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Classical Electrodynamic Systems Interacting with Classical Electromagnetic Random Radiation

Daniel C. Cole[1]

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In the past, a few researchers have presented arguments indicating that a statistical equilibrium state of classical charged particles necessarily demands the existence of a temperature-independent, incident classical electromagnetic random radiation. Indeed, when classical electromagnetic zero-point radiation is included in the analysis of problems with macroscopic boundaries, or in the analysis of charged particles in linear force fields, then good agreement with nature is obtained. In general, however, this agreement has not been found to hold for charged particles bound in nonlinear force fields. The point is raised here that this disagreement arising for nonlinear force fields may be a premature conclusion on this classical theory for describing atomic systems, because past calculations have not directed strict attention to electromagnetic interactions between charges. This point is illustrated here by examining the classical hydrogen atom and showing that this problem has still not been adequately solved.

1. INTRODUCTION

In order to gain a better understanding of the apparent split between the behavior of classical systems and quantum mechanical systems, many researchers have devoted considerable effort in the past to studying the behavior of electrodynamic oscillator systems interacting with radiation. Early in this century significant contributions to this subject were made by Einstein[1] and by Planck[2]; however, most physicists soon turned their attention strictly to quantum mechanical systems.

During the past 25 years, some of the early classical work has been reexamined due to the thought that possibly a key point may have been missing in this early work: namely, the idea that at zero temperature, a

[1] IBM General Technology Division, Essex Junction, Vermont 05452.

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nonzero classical electromagnetic radiation may be present that preserves most of our usual ideas about thermodynamic behavior, but also yields the observed nonzero fluctuating motion of particles at zero temperature.[3-6] Indeed, researchers[7-9] have speculated that the presence of this classical electromagnetic zero-point (ZP) radiation may be the important factor that provides stability to classical atomic systems, thereby resolving the classic problem of collapse for a classical charged particle orbiting an oppositely charged massive particle. Moreover, qualitative explanations for classical systems, based on the idea that classical electromagnetic ZP radiation must be present, have been proposed[7] for such quantum mechanical phenomena as tunneling, diffraction, interference, and the Heisenberg uncertainty principle.

Clearly it is desirable to quantify such interesting speculations and to address carefully the problem of the appropriate equilibrium behavior between classical charged particles and classical electromagnetic radiation. Consequently, a number of studies have been carried out over the past 14 years or so, involving the behavior of an oscillating charged point particle in a binding potential, acted upon by isotropic, homogeneous random (Gaussian) radiation.[8,10-25] These studies were directed, in particular, toward the effects of a nonlinear binding force upon an oscillating particle's (1) phase-space distribution,[8,11-14,20,21] (2) radiated and absorbed energy characteristics,[8,10,11,14,15,17,20,23,25] and (3) average energy.[18,20,22,24,25] Hydrogen-like systems were studied with special interest.[12,14,16,17,19]

This theory of classical charged particles in the presence of classical electromagnetic ZP radiation is often called stochastic electrodynamics (SED).[2] The conclusion that appears to have been reached by most researchers[3] is that SED is a better approximation to quantum mechanics (QM) than is classical physics without ZP radiation present, but that it certainly does not come close to predicting the full quantum phenomena for atomic systems that we actually observe in nature. In particular, calculations performed to date predict that a classical model of hydrogen in ZP radiation is not stable and must "ionize."[16,19] Also, upon considering anharmonic oscillator potentials, incorrect changes in energy, as compared with QM, have been predicted for such systems.[18,20,22,24,25]

In this article, I wish to raise a point that has been ignored in the above calculations in SED. Accounting for this factor may significantly alter the predicted results obtained so far in SED, possibly in such a way as to bring it closer in agreement with nature.

[2] For reviews on SED, see Refs. 7, 8, and 26–28. An extended list of references on SED can be found in Ref. 28.

[3] In particular, see Ref. 25, as well as Refs. 12, 16, 17, 19–21, and 23.

Briefly, this point is that in all of the studies carried out to date, the assumption has been made that in the small charge limit, a bound system is obtained that consists of a particle oscillating under the action of a conservative potential that is independent of the value of the charge e. In this limit of $e \to 0$, no random driving forces act on the oscillating particle and no electromagnetic energy is radiated by the oscillating particle. Hence, the random radiation driving forces and the dissipative action of radiated energy have been treated as effects that are small in the small charge limit.[4]

Certainly the above idea is a very reasonable one if the binding potential does not depend upon the charge. Blanco, Pesquera, and Santos (BPS) have carried out lengthy calculations in two significant articles,[21,23] where they claim that if the incident radiation consists of classical electromagnetic Rayleigh–Jeans (RJ) radiation, then (1) the equilibrium behavior of an oscillating charged point particle in a broad class of binding potentials follows a Boltzmann distribution function, and (2) the spectrum of the emitted radiation due to the oscillating charge is that appropriate for equilibrium with a RJ spectrum. Thus, these results agree with what, for years, most physicists have thought to be true. Moreover, these results are also significant in that they were carried out by BPS for relativistic systems, thereby removing the doubt[5] that a lack of a relativistic treatment in earlier work may have resulted in the negative results obtained for SED.

