The Logic(s) of Modal Knowledge*

Daniel Cohnitz

Abstract

This paper discusses, with the help of three examples from modal epistemology, what we can learn from formal considerations when using logic as a tool for rational reconstructions of cognitive processes. The purpose of these rational reconstructions is to explain how a certain cognitive process might eventually result in knowledge (or justified beliefs, etc.), if we pre-theoretically think that we have such knowledge or such justified beliefs. Traditionally, a rational reconstruction assumes some (more or less) unproblematic basis of knowledge and some justification-preserving inference pattern and then goes on to show how these two suffice to generate the *explicandum*.

In modal epistemology we try to apply the project of analytic epistemology to our knowledge of necessities and possibilities. The "method" of conceivability, as the cognitive process by which we arrive at knowledge of possibilities, is thus to be reconstructed as some sort of inference pattern by which we reason from (more or less) unproblematic knowledge to knowledge of possibilities. This paper discusses several ways how this could be done, including Carnap's modal logic **C** [Sch01], Diderik Batens' and Joke Meheus' *Adaptive Logic of Compatibility* [Meh00], and Timothy Williamson's "Modal Logic within Counterfactual Logic" [Wil07].

As we will see, there is no one logic, that captures all aspects of modal reasoning, but a plurality of different formal tools that help us to understand better different aspects of how we know what is possible and what is necessary.

1 Modal Epistemology

Modal epistemology tries to explain our apparent knowledge of modal propositions. In everyday reasoning, but also in scientific reasoning and especially when doing philosophy, we seem to be relying on *modal* judgments, judgments about what is *necessary*, what is *impossible* and what is *possible*. When deliberating which course of action to take for reaching a desired goal, we are taking into consideration only those courses that we think are *possible* for us to take; when distinguishing accidentally true generalizations from

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lawlike statements in the sciences, we make judgments about which of these statements state *necessities*; when reflecting on whether knowledge is true justified belief, we consider whether it is *possible* for a person to have a true justified belief that would not qualify as knowledge, etc. Of course, the modalities involved in these three examples are of different kinds. When deliberating alternative courses of action in our everyday life, we are considering possibilities in a much more restricted sense than the possibilities we take into consideration when doing philosophy. However, the epistemological question of how we can know modal propositions can be asked in the same way for all kinds of modality. We have certain ordinary methods for gaining knowledge: perception, inference, memory and introspection. For these methods we understand—at least to some degree—why they should inform us about the way the world *is*. But how could these ordinary methods also inform us about the ways the world *could be*?

Of course, in some way they do. Knowing that it is actually the case that p[1], I can know that it is possible that p. But this just provides us with "trivial" modal knowledge. Modal knowledge is *trivial* in cases of knowing that it is possible that p on the basis of knowing that p [Evn08, 665]. Thus the problem of modal epistemology is to explain modal knowledge of the form *it is necessary that* p and *it is impossible that* p, as well as instances of *it is possible that* p, where the latter cannot be inferred from knowledge that p.

In this paper I will look at different ways in which considerations from formal logic have been used to solve the problems of modal epistemology. As we will see, each of these shows us ways how we *could* arrive at non-trivial modal knowledge. In the final part of the paper I will discuss the explanatory value of these considerations.

2 Negative Conceivability

Within Modal Epistemology, one of the key notions is the notion of *conceivability*. Conceivability is—within the philosophical tradition—regarded as our main access to possibility and thus our main way of gaining modal knowledge. However, 'conceivable' can mean many different things. Since Thomas Reid (maybe even since Descartes) there is a nice intellectual game for philosophers to engage in during boring winter evenings: disambiguate the sentence 'It is conceivable that p' in so many ways that eventually one version will not obviously fail to imply 'it is possible that p'. The botany of conceivability has made considerable progress in the last decades (cf. [Evn08]). To see why this exercise is not trivial, we will briefly consider the notion of 'positive conceivability' as a candidate-interpretation of conceivability.

Positive conceivability seems in some cases to involve forming a mental image of a situation. In these cases it is (more or less) clear what 'conceiving that p' means, but much less clear why our faculty to form such an image should speak for the possibility of

[1] In this paper I will try to use small letters 'p', 'q', 'r', ... as metavariables for propositions, small greek letters, 'α', 'ϕ', 'ψ', '...' as metavariables for expressions of the formal languages discussed, and will explain all vocabularies for the formal object languages. I will make changes in all quotes to make these conform to my usage.

the situation so envisaged. It seems, at best, to be a question for empirical psychologists to figure out whether or not we are unable to perceptually imagine impossible situations.

Much worse is the fact that in very many circumstances no perceptual representation is relevant for determining the possibility of a situation. Consider Hilary Putnam's *Twin Earth* or David Chalmers' *Zombie World*. Perceptually speaking they are both indistinguishable from the actual world (at least that is the idea), but are supposed to be different nevertheless. How can I form a positive representation of such a situation? Here is David Chalmers' explication:

> In these cases, we do not form a perceptual image that represents [p]. Nevertheless, we do more than merely suppose that [p], or entertain the hypothesis that [p]. Our relation to [p] has a mediated objectual character that is analogous to that found in the case of perceptual imaginability. In this case, we have an intuition of (or as of) a world in which [p], or at least of (or as of) a situation in which [p], where a situation is (roughly) a configuration of objects and properties within a world. We might say that in these cases, one can modally imagine that [p]. [...] Modal imagination is used here as a label for a certain sort of familiar mental act. Like other such categories, it resists straightforward definition. But its phenomenology is familiar. One has a positive intuition of a certain configuration within a world, and takes that configuration to satisfy a certain description. [Cha02, 151]

The problem with this explication, unlike the interpretation in terms of mental imagery, seems to be that it is far from clear that the quoted characterization does describe a distinctive, familiar mental act. To some philosophers the notion of modal imaginability is either wholly unfamiliar or treated as identical with perceptual imaginability (discussed above) or is identified with—what Chalmers calls—negative conceivability. To the latter we shall now turn. Again Chalmers:

