This paper was written during the summer of 2021–22 for the course TEPI615: Research for Teaching, taught by Professor Mistilina Sato (see https://www.canterbury.ac.nz/courseinfo/GetCourseDetails.aspx?course=TEPI615&occurrence=21X(C)&year=2021). It was well received by Professor Sato, but aside from that has not been peer reviewed.

Integrating Mathematics With Other Curriculum Areas in Secondary Education: A Critical Review

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Introduction

Pythagoras' declaration that "all is number" highlights the wide-ranging applications of Mathematics to other fields of study and everyday experience. From its relevance within physical and social science research to the statistical underpinnings of political communication, Mathematics is powerful in areas far beyond its disciplinary confines. Mathematical knowledge can equip individuals with the cognitive tools necessary to make sense of the world around them. Indeed, during the initial months of the Covid-19 pandemic, the time that many people spent looking at exponential graphs increased exponentially, and mathematical (mis)understandings inform much current discourse around vaccinations and other epidemiological interventions. Teaching Mathematics in conjunction with other curriculum areas in secondary schools seems a promising way of enabling students to grapple with the intricacies of mathematical applications.

Curriculum integration

The practice of curriculum integration involves combining elements from multiple subject areas to form a coherent learning unit (McPhail, 2018; Thibaut et al., 2018). Arrowsmith (2013) identifies three variants as prevalent in New Zealand secondary schools: multi-, inter-, and transdisciplinary. International research corroborates the popularity of these formats (Frykholm & Glasson, 2005; Judson, 2013; Weinberg & Sample McMeeking, 2017). Multidisciplinary integration typically involves teaching disciplines in isolation but connecting the insights they provide where possible, often through an overarching theme (Arrowsmith, 2013; McPhail, 2018). Interdisciplinary integration has multiple subjects taught simultaneously, such as Science, Technology, Engineering, and Mathematics (STEM) courses (Hasni et al., 2015; Siverling et al., 2019). Transdisciplinary integration abandons disciplinary boundaries, with learning instead focused on an issue or topic and subject-specific knowledge utilised only where pertinent (Arrowsmith, 2013; de Freitas & Bentley, 2012; Treacy & O'Donoghue, 2014). In each case, integration disrupts the traditional compartmentalisation of subject areas.

Reasons given to support curriculum integration in Mathematics centre around the learning benefits it may provide over traditional instruction. It is claimed that integrated Mathematics teaching can enhance motivation and engagement (Ní Ríordáin et al., 2015; Treacy & O'Donoghue, 2014). This is often attributed to cross-disciplinary topics being more authentic and hence demonstrating the relevance of learning activities to students' lives beyond school (Aminger et al., 2021; de Freitas & Bentley, 2012; Thibaut et al., 2018). Quality of learning is also thought to increase due to the potential for integrated tasks to connect abstract mathematical ideas to real contexts and encourage peer collaboration (Judson, 2013; Thibaut et al., 2018; Treacy & O'Donoghue, 2014). McPhail (2018) notes that curriculum integration is promoted in 21st-century learning discourse as a means of enabling students to develop problem-solving abilities and other transferable skills necessary for success in a changing world. To realise these myriad benefits, however, curriculum integration must be planned and implemented judiciously, which requires an extensive range of variables to be taken into account.

Aims of this report

With this in mind, the following question was used to guide this literature review:

What factors contribute to effective integration of Mathematics with other curriculum areas in secondary schools?

The initial phase of research involved identifying approximately 300 papers through Google Scholar and searches across Education databases on EBSCO*host*. Analysis of titles and abstracts reduced this to 108 sources, which were examined and compared to produce a list of 32. These were read thoroughly and notes made on their content before identifying the 12 most suited to addressing the research question. Desiderata informally applied included rigorous research methodology, with larger samples and mixed-method approaches prioritised; focus on, or high relevance to, Mathematics integration specifically; recency of publication; and broad applicability, particularly within New Zealand educational contexts.[1] Findings are examined below with three categories – teacher, pedagogy, and institutional factors – used to structure the discussion before critically evaluating limitations prevalent in the current literature.

