#### Ontological Independence as the Mark of the Real

Jody Azzouni. Deflating Existential Consequence: A Case for Nominalism. New York: Oxford University Press, 2004. Pp. viii + 241. ISBN 0-19-515988-8

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In recent times there have been a number of proposals for a nominalistic philosophy of mathematics. These proposals divide into two quite distinct camps: those who take mathematical propositions to be true, and those who take them to be untrue.<sup>2</sup> Both options face substantial difficulties, but let's focus on the first option. The problem here is in asserting that mathematical propositions such as 'there exist infinitely many complex roots of the Riemann zeta function' are true (as this one surely is) and then to go on to deny that there are any complex numbers. To do this just seems inconsistent, or at least "intellectually dishonest" (Putnam, 1971, p. 347). One way to approach this problem is to reinterpret the mathematical claims in question so that they come out true, but do not refer to mathematical objects. So for example, Geoffrey Hellman [1989] interprets mathematical claims to be about possible structures. Such options, since they do not take mathematical claims at face value, must employ a non-uniform semantics and this is thought, by almost everyone, to be a significant price to pay for one's nominalism. The problem is particularly acute when one considers mixed mathematical and empirical statements such as 'there exists a planet with mass m and location (x, y, z) and a function G that describes the gravitational potential of the planet at time t'. Here different parts of a single sentence must be treated differently—the talk of planets (and perhaps fields) is treated literally but the mathematical parts are treated non-literally. Apparently the only alternative to reinterpreting mathematical discourse is to follow Hartry Field [1980] and deny the truth of mathematical propositions. But this option is very counterintuitive.<sup>3</sup>

In a series of papers over the past few years, Jody Azzouni [1997a, 1997b, 1998, 2000] has been toying with a novel route to nominalism. In his most recent book, *Deflating Existential Consequence*, Azzouni puts all this together

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<sup>&</sup>lt;sup>2</sup>I thus include in the latter those who deny that mathematical propositions take a truth value

<sup>&</sup>lt;sup>3</sup>Counterintuitiveness aside, this approach faces many other problems. See Burgess and Rosen [1997] for a good discussion of the problems with this approach.

and provides a detailed defence of a very interesting nominalist philosophy of mathematics. Azzouni's nominalist strategy belongs in the first camp, where mathematical propositions retain their common-sense truth values. But unlike most of his fellow campers, Azzouni does not reinterpret mathematical discourse. Instead, he parts company with almost everyone else in the debate and argues against ontological commitments being simply read off the quantifiers. As Azzouni is inclined to put it, quantifier commitments are distinct from ontological commitments.

In what follows I will outline Azzouni's proposal, before turning to a critical discussion of some of they key issues raised by his book.

# §1: Overview of Azzouni's Project

The book is divided into two sections: section I consists of chapters 1–5 and examines the connection between truth and ontology; section II consists of chapters 6–9 and presents a sustained argument for a particular (and novel) approach to determining what exists.

Azzouni begins, in chapters 1 and 2, by defending a deflationary conception of truth. He argues that the role of the truth predicate is to facilitate blind truth ascriptions—that is, assenting to some set of sentences that cannot be, or have not been, exhibited. In the former case (where the sentences in question cannot be exhibited) the use of the truth predicate is indispensable. So although the sentence ' $\Gamma$  is true' (where  $\Gamma$  is some non-empty set of sentences) is equivalent to simply asserting the conjunction of all the sentences  $s \in \Gamma$ , such deflationary use of the truth predicate does not imply its redundancy. If, for example,  $\Gamma$  is an infinite set of sentences, we cannot endorse all the sentences in  $\Gamma$  without invoking the truth predicate. Moreover, Azzouni argues that we have no choice but to take our best scientific theories, along with whatever portions of applied mathematics they employ, as true. We need to apply these theories and, according to Azzouni, to apply them we must blindly endorse to them. The device used for such blind endorsement is the truth predicate.

