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CONFIRMATION THEORY AND INDISPENSABILITY*

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ABSTRACT. In this paper I examine Quine's indispensability argument, with particular emphasis on what is meant by 'indispensable'. I show that confirmation theory plays a crucial role in answering this question and that once indispensability is understood in this light, Quine's argument is seen to be a serious stumbling block for any scientific realist wishing to maintain an anti-realist position with regard to mathematical entities.

1. INTRODUCTION

For some time now ontological discussions in the philosophy of mathematics have been dominated by three arguments. The first of these is the Benacerraf objection to the natural numbers being identified with sets (since it seems arbitrary which sets one chooses to identify them with). The second argument is the well known epistemological problem for Platonism. That is, if mathematical objects such as sets, functions and numbers have mind independent, though admittedly abstract, existence, how is it that we have knowledge of them, given a causal theory of knowledge?[1] The third argument is the Quinean argument[2] that mathematical entities are indispensable to our best physical theories[3] and therefore share the ontological status of scientific entities. The first two of these arguments are directed against Platonism, of some form or another, while the third is usually taken to be an argument *for* Platonism, perhaps, as Hartry Field suggests, "the only non-question-begging" argument for Platonism. ([10], 4)

In this paper I examine the indispensability argument and, in particular, the notion of 'indispensability' itself. I will argue (i) that dispensability cannot be "cashed out" purely in terms of eliminability, as is often assumed, and that (ii) if an entity is dispensable to a theory, all talk of that entity must be eliminable from the theory

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and the resulting theory be preferable to the original.[4] Furthermore, once this is appreciated, the indispensability argument is seen to be a serious stumbling block for any scientific realist wishing to maintain an anti-realist position with regard to mathematical entities.

By way of motivation, I should mention the work of Hartry Field.[5] Field has proposed a bold attack on Quine's argument, claiming that mathematics *is* dispensable to our best physical theories – mathematics is just a useful tool. He adopts a "fictional" account of mathematics in which all the usually accepted sentences of mathematics are literally false, but true "in the story" of accepted mathematics. He then explains why it is permissible to use this false body of sentences in physical theories in terms of mathematics' *conservativeness*. This is, very roughly, that mathematics preserves truth in the nominalist theories in which it is used, although, of course, on Field's view the mathematics itself is false.

The ambitious part of his program is to show that mathematics is in fact dispensable. He begins this task by giving a nominalistic treatment of Newtonian gravitational theory, thus allegedly showing that mathematical entities are not essential to this theory. This is the point that motivates the present paper. He does not give a clear account of what he takes 'dispensable' to mean in this context. He clearly does not take it to mean simply eliminable,[6] and yet most of his work is devoted to showing that mathematical entities are eliminable from physical theories. This failure to explicate what is meant by 'indispensable' in Quine's argument allows programs such as Field's to look more appealing than they perhaps ought.[7] This present paper, and its discussion of indispensability, should be understood in the context of this debate.

In the next section I will outline Quine's indispensability argument. After this I will look at what we mean by 'indispensable' and the role confirmation theory plays in indispensability decisions, followed in the next section, by a presentation of some examples from the history of science that suggest that mathematics is indeed indispensable to our best scientific theories. Finally, a brief conclusion in which the importance of my thesis is stressed for certain versions of nominalism.

2. THE INDISPENSABILITY ARGUMENT

Quine's indispensability argument may be stated as follows: We have good reason to believe our best scientific theories and there are no grounds on which to differentiate scientific entities from mathematical entities, so we have good reason to believe in mathematical entities, since they, like the relevant scientific entities, are indispensable to the theories in which they occur.[8] Furthermore, it is exactly the same evidence that confirms the scientific theory as a whole, that confirms the mathematical portion of the theory and hence the mathematical entities contained therein.