Teacher factors

Many studies highlight teachers' dispositions as an important precursor to effective curriculum integration. Arrowsmith (2013) investigated four New Zealand secondary schools where integrated teaching practices were widespread. She found that in each case, optimistic attitudes were a key driver of the curriculum integration programmes, though at one school several interviewees asserted that "Mathematics and Science teachers found it particularly hard to embrace" (p. 77). This was attributed to teachers of these subjects feeling that content requirements were more difficult to cover using integrated methods. Surprisingly, though, a study exploring attitudes of 245 Science, Technology, and Mathematics secondary teachers in Quebec found that 90.1% thought the use of interdisciplinary teaching was 'very important' or 'fairly important' and believed that their learning areas were among the *most* conducive for doing so, indicating that feelings towards curriculum integration held by Mathematics teachers can differ substantially (Hasni et al., 2015). Engendering positive attitudes is critical to ensuring that educators are motivated to implement curriculum integration in their practice (Thibaut et al., 2018; Weinberg & Sample McMeeking, 2017) and collaborate effectively with colleagues (Frykholm & Glasson, 2005; Ní Ríordáin et al., 2015).

Thibaut et al. (2018) conducted a thorough exploration into secondary teachers' attitudes towards STEM integration. They analysed survey responses from 135 STEM teachers and found that participation in professional development, reported personal relevance of science, and a supportive school social culture significantly predicted positive attitudes towards STEM integration. Interestingly, the length of time spent teaching Mathematics and having over 20 years of general teaching experience were associated with poorer attitudes. Perspectives about curriculum integration, therefore, appear deeply intertwined with various personal and institutional factors. Moreover, while the correlational nature of this research limits the potential for drawing robust causative inferences, the findings indicate promising areas for interventions to target, such as school culture, to enhance attitudes towards integrative teaching practices.

Attitudes alone are limited in their potential to bring about integrated Mathematics successfully, however, since educators may still have limited *knowledge* required to enact curriculum integration. New Zealand research suggests that many teachers, despite their enthusiasm for integrative methods, lack understanding of the principles behind such practices. In a qualitative analysis of the rationale given for integrated approaches, Arrowsmith (2013) identified "a common lack of secondary teachers' theoretical knowledge surrounding curriculum integration" (p. 92). Similar findings were obtained in McPhail's (2018) case study into one New Zealand secondary school emphasising curriculum integration since its inception. Despite over half of teaching time being dedicated to integrated learning, just one of 11 teachers surveyed reported having read widely about curriculum integration. Mixed levels of understanding have also been found internationally, including among Mathematics teachers. In the study by Hasni et al. (2015) with teachers in Quebec, where interdisciplinary instruction has been mandated, many teachers struggled to define 'interdisciplinarity' without using "vague expressions" (p. 161) or emphasising mainly "superficial characteristics" (p.

162). The researchers found that such explanations were consistent with teachers' descriptions of their own lessons, which tended to be limited in pedagogical rigour.

Researchers have also highlighted the importance of teachers' subject knowledge. Frykholm and Glasson (2005) examined the effects of an interdisciplinary collaborative unit-design intervention on the practices of 65 pre-service Mathematics and Science teachers. Although "participants conveyed strong convictions about the importance of connecting mathematics and science" (p. 132), many felt insecure about their content knowledge in their non-dominant area and saw this as an impediment to being able to effectively integrate the two. Other studies with Mathematics and Science teachers have also found that a lack of subject knowledge may inhibit teachers from integrating curriculum areas (Ní Ríordáin et al., 2015; Treacy & O'Donoghue, 2014; Weinberg & Sample McMeeking, 2017). Strength of teachers' disciplinary knowledge, alongside that of theory behind curriculum integration, can contribute to the effectiveness – or otherwise – of teaching practices that combine Mathematics with other subject areas.