Nevertheless, the above results may not be relevant for actual atomic and molecular systems, since these calculations did not take into account the fact that the binding force should be Coulombic in origin, and should therefore depend upon the charge. Certainly all atomic systems (in particular, atoms made up of negatively charged electrons and positively charged nuclei) consist of particles interacting via electromagnetic forces, with charge values equal to an integer times $\pm e$. The conceptual difficulty here is that in quantum mechanics, we are so used to dealing with harmonic oscillators and perturbing potentials, such as anharmonic oscillator terms, that it is easy to overlook the fact that such potentials all have their origin from Coulombic potentials and must depend upon e in atomic systems. If this fact is taken into account, then entirely different results from BPS may be obtained for the equilibrium behavior between classical charges and classical electromagnetic radiation. Moreover, as will be indicated in this article, obtaining the correct equilibrium radiated energy spectrum by a bound charge will not only require treating the binding force as being Coulombic in origin, but will also require, as argued before by Boyer,[5] that the oscillating charge be treated via relativistic dynamics.

[4] See, for example, Ref. 13, Eqs. (1.3) and (3.5). Also see Ref. 21, Eq. (3.2), and the equations at the top of p. 1260.

[5] See, in particular, Sec. VII in Ref. 10 and Sec. VI in Ref. 11.

In the following section these ideas will be clarified by considering the simplest atomic system in nature. A classical model of hydrogen immersed in classical electromagnetic ZP radiation will be analyzed using relativistic dynamics. This system will be used as a vehicle to indicate what I believe to be important points to include when comparing predicted behavior of classical atomic systems with the observed behavior of atomic systems in nature.

2. A CLASSICAL TREATMENT OF HYDROGEN IN CLASSICAL ELECTROMAGNETIC ZERO-POINT RADIATION

Consider a $-e$ charged point particle with mass m, located at position $\mathbf{z}(t)$, that is attracted by the electrostatic Coulombic binding force

$$\mathbf{F}_{\text{binding}} = -e^2 \frac{\mathbf{z}(t)}{|\mathbf{z}(t)|^3} \tag{1}$$

due to the presence of an infinitely massive $+e$ charged point particle residing at the origin $\mathbf{x} = 0$. Assume that the classical electromagnetic zero-point radiation fields $\mathbf{E}_{\text{ZP}}$ and $\mathbf{B}_{\text{ZP}}$ are present.[2] The Lorentz–Dirac equation[29 32] describing the motion of the $-e$ charge is then given by[6]

$$m \frac{d^2 z^\mu}{d\tau^2} = \frac{2}{3} \frac{e^2}{c^3} \left[\frac{d^3 z^\mu}{d\tau^3} - \frac{1}{c^2} \left(\frac{d^2 z^\lambda}{d\tau^2} \frac{d^2 z_\lambda}{d\tau^2} \right) \frac{dz^\mu}{d\tau} \right] + \frac{(-e)}{c} [\mathscr{F}^{\mu\nu}_{\text{binding}} + \mathscr{F}^{\mu\nu}_{\text{ZP}}] \frac{dz_\nu}{d\tau} \tag{2}$$

where $\mathscr{F}^{\mu\nu}_{\text{binding}}$ and $\mathscr{F}^{\mu\nu}_{\text{ZP}}$ represent the electromagnetic field contributions due to the Coulombic binding potential and to the zero-point fields, respectively.

By rewriting Eq. (2) in dimensionless units, we can expose all the independent parameters that govern the solution. A natural unit of length that should fall out of Eq. (2) is the classical radius of the electron $r_e = e^2/mc^2$, as this quantity is the only length unit that can be composed from the "material" parameters, mc^2 and e, that describe the physical properties of rest-energy (mass) and charge associated with the charged point particle. Indeed, if we consider a problem involving relativistic mechanics and the Coulomb potential, as occurs in the relativistic Kepler problem

$$m \frac{d}{dt} \left[\left(1 - \left| \frac{1}{c} \frac{d\mathbf{z}}{dt} \right|^2 \right)^{-1/2} \frac{d\mathbf{z}}{dt} \right] = -e^2 \frac{\mathbf{z}}{|\mathbf{z}|^3} \tag{3}$$

[6] The following notation is used here: Greek indices take on values 0, 1, 2, 3, a time-space point is denoted by $x^\mu = (ct; \mathbf{x})$, and the metric is $g^{\mu\nu} = \text{diag}(-1, 1, 1, 1)$.

then only the parameters m, c, and e appear. Only the length r_e can be constructed from these parameters.

The presence of the zero-point fields introduces Planck's constant $\hbar$ into the problem. As will be seen, the interaction of the zero-point fields with the $-e$ charge is governed by the dimensionless parameter of Sommerfeld's fine structure constant $\alpha = e^2/\hbar c$. The entire dynamics of our problem, as described by Eq. (2), is then parametrized by only the length r_e and the dimensionless parameter α.

The presence of the zero-point fields and the Coulombic binding force should result in a stochastically varying orbital motion of the $-e$ point charge about $\mathbf{z} = 0$. If we were to find that the dynamical motion of the particle obeyed a stationary stochastic process in time, then without knowing anything else about the details of the motion, we would be forced to conclude that the average amplitude of this oscillating motion was governed by the sole length r_e and the sole dimensionless parameter α that appear in this problem. Hence, we would have a natural prescription for finding the Bohr radius $r_B = r_e/\alpha^2 = \hbar^2/me^2$ as the only expression having a finite nonvanishing nonrelativistic limit when $c \to \infty$. Moreover, this simple scenario also allows for other physical characteristic properties of the relativistic particle's orbit to be found at other length scales, such as the Compton wavelength $\lambda_c = r_e/\alpha$, and at time scales r_e/c, $(r_e/\alpha)/c$ (i.e., λ_c/c), $(r_e/\alpha^2)/c$ (i.e., r_B/c), and $(r_e/\alpha^3)/c$ (i.e., inverse of the Bohr angular frequency).