A couple of comments are warranted here. Firstly, the reason that Quine claims we cannot differentiate between theoretical scientific entities and mathematical entities in a theory is because theories must be considered holistically, hence his view that theories are "seamless". In Quine's famous words "... our statements about the external world face the tribunal of sense experience not individually but only as a corporate body."([24], 41) Secondly, the reason we should believe in the entities postulated by our best theories is due to Quine's belief in naturalism which "sees natural science as an inquiry into reality, fallible and corrigible but not answerable to any supra-scientific tribunal, and not in need of any justification beyond observation and the hypothetico-deductive method."([26], 72) Thus naturalism justifies belief in only those entities postulated by our best scientific theories. We see then, that there are three premises on which this argument is based: *holism*, *naturalism*, and *indispensability*. This suggests three ways in which someone might resist the argument. The first way is to reject holism. For example, Penelope Maddy's recent criticisms of indispensability theory[9] are largely concerns about holism. She argues that mathematical and scientific practices don't support the kind of holism that the indispensability argument requires. In short, she claims that scientific practice doesn't warrant belief in *all* the entities of our best scientific theories.

The second way someone might resist Quine's argument is to reject naturalism, at least as Quine construes it. Such a person would claim that we should not necessarily look to science for our ontological commitments. Thus they may agree that mathematical entities are on an ontological par with other entities postulated by science, but these too lack mind independent existence. That is, they may adopt some general anti-realist position. Bas van Fraassen's constructive empiricism can be construed as such a rejection of Quinean naturalism.[10]

While I acknowledge that the two ways of disarming the Quinean argument I have just mentioned are "live" options and will not be significantly weakened by what is to follow, there is a third approach. This is to deny the indispensability of mathematics to physical science. Such an attack would typically be made by a scientific realist who is a nominalist of some kind. As I have already mentioned, Hartry Field proposes just such an approach. In order to understand exactly what is required of a program like Field's, or indeed to understand the force of the indispensability argument, we must first clear up what we mean by 'indispensable'. I shall discuss this in detail in the following section.

3. WHAT IS IT TO BE INDISPENSABLE?

In order to answer the question in this section's title I will consider a case where there should be no disagreement about the dispensability of the entity in question. I shall then analyse this case to see what leads us to conclude that the entity in question is dispensable.

Consider an empirically adequate and consistent theory Γ and let 'ξ' be the name of some entity neither mentioned nor predicted by Γ. Clearly we can construct a new theory Γ^+ from Γ by simply adding the sentence 'ξ exists' to Γ. Note, however, that ξ plays no role in the theory Γ^+, it is merely predicted by it. I propose that there should be no disagreement here when I say that ξ is dispensable to Γ^+, but let us investigate why this is so.

On one interpretation of 'dispensable' we could argue that ξ is not dispensable since its removal from Γ^+ results in a different theory, namely, Γ. This is not a very helpful interpretation though, since *all* entities are indispensable to the theories in which they occur under this reading. Another interpretation of 'dispensable' might be that ξ is dispensable to Γ^+ since there exists another theory, Γ, with the same empirical adequacy as Γ^+ in which ξ does not occur. This interpretation can also be seen to be inadequate since it may turn out that *no* theoretical entities are indispensable under this reading. This result follows from Craig's theorem.[11] If the vocabulary of the theory can be partitioned in the way that Craig's theorem requires (cf. footnote 6), then the theory can be reaxiomatised so that any given theoretical entity is eliminated.[12] I claim, therefore, that this interpretation of 'dispensable' is unacceptable since it fails to account for why ξ in particular is dispensable.

This leads me to the following explication of 'dispensable':

Definition. An entity is dispensable to a theory if there exists a modification of that theory resulting in a second theory with exactly the same observational consequences as the first, in which the entity in question is neither mentioned nor predicted. Furthermore, the second theory must be preferable to the first.

