, on my interpretation, as ⟨2, *identity*, /the even prime/, [even primes]⟩, which is clearly not trivial in the same sense. (Since there is only one even prime, [even primes] just is 2 and not an aggregate).

Clearly the driving forces behind Russell's development of the theory of denoting concepts can still be accommodated on my interpretation: he still manages to solve the issues that troubled him. Now, it is often stressed that Russell's theory of denoting concepts in *PoM* constitutes a major departure from a central and general tenet of his earlier position on propositions, namely

that what a proposition is about must be among its constituents (Hylton 1990, 206–7; Hylton 2003, 214–18; Levine 1998, 415; Noonan 1996, 72–73, 82–83). If this is correct, then denoting concepts must have seemed to Russell a defeat: he was forced to admit that some propositions do not contain what they are about and to explain how propositions can be about something not contained in them. Yet when reading *PoM*, one does not get the impression that Russell thought of his theory of denoting concepts in this way. Indeed, he does not worry about or even mention this issue at all. One important consequence of my interpretation of the *PoM* theory of denoting is that this theory no longer amounts to a major departure from the thesis that what a proposition is about must be among its constituents. On my reading, Russell manages to solve the problems that motivated his theory of denoting concepts with only a minor change to the thesis that what a proposition is about must be among its constituents: although what a proposition is about will not itself always be a constituent of the proposition, when it is not, the aggregate of terms that the proposition is about will be a constituent. The theory thus becomes a relatively natural solution to some of the problems Russell thought a naive theory of propositions faces, a solution he could accept without completely retreating from the view that propositions contain what they are about.

There is a possible worry though. Consider the famous acquaintance principle: one can grasp a proposition only if one is acquainted with every constituent of it. This principle motivated significant parts of Russell's later philosophy and it has sometimes been suggested that it was also operative in *PoM* (e.g., Levine 1998, 422–24; Stevens 2011, 53 n8). But it may be argued that we are not acquainted with the aggregate [numbers] or, for that matter, with the aggregate [men], since we are not acquainted with all of the terms that make up these aggregates. So if these aggregates are to be part of every proposition in which the denoting concepts of *number* and *man*, respectively, occur, then we cannot grasp these propositions after all. In other words, my interpretation seems to conflict with the various uses Russell puts denoting concepts to in *PoM* when we also incorporate a principle of acquaintance.

My response to this worry is that I think it is a mistake to read any strong principle of acquaintance into *PoM*. The word 'acquaintance' is used in a relevant sense only once in all of *PoM*, namely in the preface, page *xliv*. And there it is *not* used to state a version of the acquaintance principle or to express problems concerning our acquaintance with infinite classes, concepts or past people, for instance.[24] Moreover, Russell appears totally uninterested and unaware of the kind of epistemic worries which would dominate some of his later philosophy. For instance, he does not consider the fact that he is not acquainted with Socrates or Caesar when discussing 'Socrates is human' (*PoM* §48) or 'the death of Caesar' (*PoM* §52; Hylton 1990, 38), and he simply does not ask how we can be acquainted with abstract concepts. In fact, the kind of epistemological worries that lead Russell to the principle of acquaintance begin troubling him only *after* (and very soon after) the publication of *PoM*, as is evident from some of the manuscripts from that time, to which I now turn.

5. Denoting Concepts after *PoM*

In the time between *PoM* and 'On Denoting', Russell drafted some significant texts (published only posthumously) that concern denoting concepts. One of the main changes between *PoM* and these manuscripts is that he now makes a distinction between what a proposition *means* and what it *denotes*. In a manuscript of 1903, for example, he considers a theory according to which some phrases denote, most notably names, some phrases mean, most notably verbs, and some phrases both

[24] Gideon Makin (2000, 147) suggests that 'perception' and 'to perceive' are in *PoM* used for what Russell would later call 'acquaintance', but again these phrases are not in *PoM* used to introduce or discuss epistemological problems concerning for instance infinite classes, concepts or past people.

mean and denote, most notably definite descriptions (1903c). Sentences themselves are also taken to both mean and denote.[25] In the case of definite descriptions, what they denote is said to be a constituent of the denotation of the sentences in which they occur, while what they mean is said to be part of the meaning of the sentences in which they occur.

