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The ontological commitments of inconsistent theories

Mark Colyvan

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Abstract In this paper I present an argument for belief in inconsistent objects. The argument relies on a particular, plausible version of scientific realism, and the fact that often our best scientific theories are inconsistent. It is not clear what to make of this argument. Is it a reductio of the version of scientific realism under consideration? If it is, what are the alternatives? Should we just accept the conclusion? I will argue (rather tentatively and suitably qualified) for a positive answer to the last question: there are times when it is legitimate to believe in inconsistent objects.

Keywords Inconsistency · Inconsistent mathematics · Inconsistent objects · Inconsistent theories · Ontology · Scientific realism

1 Introduction

According to scientific realists, we should look to our best scientific theories for advice about what we should take to exist. There may be some caveats about the treatment of idealisations in these theories and other problem cases, but the basic advice is that science tells us what exists. Quine (1981) and others (e.g. Putnam 1971; Resnik 1997; Colyvan 2001) have taken this line of reasoning a bit further and argued that we ought to be realists about mathematics as well—at least about those parts of mathematics that are indispensable to our best scientific theories. But what should we say about cases where our best scientific theory of some domain invokes inconsistent objects? Should we take the Quinean line further still and argue for belief in inconsistent objects? Does the existence of inconsistent theories undermine the Quinean approach to metaphysics? Should we be more selective in our realism and only believe in some yet-to-be-determined portions of our best scientific

M. Colyvan (✉)
Department of Philosophy, University of Sydney, Sydney, NSW 2006, Australia
e-mail: mcolyvan@usyd.edu.au

theories (Campbell 1994, Maddy 1992)? These are difficult questions and, for the most part, I will leave them unanswered—or at least, I will just gesture at answers. My primary task in this paper is to make the case that inconsistent scientific theories present problems for scientific realists, for, on the face of it at least, an indispensability argument for belief in inconsistent objects can be mounted. I will defend the conclusion of this argument against some of the more obvious objections.

2 Inconsistency in science

There are several quite different ways in which a scientific theory might end up inconsistent. First, it might inadvertently be inconsistent: some assumptions of the theory happen to conflict with other parts of the same theory or with other well-established theories. Second, the theory might be explicitly inconsistent, again because of internal inconsistency or because it is inconsistent with other theories. Third, the theory might invoke some inconsistent mathematical tools. The third case I will deal with in the next section; in this section I will focus on the first two kinds of inconsistency.

It is a rather common situation to find that a scientific theory is inadvertently inconsistent. The conflict between our best two physical theories—general relativity and quantum mechanics—is such a case, but there are other more mundane cases. And I think it is important to see that there are mundane cases. Consider the rotation problem for spiral galaxies. Newtonian gravitational theory, applied to spiral galaxies, tells us how these object should rotate, depending on the distribution of mass. If they are very massive and, in particular, have a great concentration of mass away from the centre, gravity holds the whole thing together and it should spin like a wheel. If, on the other hand, only the centre has a high concentration of mass, the centre will spin faster than the spiral arms and there will be a decline in the radial speed as you move away from the centre. If you plot the distance from the centre against radial speed, the resulting *rotation curves* should exhibit the decline in question. In 1959 it was discovered that the Triangulum Galaxy, M33, did not exhibit the decline in radial speed predicted by theory. M33's rotation curve was found to be flat: the outer part of the galaxy was spinning at much the same radial speed as the centre. M33 spins like a wheel—bound together much more tightly than the observed mass would allow.[1]

Since then, other spiral galaxies have been observed to have flat rotation curves. Such conflicts between theory and observation are common enough in science, but it is important to bear in mind that the observations in this and many other cases are mediated by a great deal of theory. So cases like these are best viewed as conflicts between two parts of the relevant theory. Moreover, with a little work, this conflict can be seen for what it is: a contradiction. In the example at hand, the contradiction

[1] This was the first evidence that there was much more than the visible mass in the outer reaches of this galaxy—what has become known as "dark matter". Also note that moving to general relativity as the relevant gravitational theory makes no difference. This is not a relativistic effect.