> [t]he central sort of negative conceivability holds that [p] is negatively conceivable when [p] is not ruled out a priori, or when there is no (apparent) contradiction in [p]. [...]
>
> [W]e can say that [p] is ideally negatively conceivable when it is not a priori that [$\neg p$]. [Cha02, 149]

According to this clarification, what is negatively conceivable depends on our ability to detect contradictions *a priori*. A similar analysis of conceivability was given by Peter Menzies:

> [T]he mental ability to conceive something is really a complex ability, consisting in the ability to suppose that the state of affairs holds without being able to reduce this supposition to absurdity. Clearly, this complex ability presupposes a number of other more complex abilities: first, the ability to entertain suppositions; and secondly, the ability to infer other propositions, in particular absurd propositions, from suppositions. [Men98, 265]

Negative conceivability is here understood as a an episode of a priori reasoning, which can be broken down into successive steps. This sounds like much better news for a rational reconstruction of modal epistemology, because the notion of a contradiction has already a formal explication (in formal logic) and so does "inferring absurd propositions".

2.1 Towards a Rational Construction

But what is it that a rational reconstruction in formal terms could do now to further clarify the idea of negative conceivability? It seems to me that there are two epistemological problems[2] that we could at least try to clarify with the help of formal means:

(Q1) *How can we characterize the process by which we arrive at modal knowledge?*

(Q2) *Is the characterized process justification-preserving?*

If we consider negative conceivability, the process is that of supposing p, checking whether p can be ruled out a priori, and—in case of a negative result—concluding that $\neg p$ is possible. But how should that be represented in formal terms? It seems that I can "rule out a priori" that *something could be red all over and green all over at the same time*, but how am I supposed to formally represented the knowledge that goes into ruling it out? Perhaps I can rule it out because I know a priori certain meaning postulates. Perhaps I know (i) that it is a meaning postulate for color-terms that nothing can be "all-over" in two colors, if both colors are picked out by distinct basic color terms, and (ii) that 'red' and 'green' are two distinct basic color terms in English. But, surely, neither (i) nor (ii) is easily represented formally.

In order to get started we should perhaps simplify matters a bit. Let us just consider the broadest notion of possibility, logical possibility, which doesn't concern meaning postulates, etc. Logical possibilities (and necessities) will then be possible or necessary on logical grounds alone, and should—presumably—if knowable also be knowable on logical considerations alone, not requiring any further knowledge of meaning postulates, etc. If we are able to model modal epistemology for logical possibility, we might then try to develop more advanced models for more restricted notions in a second step. Perhaps we have reason to think that knowledge of necessities (of a certain modality M) is unproblematic to explain (for such an account, see [Hal02]). It might, for example, be plausible to assume knowledge of conceptual necessities on the basis of competence with the relevant concepts. But the conceptually possible is what isn't conceptually impossible, what is logically compatible with what is conceptually necessary. In that case, knowledge of conceptual possibility could be explained to arise from knowledge of conceptual necessity, combined with our account of knowledge of logical possibility. A similar story could perhaps be told for all notions of necessity, M. We'd just need an account of why we have knowledge of the relevant M-necessities, and then combine this with our account of knowledge of logical possibility to get an account of knowledge of M-possibility.

[2] An anonymous reviewer suggested that it might be helpful to see these two questions below in parallel with Hume's questions about induction. First asking for a characterization of the "nature" of the relevant reasoning, then considering its justification given the characterization.

Hence we need a way to model what it means to be "knowable on logical considerations alone". One way could be to analyze 'knowable on logical considerations alone' as what is derivable via introduction and elimination rules for logical connectives alone. According to negative conceivability, p is inferred to be possible if $\neg p$ is not a priori (hence, if $\neg p$ is not knowable on logical considerations alone). As an inference rule in a formal system (where ϕ is a variable for wffs of that system), this could be represented as something like:

$\Diamond$INTRODUCTION If $\nvdash \neg\phi$ then $\vdash \Diamond\phi$

In relation to such an inference rule we could then ask whether negative conceivability "entails" possibility (which is what Chalmers—among other things—is concerned with in his [Cha02]) more precisely: would such an inference rule be sound? *Soundness* is a metalogical property of a logic. It says that inferences made in accordance with the deductive system of this logic are in accordance with the assumed (model-theoretic) semantics of that logic. Thus negative conceivability "entails" possibility insofar as

If $\vdash \Diamond\phi$ then $\vDash \Diamond\phi$

is a metalogical validity, for a system that has $\Diamond$Intro as an admitted rule of inference. The entailment relation (between negative conceivability and possibility) is hence modeled as a relation between the deductive system and the formal semantics of that logic.

2.2 Carnap's Modal Logic

In my [Coh04] and [Coh06] it is shown that a standard system with such a rule of inference can indeed be sound, if this system is limited in its expressive power. The system I was considering as a model for inferring possibilities on the basis of their negative conceivability, was Rudolf Carnap's modal logic **C**. **C**, especially in the form I used it (which is the reconstruction of **C** in [Sch05, Sch01]), has several features that make it actually different from standard propositional modal logic, also from propositional **S5**, although Carnap's propositional modal logic is often (also by Carnap himself) mistakenly presented as equivalent with C.I. Lewis' **S5**.[3]

Why we need to depart from standard modal logic (and thus also from standard **S5**) for our purposes becomes clear when we look at the standard (Kripke-)semantics for propositional modal logic. In Kripke's *A Completeness Theorem in Modal Logic*, only those sentences are theorems that are valid[4] with respect to every subspace, $\mathscr{W}' \subseteq \mathscr{W}$, of possible worlds. Take a sentence of the form $\Diamond\phi$, ϕ being a non-modal formula of propositional modal logic. As we have said, Kripke's modal logics will have $\Diamond\phi$ as a theorem only if it is valid with respect to every subspace $\mathscr{W}' \subseteq \mathscr{W}$, but then ϕ can't be anything but a tautology for it's only these which are guaranteed to be true at at least

[3]For a discussion of correct and incorrect interpretations of Carnap's modal logic, see [Got99, HP85, Sch01].
[4]There is no possible world at which the sentence is false.

one world in every subspace. The following theorem (for a proof see [Sch01]) states this in a general way for every normal modal logic:

Theorem 2.1 *For every nonmodal formula ϕ of the language of propositional modal logic and every normal modal propositional logic* $\mathbf{L}$*: if $\Diamond\phi \in \mathbf{L}$, then: $\Box\phi \in \mathbf{L}$, and ϕ is a truthfunctional tautology provided* $\mathbf{L}$ *is consistent.*

So these semantics are unfortunately inadequate if we want to model the logic of negative conceivability. To model such logic, or at least a fair portion thereof, we need to model non-trivial[5] possibilities as valid formulas and as theorems.