Let us now turn to show that Eq. (2) may yield such a plausible result by carrying out a simple dimensional analysis. The $\mu = 0$ equation associated with Eq. (2) may be deduced from the $\mu = 1, 2, 3$ equations. Dealing only with the latter three equations, and using Eqs. (5.25)–(5.27) in Ref. 30 to express all quantities in 3-vector form, we obtain

$$\begin{aligned} m\frac{d}{dt}(\gamma\dot{\mathbf{z}}) = {} & \frac{2}{3}\frac{e^2}{c^3}\frac{d}{dt}\left[\gamma^2\ddot{\mathbf{z}} + \gamma^4\dot{\mathbf{z}}\left(\frac{\dot{\mathbf{z}}\cdot\ddot{\mathbf{z}}}{c^2}\right)\right] \\ & - \frac{2}{3}\frac{e^2}{c^5}\left[\gamma^4\ddot{\mathbf{z}}^2 + \gamma^6\left(\frac{\dot{\mathbf{z}}\cdot\ddot{\mathbf{z}}}{c}\right)^2\right]\dot{\mathbf{z}} \\ & - e^2\frac{\mathbf{z}(t)}{|\mathbf{z}(t)|^3} - e\mathbf{E}_{\mathrm{ZP}}(\mathbf{z}(t), t) - e\frac{\dot{\mathbf{z}}(t)}{c}\otimes\mathbf{B}_{\mathrm{ZP}}(\mathbf{z}(t), t) \end{aligned} \tag{4}$$

Here, $\dot{\mathbf{z}} = d\mathbf{z}/dt$, $\ddot{\mathbf{z}} = d^2\mathbf{z}/dt^2$, etc., and, as usual, $\gamma = [1 - (\dot{\mathbf{z}}/c)^2]^{-1/2}$.

To cast this equation into dimensionless form, we need to introduce a normalization unit of length l_0. Based upon the previous remarks, a natural choice for l_0 is r_e; for the moment, however, let us leave l_0 unspecified. The natural choice to make for a normalization unit of time t_0 in a relativistic theory is the unit connected to l_0 by the speed of light

c: $t_0 = l_0/c$. Hence, let $\mathbf{x}' = \mathbf{x}/l_0$ and $t' = ct/l_0$. The term on the left-hand side (LHS) of Eq. (4) can then be written in primed coordinates with an overall factor of mc^2/l_0, while the first three terms on the RHS will appear in primed coordinates with an overall factor of e^2/l_0^2.

The remaining two terms in Eq. (4) contain the zero-point fields, which we will now consider in more detail. Here, let us follow the discussion in, for example, Ref. 7, but we will need to deviate from the discussion presented there when passing from the Fourier series representation of the fields over to the Fourier integral representation in order to correct an existing error in the literature.

If we consider a cubic region of space with side length L, then any free fields in this volume can be expressed as a sum of plane waves, with periodic boundary conditions imposed. This last condition is assumed here for mathematical simplicity, since we are not attempting to describe the fields outside this volume of space; eventually, we will let $L \to \infty$. As usual, the assumption will be made that the zero-point fields obey a Gaussian stochastic process and that they satisfy conditions of homogeneity and isotropy in space. Demanding that their stochastic properties also be Lorentz invariant results in their spectral energy density being uniquely defined up to a multiplicative constant,[4,5] which we then select so as to yield agreement with experiment (such as Casimir forces between metallic parallel plates). Hence, we may write[7,28]

$$\mathbf{E}_{\mathrm{ZP}}(\mathbf{x}, t) = \sum_{\mathbf{n}} \sum_{\lambda=1,2} \left(\frac{2\pi}{L}\right)^{3/2} h_{\mathrm{ZP}}(\omega)\, \hat{\boldsymbol{\varepsilon}}_{\mathbf{n},\lambda} \cos(\mathbf{k}\cdot\mathbf{x} - \omega t + \theta_{\mathbf{n},\lambda}) \tag{5}$$

$$\mathbf{B}_{\mathrm{ZP}}(\mathbf{x}, t) = \sum_{\mathbf{n}} \sum_{\lambda=1,2} \left(\frac{2\pi}{L}\right)^{3/2} h_{\mathrm{ZP}}(\omega)(\hat{\mathbf{k}} \otimes \hat{\boldsymbol{\varepsilon}}_{\mathbf{n},\lambda}) \cos(\mathbf{k}\cdot\mathbf{x} - \omega t + \theta_{\mathbf{n},\lambda}) \tag{6}$$

where

$$\mathbf{k} = \frac{2\pi}{L}\mathbf{n}, \qquad n_x, n_y, n_z = \{0, \pm 1, \pm 2, \ldots\} \tag{7}$$

$$\hat{\boldsymbol{\varepsilon}}_{\mathbf{n},\lambda} \cdot \hat{\boldsymbol{\varepsilon}}_{\mathbf{n},\lambda'} = \delta_{\lambda,\lambda'} \tag{8}$$

$$\mathbf{k} \cdot \hat{\boldsymbol{\varepsilon}}_{\mathbf{n},\lambda} = 0 \tag{9}$$

$$[h_{\mathrm{ZP}}(\omega)]^2 = \frac{\hbar\omega}{2\pi^2} \tag{10}$$

The frequency ω is defined by $\omega = c\,|\mathbf{k}|$. The phase angle $\theta_{\mathbf{n},\lambda}$ is assumed here to be a random variable that takes on values between 0 and 2π with uniform probability; $\theta_{\mathbf{n},\lambda}$ is assumed to be independently distributed for each value of $\mathbf{n}$ and λ.