In chapter 3 Azzouni addresses the question of whether taking a theory to be true entails ontological commitment to all the entities the theory quantifies over. Quine [1948], famously, argued that there is no choice here: if a theory you take to be true says that there are Fs, then you ought to believe in Fs. Azzouni argues for what he calls the separation thesis: the thesis that what a true theory is committed to (quantifier commitments) does not need

to be the same as ontological commitments. In short, Azzouni argues for the rather controversial thesis that (existential) truth and ontology come apart. This is the really distinctive feature of Azzouni's approach to metaphysics. Ever since Quine [1948] the objectual existential quantifier of first-order logic has been taken by almost everyone to be carrying ontological commitment. Azzouni rejects this mainstream view and argues that there is at least a further question about what exists, and this question goes beyond Quine's considerations of quantifier commitments. Azzouni argues for the need for an existence predicate.

In chapters 4 and 5, Azzouni argues that, given the separation thesis, there is no definitive way to settle the question of what exists; the answers to ontological questions remain society relative. This indeterminacy thesis is argued for in two parts. The first part is that there is no philosophical argument that delivers the one true path to determining ontological commitments. The second is that there are no linguistic devices that univocally determines ontological commitments. Despite this, Azzouni suggests that we can determine what *our* society takes to be criteria for taking something to be real. This project is pursued in the second section of the book.

Section II kicks off with a chapter (chapter 6) devoted to an interesting classification of theoretical posits into *thick*, *thin*, and *ultra-thin*. (I'll have more to say about this distinction below.) In chapter 7 Azzouni argues that only the thick and thin posits should be taken as real. (Again, I'll have more to say about this issue below.) The main thrust of Azzouni's arguments in these two chapters is that only those entities with causal powers should be taken to be real. He thus argues for a version of what has become known (following Oddy [1982]) as the Eleatic Principle.

The final two chapters, chapters 8 and 9, illustrate and clarify this novel approach to ontology by an extended case study of a scientific theory—Newtonian cohesive-body mechanics—and its ontological commitments. This is followed by a brief conclusion.

As I see it, there are two really key elements in Azzouni's nominalism: the separation thesis and the notion of ontological independence. It is worth spelling these out in further detail.

#### §1.1 Quantifier Commitment Versus Ontological Commitment

On the standard (Quinean) view, ontological commitments are determined by what the first-order (objectual) existential quantifiers of our best scientific theories range over. But, according to Azzouni, we need to distinguish two things: quantifier commitments and what a discourse commits us to. The idea here is that we might agree that the first-order objectual existential quantifiers of our best physical theories range over abstract objects (that's the quantifier commitment) but it does not follow that the discourse itself is committed to abstracta. Whether or not we take abstracta to exist is a further question. Quine, in effect, took there to be no further question and thus performed what Azzouni suggests was a "sleight of hand" (p. 4) that gets Quine his preferred answer to ontological questions. But Azzouni warns us:

One can't read ontological commitments from semantic conditions unless one has already smuggled into those semantic conditions the ontology one would like to read off. (p. 55)

This portrayal of Quine is a little unfair though. It's not as though Quine was a committed platonist looking for a justification of this view. Quine's occamist sympathies sat more naturally with nominalism and, indeed, his earlier work with Goodman [1947], I think, demonstrates that Quine was a reluctant platonist. He was forced to believe in abstracta because of their indispensable role in our best scientific theories. But still Azzouni is right about there being a further question beyond the question of the quantifier commitments. And arguably, Azzouni is right that the answer to this further question need not be the trivial answer that Quine suggests.<sup>4</sup>

Some may wonder about this. How could ' $\exists$ ' mean anything other that 'there exists' (in an ontologically committing sense)? Although Azzouni doesn't quite put the point this way, we can see the issue as one involving the interpretation of logical particles. Consider the connectives of first-order classical logic: ' $\lor$ ', ' $\land$ ', ' $\supset$ ' and ' $\lnot$ '. While it is widely acknowledged that ' $\supset$ ' should not be thought of as anything more than an approximation of the English indicative conditional and it is completely inadequate as a model of the subjunctive conditional, <sup>5</sup> the natural language interpretation of other logical connectives is generally thought to be more straight forward: ' $\lor$ ' is

<sup>&</sup>lt;sup>4</sup>And indeed, the sleight of hand that Azzouni refers to is really the way in which Quine argues for the trivial answer.