The period is also marked by an increasing interest in the epistemological side of denoting. *PoM* did contain, as we have seen, the problem of infinitely complex propositions, which revolved around the basic assumption that finite beings cannot grasp propositions with infinitely many constituents. This problem is of course epistemic in nature, but is detached from epistemic worries about our access to abstract entities, for instance. In the first two manuscripts after *PoM* that concern denoting (1903a; 1903c), such epistemological considerations are also absent. Then, still in 1903, he starts a new manuscript with the observation that 'sometimes we know that something is denoted, without knowing what' (1903d, 306). He says that 'it is necessary, for the understanding of a proposition, to have *acquaintance* with the *meaning* of every constituent of the meaning, and of the whole; it is not necessary to have acquaintance with such constituents of the denotation as are not constituents of the meaning' (1903d, 307). The next two manuscripts (from 1903 and 1905 respectively) similarly introduce epistemological worries as an important motivation for sorting out denoting concepts properly:

> If we say, for instance, 'Arthur Balfour advocates retaliation', that expresses a thought which has for its object a complex containing as a constituent the man himself; no one who does not know what is the designation of the name 'Arthur Balfour' can understand what we mean: the object of our thought cannot, by our statement, be communicated to him. But when we say 'the present Prime Minister of England believes in retaliation', it is possible for a person to understand completely what we mean without his knowing that Mr. Arthur Balfour is Prime Minister, and indeed without his ever having heard of Mr. Arthur Balfour. (Russell 1903b, 315–16)

> This topic is very interesting in regard to [the] theory of knowledge, because most things are only known to use by denoting concepts. Thus Jones = the person who inhabits Jones's body. We don't have *acquaintance* with Jones himself, but only with his sensible manifestations. Thus if we think we know propositions about Jones, this is not quite right; we only know propositional functions which he satisfies, unless indeed we *are* Jones. . . we can only know an object as denoted if we are *acquainted* with the denoting concept; thus immediate acquaintance with the constituents of the denoting concept is presupposed in what we may call *denotative* knowledge. (Russell 1903b, 369)

It is in the context of such epistemological reflections that Russell first explicitly considers the question 'when a denoting phrase occurs in a proposition, does that which is denoted form a constituent of the proposition or not?' (1903d, 306).

I have argued that in *PoM*, Russell would have answered that the aggregate of the relevant terms would be a constituent of the proposition, at least when the proposition is about what is denoted. In a short posthumously published text from after *PoM*, one that does not yet contain the epistemological worries, he writes that 'generally, when a meaning occurs as denoting, another meaning with the same denotation may be substituted without changing the proposition: i.e., only the denotation, not the meaning, is a constituent of the proposition' (1903a, 298). This seems to amount to the radical view that not the denoting concept (the meaning), but only what is denoted will be a constituent of the proposition. Although this manuscript contains many preliminary and underdeveloped suggestions that are eventually rejected, Russell is clearly toying with different theories of denoting and initially appears to think that what

[25] Russell says it is *propositions* that both mean and denote, but I have already been distinguishing between sentences and propositions in a way that he does not, at least not explicitly.

is denoted, as opposed to the denoting concept, will be a constituent of the proposition.

Once epistemological considerations are on his mind, however, he feels forced to deny that when a denoting concept occurs in a proposition, what is denoted is also a constituent. For we can grasp the proposition expressed by 'Smith's wife has blue eyes' without being acquainted with Smith's wife. So on the assumption that one has to be acquainted with every constituent of a proposition in order to grasp it, it follows that Smith's wife, who is denoted by /Smith's wife/, is not a constituent of the proposition. Of course the proposition still is *about* Smith's wife in an important sense. 'Hence a difficulty', Russell now writes (1903d, 306). Again, in *PoM* this difficulty is completely absent.