As described in Ref. 26 (also see Ref. 28), by writing

$$\cos(\mathbf{k}\cdot\mathbf{x}-\omega t+\theta_{\mathbf{n},\lambda})=\tfrac{1}{2}[a_{\mathbf{n},\lambda}\,e^{-i\omega t+i\mathbf{k}\cdot\mathbf{x}}+a^{*}_{\mathbf{n},\lambda}e^{i\omega t-i\mathbf{k}\cdot\mathbf{x}}] \tag{11}$$

where

$$a_{\mathbf{n},\lambda}=e^{i\theta_{\mathbf{n},\lambda}} \tag{12}$$

we can write Eqs. (5) and (6) in a form analogous to the fields in QED, where $a_{\mathbf{n},\lambda}$ and $a^{*}_{\mathbf{n},\lambda}$ have a close connection to the photon annihilation and creation operators in QED. From the described probability distribution for $\theta_{\mathbf{n},\lambda}$ we obtain

$$\langle a_{\mathbf{n}_1,\lambda_1}a^{*}_{\mathbf{n}_2,\lambda_2}\rangle=\delta_{\mathbf{n}_1,\mathbf{n}_2}\delta_{\lambda_1,\lambda_2} \tag{13}$$

$$\langle a_{\mathbf{n}_1,\lambda_1}a_{\mathbf{n}_2,\lambda_2}\rangle=\langle a^{*}_{\mathbf{n}_1,\lambda_1}a^{*}_{\mathbf{n}_2,\lambda_2}\rangle=0 \tag{14}$$

where the angular brackets represent the operation of taking the expectation value.

To pass over to a Fourier series representation, let

$$a_{\lambda}(\mathbf{k})\equiv\left(\frac{L}{2\pi}\right)^{3/2}a_{\mathbf{n},\lambda} \tag{15}$$

In these new variables, Eq. (14) remains in the same form, but

$$\begin{aligned}\langle a_{\lambda_1}(\mathbf{k}_1)\,a^{*}_{\lambda_2}(\mathbf{k}_2)\rangle&=\left(\frac{L}{2\pi}\right)^{3}\delta_{\mathbf{n}_1,\mathbf{n}_2}\delta_{\lambda_1,\lambda_2}\\&\approx\delta^{3}(\mathbf{k}_1-\mathbf{k}_2)\,\delta_{\lambda_1,\lambda_2}\end{aligned} \tag{16}$$

where the second line becomes an equality in the limit $L\to\infty$.[7]

Now making the usual substitution of

$$\left(\frac{2\pi}{L}\right)^{3}\sum_{\mathbf{n}}\cdots\to\int d^{3}k\cdots \tag{17}$$

[7] More explicitly, consider $G(\mathbf{k}_1,\mathbf{k}_2)=(L/2\pi)^3\,\delta_{\mathbf{n}_1,\mathbf{n}_2}$. Obviously, if $\mathbf{k}_1\neq\mathbf{k}_2$, then $G(\mathbf{k}_1,\mathbf{k}_2)=0$. Secondly, consider any arbitrary volume V_k in $\mathbf{k}$-space such that $\mathbf{k}_2\in V_k$. Then,

$$1=\sum_{\substack{\mathbf{n}_1\text{ such that}\\ \mathbf{k}_1\in V_k}}\delta_{\mathbf{n}_1,\mathbf{n}_2}\approx\int_{V_k}d^{3}k_1\left\{\left(\frac{L}{2\pi}\right)^{3}\delta_{\mathbf{n}_1,\mathbf{n}_2}\right\}$$

where Eq. (17) was used. Hence, $G(\mathbf{k}_1,\mathbf{k}_2)\to\delta^{3}(\mathbf{k}_1-\mathbf{k}_2)$ for $L\to\infty$.

we obtain Eqs. (5) and (6) in the form

$$\mathbf{E}_{ZP}(\mathbf{x},t)=\sum_{\lambda=1,2}\int d^3k h_{ZP}(\omega)\,\hat{\boldsymbol{\varepsilon}}_\lambda(\mathbf{k})\frac{1}{2}\{a_\lambda(\mathbf{k})\,e^{-i\omega t+i\mathbf{k}\cdot\mathbf{x}}+a_\lambda^*(\mathbf{k})\,e^{i\omega t-i\mathbf{k}\cdot\mathbf{x}}\} \tag{18}$$

$$\mathbf{B}_{ZP}(\mathbf{x},t)=\sum_{\lambda=1,2}\int d^3k h_{ZP}(\omega)(\hat{\mathbf{k}}\otimes\hat{\boldsymbol{\varepsilon}}_\lambda(\mathbf{k}))\frac{1}{2}\{a_\lambda(\mathbf{k})e^{-i\omega t+i\mathbf{k}\cdot\mathbf{x}}+a_\lambda^*(\mathbf{k})e^{i\omega t-i\mathbf{k}\cdot\mathbf{x}}\} \tag{19}$$

Equations (15)–(19) correct an annoying existing error in the literature that has been repeated many times by a number of researchers, including myself.[8] Surprisingly, this simple error has apparently never been noted and corrected elsewhere in the literature, which is undoubtedly why it continues to be made. Fortunately, however, the error does not effect previous ensemble average calculations, but it needed to be corrected in order to clarify the present dimensional analysis of Eq. (4).