is between Newtonian gravitational theory, the theory involved in calculating the mass-density of the galaxy in question, and the theory involved in determining the radial speeds of galaxies (Doppler shifts). The first two pieces of theory taken together, suggest that the galaxy will have a rotation curve like the theoretically-predicted one, with the decline in speeds as you move away from the centre of the galaxy. But the third piece of theory suggests that there is no such decline and that the rotation curve is flat. To perhaps labour the point, the conjunction of these three pieces of theory suggest that M33 both does and does not spin according to the predicted rotation curve.[2]

The contradiction in this case was revealed by some hard science, it wasn't deliberately built in to any of the relevant theories. But this is not always the way. Various idealisations in science, when seen in the right light, deliberately introduce contradictory assumptions. Maddy (1992) presents a nice example of the assumption of infinitely deep oceans and how this assumption is required by the theory of waves on the open ocean. Maddy points out that the assumption of infinite depth raises problems for Quinean realists because this assumption would seem to be indispensable to our best theory of ocean waves. By Quine's line of reasoning, it would seem that we ought to believe in infinitely deep oceans. But Maddy might have pushed her point even further: there are other relevant pieces of theory that give finite depths for the oceans. The standard sonar techniques (and the theory invoked in sonar), for example, tell us that oceans have finite depths.[3] So what we are really dealing with here is a contradiction between two pieces of theory. Taking the conjunction of the two pieces of theory, we have it that oceans are both infinitely deep and not infinitely deep.[4]

What these examples show is that whenever we have a conflict between theory and observation or whenever we make idealisation in science, we may in fact be thought to be confronting inconsistency. Inconsistency might not be confined to a few critical episodes in the history of science. It might be quite common, and that gives us all the more reason to pursue a decent theory of how to treat it. Most importantly, for present purposes, we need a story about what ontological pronouncements we should make when faced with inconsistent theories. In the cases above, the theory involves an inconsistent object—a spiral galaxy and an ocean, respectively. More specifically, the theory attributes inconsistent properties to the objects in question and we need to know how to deal with such cases.

[2] I am not suggesting that the contradiction here presents insurmountable problems, just that there is a contradiction—at least until an adequate theory of dark matter is developed. See Abell et al. (1987) for further details.

[3] Not to mention the theory that the earth is roughly spherical with a finite diameter!

[4] Again there is nothing too troubling here. The sonar theory is surely correct and the assumption of infinitely deep oceans is a mere idealisation. It is clear what we ought to believe here (and that was Maddy's point). It is just that hard-nosed Quineans, she suggests, have trouble delivering the right answer.

3 Inconsistency in mathematics

The third kind of inconsistency I mentioned earlier involves employing inconsistent mathematics.[5] Take the case of the early calculus. This theory was straightforwardly inconsistent: infinitesimals were taken to be non-zero (by stipulation and by the fact that one needed to divide by them) and yet at other times they were taken to be equal to zero. Moreover, within one proof (such as a standard proof from first principles of the derivative of a polynomial, for example) both these contradictory properties were invoked. Newton and Leibniz also seemed to have contradictory interpretations of the infinitesimals. Newton, at least took infinitesimals to be changing quantities, approaching zero and yet these changing quantities could appear in equations where other terms were not changing (Gaukroger forthcoming). But appearing in such equations implies that infinitesimals are not changing at all. For example, the equation $2x = 2x + \delta$, where δ is an infinitesimal, implies that since all other terms are not changing, either δ is not changing (and is in fact equal to zero) or that the equation in question does not hold because $2x + \delta$ does not equal anything fixed at all, let alone $2x$.

So here we have an inconsistent theory, that remained inconsistent from its origins with Newton and Leibniz in the latter part of the 17th century until well into the 19th century.[6] For over a century this mathematical theory was inconsistent, but indispensable to mechanics, gravitational theory, electromagnetic theory, and the list goes on. Another example of an inconsistent mathematical theory is naïve set theory. Famously, naïve theory gave rise to Russell's and other paradoxes.[7] But once again, this did not stop the theory from being useful in applications—both applications elsewhere in mathematics and in empirical science. And, like the calculus, these were not isolated or obscure applications. Early calculus and naïve set theory both find wide and varied applications. This is interesting in itself and bears on more general questions about the applicability of mathematics (Colyvan 2008 and forthcoming).