Russell's initial solution to the epistemic troubles uses the distinction between the meaning and the denotation of a proposition. Smith's wife is contained in the denotation of the proposition expressed by 'Smith's wife has blue eyes', but is not part of the proposition's meaning. The denoting concept /Smith's wife/ is conversely part of the meaning, but not of the denotation. The proposition is about (the constituents of) its denotation, but to grasp the proposition, we need to be acquainted with (the constituents of) its meaning. Reading the various unpublished manuscripts between *PoM* and 'On Denoting' that concern denoting, one gets the sense that Russell keeps running into further subtleties and issues working out this solution until he finally introduces what will become the theory of 'On Denoting' (1905a) in the manuscript 'On Fundamentals' (1905b).

A particularly nasty problem that troubled Russell in this period after *PoM* relates to propositions that are about denoting concepts themselves. Before the epistemological considerations set in, Russell happily writes that 'every phrase which has meaning and denotation [including denoting phrases] can be, and in logic-books commonly is, turned into one having the former meaning as denotation, by the device of italics or inverted commas' (1905c, 284). He does not yet see any problems arising from this, which should not be surprising: as I have argued, the *PoM* theory of denoting concepts can make sense of propositions about denoting concepts.

But once Russell distinguishes between the meaning and denotation of propositions (not just of denoting concepts) and, in light of epistemological worries, denies that what a denoting concept denotes is part of the meaning of a proposition in which it occurs, it becomes increasingly difficult to explain how there can be propositions *about* a denoting concept itself, as opposed to about what it denotes. It is perhaps mainly this problem, which in 'On Denoting' (1905a) becomes the Gray's Elegy argument, that drove Russell into the direction of the theory of denoting phrases adopted in that paper.

The (partial) account of the changes to the theory of denoting concepts from *PoM* to 'On Denoting' that I advocate, then, is roughly as follows. In *PoM*, Russell accepted the rule: if a denoting concept of *C* is a constituent of a proposition *and* shifts what the proposition is about, then the aggregate of *C*s (or the *C*, in case there is only one) is also a constituent of the proposition. With this rule he could understand the occurrences of the denoting concepts of *C* in which aboutness is not shifted—i.e., those propositions which are about these denoting concepts—as those in which this aggregate is not also a constituent of the proposition. This theory of denoting concepts constitutes only a very minor departure from the thesis that what a proposition is about is always a constituent of it: although there are now some propositions that do not contain the terms they are about as constituents, in these cases the aggregate of the relevant terms will be a constituent. After *PoM*, Russell started to distinguish between meaning and denotation and epistemological worries led him to deny that what a denoting concept denotes is in any form part of the meaning of propositions in which said concept occurs. In developing this new theory, he realised that he could not anymore explain how there can be propositions about denoting concepts themselves. Because of this seemingly inescapable

problem, he began looking for alternatives and finally adopted the quantificational theory of the denoting phrases that is developed in 'On Denoting' (1905a) and already considered in 'On Fundamentals' (1905b).[26]

6. Empty Denoting Concepts

Now how does what admittedly seems like a relatively minor interpretational issue fit into a larger picture of Russell's philosophical development? I want to connect the above discussion of denoting concepts to the ontology of *PoM* by examining in detail a particularly tricky passage, namely §73. I first introduce the passage and its central topic, namely denoting concepts that do not denote anything, which I will call *empty* denoting concepts. (An *empty denoting phrase* is a denoting phrase that indicates an empty denoting concept.) In the next section, I then explain how I think the passage fits into Russell's overall ontology in *PoM*.

The problem that Russell addresses in §73 is how to make sense of the proposition 'nothing is not nothing', which, he says, is 'undoubtedly capable of an interpretation which makes it true'. He claims that

> It is necessary to realize, in the first place, that a concept may denote although it does not denote anything. This occurs when there are propositions in which the said concept occurs, and which are not about the said concept, but all such propositions are false. (*PoM* §73)

So there can be no doubt that he thinks that there are empty denoting concepts. His first characterisation of these is that they are the concepts such that all propositions in which they occur are false. This would of course immediately lead to problems concerning negation: there would be a false proposition that also has a false negation. In fact, this would be the case for every proposition that has as a constituent a denoting concept that does not denote anything and yet is not about the concept itself.