In the preceding example then, ξ is dispensable since Γ makes no mention of ξ and Γ is preferable to Γ^+ in that the former has less ontological commitment than the latter, all other things being equal (and assuming, of course, that less ontological commitment is better).[13]

Now it might be argued that on this reading once again every theoretical entity is dispensable, since by Craig's theorem we can eliminate any reference to any entity and the resulting theory will be better, since it doesn't have ontological commitment to the entity in question. This argument is flawed though, since the reason for preferring one theory over another is a complicated question – it is not simply a matter of empirical adequacy combined with a principle of ontological parsimony. In the next section I will discuss some aspects of confirmation theory and what role it plays in indispensability decisions.

4. THE ROLE OF CONFIRMATION THEORY

Confirmation theory is the study of those principles which guide scientific reasoning other than reasoning of the deductive kind. In particular, it will help us in deciding whether one theory is better than another by giving us some desiderata for "good theories".[14]

Firstly and foremost, a "good theory" must be empirically adequate, that is, it must agree with all (or at least most) observation. Secondly, it must be consistent, both internally and with other major theories. This is not the whole story though. As we have already seen Γ and Γ^+ have the same degree of empirical adequacy and consistency (by construction) and yet we are inclined to prefer the former over the latter. I am in agreement with many authors here[15] that amongst the additional features we require are the following:

(i) **Simplicity:** Given two theories with the same empirical adequacy, we generally prefer that theory which is simpler in both its statement and in its ontological commitments. For example, Einstein, in his formulation of the special theory of relativity, refused to admit an undetectable luminiferous ether, as some rival theories did, to jointly explain the propagation of electromagnetic radiation through apparently empty space and the failure of the Michelson-Morley experiment to detect such an ether.([9], 38)

(ii) **Unificatory/Explanatory Power:** Philip Kitcher argues, rather convincingly, in [16] for scientific explanation being unification. That is, accounting for a maximum of observed phenomena with a minimum of theoretical devices. Whether or not you accept Kitcher's account, we still require that a theory not simply predict certain phenomena, but explain why such predictions are expected. Furthermore, the best theories do so with a minimum of theoretical devices. For example, the success of Newtonian gravitational theory was in no small way due to its ability to explain such diverse phenomena as tides, planetary orbits and projectile motion (among other things) from a small stock of theoretical "machinery".

(iii) **Boldness:** We expect our best theories not to simply predict everyday phenomena, but to make bold predictions of novel entities and phenomena which lead to future research. The prediction of gravitational waves by general relativity is an example of such a bold prediction that is still being actively investigated.

(iv) **Formal Elegance:** This is perhaps the hardest feature to characterise (and no doubt the most contentious). However, there is at least *some* sense in which our best theories have aesthetic appeal. For instance, it may well be on the grounds of formal elegance that we rule out *ad hoc* modifications of a failing theory.

I will not argue in detail for each of these, except to say that despite the notorious difficulties involved in explicating what we mean by such terms as ‘simplicity’ and ‘elegance’, we do look for such virtues in our best theories. Otherwise we could never choose between two theories such as Γ and Γ^+. I do not claim that this list is comprehensive nor do I claim that it is minimal,[16] I merely claim that these sorts of criteria are typically appealed to in the literature to distinguish “good theories”, and I have no objection to such appeals.

In the light of the preceding discussion then, we see that a claim that an entity is dispensable is a claim that a modification of the theory in which it is posited can be made in such a way as to eliminate the entity in question and result in a theory which is better overall in terms of simplicity, elegance and so on. Thus we see that the argument I presented at the end of the previous section that *any* theoretical entity is dispensable, does indeed fail, as I claimed. This is because in most cases the benefit of ontological simplicity obtained by the elimination of the entity in question will be more than offset by losses in other areas.

While it seems reasonable to suppose that the elimination of talk of physical entities such as electrons, from the body of scientific theory, would cause an overall reduction in the previously described virtues of that theory, it is not so clear that the elimination of talk of mathematical entities would have the same impact. Someone might argue that mathematics is certainly a very effective language for the expression of scientific ideas, in that it simplifies the calculations and statement of much of science, but to do so at the expense of introducing into one’s ontology the whole gamut of mathematical entities is just not a good deal.