This problem does not occur in Carnap's modal logic. Instead of having a variable subspace of $\mathscr{W}$ determine validity, we assume a fixed space $\mathscr{W}$, containing all possible interpretations[6] of our language and identify interpretations with possible worlds. This way our modal logic is built up just like standard propositional logic, but with two further logical operators in its vocabulary ($\Box$ and $\Diamond$)[7], the respective clauses for the construction of well-formed formulae, and the following definition of valuation, $\mathrm{V}_{\mathscr{W}}$:

Valuation for Carnap's Propositional Modal Logic Where α is any sentence letter, ϕ and ψ are any wffs, $\mathscr{W}$ the set of all possible interpretations $\mathscr{I}_n$, $n \in \mathbb{N}$:

$$\begin{aligned}
\mathrm{V}_{\mathscr{W}}(\alpha, \mathscr{I}_i) &= \mathscr{I}_i(\alpha) \\
\mathrm{V}_{\mathscr{W}}(\neg\phi, \mathscr{I}_i) &= 1 \text{ iff } \mathrm{V}_{\mathscr{W}}(\phi, \mathscr{I}_i) = 0 \\
&\vdots \\
\mathrm{V}_{\mathscr{W}}(\Box\phi, \mathscr{I}_i) &= 1 \text{ iff for every } \mathscr{I}_j \in \mathscr{W},\ \mathrm{V}_{\mathscr{W}}(\phi, \mathscr{I}_j) = 1 \\
\mathrm{V}_{\mathscr{W}}(\Diamond\phi, \mathscr{I}_i) &= 1 \text{ iff for at least one } \mathscr{I}_j \in \mathscr{W},\ \mathrm{V}_{\mathscr{W}}(\phi, \mathscr{I}_j) = 1
\end{aligned}$$

Hence a sentence is necessarily true, iff it is logically true, iff it is true in all "possible worlds". Choosing the valuation function like this validates all theorems of **S5**. But these semantics also determine that for every sentence letter α, $\Diamond\alpha$ is a validity (is true in all "possible worlds"/interpretations), and the same holds for every (merely) consistent sentence of that language. However, this is now exactly as it should be, because these validities model the non-trivial possibilities we wanted from our semantics. A sentence is possibly true iff it is true at at least one "possible world", and sentences can be true at a possible world without being also true in all other possible worlds.

It is worth noting, that this logic is not closed under homomorphic substitutions. Although, say, $\Diamond P_1$ is a validity, we can't replace P_1 in it by arbitrary other wffs. If, for

[5] I here call a possibility, $\Diamond p$ "trivial" iff $\Box p$ is also true. Note the difference between this notion and the notion of "trivial modal knowledge" we used above (which was knowledge of the possibility of p that was derived from it actually being the case that p).

[6] Functions from the set of sentence letters $\{P_1, P_2, \dots\}$ into the set $\{1, 0\}$.

[7] Of course, the vocabulary also includes the usual logical connectives of propositional logic, sentence letters $P_1, P_2 \dots$, and parentheses.

example, we would substitute $P_1 \wedge \neg P_1$ for P_1, the result, $\Diamond(P_1 \wedge \neg P_1)$ would be invalid. But, of course, our logic is still closed under syntactically isomorphic substitution.[8] Other than that, it is unproblematic to state natural deduction rules for **C** that are sound and complete with respect to the semantics sketched, and include our rule of Negative Conceivability, $\Diamond$INTRODUCTION, and a rule of necessitation:

$\Box$INTRODUCTION If $\vdash \phi$ then $\vdash \Box\phi$

as admissible rules of inference. Such a system of rules is provided in [Sch01]. Thus, with respect to **C**, we can give a positive answer to the question whether Negative Conceivability entails possibility: at least our formal model seems to show that inferring the (logical) possibility of p on the basis of not being able to arrive a priori at $\neg p$ is indeed a sound inference.

However, the formal model quickly ceases to inform us much about Negative Conceivability, when we move on to richer languages than modal propositional logic. Let us first introduce the notion of a "**C**-modal expansion" of a non-modal logic:

Definition We call a logic a **C**-modal expansion, $\mathbf{C_L}$, of a logical System **L** with an interpretational semantics (in terms of **L**-structures) iff we add to **L** the two modal operators $\Box$ and $\Diamond$, and their respective semantical interpretations (identifying **L**-structures with possible worlds).

In the case above we considered a **C**-modal expansion of standard propositional logic. What happens if we look at the **C**-modal expansions of richer logics? What happens, in particular, when we look at a **C**-modal expansion of First-Order Predicate Logic (**FOL**)?

2.3 The Limits of Carnap's Modal Logic

Unfortunately, one can prove an interesting connection between decidability, completeness, and soundness of **C**-modal expansions (cf. [Sch01], [Coh04]):

Theorem 2.2 *A* **C***-modal expansion of an undecidable logic* **L**, $\mathbf{C_L}$, *is incomplete or not algorithmic.*

Proof For *reductio* we assume $\mathbf{C_{FOL}}$ (**FOL** being undecidable) to be algorithmic (its theorems are enumerable by a deterministic turing machine) and complete. For every formula ϕ of $\mathbf{C_{FOL}}$ either $\Box\phi$ or $\Diamond\neg\phi$ is valid in $\mathbf{C_{FOL}}$ given its semantics. If $\mathbf{C_{FOL}}$ is complete, one of them will be derivable after finite time. This is a decision procedure for theoremhood in **FOL** (since all formulae of **FOL** are a subsystem of $\mathbf{C_{FOL}}$. There is no such decision procedure (**FOL** was assumed to be undecidable). Thus $\mathbf{C_{FOL}}$ is incomplete or not algorithmic. Q.E.D

[8] For a discussion of this feature and the question whether closure under homomorphic substitution should be considered a necessary property of logics proper can be found in [Mak66, Sch01].