Russell rejects his first characterisation for another reason, however:

> Consider, for example, the proposition 'chimaeras are animals' or 'even primes other than 2 are numbers'. These propositions appear to be true, and it would seem that they are not concerned with the denoting concepts, but with what these concepts denote; yet that is impossible, for the concepts in question do not denote anything. (*PoM* §73)

In his initial definition, a proposition containing an empty denoting concept, but which is nonetheless *not* about the concept, is false. But that cannot be right, Russell claims, because there are propositions of that kind that appear true. (The examples at first do not seem to contain denoting concepts. Earlier in the book, however, he says that bare plurals, such as 'chimeras', are synonymous with the corresponding denoting phrase using 'all', such as 'all chimeras' (*PoM* §67). So the denoting concepts in his examples are meant to be /all chimeras/ and /all even primes other than 2/.) Now, Russell says that they 'appear' true because, ultimately, he does not think that they are true. But he is clearly not happy calling them false either. Instead, he insists, 'it is most correct to reject the proposition ["chimeras are animals"] altogether' (*PoM* §73).

The conclusion thus seems to be that there are no propositions that have empty denoting concepts as constituents: Russell concludes that such propositions must be rejected. But that cannot be right either because the notion of a term is tied to the possibility of occurring in propositions: 'whatever. . . may occur in any true or false proposition. . . I call a *term*' and again 'every term, to begin with, is a logical subject: it is, for example, the subject of the proposition that itself is one' (*PoM* §47). In other words, if there are no propositions containing the empty denoting concept /all even primes other than 2/, then the denoting concept itself cannot be a term at all. So Russell's introduction of empty

[26] Note that on this account, the target of the Gray's Elegy argument is not the theory of denoting concepts of *PoM*.

denoting concepts seems to be undermined by his rejection of propositions with empty denoting concepts as constituents.

The way out of this tangle is, I think, that Russell wants to reject all propositions that contain empty denoting concepts *and are about what the denoting concept denotes*. The conclusion then would be that empty denoting concepts are fine, but they can never shift aboutness. The sentence '*all even primes other than* 2 is an empty denoting concept' is perfectly in order and expresses a proposition that is about /all even primes other than 2/. The sentence 'all even primes other than 2 are numbers' is meaningless, however, because it purports to express a proposition which contains an empty denoting concept that shifts aboutness. This proposition must, on my interpretation, be rejected because there is no aggregate of the even primes other than 2 which can be a constituent of it.

This way of reading Russell also accords with what he concludes about 'nothing is not nothing':

> We may now reconsider the proposition 'nothing is not nothing'—a proposition plainly true, and yet, unless carefully handled, a source of apparently hopeless antinomies. *Nothing* is a denoting concept, which denotes nothing. The concept which denotes is of course not nothing, *i.e.* it is not denoted by itself. The proposition which looks so paradoxical means no more than this: *Nothing*, the denoting concept, is not nothing... (*PoM* §73)

Russell deals with the paradoxical proposition by reinterpreting it as being about the denoting concept /nothing/. In this analysis, /nothing/ is *not* used to shift what the proposition is about. It could not be so used because Russell rejected all propositions in which an empty denoting concept purports to shift aboutness. In short, one cannot use empty denoting phrases to talk about what there is not.[27]

That Russell reinterprets the proposition expressed by 'nothing is not nothing' as being about the denoting concept /nothing/ clearly harmonises with my interpretation of the theory of denoting concepts. For on my interpretation, a denoting concept of *C* can only shift what a proposition is about if the aggregate of *C*s is a constituent of the proposition. If this aggregate is not a constituent, then the proposition will be about the concept instead. Now, in the case of empty denoting concepts, there cannot be a proposition in which both it and the corresponding aggregate occur, simply because there is no corresponding aggregate in the first place. After all, if *C* is a class-concept for which 'x is a *C*' is false for all values of x, then there is no class-as-one of all the *C*s. Hence all propositions in which an empty denoting concept occurs will be about the concept, just as in Russell's analysis of 'nothing is not nothing'. This also explains why Russell rejects the purported proposition expressed by 'chimeras are animals': if there was a proposition expressed by that sentence, then it would clearly not be about /all chimeras/ but about chimeras. Because /all chimeras/ is empty, that can never be the case: there is no [chimeras] that can be a constituent of the proposition.