One response to this is to simply deny that it is a high price at all. After all, a powerful and efficient language is the cornerstone of any good theory. If you have to introduce a few more entities into the theory to get it, then so be it. Although I have considerable sympathy with this response, in this paper I wish to pursue a different and, I think, more convincing line. I wish to argue that mathematics plays an *active* role in many of the theories which make use of it. That is, it is not just a tool which makes calculations easier or simplifies the statement of the theory – it makes important contributions to all of the desiderata of good theories I mentioned earlier.[17]

My strategy from here is to show that there is good reason to believe that the mathematised version of a theory is more “virtuous” than the original theory, and so there is good reason to believe mathematics is indispensable, in the sense I have previously outlined, to our best physical theories. I shall demonstrate this in the next section by appealing to a number of examples of physical theories and showing how the mathematised theory is seemingly capable of more than what one would expect from a nominalised version. Notice that it is not necessary for me to show that this is the case for *all* our best physical theories, as the indispensability argument goes through in case mathematics is indispensable to some non-empty subset of our best physical theories.

I also note here that I am not proposing to demonstrate beyond all doubt the indispensability of mathematics in the cases I consider, merely to suggest that if these theories were stripped of their mathematical content it seems that they would lose much of their appeal. If I succeed, the burden of proof will lie with anyone who claims that mathematics *is* dispensable, for they must show, firstly, how it is possible to remove all commitment to mathematical entities from all physical theories and, secondly, how this removal does not result in a reduction of virtue of these theories. This problem will be particularly evident in the cases I consider in the next section.

5. THE ROLE OF MATHEMATICS IN PHYSICAL THEORIES

In his book, *Philosophical Naturalism*, David Papineau suggests that “the incorporation of pure mathematics into scientific theories [...] might make it easier to do calculations, but [...] receives no backing from principles of scientific theory choice.” ([21], 196) In this section I will show that this view is, at the very least, extremely controversial. I will demonstrate this by appealing to a number of examples in which mathematics contributes to the unification and boldness of the physical theory in question, and therefore *is* supported by well recognised principles of scientific theory choice.

5.1. *The Complex Numbers*

The first example is the complex numbers. I will discuss how the introduction of these was responsible for a great deal of unification within both pure mathematics and in more applied fields such as differential equations in physics. In particular, I wish to look at the way in which the complex numbers unify the exponential function and the trigonometric functions, and how this has direct applications in the study of second-order ordinary differential equations which arise in almost all branches of science, including fluid mechanics, heat conduction and population dynamics.

We begin by introducing the number $i = \sqrt{-1}$ and defining a complex variable $z = x + yi$ where x and y are real. Once we have extended the operations '$+$' and '$\cdot$' and the relation '$=$' from the reals to the complex in the natural way, we can introduce complex exponentiation via the following equation known as Euler's formula:

$$e^{i\theta} = \cos\theta + i\sin\theta \qquad \theta \in \mathbb{R}.$$

From this we can define the trigonometric functions for a complex variable z as

$$\sin z = \frac{e^{iz} - e^{-iz}}{2i} \text{ and } \cos z = \frac{e^{iz} + e^{-iz}}{2}.$$

The usual real-valued sine and cosine functions are seen to be special cases of the more general definitions. Thus the complex numbers are instrumental in the unification of the trigonometric and the exponential functions. This unification, being within mathematics itself, may seem somewhat irrelevant to the matter at hand, so I shall demonstrate how this unification "flows through" to physics.