We can now rewrite Eqs. (18) and (19) in dimensionless coordinates by letting $\mathbf{k}'=l_0\mathbf{k}$ and by writing

$$\mathbf{E}_{ZP}(\mathbf{x},t)=\frac{e}{l_0^2}\sum_{\lambda=1,2}\int d^3k' h'_{ZP}(\omega')\,\hat{\boldsymbol{\varepsilon}}'_\lambda(\mathbf{k}')$$
$$\times\frac{1}{2}\{a'_\lambda(\mathbf{k}')\,e^{-i\omega' t'+i\mathbf{k}'\cdot\mathbf{x}'}+a'^*_\lambda(\mathbf{k}')\,e^{i\omega' t'-i\mathbf{k}'\cdot\mathbf{x}'}\} \tag{20}$$

[8] The error being referred to was apparently first made in Ref. 5, and has been repeated ever since in Boyer's work, my own work, as well as in the work of several others (see, for example, Refs. 9, 21, and 22). To clarify the situation, the expressions used in the past for the zero-point fields of

$$\mathbf{E}_{ZP}(\mathbf{x},t)=\sum_\lambda\int d^3k\left[\frac{\hbar\omega}{2\pi^2}\right]^{1/2}\hat{\boldsymbol{\varepsilon}}_\lambda(\mathbf{k})\cos(\mathbf{k}\cdot\mathbf{x}-\omega t+\theta_\lambda(\mathbf{k})) \tag{a}$$

$$\mathbf{B}_{ZP}(\mathbf{x},t)=\sum_\lambda\int d^3k\left[\frac{\hbar\omega}{2\pi^2}\right]^{1/2}(\hat{\mathbf{k}}\otimes\hat{\boldsymbol{\varepsilon}}_\lambda(\mathbf{k}))\cos(\mathbf{k}\cdot\mathbf{x}-\omega t+\theta_\lambda(\mathbf{k})) \tag{b}$$

are incorrect, as can be seen immediately by noting the incorrect dimensions associated with the above fields. Equations (a) and (b) above should be replaced by Eqs. (18) and (19). Likewise, expressions used in the past for ensemble averages involving the phases, such as

$$\langle\cos(A+\theta_{\lambda_1}(\mathbf{k}_1))\cos(B+\theta_{\lambda_2}(\mathbf{k}_2))\rangle=\delta_{\lambda_1,\lambda_2}\delta^3(\mathbf{k}_2-\mathbf{k}_1)\,\tfrac{1}{2}\cos(B-A) \tag{c}$$

are also incorrect, as can also be seen immediately by noting that the LHS of Eq. (c) is dimensionless, while the RHS has the dimensions of (length)^3 due to $\delta^3(\mathbf{k}_2-\mathbf{k}_1)$. Instead, the transformation of Eq. (15) should be used, so that Eq. (16) is obtained. Fortunately, the above two errors cancel each other out when computing ensemble averages involving products of incident radiation fields.

and similarly for $\mathbf{B}_{ZP}(\mathbf{x}, t)$, where $\hat{\varepsilon}'_\lambda(\mathbf{k}') = \hat{\varepsilon}_\lambda(\mathbf{k})$, and

$$a'_\lambda(\mathbf{k}') = \frac{1}{l_0^{3/2}} a_\lambda(\mathbf{k}) \tag{21}$$

$$h'_{ZP}(\omega') = \left(\frac{l_0}{e^2}\right)^{1/2} h_{ZP}(\omega)$$

$$= \frac{1}{\alpha^{1/2}} \left(\frac{\omega'}{2\pi^2}\right)^{1/2} \tag{22}$$

In Eq. (22), the fine structure constant α appears. Note that

$$\langle a'_{\lambda_1}(\mathbf{k}'_1)\, a'^{*}_{\lambda_2}(\mathbf{k}'_2)\rangle = \delta^3(\mathbf{k}'_1 - \mathbf{k}'_2)\, \delta_{\lambda_1,\lambda_2} \tag{23}$$

If we combine the above results, Eq. (4) can now be written in dimensionless coordinates,[9] with $\gamma = (1 - |\dot{\mathbf{z}}'|^2)^{-1/2}$:

$$\frac{d}{dt'}(\gamma \dot{\mathbf{z}}') = \frac{\left(\frac{e^2}{mc^2}\right)}{l_0} \left\{ \frac{2}{3} \frac{d}{dt'} [\gamma^2 \ddot{\mathbf{z}}' + \gamma^4 \dot{\mathbf{z}}' (\dot{\mathbf{z}}' \cdot \ddot{\mathbf{z}}')] \right.$$

$$- \frac{2}{3} [\gamma^4 (\ddot{\mathbf{z}}')^2 + \gamma^6 (\dot{\mathbf{z}}' \cdot \ddot{\mathbf{z}}')^2] \dot{\mathbf{z}}' - \frac{\mathbf{z}'(t')}{|\mathbf{z}'(t')|^3}$$

$$- \sum_\lambda \int d^3k' h'_{ZP}(\omega')\, \hat{\boldsymbol{\varepsilon}}'_\lambda(\mathbf{k}') \frac{1}{2} [a'_\lambda(\mathbf{k}')\, e^{-i\omega' t' + i\mathbf{k}' \cdot \mathbf{z}'} + \text{c.c.}]$$

$$\left. - \dot{\mathbf{z}}' \otimes \sum_\lambda \int d^3k' h'_{ZP}(\omega') (\hat{\mathbf{k}}' \otimes \hat{\boldsymbol{\varepsilon}}'_\lambda(\mathbf{k}')) \frac{1}{2} [a'_\lambda(\mathbf{k}')\, e^{-i\omega' t' + i\mathbf{k}' \cdot \mathbf{z}'} + \text{c.c.}] \right\} \tag{24}$$

Thus, two dimensionless parameters appear in Eq. (24): $(e^2/mc^2)/l_0$ and α (in $h'_{ZP}(\omega')$). Since the only length given to us prior to solving this problem is the length r_e associated with the material properties of the $-e$

[9] Note that the Fourier series form of $\mathbf{E}_{ZP}$ and $\mathbf{B}_{ZP}$ in Eqs. (5) and (6) can certainly still be used here; however, an additional dimensionless parameter of $(l_0/L)^{3/2}$ will appear, which is taken to be infinitesimally small here.

charge, then $l_0 = r_e = e^2/mc^2$ seems a natural choice to make. Nevertheless, there are other choices that one might want to consider, such as $l_0 = \lambda_c = r_e/\alpha$ or $l_0 = r_B = r_e/\alpha^2$. Regardless of which of these choices is made, the above equation of motion for a classical hydrogen atom becomes parametrized by only one constant: namely, the fine structure constant α, which then appears in $(e^2/mc^2)/l_0$, as well as in the $(1/\alpha)^{1/2}$ factor in the $h'_{ZP}(\omega')$ part of the zero-point field terms.