Pedagogy factors

Largely dependent on teachers possessing robust subject knowledge, pedagogical approaches to curriculum integration must ensure that topics chosen for integration coalesce appropriately. Frykholm and Glasson (2005) utilised the term 'pedagogical context knowledge' to describe teachers' awareness of connections between learning areas, arguing that "*any* effort to connect science and mathematics with meaning must be situated in authentic contexts" (p. 139). Perhaps for this reason, most research into integrated Mathematics teaching connects the subject with scientific disciplines, which are typically thought most suitable for learning in both areas to occur simultaneously. As Treacy and O'Donoghue (2014) write, "science is the logical partner of mathematics for such integration" (p. 705). Studies indicate that secondary teachers tend to endorse this perception (Hasni et al., 2015; Frykholm & Glasson, 2005; Ní Ríordáin et al., 2015).

Subject compatibility is essential for ensuring that curriculum integration activities use disciplinary concepts to meaningfully advance one another and promote learning. Judson (2013) developed the Mathematics Integrated into Science: Classroom Observation Protocol (MISCOP) scale to gauge lesson effectiveness. Factor analysis revealed that the construct accounting for the most variance (22.1%) corresponded to 'Meaning and Purpose', which included items such as "mathematics allowed students to learn science in ways not otherwise possible" and "mathematics strengthened conceptual understanding of science content" (p. 65). Aminger et al. (2021) found that purposeful content structuring was variable among six pre-service science teachers in California. While all participants integrated both subjects, two included mathematical content in ways that were "low in cognitive demand" (p. 196), often using numerical displays but not utilising them to derive formulae relevant to the scientific phenomena being explored. The authors concluded that this impaired students from connecting subject content: "although the mathematics was prescribed, there was still evidence that students were unsure of the mathematics they were expected to use" (p. 204).[2]

Project-based learning has been proposed as a means of ensuring that Mathematics is explored in depth alongside other curriculum areas. Through a naturalistic inquiry of seven United States classrooms, Siverling et al. (2019) identified that collaboration and dialogue supported learning during integrated engineering design group projects. They found that students often drew on mathematical content *beyond* the intended foci of the unit to justify choices made to their peers. Hence, integrated projects extended students' mathematical thinking and reasoning further than more structured formats might achieve. Treacy and O'Donoghue (2014) obtained similarly positive findings after introducing their Authentic Integration model for combining Mathematics and Science to four Irish secondary schools. Learners completed various tasks requiring them to apply subject knowledge to real-world scenarios through group activities. Their understanding of content by the end of the unit tended to be high and teachers claimed that tasks were engaging and worthwhile. Group projects, therefore, look to be an effective pedagogical approach for integrating Mathematics with other subjects.

Lending further support to this idea, de Freitas and Bentley (2012) conducted a case study into integrated Mathematics and Physics lessons at museums in New York. The researchers followed six first-year secondary students while they completed a five-lesson unit on aircraft wing design that involved learning physics concepts about airflow, studying aeroplane exhibits, then constructing a foam plane and investigating its flight. Like Siverling et al. (2019), the authors found that dialogue between group members enabled them to develop and share mathematical knowledge. Additionally, they observed that many individuals used gestures and movement to communicate mathematical concepts during their discussions (e.g., by using their arms to represent various shapes). The authors concluded that the aeronautical project enabled students to learn abstract content in a highly effective way by "developing a strong embodied and material sense of mathematics through their participation in the program" (p. 46).