Consider the second-order linear homogeneous ordinary differential equation with constant coefficients:

(1) $\quad y'' + y' + y = 0$

where y is a real-valued function of the single real variable x. Equations such as these are solved by considering their characteristic equations, which are quadratics and so, by the fundamental theorem of algebra, always have two (complex) roots (counting multiplicity). In this case the characteristic equation is

$$r^2 + r + 1 = 0$$

which has (complex) roots $r = -\frac{1}{2} \pm i\frac{\sqrt{3}}{2}$. Now the general solution to an equation with unequal roots to its characteristic equation is:

$$(2) \qquad y = c_1 e^{r_1 x} + c_2 e^{r_2 x}$$

where c_1 and c_2 are arbitrary real constants and r_1 and r_2 are the two distinct roots of the characteristic equation.[18] Note that (2) is indifferent as to whether r_1 and r_2 are real or complex. Thus the solution to (1) is given by

$$y = c_1 e^{(-\frac{1}{2}+\frac{\sqrt{3}}{2}i)x} + c_2 e^{(-\frac{1}{2}-\frac{\sqrt{3}}{2}i)x}$$

from which we can obtain the *real* solution

$$y = e^{\frac{-x}{2}} \left(c_1 \cos\left(\frac{\sqrt{3}}{2}x\right) + c_2 \sin\left(\frac{\sqrt{3}}{2}x\right) \right).$$

Thus we see that without the use of complex numbers we would have to treat the equations $y'' - y = 0$, which has real roots to its characteristic equation, quite differently from $y'' + y = 0$, which has complex roots to its characteristic equation. Furthermore, the connection between the exponential function, which is a solution to the first, and the cosine and sine functions, which are solutions to the second, is spelled out via the definitions of trigonometric functions of a complex variable given earlier. This I see as a fine example of the unity which a mathematical theory may bring to both other mathematical theories and also scientific theory generally.

What is more, I take this unity to be not simply an algorithmic unity, that is, a single method for finding solutions to these equations. Rather, I take it that the algorithmic unity arises out of deep structural similarities between the systems portrayed by these equations. For example, if two different physical systems are governed by the same differential equation, it's clear that there is some similarity between these systems, no matter how disparate the systems may seem (just as a red planet and a red dog have something in common). It seems plausible, at least, that this similarity is structural and is captured by the relevant differential equation. Even when the systems are governed by different differential equations, structural similarities may still be revealed in the mathematics. In the case of the two equations in the previous paragraph, the structural similarity of any two systems governed by these equations is revealed by the connection between the equations' respective solutions. Mathematics, because of its abstract nature, is extremely well suited to providing unification in this very important sense.

5.2. *The Dirac Equation*

In the next example I will show that mathematics may contribute to the boldness of theories by playing an important role in the prediction of novel phenomena – in this case the discovery of antimatter.

In classical physics one occasionally comes across solutions to equations which are discarded because they are "non-physical". Examples include negative energy solutions to dynamical systems. Such a situation arose for Paul Dirac in 1928 when he was studying the solutions of the equation of relativistic quantum mechanics which now bears his name. This equation describes the behaviour of electrons and hydrogen atoms, but was found to also describe particles with negative energies. It must have been tempting for Dirac to simply dismiss such solutions as "non-physical", however, strange things are known to occur in quantum mechanics and intuitions about what is "non-physical" are not so clear. Instead, Dirac trusted the mathematics and investigated the possibility of negative energy solutions. In particular, he sought to give an account of why a particle cannot make a transition from a positive energy state to a negative one.

Dirac realised that the Pauli exclusion principle[19] would prevent electrons from dropping back to negative energy states if such states were already occupied by negative energy electrons. Furthermore, if a negative energy electron were raised to a positive energy state, it would leave behind an unoccupied negative energy state. This unoccupied negative energy state would act like a positively charged electron or a "positron". Thus Dirac, by his faith in the mathematical part of relativistic quantum mechanics and his reluctance to discard what looked like non-physical solutions, predicted the positron.[20]

This story is even more remarkable for the fact that Dirac was trying to reconcile quantum mechanics with special relativity by reworking Schrödinger's wave mechanics in terms of particle waves. This point of view is, as we now know, largely mistaken. The proper context for reconciling special relativity and quantum mechanics is via quantum field theory, and yet the mathematical component of Dirac's theory has survived, indeed it is an important part of modern quantum field theory.([30], 120–121) So not only did Dirac's equation play a significant role in predicting a novel entity, despite the relevant solutions seeming non-physical, it did so based largely on false assumptions. It is hard to see how a nominalised version of Dirac's theory would have had the same predictive success.[21]