If perturbation methods are appropriate for analyzing the solution to Eq. (24), then clearly these methods should be based on the magnitude of α, since α is the single dimensionless parameter remaining in the problem. In the past, perturbation methods have been used by researchers when trying to deduce the stochastic behavior of a charged point particle in a nonlinear force field, while acted upon by classical electromagnetic random radiation. The above hydrogen atom problem was one of these attempted nonlinear force problems.[12,14,16,17,19] The assumptions made by these researchers have generally been that[10] (a) the incident radiation force terms depended on Ω, where Ω was some small dimensionless parameter that was usually stated as being proportional to e, (b) the radiation reaction terms depended on Ω^2, and (c) the binding force term could be treated in the perturbation analysis as being independent of Ω. Thus, the terms (a) and (b) were treated as though they were perturbation terms in the equation of motion.

Certainly Eq. (24), as it stands, does not follow the assumptions just mentioned, regardless of the choice for l_0. Moreover, suppose the choice is not made that ct_0 and l_0 are equal, but rather that l_0 and ct_0 differ by some power of α. Then neither the radiation reaction terms nor the ZP force terms will be simply proportional to some power of α, but will depend upon α in a more complicated manner. Consequently, assumptions (a) and (b) cannot possibly be satisfied under this condition.

In order to clarify fully this point, one particular choice of normalization units is worth mentioning. If $l_0 = r_B$ and $ct_0 = r_B/\alpha$, then the equation of motion comes closest to following the assumptions of (a, b, c), but with $\Omega = \alpha^{3/2}$. To see this result quickly from Eq. (24), first let $l_0 = r_B/\alpha$ $(= ct_0)$, and then let $\mathbf{z}' = \alpha \mathbf{z}''$. One obtains

[10] References 8, 11–16, 19–21, and 23 all use an "approximate" Fokker–Planck equation approach. Clearly, Refs. 13, 21, and 23 use precisely the classification (a, b, c) of the perturbation terms as described here. Although not all of the other references use quite this classification, supposedly all the methods are equivalent in the small charge limit: see Ref. (12b) and, in particular, the unpublished work by Pesquera and Claverie cited in Ref. 21 as Ref. 19.

$$\frac{d}{dt'}\left(\gamma\frac{d\mathbf{z}''}{dt'}\right)=\frac{-\mathbf{z}''}{|\mathbf{z}''|^3}-\alpha^{3/2}\sum_{\lambda}\int d^3k'\left(\frac{\hbar\omega'}{2\pi^2}\right)^{1/2}\hat{\boldsymbol{\varepsilon}}'_{\lambda}(\mathbf{k}')$$

$$\times\frac{1}{2}\{a'_{\lambda}(\mathbf{k}')\exp[-i(\omega't'-\alpha\mathbf{k}'\cdot\mathbf{z}'')]+\text{c.c.}\}$$

$$-\alpha^{5/2}\frac{d\mathbf{z}''}{dt'}\otimes\sum_{\lambda}\int d^3k'\left(\frac{\hbar\omega'}{2\pi^2}\right)^{1/2}(\hat{\mathbf{k}}'\otimes\hat{\boldsymbol{\varepsilon}}'_{\lambda}(\mathbf{k}'))$$

$$\times\frac{1}{2}\{a'_{\lambda}(\mathbf{k}')\exp[-i(\omega't'-\alpha\mathbf{k}'\cdot\mathbf{z}'')]+\text{c.c.}\}$$

$$+\alpha^3\frac{2}{3}\frac{d}{dt''}\left[\gamma^2\frac{d^2\mathbf{z}''}{dt'^2}+\gamma^4\alpha^2\frac{d\mathbf{z}''}{dt'}\left(\frac{d\mathbf{z}''}{dt'}\cdot\frac{d^2\mathbf{z}''}{dt'^2}\right)\right]$$

$$-\alpha^5\frac{2}{3}\left[\gamma^4\left(\frac{d^2\mathbf{z}''}{dt'^2}\right)^2+\gamma^6\alpha^2\left(\frac{d\mathbf{z}''}{dt'}\cdot\frac{d^2\mathbf{z}''}{dt'^2}\right)^2\right]\frac{d\mathbf{z}''}{dt'} \tag{25}$$

where

$$\gamma=\left[1-\alpha^2\left|\frac{d\mathbf{z}''}{dt'}\right|^2\right]^{-1/2} \tag{26}$$

$$\mathbf{z}''=\frac{\mathbf{z}}{r_B} \tag{27}$$

$$t'=\frac{t}{((r_B/c)/\alpha)} \tag{28}$$

Note that although the form of the resulting equation involving $\mathbf{z}''$ and t' is close to the assumed (a, b, c) form, there are significant differences. For an explicit comparison, consider the nonrelativistic equation of motion assumed in Ref. 14 (Eq. 3.1) for tackling this classical hydrogen problem:

$$m\ddot{\mathbf{z}}=-e^2\frac{\mathbf{z}}{|\mathbf{z}|^3}-e\mathbf{E}_{\text{ZP}}(0,t)+\frac{2}{3}\frac{e^2}{c^3}\dddot{\mathbf{z}} \tag{29}$$