Institutional factors

To coordinate project-based learning, field trips, or other curriculum integration activities, teachers require time for planning. However, researchers argue that schools often do not allocate sufficient time for doing so. Studies from Ireland highlight this, with Treacy and O'Donoghue (2014) stating that "'time'…[was] consistently referred to by the teachers" (p. 714) and Ní Ríordáin et al. (2015) noting that educators who implemented an intensive Mathematics and Science integration programme were not given reduced teaching hours, instead being required to use their personal time. Hasni et al. (2015) identified similar reported time constraints for planning integrated lessons in Canada, as did Weinberg and Sample McMeeking (2017) in the United States. New Zealand teachers face comparable difficulties. Even at institutions selected due to their emphasis on curriculum integration, Arrowsmith (2013) found variable time allocated for preparing lessons, especially collaborative planning: one school had fortnightly meetings, one had sessions twice per term, another met once each term to brainstorm activities, and the fourth did not report regular arrangements. McPhail (2018) also listed time limitations as a barrier to teachers preparing integrated lessons. Insufficient planning risks diminishing lesson quality in general. However, given that teachers involved with Mathematics integration often report lower subject knowledge outside their main area (Frykholm & Glasson, 2005; Treacy & O'Donoghue, 2014; Weinberg & Sample McMeeking, 2017) and place high value on collegial support (Arrowsmith, 2013; Ní Ríordáin et al., 2015; Thibaut et al., 2018), providing time for collaboration and planning appears to be a crucial requisite for high-quality curriculum integration.

Access to professional development is also important for ensuring teachers have the skills and confidence to integrate Mathematics effectively (Judson, 2013; Ní Ríordáin et al., 2015). Participants from every school that Arrowsmith (2013) studied considered this a valuable resource. Several believed that professional development would be particularly worthwhile for teachers not supportive of curriculum integration in order to challenge their resistance. This accords with the regression analysis in Thibaut et al. (2018) showing that secondary teachers' attitudes to STEM curriculum integration were significantly predicted by participating in professional development during the past year ($\beta = .20$, $p < .01$). McPhail (2018) noted that foci of professional development for curriculum integration can vary, with much of that conducted in one New Zealand secondary school being centred around pedagogical methods with "little focus on the more complex issue of conceptual integration" (p. 61). Attention to this dimension, McPhail argued, is essential for ensuring that integrative learning extends beyond "thematic, common sense knowledge to abstract interdisciplinary thinking" (p. 61).

Potential for exploring conceptual links between subject areas can be influenced by national-level curriculum and assessment systems. Secondary educators in countries with more prescriptive Mathematics curricula often report difficulties balancing the need to cover subject-specific content with their desire to integrate learning areas (Ní Ríordáin et al., 2015; Treacy & O'Donoghue, 2014). In contrast, teachers in Quebec surveyed by Hasni et al. (2015) identified the mandated interdisciplinarity of the curriculum as a key motivator of integrative practices, while Aminger et al. (2021) found that the *Next Generation Science Standards* expectation that science lessons allow opportunities for "using mathematics and computational thinking" (p. 188) underpinned many attempts made by pre-service teachers in the United States to incorporate mathematical content and processes. Curricular requirements, then, can enable or inhibit robust integrated teaching practices. New Zealand likely occupies a favourable position given the broad, conceptual nature of the national curriculum.

The National Certificate of Educational Achievement (NCEA), however, imposes assessment requirements that may limit subject integration in New Zealand. Teachers interviewed by McPhail (2018) identified challenges with locating NCEA standards that aligned well with integrated units since these tend to be "firmly subject-based" and hence "intrude into the curriculum design" (p. 62). Arrowsmith (2013) found that many principals also felt NCEA assessments were more suited to non-integrated teaching. Directions taken in the curriculum refresh and NCEA review over the coming years will likely be a decisive factor in determining how effectively Mathematics and other learning areas are integrated in New Zealand secondary schools.