5.3. *The Lorentz Transformations*

This final example is similar to the previous one in some ways. I will discuss how a set of equations known as the Lorentz transformations were part of a paper written in 1904 by H.A. Lorentz based on some fairly strange assumptions, and yet these same transformations became an integral part of Einstein's special relativity a year later, based on entirely different assumptions. So again we see an example of some mathematical equations surviving the death of the theory that spawned them, thus suggesting that the mathematics is capturing something that the original theory did not.

The "luminiferous ether" was postulated by physicists in the middle of the nineteenth century[22] as the medium through which Maxwell's electromagnetic radiation must be transmitted, since a wave propagating through a vacuum seemed altogether too strange. Indeed, even Maxwell seemed to support such a theory.([2], 39) Furthermore it was proposed that this ether may provide the "absolute rest" frame for Newtonian mechanics. Granted these assumptions, it was then reasonable to assume that the earth should be moving relative to the frame of the ether, so we ought to be able to detect an "ether wind" as a result of this motion.

In 1887 Albert Michelson and Edward Morley conducted an ingenious experiment designed to detect the “ether wind”. This experiment made use of a piece of equipment known as an interferometer, which consisted of two arms of equal length set at right angles to one another along which two beams of light were “raced”. By observing interference patterns (if any) between the two light beams, very small differences in the average velocities (relative to the earth) of the two beams of light could be detected. Indeed, one would expect a difference since, by elementary physics, the beam of light travelling into the ether wind (and back again) should travel slower than the beam travelling across the ether wind. The fact that no difference was ever detected, despite exacting levels of precision and many repeats of the experiment, was one of the great problems for physicists in the latter part of the nineteenth, and early twentieth centuries.

One explanation for the failure of the Michelson-Morley experiment to detect any such velocity difference was offered by George FitzGerald in 1892. FitzGerald proposed that the arm of the interferometer travelling into the ether was shortened by exactly the amount required for the two light beams to take the same time for their respective journeys. This seemingly *ad hoc* idea was given support by Lorentz in his 1895 paper ([17]), and in his 1904 paper ([18]) he offered an explanation for this phenomenon in terms of his theory of electromagnetic forces. In this latter paper, Lorentz gives a mathematical statement of this shortening as a function of the velocity of the interferometer relative to the “stationary frame of reference” of the ether. The resulting equations are now known as the Lorentz transformations. These four equations state that there will be a contraction of the length of the arm of the interferometer travelling into the ether but no corresponding contraction in either of the two directions perpendicular to this, and so, in particular, there will be no shortening of the length of the other arm. The fourth equation was a time-dilation equation. This stated that a clock in motion runs more slowly than a stationary one. Lorentz, however, insisted that there was only one “true” time and that the “local time” introduced was just a mathematical device to simplify Maxwell’s equations for bodies in motion – it had no physical significance. ([2], 71–79)

It is now well known that these equations are an integral part of special relativity. In special relativity, however, the equations are based on much more reasonable assumptions – namely the principle of relativity[23] and the principle of constancy.[24] ([9], 37–38) In fact, it might be argued that in Einstein's theory, the Lorentz transformations are derived by careful attention to what we *mean* by length and time, rather than being motivated by an *ad hoc* defence of the ether.[25]

The point this example illustrates is that whatever merit Lorentz's theory had, and this merit was quite considerable (since his theory predicted the novel phenomenon of the FitzGerald-Lorentz contraction amongst other things) it was surely largely in the transformations themselves, since presumably the underlying theory was false. It is hard to see how a nominalistic version of this theory would have made such predictions, since it is apparent from the above discussion that the predictive success of the theory came from the fact that the extremely abstract nature of mathematics allowed abstraction away from the false underlying principles.[26]

6. CONCLUSION

To sum up then. I have argued that an entity is dispensable to a theory if a *better* theory can be constructed from the first, in which the entity in question plays no part. Confirmation theory was seen to be important here, in that it provided grounds on which to base decisions about which of two competing theories is the better. Thus confirmation theory plays a crucial role in indispensability decisions. Furthermore, it is around this sense of 'dispensable' that Quine's indispensability argument revolves. It follows then, that any critic of the indispensability argument who wishes to deny that mathematics is indispensable to our best physical theories, is obliged to not only give an account of how scientific theories may be constructed without reference to mathematical entities of any kind, but also show that the resulting theory is preferable to the original.