(What about the second occurrence of 'nothing' in 'nothing is not nothing'? In the case of 'chimeras are animals', there are several closely related propositions that Russell thinks are perfectly acceptable, for instance 'x is a chimera implies x is an animal for all values of x'. Here we must interpret 'x is a chimera' as 'x is-a chimera' so that it expresses a relation between x and the class-concept *chimera*. It is because 'x is-a chimera' is false for

[27] A reviewer suggests a possible interpretation according to which /nothing/ denotes the class of null class-concepts. The evidence for this is the last sentence of Russell's discussion of /nothing/, in which he writes that 'it by no means follows from this that there is an actual null-class: only the null class-concept and the null concept of a class are to be admitted' (*PoM* §73). Russell does not, I think, mean to suggest that null class-concepts are doing work in the analysis of 'nothing is not nothing'. Instead, he simply stresses that this sentence does not force us to accept a null-class. Although the idea of the class of null class-concepts is important in the paragraph *after* the discussion of /nothing/, Russell is there concerned with 'a new difficulty' and not with denoting.

all values of x that the denoting concepts of *chimera* are empty.[28] In 'nothing is not nothing', the second occurrence of 'nothing' similarly indicates an empty class-concept. The right interpretation of 'nothing is not nothing' would, I think, therefore be this: when the denoting concept /nothing/ is substituted for x in 'x is nothing', the resulting proposition is about /nothing/ and false. Hence 'nothing is not nothing' is true.)

7. The Ontology of *PoM*: the world is filled...

It is often argued that Russell could have avoided some of the ontological extravagance of *PoM* while retaining his theory of propositions and denoting.[29] In *PoM*, Russell more than once lists some of the entities he believes in, for instance here:

> Numbers, the Homeric gods, relations, chimeras and four-dimensional spaces all have being, for if they were not entities of a kind, we could make no propositions about them... to mention anything is to show that it is. (*PoM* §427)

The list is accompanied by an argument: we must accept that numbers, Homeric gods, etc. have being because otherwise we could make no propositions about them. The assumption behind this argument is thus that we do make propositions about them. Some interpreters, relying on the standard interpretation, have argued that Russell could have escaped the argument by using his notion of an empty denoting concept (Hylton 1990, 211–21; Hylton 2003; Griffin 1996; Candlish 2007, 112; Stevens 2011). They argue he could have maintained (i) that every logically significant phrase of a meaningful sentence indicates an entity that is a constituent of the proposition expressed, (ii) that there are propositions that are about, say, chimeras, and (iii) that there are no chimeras. The reasoning is this. Suppose that there are indeed no chimeras: they neither exist nor have being. The sentence 'every chimera exists' is now thought to expresses the (false) proposition ⟨/every chimera/, *existence*⟩. Every logically significant phrase still corresponds to an entity in the proposition and the proposition is still supposed to be *about* chimeras (according to the standard interpretation of Russell's theory of denoting concepts). The concept /every chimera/ is simply an empty denoting concept which shifts aboutness, so that the proposition supposedly is still about chimeras, but because it is empty the proposition is false. So when denoting concepts that do not denote anything are taken into account, it seems that Russell's commitment to there being propositions about chimeras is compatible with there not being any.

This reasoning must of course be rejected if my interpretation of the theory of denoting concepts is correct. On my reading, if /every chimera/ is an empty denoting concept, then whenever it occurs in a proposition, the proposition is not about chimeras, but about the concept itself. As we have seen, this accords with Russell's understanding of 'nothing is not nothing'. It follows that empty denoting phrases cannot be used to express propositions that are about what there is not: they can only be used to express propositions that are about the empty denoting concepts themselves. It would then not be an oversight of Russell that he could have escaped his own argument, but rather an oversight of recent commentators that Russell couldn't.

An important interpretative issue remains, however. On the one hand, as we have seen, Russell argues that Homeric gods, chimeras, and so on, all must have being. On the other hand, as we have also seen, in the passage on empty denoting concepts, he says that /all chimeras/ is an empty denoting concept. So are there chimeras or not?