OK, then perhaps $\mathbf{C_{FOL}}$ is not complete. But that alone would not mean that Negative Conceivability does not entail possibility, because for the latter one would need to show that admitting the rule of $\Diamond$INTRODUCTION leads to invalid conclusions and hence to a system of rules that isn't anymore sound. Unfortunately, this argument breaks down in light of the next theorem:

Theorem 2.3 *For every consistent axiomatisation of a logic of kind* $\mathbf{C_L}$ *in which the rules* $\Diamond$INTRODUCTION *and* $\Box$INTRODUCTION *are admissible, the following holds:* $\mathbf{C_L}$ *is semantically complete iff it is semantically sound.*

Proof Let us assume that $\mathbf{C_{FOL}}$ is incomplete (so it might still be algorithmic). We also assume that the rules are admissible. Incompleteness means that there is some formula ϕ of $\mathbf{C_{FOL}}$ such that $\vDash \phi$ but $\nvdash \phi$. By $\Diamond$INTRODUCTION, $\nvdash \phi$ allows us to infer $\Diamond\neg\phi$, which is equivalent with $\neg\Box\phi$, hence $\vdash \neg\Box\phi$. Since $\vDash \phi$ (assumption), the semantics of the modal operators guarantee $\vDash \Box\phi$. Therefore incompleteness would lead to an unsound logic if we kept the rules.

Now we prove the other direction: Assume that $\mathbf{C_{FOL}}$ is not sound, but complete and consistent. Thus, by unsoundness, there is a formula ϕ such that $\nvDash \phi$, but $\vdash \phi$. By $\Box$INTRODUCTION, $\vdash \Box\phi$, but by the semantics of the modal operators and $\nvDash \phi$, $\vDash \Diamond\neg\phi$, which is equivalent with $\vDash \neg\Box\phi$. Q.E.D

Thus as a rational reconstruction of what is going on in negative conceivings, this reconstruction is a failure. Negative Conceivability does not entail possibility, if we are dealing with beliefs the sentential representations of which stand in inferential relations, such that a logic with the expressive power of First-Order Logic is needed to adequately represent the intuitively obtaining logical relations. In other words: if we confine ourselves to the standards of classical logic, negative conceivability does not entail possibility even if we are dealing with mere first-oder logical possibilities (without discussing the problem of a posteriori necessities or second-oder logical truths, etc).

2.4 An Adaptive Alternative

The question now is whether there is a somewhat friendlier interpretation of negative conceivability in a somewhat different system, for which negative conceivability *would* entail possibility. Looking again at Chalmers' definition of *ideal* negative conceivability, there is a hint how to improve the rational reconstruction. Menzies had in his conception of (negative) conceivability the notion of an *ideal reasoner*. Chalmers is worried that this notion might be incoherent and suggests an alternative construction:

> [O]ne can dispense with the notion of an ideal reasoner and simply invoke the notion of undefeatability by better reasoning. Given this notion, we can say that p is ideally conceivable when there is a possible subject for whom p is prima facie conceivable, with justification that is undefeatable by better reasoning. The idea is that when prima facie conceivability falls short of ideal conceivability, then either the claim that the relevant tests are passed will be

> unjustified, or the justification will be defeatable by further reasoning. For ideal conceivability, one needs justification that cannot be rationally defeated. [Cha02, 148]

The idea of "undefeatability by better reasoning" is not modeled properly in the rational reconstruction above. The reason is that the standard-logical picture we have drawn is *too static*.

2.4.1 Internal and External Dynamics

In fact, standard logic allows for only two sorts of *dynamics*: the first is a form of *external* dynamic: if we are reasoning by some logic $\mathbf{L}$ from a set of data Γ and, at some point in time, are supplied with a supplementary set of data Γ', we are in general able to derive more consequences from that point in time on: $Con_{\mathbf{L}}(\Gamma) \subseteq Con_{\mathbf{L}}(\Gamma \cup \Gamma')$.

This form of external dynamic contrasts with non-standard external dynamics: a conclusion may be withdrawn in view of new information. In this case, the consequence relation is *non-monotonic*. This latter form of external dynamics is highly relevant for an adequate reconstruction of our modal reasoning: newly discovered necessities might lead to the revision of possibility claims which were made on a limited information base. If our logic of modal reasoning is supposed to reflect the dynamics of modal inquiry properly, this non-standard external dynamic needs to be represented.

The second form of standard-dynamics is the *internal* dynamic, which is not represented in standard metalogic even. This dynamic obtains because given a set of rules of inference, not all formulas eventually derivable from a set of data are derivable already in the first step by a single application of one of the rules. The set of formulas derived (and thus shown to be derivable) monotonically increases when a proof proceeds. This is, however, without metalogical or proof theoretic consequences:

> The derivability of a statement, however, does not depend on the question whether one sees that it is derivable. So, this form of internal dynamics is related to logical heuristics and to computational aspects, rather than of the logic properly. To be more precise: the formulation of the proof theory is fully independent of it.[Bat06]

Derivability is thus an all or nothing affair in classical logic. Neither does a formula become derivable if it wasn't before, nor does—in standard logic—a formula cease to be derivable. Especially this second form of an internal dynamic is of interest for a logic of modal reasoning: inferences that seem valid at a time might be shown to be invalid later as the proof continues. Although something seems possible, because its negation is not yet proved, it might turn out impossible later, when its negation is found derivable after all. This might be what Chalmers was after when speaking of "undefeatability by better reasoning"; instead of derivability by some ideal thinker, we need derivability that cannot be trumped by further reasoning from the premises. This way negative conceivability can indeed be defined for quantified logic such that it *entails* possibility. What such a logic should then provide is a formal criterion to tell when the stage of undefeatability by further reasoning is reached (and maybe also some criterion to tell whether one is getting closer to such a stage). A logic that provides all this for the internal dynamics mentioned and which can be easily turned into a logic that would also capture the external dynamics mentioned, is Diderik Batens' and Joke Meheus' *Adaptive Logic of Compatibility*, **COM** (see [Meh00]). We will not go into the details here, but will briefly look at its main properties.