<sup>&</sup>lt;sup>5</sup>Consider the indicative conditional (uttered, if you like, by someone who doesn't know that Palo Alto is in California) 'If Palo Alto is in New Mexico then it is on Central Time'. This sentence is surely false (since New Mexico is on Mountain Time), yet it comes out true if we take the conditional to be the material conditional (since the antecedent is false). Similar examples suffice to show the inadequacy of the material conditional for subjunctive conditionals.

'inclusive or', ' $\land$ ' is 'and', and ' $\neg$ ' is 'not'. But even these interpretations have been questioned. Nor should we be fooled by what these connectives are called. The fact that we commonly call the symbol ' $\neg$ ' of classical logic 'negation' does not settle the dispute between intuitionistic and classical logicians over the correct formal representation of natural language negation. The point is that while the logical particles often have natural language interpretations, these interpretations are not written in stone—they need to be argued for. So too for the existential quantifier. If it is to be interpreted as carrying ontological commitment, this needs to be argued for. ' $\exists$ ', after all, is simply a symbol in a formal language; its natural language interpretation is not a matter determined by logic alone.

#### §1.2 Ontological Independence

Azzouni [1998] once argued for a position that Penelope Maddy calls "ontological nihilism". This is the view that "there are no philosophically conclusive ways to argue for our criterion for what exists" (p. 5). In the book under review, Azzouni stands by this earlier position but invokes an appeal to practices in place in both the scientific and general communities that determine what we take to exist. But, according to Azzouni, strictly speaking, whatever criterion we adopt will be society relative. He goes on to provide an account of what our society takes to be the criterion for existence: ontological independence. The idea here is that we do not take as real entities in dreams, entities in myth and entities in fiction. For an entity to be taken to be real it must enjoy independence from human minds, works of fiction and the like. To help get a grip on what entities we take to be ontologically independent, Azzouni distinguishes three kinds of posits, based on the epistemic access we have to them: thick, thin, and ultra-thin.

Lets begin with ultrathin posits. These are posits that don't have any epistemic burdens. They are, if you like, *mere* posits. They are the posits of fiction and of (unapplied) pure mathematics. No epistemic access is required. As Azzouni puts it "[a] mathematical subject with its accompanying [ultrathin] posits can be created ex nihilo by simply writing down a set of axioms" (p. 127). Next we have, thin posits. These are posited by empirical theories that enjoy the five Quinean [1955] virtues of simplicity, familiarity, scope, fecundity, and success under testing. Thin posits thus pay their epi-

<sup>&</sup>lt;sup>6</sup>For example, although the logical connective ' $\wedge$ ' is symmetrical, English conjunctions need not be. (Consider the two sentences 'he ran down the flank and kicked the goal' and 'he kicked the goal and ran down the flank'.)

stemic dues; As Azzouni puts it, they pay their *Quinean rent* (p. 128). But thin posits must do more; they must also have a story about why there is no thick epistemic access to them. Azzouni highlights the difference between thin and ultrathin posits in the following passage.

The difference between thin posits and ultra-thin posits (which live free of charge) is striking. Should one of the former fail to pay its Quinean rent when due, should an alternative theory with different posits do better at simplicity, familiarity, fecundity, and success under testing, then we have a reason to deny that the thin posits, which are wedded to the earlier theory, exist—thus, the eviction of centaurs, caloric fluid, ether, and their ilk from the universe. (p. 129)

This brings us to the final kind of posit: thick posits. These are those that we have the most secure epistemic access to. These are posits whose epistemic access has the following features (pp. 129–136):

- 1. Robustness: The epistemic access does not depend on what the epistemic agent expects from that access. Our theory about the posits in question might be incorrect or our theory of how the thick epistemic access operates may be incomplete or wrong. Posits can surprise us in ways fictional entities, say, cannot.
- 2. Refinement: There are ways of adjusting and refining the thick epistemic access we have to the posit in question: we can use more powerful telescopes to view distant galaxies and we can bring a rose closer to our nose to better smell it.
- 3. Monitoring: We can track the posits in question by either detecting their behaviour through time or by exploring different aspects of the posits in question. So, for example, we can follow a particle via its track in a cloud chamber or we can walk around a barn to ensure that it's not a Hollywood prop.
- 4. Grounding: Certain properties of the object can be used to explain how the thick epistemic access we have enables the discovery of those and other properties of the object. So, for example, I can tell that a jet is moving across the sky by observing its vapour trail and seeing that the leading edge of the trail is advancing across the sky.