While I admit that I have remained rather vague about the details of how to compare theories, nevertheless, I *have* presented a case for accepting that mathematical entities directly contribute towards qualities such as boldness and unificatory power, which we see as properties of good theories. The above mentioned critic may argue about degrees of unification and boldness and the like, but at the very least they must demonstrate that scientific theory stripped of all its mathematical entities has some degree of virtue comparable to the original theory (if indeed science can be stripped of all its mathematical entities in the first place). It is difficult to see how this could be done, given that we don't, in general, understand why it is that mathematics contributes to the virtue of those portions of science that make use of it. In the words of the renowned mathematical physicist Eugene Wigner:

> The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve. ([31], 14)

It is perhaps being altogether too unreasonable to expect such a "miracle" of the nominalist's formalisation of science as well.

Finally, I wish to state explicitly that I am not claiming to have dealt a fatal blow to nominalist programs such as Hartry Field's, nor was that my intention. I merely want to point out *what is required of them* in the light of the indispensability argument, something which the participants of these programs all too often overlook,[27] or at least fail to fully appreciate.

NOTES

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[1] Although Benacerraf made explicit reference to the causal theory of knowledge in his formulation of the argument ([4]), the argument can be reconstructed so that no such reference is required (as in ([11], 25–26)).

[2] This argument goes back at least as far as Frege (see [12], 187) but owes its modern formulation to Quine (see, for example, [23] and [27]). Perhaps the most thorough presentation of this argument is by Hilary Putnam (see [22], 57–74).

[3] Strictly speaking it's not the entities themselves that are dispensable or indispensable, but rather it's the the postulation of the entities in question that may

be so described. Having said this, though, I will, for the most part, continue to talk about *entities* being dispensable or indispensable, occurring or not occurring, and eliminable or not eliminable. I do this for stylistic reasons but I apologise in advance to any reader who is is irritated by this.

4 John Burgess in [6] argues for a similar conclusion in his discussion of revolutionary nominalism. The difference though, is that he sees the question of theory choice, between Platonistic and nominalistic scientific theories, as a separate issue to that of indispensability, whereas I see it as part of what we mean by 'indispensability'.

5 See [10] and [11] for details of Field's program.

6 For example, in *Science Without Numbers* Field suggest that mathematical entities are not theoretically indispensable since "we can give *attractive* reformulations of [the theories of modern physics] in which mathematical entities play no role" (my emphasis). ([10], 8)

7 Quine actually speaks of entities existentially quantified over in the canonical form of our best theories, rather than indispensability. (See [23] for details.) Still, the debate continues in terms of *indispensability*, so we would be well served to clarify this latter term.

8 It might be questioned whether physical theories actually refer to mathematical entities such as numbers and functions in the same way as they do to electrons and muons, for instance. Perhaps they treat the physical systems as a model of the *uninterpreted* mathematical structure for which the real numbers, say, are also a model. I think this view is mistaken, however, since there are at least some instances where theories refer explicitly to numbers. One such case is the use of dimensionless constants such as the fine structure constant and the electron/proton mass ratio in physics. It is difficult to see how scientists talking about such numbers could be construed to be talking about anything other than the numbers themselves, since the fact that these numbers are dimensionless rules out interpreting them as numerical quantifiers, quantifying over some portion of the physical system in question. This, of course, doesn't justify belief in, say, all of the real numbers (because science doesn't require that many dimensionless constants). My point is simply that there appears to be *some* reference to numbers in our best scientific theories. I am indebted to Peter Forrest for discussion on this point.