2.4.2 The Adaptive Logic of Compatibility

The idea basically stems from research on Paraconsistent Logic. Paraconsistent Logic is designed to allow "rational" inferences and thus information extraction even in cases in which we reason from an inconsistent set of data. In these cases, a Paraconsistent Logic blocks "explosion" (*ex falso quodlibet*) by restricting the inference rules of Classical Logic. If we would use Paraconsistent Logic in all cases though, we would not be able to extract *all* information from a set of data (or a subset of a larger set of inconsistent data) that would turn out to be consistent. In general, the set of consequences derivable from a set of assumptions Γ by paraconsistent rules, is a subset of the consequences derivable by classical rules. $Con_{\mathbf{PL}}(\Gamma) \subseteq Con_{\mathrm{CL}}(\Gamma)$. To avoid this, an Adaptive Logic assigns a *Lower Limit Logic* (LLL), in the usual case a Paraconsistent Logic, and an *Upper Limit Logic*, usually Classical Logic, (ULL). Now, instead of simply reverting to LLL when the set of data (Δ) it is reasoning from is inconsistent, an adaptive logic interprets Δ as normal as possible, such that it might be that $Con_{LLL}(\Delta) \subset Con_{AL}(\Delta) \subset Con_{ULL}(\Delta)$. One of the logics sets the *standard of normality* relative to which the Adaptive Logic is *corrective* or *ampliative*. Inconsistency-adaptive logics are usually interpreted as corrective.[9]

In the case of **COM**, we are dealing with an ampliative adaptive logic. The LLL of **COM** is classical modal logic (**S5**). The ULL, $S5^P$, is obtained by extending **S5** with the rule "If $\nvdash_{S5} \neg A, then \vdash_{S5^P} \Diamond A$". This is already familiar from the considerations above. Of course, $S5^P$ is also not closed under Uniform Substitution, and if A is a fully modal wff then either A or $\neg A$ is a theorem of $S5^P$.

This logic is now equipped with a dynamic proof theory. Dynamic proof theories use a marking rule, that allows the application of conditional derivation rules if the lines the rules are applied to are marked with conditions. In every step of the proof it is first checked whether all conditions are still satisfied. If a condition fails to be satisfied, its lines and all lines derived from it are removed from the proof. The (as yet) unmarked lines at a stage in a proof are the lines derived at that stage.

Definition A is finally derived in a proof from $\Gamma^{\Box}$ iff A is derived on a line that is not marked and will not be marked in any extension of the proof.

Definition $\Gamma^{\Box} \vdash_{COM} A$ (A is finally derivable from $\Gamma^{\Box}$) iff A is finally derived in a proof from $\Gamma^{\Box}$.

[9] Unless one is a dialetheist and considers Paraconsistent Logic the standard of normality.

With such a dynamic understanding of derivability, soundness and completeness of final derivability with respect to the semantics of **COM** can be proved. The idea of final derivability accords to Chalmers' notion of "undefeatability by better reasoning". Moreover, although there is no positive test for possibility for an undecidable logic **COM**, there are criteria applying to specific situations that we might use to tell whether a given formula is finally derivable (see [Meh00, 345]), and, more importantly, a semantic way by which one can arrive at a (provisional) estimate about how much information has been extracted from the premises in a proof. The latter is achieved with the help of a "block semantics" for **COM** that assures that the dynamics of the proof-theory are real. What is derivable at a stage is finally derived with respect to the insights gained at that stage of the proof. Unfortunately here is not the space to characterize the block-approach in more detail. Details can be found in [Meh00] and [Bat06].

3 Modal Logic in Counterfactual Logic

In the previous examples, the focus was on knowledge of possibilities and such knowledge might arise inferentially. What we considered unproblematic was knowledge of logical truth, and it was shown how there could be ways to extend a model of our inferential access to logical truth (and thus to logical necessity) such that the model would encompass also our knowledge of logical possibility as inferentially accessible. The third example that I want to discuss, Timothy Williamson's Counterfactual Logic, has a different starting point. Also Williamson wants to show that one part of our modal knowledge can be logically reduced to another part of our modal knowledge. However, the part of our modal knowledge that serves as the unproblematic (or less problematic) reduction base, is our knowledge of counterfactuals.

Williamson considers both, our knowledge of necessity and our knowledge of possibility, to be equally problematic. This holds in particular when it comes to knowledge of metaphysical possibility and necessity, which philosophers seem to be especially interested in. What is metaphysically necessary and possible seems to be mind-independent. However, philosophers (as we have seen above in Chalmers' description of "modal imagination") try to explain our knowledge of this modality via a cognitive process that seems at best to be able to inform us about our mental ability to imagine something (hence something mind-dependent). In all other cognitive projects, in science as well as in everyday matters, we do not trust conceivability as a guide to relevant, mind-independent possibility. If this capacity (to imagine metaphysical possibilities reliably) is an extra-feature of our cognitive apparatus (of value only for figuring out what is metaphysically possible and necessary), then a pressing question will be why we should have this extra capacity? An ability to philosophize was presumably not relevant in our evolutionary history [Wil07, 136].

However, Williamson argues, there is a form of modal knowledge that is perhaps easier to explain, namely our knowledge of *counterfactuals*. Counterfactual thinking is deeply integrated in our everyday reasoning, and it might even be constitutively linked to our ability to reason causally. Thus, although our ability for counterfactual reasoning will have to be explained by a full-fledged epistemology, it will have to do that anyway, not just for the purpose of explaining the epistemology that makes philosophy possible. Our knowledge of metaphysical possibility and necessity can perhaps be explained to be just a by-product of our capacity to evaluate counterfactuals.