One initially attractive solution is to think that denoting concepts cannot denote what does not exist. That would, after all, explain the tension between, on the one hand, Russell's argu-

[28] In fact, this is the definition of an empty denoting concept that Russell settles on in §73. I take up the question whether *chimera* really is empty in the next section.

[29] See footnote 5.

ment that chimeras must have being and, on the other hand, his claim that /all chimeras/ does not denote anything: if /all chimeras/ denotes, then it must denote something existing (or a combination of existing terms). Since chimeras do not exist, it follows that /all chimeras/ does not denote anything, which is still compatible with there being non-existent chimeras. But this explanation fails because Russell did accept denoting concepts that denote (combinations of) non-existent terms, for instance numbers. The original question therefore still stands: are there chimeras or not?

Nicholas Griffin (1996) has argued that Russell did not think that chimeras have being at all. So he could take §73 at face value: /all chimeras/ is empty, there are no chimeras. The question then is how to interpret §427, in which Russell argues that chimeras are 'entities of a kind' because we can make propositions about them, and §47, which contains the sentence: 'a man, a moment, a number, a class, a relation, a chimaera, or anything else that can be mentioned, is sure to be a term; and to deny that such and such a thing is a term must always be false'.

Griffin reinterprets these statements that, when taken at face value, commit Russell directly to there being chimeras, as follows:

> On my interpretation, however, we must conclude that Russell is referring here to the denoting concepts themselves, not to the terms they appear to denote. For the denoting concept itself is always a term, though, in my view, there will often be no further term denoted by it. This reading of the passage will, I concede, seem strained in the absence of any explicit indication that Russell is mentioning denoting concepts. Yet it is in fact what I think he meant.[30] (Griffin 1996, 54)

[30]Griffin takes a similar position about the following passage: 'Every pair of terms, without exception, can be combined in the manner indicated by *and*, and if neither *A* nor *B* be many, then *A* and *B* are two. *A* and *B* may be any conceivable entities, any possible objects of thought, they may be points or numbers or true or false propositions or events or people, in short anything that can be counted. A teaspoon and the number 3, or a chimaera and a four-dimensional space, are certainly two' (*PoM* §71).

Griffin acknowledges that his reading will seem strained, but it might be worth stressing just how strained it is.

The first strange consequence of Griffin's reading is that Russell would have failed to assert that there are men or moments or the other things in his list: it would surely be incredible to reinterpret 'a chimera' but not 'a man' and 'a moment'. But if every item on the list is meant to indicate only a denoting concept, then why would Russell have bothered to list six items: men, moments, numbers, classes, relations and chimeras? Moreover, why list things that are clearly going from concrete to abstract to mythical? And why only mention denoting concepts with 'a' and not any of the other kinds? Note also that the list in §47 occurs before denoting concepts have been introduced. He would have been misleading his readers if he intended to be saying that the denoting concept /a chimera/ is a term when the reader cannot at this point in the book know what a denoting concept is.

Perhaps most strikingly, Russell later (in *My Philosophical Development*, first published in 1959) explicitly describes himself as accepting, before discovering the theory of descriptions of 'On Denoting', the argument that the truth of 'the golden mountain does not exist' implies that the golden mountain has being (1959, 84). This has sometimes been explained away as an instance of Russell's memory failing him in the later years of his life (Stevens 2011, 58). But he also describes himself in this way much earlier, namely in *Sceptical Essays*, first published in 1928:

> A good deal of modern pluralist philosophy has been inspired by the logical analysis of propositions. At first this method was applied with too much respect for grammar; Meinong, for example, maintained that, since we can say truly 'the round square does not exist', there must be such an object as the round square, although it must be a non-existent object.*The present writer was at first not exempt from this kind of reasoning, but discovered in 1905 how to escape from it* by means of the theory of 'descriptions', from which it appears that

> the round square is not mentioned when we say 'the round square does not exist'. (Russell 2004 (1928, 57, my emphasis)

It is one thing to argue that Russell's memory sometimes fails concerning the details of the genesis of *PoM* (Grattan-Guinness 1996, 103), but it is quite another to argue that his memory fails (both in 1959 *and in 1928*) concerning the broader development of his thinking around that time, which Griffin is committed to.