The canonical example of such privileged access is observation, but it is important to note that observation is not the only such form of access. Epistemic access through various kinds of instruments can also have these four features. Azzouni is not offering an instrumentalist account of theoretical entities.

So how does this help us determine what we ought to take as real? Azzouni argues that we are only ontologically committed to the thick and thin posits—it is only these that enjoy ontological independence. As Graham Oddy once put it (without endorsement) "[r]espectable entities work for their living" (Oddy, 1982, 285–286). The ultrathin are freeloaders and should not be admitted to our ontology.

## §2: Critical Discussion

I will now turn to the task of offering some critical discussion of Azzouni's nominalist project. There is no doubt that Azzouni's project is a large one and there are many parts one might choose to question. I will focus on just a couple of issues that I think are in need of further work.

The first thing to note about the thick, thin and ultrathin distinction is that it does not cover an important class of posits: those posits that would be thin except that they do not satisfy the defeasibility condition. The posits I have in mind, are more than ultra-thin, because they pay their Quinean rent—they are part of a well-confirmed empirical theory exhibiting the Quinean virtues—but they are not thin, because they do not come equipped with an excuse as to why they are not thick posits. Let's call such posits, very thin. Are there any such posits? According to Azzouni there aren't, but I think he is wrong about this. As I've argued elsewhere (Colyvan 2001), mathematical entities invoked by physics (real numbers, complex numbers, functions and the like) are very thin. But there are also other posits that are very thin. Consider various gaps in the fossil record. These gaps represent creatures for which there are no fossil traces but whose existence we can be certain of from evolutionary considerations. It would seem that these posits (the creatures themselves, that is—not their missing fossils) are clearly not thick, since there is no trace of them, they are not thin because they do not satisfy the defeasibility condition (there's no story in place about why they are not thick posits). But these posits are well

<sup>&</sup>lt;sup>7</sup>These creatures, it turns out, just weren't fortunate enough to fall into tar pits or die in any other fossil-rich environments. But surely that won't suffice as an excuse, for

supported by current evolutionary theory (or so we are supposing for the purpose of the example). Gaps in the fossil record (again, the creatures not the missing fossils) are very thin posits. What do we say about such entities? I think we have very good reason to accept that such very thin posits are real. At the very least, there is no clear intuition against taking them to be real. If this is right, we have a case for being ontologically committed to the thick, the thin and the very thin. But at least a large number of mathematical posits are very thin, so for all Azzouni has said, mathematical realism is still a live option.

Now Azzouni might reply (and, indeed, has replied in private correspondence) that all creatures that went extinct long ago are thin—whether they left behind fossils or not. They all pay their Quinean rent and they all have the same story in place as to why they are not thick posits: they didn't overlap with us temporally. This strikes me as a reasonable response but now I start to lose my grip on what counts as thick access. A jet flying above counts as thick because it leaves behind a vapour trail (and the epistemic access to the jet via the vapour trail satisfies the four crucial features). But aren't dinosaur footprints epistemically very similar to a vapour trail? The latter is the "footprint" of the jet. Moreover, some fossils even involve tissue from the extinct creature in question, so these are more like finding a jet wreckage than looking at footprints. Surely we have thick access to jets via jet wreckages and to extinct creatures via tissue remains.