If we think about the semantics of counterfactuals and the semantics of modal operators, the following two principles suggest themselves:

(N) The necessary is that whose negation counterfactually implies a contradiction.

(P) The possible is that which does not counterfactually imply a contradiction.

Based on these two equivalences[10], Williamson tries to show that modal reasoning reduces to a special case of counterfactual reasoning. Here is, how the idea is developed formally (cf. [Wil07, 293–304]):

Williamson shows that **S5** can be obtained via suitable definitions as a subsystem of a (relatively weak) logic of counterfactuals. The counterfactual logic is characterized proof-theoretically in the following way:

PC	If ϕ is a truth-functional tautology, then $\vdash \phi$
Reflexivity	$\vdash \phi \mathrel{\Box\!\!\to} \phi$
Vacuity	$\vdash (\neg\phi \mathrel{\Box\!\!\to} \phi) \to (\psi \mathrel{\Box\!\!\to} \phi)$
MP	If $\vdash \phi \mathrel{\Box\!\!\to} \psi$ and $\vdash \phi$ then $\vdash \psi$
Closure	If $\vdash (\psi_1 \wedge \cdots \wedge \psi_n) \to \phi$ then $\vdash ((\chi \mathrel{\Box\!\!\to} \psi_1) \wedge \cdots \wedge (\chi \mathrel{\Box\!\!\to} \psi_n)) \to (\chi \mathrel{\Box\!\!\to} \phi)$
Equivalence	If $\phi \equiv \phi^*$ then $\vdash (\phi \mathrel{\Box\!\!\to} \psi) \equiv (\phi^* \mathrel{\Box\!\!\to} \psi)$
MP$\Box\!\!\to$	$\vdash (\phi \mathrel{\Box\!\!\to} \psi) \to (\phi \to \psi)$
ES	$\vdash (\phi \mathrel{\Box\!\!\to} (\psi \mathrel{\Box\!\!\to} \bot)) \to ((\phi \mathrel{\Box\!\!\to} \bot) \vee (\psi \mathrel{\Box\!\!\to} \bot))$

The modal operators $\Box$ and $\Diamond$ can then be introduced via definitions:

[10] Williamson argues that there are equivalent formulations for (N) and (P) that lend additional support for believing that these equivalence are (necessarily) true. One is that, given Closure and Reflexivity (to be introduced below), (N) and (P) are equivalent with

(N′) $\Box\phi \equiv (\neg\phi \mathrel{\Box\!\!\to} \phi)$

(P′) $\Diamond\phi \equiv \neg(\phi \mathrel{\Box\!\!\to} \neg\phi)$.

According to Williamson's reading, (N') and (P') express the principles that the necessary is that which is counterfactually implied by its own negation, and that the possible is that which does not counterfactually imply its own negation.

Moreover, if we add propositional quantification (quantification into sentence position), in which it is plausible to consider $\neg\phi \mathrel{\Box\!\!\to} \phi$ to be equivalent with $\forall P(P \mathrel{\Box\!\!\to} \phi)$, we arrive at two further equivalences that seem intuitively plausible:

(N″) $\Box\phi \equiv \forall P(P \mathrel{\Box\!\!\to} \phi)$

(P″) $\Diamond\phi \equiv \exists P\neg(P \mathrel{\Box\!\!\to} \neg\phi)$

Something is necessary iff whatever were the case, it would still be the case, and something is possible iff it is not such that it would fail in every eventuality.

Definition Where ϕ is a wff of our language of counterfactual logic, and $\perp$ is a constant for a logical falsehood, then $\Box\phi$ is a metalogical abbreviation for $\neg\phi \mathrel{\Box\!\!\rightarrow} \perp$. $\Diamond\phi$ is a metalogical abbreviation for $\neg(\phi \mathrel{\Box\!\!\rightarrow} \perp)$.

One can see the two principles (N) and (P), discussed above, reflected in this definition of the modal operators. Without the last two axioms above, we obtain **K**; with axiom MP$\Box\!\!\rightarrow$ we get **T**. Since ES is equivalent with E ($\vdash \Diamond\phi \rightarrow \Box\Diamond\phi$), we get **S5** when taking all axioms together. Thus, Modal Logic can be based in Counterfactual Logic, and reasoning about necessities and possibilities reduced to reasoning with counterfactuals:

> Given that the equivalences [(N)] and [(P)] and their necessitations are logically true, metaphysical modal thinking is logically equivalent to a special case of counterfactual thinking. Thus, modulo the implicit recognition of this equivalence, the epistemology of metaphysically modal thinking is tantamount to a special case of the epistemology of counterfactual thinking. Whoever has what it takes to understand the counterfactual conditional and the elementary logical auxiliaries $\neg$ and $\perp$ has what it takes to understand possibility and necessity operators. [Wil07, 158]

This concludes my third example for a logical reconstruction in modal epistemology. In the next section I want to briefly discuss the explanatory (or at least explicatory) value that such reconstructions have for epistemology.

4 The Role(s) of Logic in Philosophy

When teaching introductory courses to philosophy students, the main story one seems to be telling in order to "sell" logic to the perplexed kids that find themselves all of a sudden confronted with something that looks suspiciously like mathematics and therefore quite unrelated to what they thought philosophy was all about, is the one about argument reconstruction. Arguments, it is said, are what philosophers study (and perhaps produce) most of their time, and logic, and first-oder predicate logic in particular, is a method that one needs to apply for doing that properly. Over the years of teaching introductory logic courses I found this story more and more unconvincing. True, sometimes we use the formalism of first-order predicate logic for analyzing arguments, but the philosophers that do so are (to some extent, unfortunately) in the minority, and, moreover, it often even seems besides the point to reconstruct an argument in first-order predicate logic. As is well known, the fact that an argument has a formal representation in first-order predicate logic is neither sufficient nor necessary for the argument being good, so sometimes it isn't clear that we could learn much about an argument from formalizing it. But while this overstates the usefulness of formal logic for the everyday analysis of arguments, it—at the same time—unjustifiedly overshadows all other important ways in which philosophy makes use of formal logic. For example, the quite straightforward story, that logic is a major sub-discipline of philosophy and has studied such notions as "logical consequence" and "logical truth" since Aristotle, seems to me to be already a great justification for why one should study logic when one studies philosophy. At least this story seems sufficient for all other obligatory introductory courses into sub-disciplines of philosophy (most of which can't argue for *their* methodological usefulness, and many of which can't present as many and as fascinating and as widely accepted *results* as logic can).