9 See [19] and [20] for details of her arguments against indispensability theory. See also [7] for a reply to her arguments.

10 See [29] for further details of constructive empiricism.

11 This theorem states that relative to a partition of the vocabulary of an axiomatisable theory T into two classes, τ and ω (theoretical and observational say) there exists an axiomatisable theory T', in the language whose only non-logical vocabulary is ω, of all and only the consequences of T that are expressible in ω alone.

12 Naturally the question of whether such partitioning is possible is important and somewhat controversial. It seems that Quine would deny that such a partition is possible. If he is right about this, it will be considerably more difficult to eliminate

theoretical entities from scientific theories. I'm willing to grant for the sake of argument, at least, that such a partitioning *is* possible.

[13] One way in which you might think that less ontological commitment is *not* better, is if ξ actually exists. In this case it seems that Γ^+ is the better theory since it best describes reality. This, however, is to gloss over the important question of how we come to know that ξ exists. If there is some evidence of ξ's existence then Γ^+ will indeed be the better theory, since it will be empirically superior. If there is no such evidence for the existence of ξ then it seems entirely reasonable to prefer Γ over Γ^+ as I suggest. It is the latter I had in mind when I set up this case. Indeed, the former case is ruled out by construction. I am not concerned with whether ξ actually exists or not – just that there be no evidence for it.

[14] One might question whether these principles are deserving of the status of theory, given that they are are at best rather vague methodological principles whose application is poorly understood. This need not concern us though, since nothing I say depends on these principles being well understood.

[15] See, for instance, Hempel [14], 203–206, Horwich [15], 1–15, Weinberg [30], 105–131, Glymour [13], 152–155, and Quine in [25]. While all these authors wouldn't agree with my characterisation of the additional features entirely, I think I have captured what are the most common elements in their accounts.

[16] For instance, it may be possible to explain formal elegance in terms of simplicity and unificatory power.

[17] The mathematical physicist Freeman Dyson makes a similar point when he says "…mathematics is not just a tool by means of which phenomena can be calculated; it is the main source of concepts and principles by means of which new theories can be created."([8], 129)

[18] See [5] for further details.

[19] This principle states that no two particles of spin one half can occupy the same energy state at the same time, thus all electrons must occupy different specific energy levels, from the lowest upwards.

[20] The positron was subsequently discovered in 1932 as an ingredient of cosmic rays.

[21] Mark Steiner discusses this example in a slightly different context in his paper [28]. See his paper for other excellent examples of the important role mathematics plays in physical theory. See also [1] for further discussion of the role mathematics plays in scientific discovery.

[22] The concept of the ether, however, goes back at least as far as Descartes.

[23] The laws of physics are the same for all inertial reference frames.

[24] The speed of light (in a vacuum) is a constant for all inertial reference frames.

[25] I don't mean to denigrate Lorentz's theory, indeed Lorentz and Henri Poincaré very nearly produced the special theory of relativity between them. In fact the renowned physicist Sir Edmund Whittaker thought Einstein's role in the formulation of special relativity was overemphasised when he summarised Einstein's contribution in his 1910 book on the history of the theories of the ether and electricity in the single sentence: "…Einstein published a paper [the 1905 paper] which set forth the relativity theory of Poincaré and Lorentz with some ampli-

fications, and which attracted much attention."([2], 72) Needless to say most physicists would not agree with Whittaker's deflationary account of Einstein's contribution!

[26] To put it another way, there are bridge laws between the mathematised Lorentz theory and the nominalised version of the same, but also between the mathematised Lorentz theory and special relativity.

[27] In fairness to Field though, he does seem to recognise some of the requirements I have outlined in that he argues that nominalistic theories can be "more illuminating" than their Platonistic counterparts.([10], 44) Although I think he is wrong about this I will not pursue the issue here.

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