The point I'm pushing at here might be expressed in terms of theory ladeness of observation. Some observation (or epistemic access, more generally) requires less theory to interpret than others but there is theory involved in almost all epistemic access. The boundary between thick and thin is at best a fuzzy boundary. It is also worth noting that in light of this response of Azzouni's, very few posits are going to be thick, almost everything in the past that disappeared before we were around will be (at best) thin. Azzouni (again in private correspondence) suggests that even photographic evidence from the past will not satisfy the crucial four conditions for thick access. This, I think, highlights how important the defeasibility condition is—it's

this is very close to the trivial excuse of "we don't have thick epistemic access because things didn't work out that way". If Azzouni accepts such excuses, the whole defeasibility condition is rendered useless. At the very least, I think a great deal more needs to be said about what sort of story will satisfy the defeasibility condition. More on this shortly.

<sup>&</sup>lt;sup>8</sup>Other examples might include the predicted gravitational waves of general relativity, the dark matter of cosmology, and various gaps in the periodic table of elements (see Colyvan 2001, pp. 44–45 for a discussion of the latter).

the only thing between Azzouni and a rather widespread anti-realism.

It's worth elaborating on this last point and reflecting on why Azzouni needs the defeasibility condition. Consider stars and planets outside our light cone. These pay their Quinean rent, but fail to be thick. Azzouni does not want to have to exclude stars and planets outside our light cone from our ontology. Indeed, many people share the view that there are such entities, so Azzouni does not want to accept only thick posits as realthat's clearly too restrictive. But nor does he want to allow all Quinean rent payers—that, for Azzouni at least, is too liberal. He wants to drive a wedge between cases like the stars outside our light cone and Quinean rent paying mathematical entities. He does this by invoking the defeasibility condition: stars and planets outside our light cone come equipped with an excuse for not being thick—they're too far away to allow the appropriate epistemic access. But, according to Azzouni, mathematical entities do not come equipped with such an excuse. But it's not clear that this is right. What's wrong with the excuse that we can't have thick epistemic access to the mathematical entities posited by our best scientific theories because these entities are abstract? Now this might sound like a cheat but I'm not sure it is (Colyvan 2001, p. 62). The fact that mathematical entities are abstract explains not only the fact that we cannot have thick epistemic access to them, but it also explains why we don't even try to forge such access. Azzouni needs to tell us more about what counts as satisfying the defeasibility condition. Without this, the door is left open for mathematical realists to argue for mathematical entities being thin and so deserving of ontological commitment. Or, as I've already mentioned, if Azzouni is too restrictive about the defeasibility condition a rather unpalatable anti-realism beckons.

# 3: Conclusion

In some of his previous articles Azzouni was a little coy about his commitment to the view he was advancing and he even seemed reluctant to call himself a nominalist. The coyness is now gone. Azzouni has written a lively and provocative book advancing a novel approach to the philosophy of mathematics. Each conclusion is supported by tight argumentation and cannot be easily dismissed. If Azzouni is right, a great deal of contemporary philosophy of mathematics needs to be rethought. At the very least Azzouni offers another position to be considered: the nominalist position of taking

mathematics at face value (with the usual semantics), taking the usually accepted mathematical claims employed in empirical science to be true, yet denying that this commits one to the existence of mathematical objects. Looked at in this light, Azzouni has offered a considerable challenge to all philosophers of mathematics. Platonists and nominalists alike are likely to disagree with some of the central theses of Azzouni's account. In particular, I suspect that many philosophers from both camps will take issue with the separation thesis.<sup>9</sup>

Part of what makes a good philosophy book good is that it advances the debate. Questioning previously unquestioned assumptions, providing compelling arguments for unpopular theses, looking at old debates from new perspectives, and issuing new challenges for future research are the hallmarks of a good philosophy book. Deflating Existential Consequence: A Case for Nominalism is a very good book. As I've indicated above, I doubt that Azzouni is right in every detail, but few good philosophy books enjoy such a luxury. What matters is that there is now a new position in the philosophy of mathematics landscape. This position needs to be shown to be untenable, or defended and advanced. Either way, Azzouni has helped move forward the realist—anti-realist debate in both the philosophy of mathematics and metaphysics more generally. 10

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 $<sup>^9\</sup>mathrm{See}$  Bueno and Zalta [forthcoming] for other concerns about Azzouni's version of nominalism.

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