But logic is more than that. The formal considerations above also seem to be philosophically enlightening to some extent, although we were neither reconstructing arguments, nor did we look at these models to better understand the notion of logical consequence[11]. We looked at these formal models to learn something about epistemology. *What* we learn from models like these is controversial. In the last sections of this paper I want to argue that our models can at least answer the modal *skeptic*.

4.1 Logic in Epistemology

Analytic epistemology is traditionally interested in *rational reconstructions* of cognitive processes. The purpose of these rational reconstructions is to make plain how a certain cognitive process might eventually result in knowledge or justified beliefs, etc., if we pre-theoretically think that we have such knowledge or such justified beliefs. Typically a rational reconstruction assumes some (more or less) unproblematic basis of knowledge and some justification-preserving inference pattern and then goes on to show how these two suffice to generate the *explicandum*.

The role of these justification-preserving inference patterns seems crucial. It is not enough just to know that so far we have been quite successful in reasoning from basis X with pattern Y; the philosophical analysis should tell us *why* that is so. This explanatory function is usually satisfied by delimiting the choice of inference patterns (based on *a priori* considerations).

In modal epistemology we try to apply the project of analytic epistemology to our knowledge of necessities and possibilities. For example, the "method" of conceivability, as the method by which we infer possibilities, is thus to be reconstructed as some sort of inference pattern by which we reason from (more or less) unproblematic knowledge to knowledge of possibilities. Whether these inferences are psychologically real, i.e. whether our modal reasoning in fact proceeds in the way the rational reconstruction would reconstruct it, seems traditionally somewhat irrelevant for analytic epistemology.

When saying "traditionally", I'm referring to traditional analytic projects as, for example, Carnap's *Aufbau*, [Car28]). Let us distinguish between a "pre-theoretic level" as the level at which we would describe ourselves as knowing that, for example, it is possible that p because we find it conceivable, and the level of "rational reconstruction" at which the notion of "conceivability" gets analyzed as the complex notion of supposing that p, failing to reduce p to absurdity, and concluding that p is possible. If the traditional picture were correct, then showing that the latter process can be modeled as a valid inference should explain how the former process leads to knowledge:

[11] Although this is something we could have done with these formal models. But it wasn't the purpose of our discussion of these.

Pre-theoretic Level	I find p conceivable (believing Γ to be relevant constraints)	$\longrightarrow$	I find p possible
			$\uparrow$ If valid the inference at the level of reconstruction explains and justifies the reasoning at the pre-theoretic level.
Level of Rational Reconstruction	p is not proved to be inconsistent (with Γ)	$\longrightarrow$	p is possible

But there is good reason to think that this "explanation" would be far too quick. As Carrie Jenkins has argued against Williamson's argument discussed above, showing that modal logic can, via the definition of modal operators, be treated as a subsystem of counterfactual logic does not by itself show that the epistemology of metaphysical modality is really nothing but a special case of the epistemology of counterfactuals.

The problem is that the logical reconstruction only shows that there is *a* way to arrive at knowledge of possibilities and necessities from less problematic knowledge, but it does not show that it is *our* way [Jen08]. Since the equivalences (N) and (P), are only logical equivalences, not synonymies, knowing one side of the equivalence is not just the same as knowing the other. Thus, in this sense, Williamson has not shown that modal epistemology just reduces to counterfactual epistemology. It still could be that in fact we are using a different (or perhaps several different) cognitive mechanism to arrive at knowledge of possibilities and necessities. Perhaps that would not be very economical, and it seems that Williamson is offering a simplicity-argument, suggesting that we should not assume an extra cognitive mechanism for something that was not relevant for our evolutionary history (knowledge of metaphysical possibility and necessity), when we can give an explanation of that knowledge in terms of a cognitive mechanism that primarily produces knowledge which was evolutionary relevant (e.g. knowledge of causal dependencies), if we add to it knowledge of logically necessary truths (like (N) and (P)), which we plausibly have (c.f. [Wil07, 162]. Jenkin's does not think that this simplicity argument is sufficient:

> [T]he envisaged application of Ockham's razor looks inappropriate. For it seems very unlikely that many of us are using this route to modal knowledge much of the time. Most people could not work through the relevant derivations if they tried, and even those who could certainly do not seem to be doing that kind of thing very often. Hence there must be some other way of knowing about modality which most of us use most of the time, and the epistemological puzzles raised by the existence of *this* route are left untouched by Williamson's discussion. [Jen08, 697]

I'm not sure that I understand which "derivations" Jenkin's has in mind, that could not be worked through by most people. Of course, most people could not derive **S5** from counterfactual logic, but the point of the rational reconstruction that Williamson offers does not seem to be that people work through the rational (formal) reconstruction of their reasoning in order to assure themselves that it is OK to reason from counterfactual knowledge to modal knowledge. Here is how Williamson describes the inference he has in mind:

> [By (N)], we assert $\Box[p]$ when our counterfactual development of the supposition $\neg[p]$ robustly yields a contradiction; we deny $\Box[p]$ when our counterfactual development of $\neg p$ does not yield a contradiction (and we do not attribute the failure to a defect in our search). Similarly by [(P)], we assert $\Diamond[p]$ when our counterfactual development of the supposition $[p]$ does not robustly yield a contradiction (and we do not attribute the failure to a defect in our search); we deny $\Diamond[p]$ when our counterfactual development of $[p]$ robustly yields a contradiction. Thus our fallible imaginative evaluation of counterfactuals has a conceivability test and an inconceivability test for impossibility built in as fallible special cases. [Wil07, 163]

The way Williamson describes these "derivations" should sound familiar from the discussion of negative conceivability above. Thus, at least to some epistemologists (like, for example, Chalmers and Menzies) who tried to explicate conceivability as our way to arrive at knowledge of possibility, the cognitive machinery required was at least as demanding as the one required by Williamson's account. Moreover, at least these authors who emphasize the role of negative conceivability for knowledge of possibility seem to take many of us to be working through exactly these derivations sufficiently often.