Research limitations

Several limitations are prevalent in the research literature on Mathematics curriculum integration, among them a narrow focus on connecting the subject with scientific disciplines. This is often justified by the high compatibility of disciplinary knowledge in these fields (Frykholm & Glasson, 2005; Treacy & O'Donoghue, 2014). Funding availability may also encourage this limited scope, with many studies supported by grants from the National Science Foundation (e.g., Aminger et al., 2021; de Freitas & Bentley, 2012; Weinberg & Sample McMeeking, 2017) or similar organisations (e.g., Ní Ríordáin et al., 2015; Siverling et al., 2019; Thibaut et al., 2018). Research in this area is also frequently justified through fiscal profitability. Ní Ríordáin et al. (2015) describe the benefits of Mathematics and Science integration "for Ireland's economic future" (p. 5) while Thibaut et al. (2018) begin their paper by lamenting "the current shortage of graduates" (p. 632) in STEM fields. Effective methods to integrate Mathematics with humanities disciplines, for instance, remain largely uninvestigated. McPhail (2018) reports how one student felt pseudoscience was ably explored in a Chemistry–English module. The use of statistics in journalism and media texts to inform, bemuse, and coax readers may be a fruitful avenue for Mathematics curriculum integration, though research is yet to delve into what factors might best support this type of crossover.

Many studies are limited by reliance on qualitative methods and small sample sizes. Judson's (2013) construction of the MISCOP scale used a quantitative approach, and some researchers include quantitative results to supplement qualitative findings (e.g., Thibaut et al., 2018; Treacy & O'Donoghue, 2014), though most studies only utilise qualitative data forms. While this can permit nuanced insights to be gathered – for example, using interviews rather than Likert scale responses to measure teacher attitudes – it also inhibits being able to assess integration practices through inferential statistical methods. Moreover, dependence on small samples may heighten the risk of selection bias. Aminger et al. (2021) examined six pre-service teachers out of 16 who agreed to take part by "select[ing] all of those who provided their students substantive opportunities to engage in the practice of *using mathematics and computational thinking*"

(p. 194). Similarly, de Freitas and Bentley (2012) chose to "follow one group of [six] boys" (p. 40) out of the 130 students available. In each case, the extent to which results can be generalised beyond the specific participants studied remains largely uncertain.

Similarly, the applicability of international research to curriculum integration in New Zealand is questionable. Studies considered in this review were largely conducted in the United States (Aminger et al., 2021; de Freitas, 2012; Frykholm & Glasson, 2005; Judson, 2013; Siverling et al., 2005; Weinberg & Sample McMeeking, 2017), though some came from Ireland (Ní Ríordáin et al., 2015; Treacy & O'Donoghue, 2014), Canada (Hasni et al., 2015), and Belgium (Thibaut et al., 2018). These countries have different curricula, institutional practices, and teaching methods to New Zealand. Although the local findings from Arrowsmith (2013) and McPhail (2018) revealed no major discrepancies with international research, these studies focused on curriculum integration generally rather than the specific combination of Mathematics with other subjects, making direct comparisons difficult. Given that New Zealand's conceptual Mathematics curriculum contrasts with the "very descriptive syllabi" (Ní Ríordáin et al., 2015, p. 5) in many educational contexts overseas, additional research is required to further unpack the factors that contribute to effective integration of Mathematics with other disciplines, especially in light of upcoming changes to the *New Zealand Curriculum* and NCEA.

Conclusion

Despite these limitations, this review has identified numerous factors relevant to the success of Mathematics curriculum integration in secondary education, which fall into three interrelated categories. Teacher factors include having supportive attitudes towards integrated methods as well as sufficient knowledge of curriculum integration and subject content in order to put this into practice. Pedagogy factors relate to the use of suitable learning activities to support the development of robust disciplinary knowledge. It is crucial that the mathematical concepts incorporated go beyond using displays of numerical data and instead extend students' learning of Mathematics and the subject(s) with which it is integrated. Projects, especially those done in groups, appear highly suited towards this end. Institutional factors include national-level curricula and assessment requirements as well as school-level support for curriculum integration, especially through provision of adequate time for collaborative planning alongside professional development. These various factors are deeply interwoven: dispositions can inform pedagogical choices, school culture may affect teachers' attitudes regarding curriculum integration, and so forth. Regardless of the difficulty of integrating Mathematics with other subjects in a complex educational landscape, the benefits that curriculum integration offers students make it a goal worth striving for.