Another kind of inconsistent mathematics that is worth mentioning is explicitly inconsistent mathematics. Such theories are usually developed in the context of a paraconsistent logic and are a relatively recent development. Examples here are finite models of arithmetic (Priest 1997) and explicitly inconsistent theories of infinitesimals (Mortensen 1995). Although, these theories very interesting they are not relevant for the present purposes. After all, we are interested in inconsistent mathematical theories that are indispensable to science. It is hard to make the case

[5] This might to be just a special case of one of the other two—a case where one of the theories in question is a mathematical theory. Be that as it may, the mathematical case is different enough to warrant separate treatment.

[6] The calculus was eventually put on a firm foundation with the work on limits by Bolzano, Cauchy, Weierstrass, and others. (See Kline (1972) for details of the history.) Later still, the original infinitesimal theory was made rigorous by Robinson (1966) with non-standard analysis, and (in a different way) by Conway (1976) with surreal numbers. Finally, Mortensen (1995) provided a paraconsistent treatment (invoking paraconsistent logic) of what was arguably the original inconsistent theory of infinitesimals.

[7] See Giaquinto (2002) for a very good discussion of the paradoxes and the foundations of modern set theory.

that either finite models of arithmetic or modern inconsistent theories of infinitesimals are indispensable in the required sense. While inconsistent calculus can be argued to have wide-ranging applications, it does not seem to have any advantages in this regard over its consistent rivals. And it is not clear that finite models of arithmetic have much by way of applications at all, let alone turning out to be indispensable to science. In any case, we have a couple of good candidate inconsistent mathematical theories—early calculus and naïve set theory—and that's all we really need.

4 An argument for inconsistent objects?

It is not too hard to see how a Quinean argument for inconsistent mathematical objects would go. It would proceed in much the same way as the regular one for realism about mathematical objects: given the indispensability of calculus in the 18th century, 18th century metaphysicians (if not scientists) ought to have believed in the existence of inconsistent objects, namely, infinitesimals. The case for belief in other contradictory objects, such as inconsistent spiral galaxies and oceans (discussed in Sect. 2), is much the same.

The crucial premise of such indispensability arguments is that we ought to be ontologically committed to all and only the entities indispensable to our best scientific theories. A couple of remarks are in order here. First, I take an entity to be indispensability to a theory *T* if there is no competing theory that is at least as good as *T* and which does without the entity in question (Colyvan 2001, pp. 76–78). This leads to the question of how to make judgements about ranking theories and, in particular, what it means to be the best theory. These, of course, are very difficult questions, the answers to which will involve an account of the weighing of empirical adequacy, explanatory power, simplicity, and many other theoretical virtues (Colyvan 2001, pp. 78–81). I have nothing new to say here and no guidance on how to conduct this juggling act. But, fortunately, we don't need to resolve such issues now! All that matters for present purposes is that somehow, scientists *do* manage to make determinations about whether one theory is better than another, and such determinations tend to be made on the basis of considerations of a variety of theoretical virtues. Moreover, being the best theory is simply to be better (in this sense) than all extant competitors. I should add that while consistency is one of the virtues, it is not clear that it holds any privileged position among the theoretical virtues. It may be more important than some, but the fact that scientists seriously entertain inconsistent theories, shows that consistency does not trump all other theoretical virtues. Consistency, it seems, is just one virtue among many.

So what should we say about the argument for inconsistent objects? My tentative conclusion is that anyone persuaded by the indispensability argument for scientific and mathematical realism, should also (perhaps reluctantly) sign up for belief in inconsistent objects. Note that this is not an argument that the world is inconsistent or that the world contains inconsistent objects, just that *there were times when we had warrant to believe in such inconsistent objects*. So the conclusion is not as radical as it might first seem. But even this conclusion many will find hard to swallow. That it should be even rational to believe in inconsistent objects will strike many as unacceptable, so let me very briefly run through some alternatives and some arguments against the tentative conclusion just advanced.

The first objection is that belief in even one inconsistent object leads to trivialism, where everything is taken to be true, including every other contradiction. This is right, but only if one holds onto classical logic or some other explosive logic in which *ex contradictione quodlibet* holds. If one is to entertain inconsistency, one needs to adopt a non-explosive logic: one in which there is some Q for which $P, \neg P \nvdash Q$.[8] Such paraconsistent logics block trivialism. They stop the rot. But even belief in one inconsistent object is too much rot for some tastes.