4.2 Answering the Skeptic

But even if we would grant to Jenkins that more would need to be done in order to show that reasoning through counterfactuals really encompasses *all* our ways to arrive at modal knowledge, showing that this is at least one of the ways open to us to arrive at modal knowledge is already an important result. Although Jenkins suggests that the question of how we *might* know about modality is less interesting for epistemologists of modality than the question how we *do* know about modality, skepticism about modal knowledge precisely because of its problematic epistemology is not unpopular among epistemologists of modality. On the one hand there are extreme skeptics who believe that talk of metaphysical or logical possibility is literally nonsensical, because there is no reasonably way of knowing them. One example of an epistemologically motivated modal skeptic is Peter Van Inwagen:

> [T]here is no such thing as logical possibility—not, at least, if it is really supposed to be a species of possibility. Belief in the reality of "logical possibility" may be based, at least in part, on a faulty inference from the reality of logical *impossibility*, which is real enough. Logical impossibility is an epistemological

> category: the logically impossible is that which can be seen to be impossible on the basis of logical considerations alone—or, to be liberal, logical and semantical considerations alone. [...] What I dispute is the contention that if a concept or state of affairs is not logically impossible, then it is "logically possible." It hardly follows that, because a certain thing cannot be proved to be impossible by a certain method, it is therefore possible in any sense of 'possible' whatever. [VI98, 71]

On the other hand there are epistemologists who take the fact that our modal epistemology seems unsuited for knowledge of mind-independent metaphysical possibility and necessity to be motivation for a non-cognitive construal of these modal notions (cf. [Bla93, Fuh02]).

But these skeptics can be answered with the formal arguments presented. We can answer to skeptics of logical possibility that there is a clear sense in which we can say of a proposition that it is logically possible, and there is also a clear sense in which we can know of such possibilities on the basis of logical considerations alone. This was shown in our discussion of **C** and **COM**.

And we can, on the basis of Williamson's consideration, answer to non-cognitivists about metaphysical modality, that unless they are ready to treat also knowledge of counterfactuals to be a mere projection, there is no special reason to consider our knowledge of metaphysical modality to be any more problematic.

Of course, there could be skeptics who offer a different argument. Skeptics, who offer reasons to believe that the actual ways in which we come to know mundane possibilities are just unfit for providing us with knowledge of a philosophically more interesting kind. But it seems to me that here the ball is still in the field of the skeptic, who owes us reasons to believe that the ways for arriving at beliefs about necessity and possibility that he finds problematic are indeed our ways. In *this* case it would not be sufficient just to point out that there are problematic ways to form such beliefs.

References

[Bat06] Diderik Batens. Adaptive logics. Technical report, Centre for Logic and Philosophy of Science, Ghent, 2006.

[Bla93] Simon Blackburn. *Essays in Quasi-Realism.* Oxford University Press, 1993.

[Car28] Rudolf Carnap. *Der logische Aufbau der Welt.* Weltkreis Verlag, Berlin, 1928.

[Cha02] David Chalmers. Does conceivability entail possibility? In Tamar Szab Gendler; John Hawthorne, editor, *Conceivability and Possibility*, pages 145–200. Oxford University Press, 2002.

[Coh04] Daniel Cohnitz. Why consistentism won't work. In Erik Weber; Tim De Mey, editor, *Modal Epistemology*, pages 33–51. Koninlijke Vlaamse Academie van Belgie voor Wetenschappen en Kunsten, 2004.

[Coh06] Daniel Cohnitz. *Gedankenexperimente in der Philosophie.* Mentis, 2006.

[Evn08] Simon Evnine. Modal epistemology: Our knowledge of necessity and possibility. *Philosophy Compass*, 3/4:664–684, 2008.

[Fuh02] André Fuhrmann. Das Mögliche und das Vorstellbare. *Logos*, N.S. 7:343–360, 2002.

[Got99] Georg Gottlob. Review of a Carnapian extension of S5. In Ewa Orłowska, editor, *Logic at Work. Essays dedicated to the memory of Helena Rasiowa*, pages 55–78. Physica-Verlag, Heidelberg, 1999.

[Hal02] Bob Hale. Knowledge of possibility and knowledge of necessity. *Proceedings of the Aristotelian Society*, CIII:1–20, 2002.

[HP85] Herbert E. Hendry and M. L. Pokriefka. Carnapian extensions of S5. *Journal of Philosophical Logic*, 14:111–128, 1985.

[Jen08] C. S. Jenkins. Modal knowledge, counterfactual knowledge and the role of experience. *The Philosophical Quarterly*, 58:693–701, 2008.

[Mak66] David Makinson. How meaningful are modal operators. *Australasian Journal of Philosophy*, 44:331–337, 1966.

[Meh00] Diderik Batens; Joke Meheus. The adaptive logic of compatibility. *Studia Logica*, 66:327–348, 2000.

[Men98] Peter Menzies. Possibility and conceivability: A response-dependent account. *European Journal of Philosophy*, 3:255–277, 1998.

[Sch01] Gerhard Schurz. Rudolf Carnap's modal logic. In Werner Stelzner and Manfred Stöckler, editors, *Zwischen traditioneller und moderner Logik*, pages 365–380. Mentis, Paderborn, 2001.

[Sch05] Gerhard Schurz. Logic, matter of form, and closure under substitution. In L. Behounek and M Bilkova, editors, *The Logica Yearbook 2004*, pages 33–46. Filosofia, Prag, 2005.

[VI98] Peter Van Inwagen. Modal epistemology. *Philosophical Studies*, 92:67–84, 1998.

[Wil07] Timothy Williamson. *The Philosophy of Philosophy*. Blackwell, 2007.