In support of his reinterpretation, Griffin cites a letter Russell sent to Victoria Welby. In this letter, written on 3 Febuary 1904, Russell explains some his distinctions as follows:

> We have to distinguish (1) the relation of a word to the thought it *expresses*: this is the sense of a word as given in dictionaries and preserved in translation; (2) the relation of a thought (idea) to that of which it is the idea; (3) in certain cases, like that of the Prime Minister, a further relation of the object of the idea (which object, in such cases, I call a *concept*) to another object or collection of objects: this is this third relation that I call *denoting*. (cited in Griffin 1996, 43)[31]

The denoting concept /the Prime Minister/ is here said to be 'the object of the idea (thought)' associated with the words 'the Prime Minister'. Now in *PoM*, Russell wrote that 'whatever may be an object of thought. . . is a term' (*PoM* §47). There is thus a similarity in terminology: 'thoughts' and their 'objects'. Given this similarity and the fact that in the letter, Russell calls /the Prime minister/ the object of the thought associated with 'the Prime Minister', Griffin thinks that what Russell means when he writes that 'whatever may be an object of thought. . . is a term' is that denoting concepts are terms. If this is granted, then it is more plausible that the list itself, occurring just after this characterisation, must be a list of denoting concepts, not what they denote.

Against Griffin's appeal to this letter, we have already noted that Russell starts toying with different theories of denoting concepts almost immediately after *PoM*. So it is unclear what value the letter from 1904, from after the publication of *PoM*, has for understanding that book. On the other hand, Welby had asked Russell for clarification of some elements of *PoM*, so perhaps we should be charitable and indeed take Russell to be explaining what he thought in *PoM*. But even when we are charitable in this way, the letter still fails, I think, to support Griffin's interpretation. In the letter, Russell says that the denoting concept /the Prime Minister/ is *the object of the thought* associated with the phrase 'the Prime Minister'. But it is, I think, a mistake to equate *an object of thought*, which is the phrase as it occurs in *PoM*, with *the object of a thought*, which is the phrase as it occurs in the letter to Welby. The latter is clearly a technical term that depends on the technical term *thought* that Russell introduces in the letter. The former is not a technical term at all, which is plain from the fact that *thought* was not given a technical sense in *PoM*.

If Griffin's interpretation of the contrast between §73 and §427 is unconvincing, then how *are* we to understand that contrast? How can /all chimeras/ be an empty denoting concept if Russell explicitly argues that chimeras must be entities of some kind? The clue, I think, can be found in a different section of *PoM*. In §62, Russell discusses the sentence 'I met some man', saying that 'the actual man whom I met forms no part of the proposition in question'. He goes on to explain how to interpret the proposition using denoting concepts. In the process he remarks that 'the whole human race is involved in my assertion: if any man who ever existed or will exist had not existed or been going to exist, the purport of my proposition would have been different' (*PoM* §62).

The significance of this remark in the present context is that Russell clearly thinks that in the proposition expressed, /some man/ only denotes men who at some point in time exist, whether in the past or now or in the future. That precludes Sherlock Holmes from being denoted: although he is a man, he is presumably a 'pseudo-existent of a novel', one of the 'many terms

[31] The letter is also given in full in Griffin (1996, 58–60).

which do not exist' (*PoM* §48). Because Russell accepted denoting concepts that denote non-existents, Sherlock Holmes is not excluded as being among the men denoted per definition. The upshot must be that in the proposition expressed by 'I met some man', /some man/ is tacitly restricted to past, present or future existent men and perhaps better represented as /some existent man/.

Let us now return to §73 in which Russell claims that /all chimeras/ is an empty denoting concept, seemingly contradicting his claim that chimeras must be entities of some kind, which occurs both before and after §73. If we do not want to attribute to Russell a simple inconsistency, and also do not want to take Griffin's route, we should take Russell's claim that there are no chimeras in §73 to be tacitly restricted in the same way that /some man/ is in §62. So when he writes that 'chimeras are animals' appears true, which is problematic because the denoting concept in question does not denote anything, we should take the denoting concept to be /all existent chimeras/. If we adopt this reinterpretation consistently throughout §73, then the section does not give rise to interpretational problems.