This suggests another, related line of attack: the law of non-contradiction (LNC) takes priority over other logical, mathematical, and scientific considerations and this law tells us, loud and clear, that there are no inconsistent objects. Or so the objection goes.[9] A complete reply to this objection would take a bit more work, but the short answer is that it is very hard to motivate such a privileged place for LNC (Bueno and Colyvan 2004 and Priest 1998).[10] After all, why should LNC trump other considerations such as those entertained here. In any case, anyone holding a holist, falliblist epistemology seems unable to reserve such a privileged position for LNC. But worse still for the objection under consideration, it is not clear that LNC does give such loud, clear advice. The problem is that if LNC is taken to be the statement that $\neg(P \wedge \neg P)$ is a theorem, then paraconsistent logics such as *LP* respect LNC—they have this version of LNC as a theorem. (How do such logics allow contradictions, you might wonder. They allow contradictions because although all instances of the schema $\neg(P \wedge \neg P)$ are true in *LP*, some instances are also false. Take P to be a sentence that is both true and false, then its negation is also true and false and so too is the negation of the conjunction of P and its negation. So although there are no cases where $\neg(P \wedge \neg P)$ is false and false only, there are cases where it is at least false, See Priest (2001) for the details.)

Let's leave general worries about inconsistency aside and focus on an objection to inconsistent objects, in particular. It might be argued that the notion that the world itself could be inconsistent, populated with inconsistent objects, is barely intelligible, let alone plausible, so we should resist this conclusion. We might even invoke inductive support for the consistency of the world: most of the objects we find are consistent and (more controversially) most scientific theories are consistent. But now, the objection continues, we note that the argument I've provided for inconsistent objects is in fact an argument that we *ought to believe* in inconsistent objects, not that the world contains any. So we have a Moore-like conflict here: the world is not populated with inconsistent objects and yet we ought to believe that it is so populated. This objection points out that my argument for belief in inconsistent objects is only one conjunct here and we cannot ignore the other. The upshot is that

[8] See Priest (2001) for an introduction to some of these logics and Priest 1998 for replies to other general worries about believing in contradictions.

[9] Everett (2005) makes an interesting argument against fictional realism based on the inconsistency found in fiction. Everett's argument has some similarity to the objection I am considering here. Thanks to Peter Godfrey-Smith for drawing my attention to Everett's paper.

[10] Although, see other essays in Priest et al. 2004 for arguments in favour of LNC and its priority.

there is no clear argument for inconsistent objects. Perhaps the correct ontological conclusion to draw is an agnostic one. I think this is an interesting objection but it is one that pushes away from scientific realism; a scientific realist ought to exercise great caution in advancing such a line of thought. Why? Well, for a start, the contradictions I've outlined are fairly widespread, so this line of thought, if not carefully qualified, could end up with agnosticism about spiral galaxies, oceans and the like. And such an agnosticism is in some ways less acceptable to a scientific realist than standard versions of instrumentalism such as van Fraassen's (1980) constructive empiricism. For unlike constructive empiricism, this agnosticism recommends agnosticism about some observables.[11]

It is also worth noting that the line of thought underwriting the objection in the last paragraph closely resembles the pessimistic meta-induction against scientific realism (Laudan 1981). The meta-induction tells us that meta-level reflection gives us reason to believe that our current scientific theories are false, yet at the base-level—at the level of scientific theorising and weighing of evidence—we have reason to believe that these theories are true. This Moore-like situation is resolved by scientific anti-realists in favour of the meta-level reflection. Although it might be tempting for the scientific realist to reject inconsistent objects by appeal to a meta-theoretic induction, such a concession to antirealism is fraught with danger. The general problem is that the kind of selective realism many philosophers would like to subscribe to is unstable. The indispensability argument encourages them to take their realism a bit further and embrace mathematical realism or else give up on their scientific realism (Colyvan 2006). The case of inconsistent objects just raises the stakes but the basic instability in selective realism is the same. The bottom line is that some moves are simply not available to the scientific realist because such moves are tantamount to giving up on realism. If such moves are consistently applied to nearby cases, the realism slides into a kind of widespread agnosticism. It seems to me that invoking agnosticism in the face of pessimistic meta-inductions is such a move.[12]