Substituting in Eq. (18), and using the same normalization units of the Bohr radius and the inverse of the Bohr angular frequency as in Eqs. (27) and (28), we obtain

$$\frac{d^2\mathbf{z}''}{dt'^2}=-\frac{\mathbf{z}''}{|\mathbf{z}''|^3}+\alpha^3\frac{d^3\mathbf{z}''}{dt'^3}$$

$$-\alpha^{3/2}\sum_{\lambda}\int d^3k'\left(\frac{\hbar\omega'}{2\pi^2}\right)^{1/2}\boldsymbol{\varepsilon}'_{\lambda}(\mathbf{k}')\frac{1}{2}\{a'_{\lambda}(\mathbf{k}')\,e^{-i\omega't'}+\text{c.c.}\} \tag{30}$$

Thus, this assumed equation *does* follow assumptions (a, b, c), provided Ω is identified as $\alpha^{3/2}$. However, upon comparing Eq. (30) with

the above Lorentz–Dirac equation of Eq. (25), where both equations are normalized in the same way, we see that Eq. (30) is missing terms that cannot be justified simply on the basis of the magnitude of α. Indeed, the second line of Eq. (25) is of order $\alpha^{5/2}$, but it has been neglected in Eq. (30) despite the inclusion in Eq. (30) of the radiation reaction term of order α^3. Likewise, if we expand γ in the term on the LHS of Eq. (25) via

$$\gamma = 1 + \frac{1}{2}\alpha^2 \left|\frac{d\mathbf{z}''}{dt'}\right|^2 + \frac{3}{8}\alpha^4 \left|\frac{d\mathbf{z}''}{dt'}\right|^4 + \cdots \tag{31}$$

we see that the first-order correction term is of order α^2, yet this term is missing in Eq. (30).

Moreover, it does not seem at all obvious that one can justify the dipole approximation in Eq. (30) consisting of neglecting the $\alpha\mathbf{k}' \cdot \mathbf{z}''$ term in the argument of the ZP fields; the approximation may very well produce an error of significance. For example, if we naively rewrite the last term in the first line in Eq. (25) by making a Taylor's expansion in the spatial argument, then we obtain

$$-\alpha^{3/2} \sum_{\lambda} \int d^3k' \left(\frac{\hbar\omega'}{2\pi^2}\right)^{1/2} \boldsymbol{\varepsilon}'_\lambda(\mathbf{k}') \frac{1}{2} \{a'_\lambda(\mathbf{k}')\, e^{-i\omega' t'} [1 + i\alpha\mathbf{k}' \cdot \mathbf{z}'' + \cdots] + \text{c.c.}\}$$

The first term in the expansion represents the dipole approximation and appears with an overall factor of $\alpha^{3/2}$, while the next term in the expansion contains a factor of $\alpha^{5/2}$. Neglecting this $\alpha^{5/2}$ term in Eq. (30) does not appear consistent with retaining the radiation reaction term of order α^3.

Unfortunately, previous research[12,14,16,19] on the classical hydrogen atom in classical electromagnetic ZP radiation has been based on an equation of motion that is either identical or else very similar in form to Eq. (29). As can be seen from the above comments, Eq. (29) certainly appears to be an inconsistent and inaccurate approximation to the full equation of motion. Consequently, the previous analysis reported[16] that the classical hydrogen atom in ZP radiation will result in the self-ionization of the electron must be viewed as being unsupported at the present time.

Before leaving our discussion of the hydrogen atom to make a few comments about how the above ideas carry over to other atomic systems, let us briefly discuss how Eq. (24) generalizes to the case where the temperature T is nonvanishing.

When $T \neq 0$, an additional parameter enters into Eq. (24). Here, zero-point plus Planckian (ZPP) radiation must be considered,[11] so the fields

[11] See, for example, Refs. 26 or 28.

$\mathbf{E}_{ZP}$ and $\mathbf{B}_{ZP}$ need to be changed to $\mathbf{E}_{ZPP}$ and $\mathbf{B}_{ZPP}$. Equations (20) and (24) then change by simply replacing $h'_{ZP}(\omega')$ by

$$h'_{ZPP}(\omega') = \left(\frac{l_0}{e^2}\right)^{1/2} \left[\frac{\hbar\omega}{2\pi^2} \coth\left(\frac{\hbar\omega}{2kT}\right)\right]^{1/2}$$
$$= \frac{1}{\alpha^{1/2}} \left(\frac{\omega'}{2\pi^2}\right)^{1/2} \left[\coth\left(\frac{\omega'\beta}{2\alpha}\right)\right]^{1/2} \tag{32}$$

where

$$\beta \equiv \left[\frac{e^2/l_0}{kT}\right] \tag{33}$$

Thus, for the ZPP case, an extra factor of

$$\left[\coth\left(\frac{\omega'\beta}{2\alpha}\right)\right]^{1/2}$$

must be included in the integrands of the last two terms in Eq. (24). Hence, β becomes the additional dimensionless factor that parametrizes the equation of motion in the nonzero-temperature case. If $l_0 = (e^2/mc^2)(1/\alpha^n)$, for some value of n, then $\beta = (mc^2/kT)\,\alpha^n$. Hence, in a very natural way, the ratio of the rest energy to kT becomes the additional parameter of importance, aside from α, for the nonzero temperature case.