Another more global interpretational challenge remains, however. Russell repeatedly stresses the importance of existence-proofs in mathematics.[32] If, for example, we want to talk about the limit of some infinite series, then we first have to show that there is one (*PoM* §267). But suppose that there is in fact no limit of the particular series. Then it seems that the statement 'the limit of this infinite series is not' is both meaningful and about the limit of the series. By Russell's lights, however, it must therefore be false, because if there is a proposition about the limit, then the limit must have being (*PoM* §427). Conclusion: there is a limit after all. It is not clear how he could avoid this reasoning, yet he must be able to, otherwise we can *always* prove that a given series (even a divergent one) has a limit.

This difficulty is sometimes explained away with the same story, introduced at the beginning of this section, that is used to argue that Russell could have avoided his ontological extravagance: what the meaningfulness of 'the limit of this infinite series is not' forces on Russell is not any limit, but the denoting concept /the limit of this infinite series/ (Stevens 2011, 70–71). The proposition expressed by 'the limit of this infinite series is not' is also still about the limit because the denoting concept shifts aboutness. Yet it is possible that the concept is empty and there is no limit of the relevant kind.

Of course I cannot adopt this explanation because I think it rests on a mistaken understanding of the theory of denoting concepts. What I would suggest instead, is that Russell's argument—that if there are meaningful sentences about x, then x has being—is a two-way street. If we take it as given that a certain sentence about x is meaningful, then we must conclude that x has being. (I have argued that this conclusion cannot be avoided by an appeal to empty denoting concepts, at least not if my interpretation of the theory of denoting concepts in *PoM* is accepted.) Russell clearly assumes that there are meaningful sentences about chimeras, 'chimeras do not exist' is presumably an example. It then follows by Russell's argument that chimeras have being. In this case, then, we start with a meaningful sentence about chimeras, from which it follows that it expresses a proposition that is about chimeras, and so we conclude that there must be chimeras.

But in the case of mathematics it is not always clear that a sentence is meaningful. In particular, a sentence about the limit of some infinite series may very well be meaningless, namely if there is no such limit. *That* is why existence-proofs are so important: anyone can make up a denoting concept and attempt to say something using it. But they will only be successful in their attempt if the denoting concept in fact denotes something. In

[32] As pointed out by Hylton (1990, 211–12) and Griffin (1996, 51). Note that existence in the mathematical sense is different from existence in the philosophical sense: a class exists when it has at least one member (*PoM* §25).

other words, unless we show that /the limit of this infinite series/ denotes something, 'the limit of this infinite series is not' may very well be meaningless.[33] Whereas Russell already accepts that there are meaningful sentences about chimeras, he does not already accept that there are meaningful sentences about, say, the limit of this-or-that series. To show that there are meaningful sentences about the limit in question, we have to give an existence proof.

8. Conclusion

This essay contains two main contentions. The first is that the theory of denoting concepts in *PoM* has generally been misunderstood. In section 3, I presented what I have called the standard interpretation and presented three passages from *PoM* that directly contradict it. I also argued that there is a more philosophical reason to reject the standard interpretation, namely that it cannot make sense of propositions that are about denoting concepts. I then developed in section 4 what I take to be a better interpretation, according to which a denoting concept of *C* only shifts what a proposition is about if the aggregate of the *C*s or the *C* (in case there is only one) is also a constituent of the proposition. This interpretation is directly supported by a central passage in *PoM* in which Russell shows how the theory of denoting concepts explains that finite human beings can grasp propositions about infinite classes.

The second main contention is that Russell's theory of propositions and denoting concepts in *PoM*, together with the assumption that there are meaningful sentences about chimeras, entails that there are chimeras. This goes against a recently popular way of understanding the relationship between *PoM*'s theory of denoting concepts and ontology, according to which empty denoting concepts can be used to maintain (i) that there are meaningful sentences about chimeras and (ii) that nonetheless there are no chimeras.

[33] Unless, of course, the sentence is interpreted as being about the denoting concept /the limit of this infinite series/, in which case it would simply be false.

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