Notes

[1] Two sources examining New Zealand secondary schools were retained due to this final criterion, despite focusing on curriculum integration generally rather than Mathematics integration specifically.

[2] McPhail (2018) elaborates further on the potential detriments to learning when insights from constituent subjects are not synthesised. Observing that integration in New Zealand often relies on thematic connections, McPhail identifies two primary issues: discipline-specific concepts being fragmented and knowledge gains from combining subjects failing to progress "beyond an everyday, generalized level" (p. 64). The applicability of Mathematics to virtually all topics with quantitative elements may heighten the risk of disciplinary content being diluted, thereby inhibiting students' opportunities to learn.

References

Aminger, W., Hough, S., Roberts, S. A., Meier, V., Spina, A. D., Pejela, H., McLean, M., & Bianchini, J. A. (2021). Preservice secondary science teachers' implementation of an NGSS practice: Using mathematics and computational thinking. *Journal of Science Teacher Education, 32*(2), 188–209. https://doi.org/10.1080/1046560X.2020.1805200

Arrowsmith, S. (2013). *New Zealand secondary school teachers' perceptions and practices toward curriculum integration* [Master's thesis, Victoria University of Wellington]. ResearchArchive. http://hdl.handle.net/10063/2841

de Freitas, E., & Bentley, S. J. (2012). Material encounters with mathematics: The case for museum based cross-curricular integration. *International Journal of Educational Research, 55*, 36–47. https://doi.org/10.1016/j.ijer.2012.08.003

Frykholm, J., & Glasson, G. (2005). Connecting science and mathematics instruction: Pedagogical context knowledge for teachers. *School Science and Mathematics, 105*(3), 127–141. https://doi.org/10.1111/j.1949-8594.2005.tb18047.x

Hasni, A., Lenoir, Y., & Froelich, A. (2015). Mandated interdisciplinarity in secondary school: The case of science, technology, and mathematics teachers in Quebec. *Issues in Interdisciplinary Studies, 33*, 144–180. https://files.eric.ed.gov/fulltext/EJ1117890.pdf

Judson, E. (2013). Development of an instrument to assess and deliberate on the integration of mathematics into student-centered science learning. *School Science and Mathematics, 113*(2), 56–68. https://doi.org/10.1111/ssm.12004

McPhail, G. (2018). Curriculum integration in the senior secondary school: A case study in a national assessment context. *Journal of Curriculum Studies, 50*(1), 56–76. https://doi.org/10.1080/00220272.2017.1386234

Ní Ríordáin, M., Johnston, J., & Walshe, G. (2015). Making mathematics and science integration happen: Key aspects of practice. *International Journal of Mathematical Education in Science and Technology, 47*(2), 233–255. https://doi.org/10.1080/0020739X.2015.1078001

Siverling, E. A., Suazo-Flores, E., Mathis, C. A., & Moore, T. J. (2019). Students' use of STEM content in design justifications during engineering design-based STEM integration. *School Science and Mathematics, 119*(8), 457–474. https://doi.org/10.1111/ssm.12373

Thibaut, L., Knipprath, H., Dehaene, W., & Depaepe, F. (2018). How school context and personal factors relate to teachers' attitudes toward teaching integrated STEM. *International Journal of Technology and Design Education, 28*(3), 631–651. https://doi.org/10.1007/s10798-017-9416-1

Treacy, P., & O'Donoghue, J. (2014). Authentic integration: A model for integrating mathematics and science in the classroom. *International Journal of Mathematical Education in Science and Technology, 45*(5), 703–718. https://doi.org/10.1080/0020739X.2013.868543

Weinberg, A. E., & Sample McMeeking, L. B. (2017). Toward meaningful interdisciplinary education: High school teachers' views of mathematics and science integration. *School Science and Mathematics, 117*(5), 204–213. https://doi.org/10.1111/ssm.12224