For completeness and for future use, it is of some interest to indicate how Eq. (24) changes if the incident classical electromagnetic radiation is assumed to have the traditional Rayleigh–Jeans spectrum.[12] Now Eqs. (20) and (24) change by replacing $h'_{ZP}(\omega')$ by

$$h_{RJ}(\omega') = \left(\frac{l_0}{e^2}\right)^{1/2} \left[\frac{kT}{\pi^2}\right]^{1/2} = \frac{1}{\pi} \frac{1}{\beta^{1/2}} \tag{34}$$

Hence, β enters in here as well, which will be helpful in making comparisons between the ZPP and RJ cases for the physical behavior of the oscillating charge. Of course, here the only reasonable choice for l_0 is where $l_0 = r_e$, since α cannot enter the problem in the RJ case. For this choice, $\beta = (mc^2/kT)$.

[12] For an interesting recent comparison of the physical behavior of (1) a classical anharmonic oscillator in the presence of ZPP radiation, as well as RJ radiation, to (2) a quantum mechanical anharmonic oscillator, see Ref. 25. Note, however, that the assumption is made here that the binding force does not depend upon the charge.

3. CONCLUDING REMARKS

If classical electrodynamics with classical electromagnetic ZP radiation does correctly describe nature at some level of physics, then the most likely level for this connection is within the domain of atomic and molecular physics. Here, electrodynamic interactions are clearly dominant. At physical dimensions of smaller and larger scale than the atomic domain, the strong, weak, and gravitational interactions become important. No serious attempt has yet been made in the literature to extend stochastic electrodynamics to these areas of physics.

The interactions in atomic and molecular physics occur between molecules, atoms, electrons and nuclei, electrons and positrons, etc. These interactions obey the well-defined rules of electromagnetic theory. Any comparisons between (1) the predicted behavior of the theory of classical electrodynamics with classical electromagnetic ZP radiation, and (2) the observed behavior of real atomic systems must pay strict attention to this interaction; any perturbation analysis must be based closely upon the magnitude of the physical parameters governing the interaction.

In this article, the classical hydrogen atom in classical electromagnetic ZP radiation was shown to be parametrized in a very natural way by only one dimensionless parameter: $\alpha = e^2/\hbar c$. If a more complicated atomic system is considered, such as a many-electron atom, then the Lorentz force due to each electron acting on the other ones must be included in the Lorentz-Dirac equation of motion for each electron. These Lorentz forces involve the retarded electromagnetic fields associated with each electron:

$$\mathbf{E}(\mathbf{x}, t) = (-e)\left[\frac{(\mathbf{R} - R\boldsymbol{\beta})(1-\beta^2)}{(R - \mathbf{R}\cdot\boldsymbol{\beta})^3}\right]_{t_r} + \frac{(-e)}{c}\left[\frac{\mathbf{R}\otimes\{(\mathbf{R} - R\boldsymbol{\beta})\otimes\dot{\boldsymbol{\beta}}\}}{(R - \mathbf{R}\cdot\boldsymbol{\beta})^3}\right]_{t_r} \tag{35}$$

$$\mathbf{B}(\mathbf{x}, t) = \left[\frac{\mathbf{R}}{R}\right]_{t_r} \otimes \mathbf{E}(\mathbf{x}, t) \tag{36}$$

where $\mathbf{R}(t) \equiv \mathbf{x} - \mathbf{z}(t)$, $\boldsymbol{\beta} \equiv \dot{\mathbf{z}}/c$, and $t_r \equiv t - R$. Using the same reasoning as for the one-electron atom, one can show that α, as well as Z, where $+Ze$ is the charge on the nucleus, arise as the sole dimensionless parameters governing the n-electron atom at temperature $T = 0$.

Unfortunately, the perturbation analyses that have been carried out in the past on the stochastic behavior of oscillating bound classical charges, have not treated the binding force as being dependent upon the charge, or at least not in a consistent way. Consequently, the conclusions of these studies do not apply to molecular and atomic systems, where the binding forces are completely electromagnetic in origin. Indeed, past research in

SED has largely missed the connection that the fine structure constant α falls out as the fundamental parameter of importance for such systems.

To see what can go wrong if the above idea is not taken into account, consider the following system that has been considered numerous times within SED: namely, a point charge oscillating about the center of an arbitrarily assumed anharmonic oscillator potential. Assume this potential is *not* electromagnetic in origin. Now the assumptions discussed in Sec. 2 can be made, where the incident radiation force terms and the radiation reaction terms are proportional to e^1 and e^2, respectively, while the binding force is independent of e. A perturbation expansion scheme in the charge e then results that conflicts with what should be used for actual atomic systems. The predicted result from this analysis may very well be (1) a Boltzmann equilibrium distribution in phase space for the oscillating particle, and (2) an equilibrium incident radiation field with a RJ spectrum, precisely as has been envisioned by physicists for years, and as has been recently analyzed in detail by BPS.[21,23]

However, the anharmonic oscillators that arise in atomic systems, such as the diatomic molecule, and the ones that would be taken seriously within the domain of QED, are ones that have binding potentials due to electromagnetic charges. Consequently, such a system should *not* be treated via the perturbation scheme indicated above. Consequently, the conclusions mentioned above that have been reached by other researchers on the equilibrium behavior of oscillating classical charged particles and classical electromagnetic incident radiation, are *not applicable* for atomic systems, or, at the very least, have not yet been demonstrated to hold for such systems.

Hence, the correct equilibrium behavior for such classical electrodynamic systems still remains an open question in physics. Only upon resolving this question can a close comparison be carried out between SED and the observed behavior of atomic and molecular physical systems.

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