Another objection might be to deny that an inconsistent theory can ever be one of our best. We might admit that some inconsistent theory is the best we currently have available but that the inconsistency of the theory in question is sufficient to think that the theory should not be treated realistically. It might even be argued that there is an equivocation on the word 'best' here: epistemically best versus instrumentally best. It is only the epistemically best theories that we ought to believe, but inconsistent theories can only every be instrumentally best—useful models for certain purposes. I must admit that something like this sounds right, but it is very hard to spell out the details in a non-ad-hoc way, and in a way that does not render a great deal of science merely instrumental. As I have already suggested, many of our best scientific theories have been and continue to be inconsistent. Are we to always classify such theories as merely instrumental? Again this seems like a huge

[11] See Beall and Colyvan (2001) for an argument for observable contradictions.

[12] Of course, the scientific realist always has the option of arguing that there are relevant differences between the two cases. In the case of the standard pessimistic meta-induction, take the side of the base-level theory, but in the case of the pessimistic meta-induction involving inconsistent objects, side with the meta-induction (and reject inconsistent objects). Such a move needs to be independently motivated though.

concession to anti-realism. Also, this line of objection seems to give a privileged role to consistency among the theoretical virtues. This seems both ad hoc and at odds with scientific practice, where inconsistent theories and theories with persisting anomalies (which usually amounts to the same thing) are seriously entertained and do not seem excluded from realistic interpretation.

Along similar lines, we might distinguish between believing in the entities of a theory and believing in the theory itself. We might, for example, accept the entities of our best scientific theories but remain agnostic about some of the properties ascribed to the entities in question, especially if those properties are inconsistent properties. Again we might invoke a notion of 'instrumentally best' and allow that entities posited with inconsistent properties should be looked upon as useful fictions, or the like. But this response requires privileging consistency in a way that I have already noted as problematic. More importantly, I find it hard to understand how commitment to a theoretical entity can be detached from the theory positing it. In some straightforward cases perhaps there is no problem—with medium-sized objects of our common experience, for example. But in general it seems that scientists and scientific realists alike believe in theoretical entities such as electrons because of their role in the scientific theories which posit them. Perhaps sense can be made of believing in theoretical entities without believing in the theories positing them, but at the very least some detailed work is required to give a plausible account of how this might work. In any case, such a proposal marks a significant departure from Quinean realism (and arguably other standard forms of scientific realism).

The final objection to my argument for inconsistent objects is that what we have, in effect, is a reductio of Quinean naturalised metaphysics. In order to establish this, however, it would need to be established that other versions of scientific realism escape the problems associated with inconsistent theories, for otherwise we might end up with a reductio of scientific realism, or even metaphysics more generally.[13] Although I haven't argued for this here, I think a similar problem can be presented for scientific realists of most stripes. Indeed, anyone who takes our best science to be giving us guidance on ontological matters, has a prima facie problem whenever those best scientific theories are inconsistent. A great deal of work has gone into understanding the role of idealisations (although they are usually not thought of as involving contradictions),[14] but the use of inconsistent mathematics in our best science has received very little attention. The ontological significance of inconsistent theories, such as the spiral-galaxy case, has also received little attention. In any case, it seems rather implausible that the same strategy for dealing with all three kinds of inconsistency would work. As I hope I've made clear, the three cases presented in this paper are quite different. I have used the Quinean framework to make the case for inconsistent objects, but I don't think that Quinean realists are alone here. Inconsistency in science is a topic that realists of all stripes would be well advised to think more about.

[13] For different reasons, Yablo (1998) suggests that the Quinean approach to metaphysics is metaphysics' last stand; if the Quinean project doesn't succeed, then that's the end of metaphysics. Mortensen (forthcoming) suggests that the problem here is for Platonists of all kinds. He suggests that Platonists face the following dilemma: either give up Platonism or admit inconsistent mathematical objects.

[14] See, for example, Batterman (2002), Cartwright (1983), and Godfrey-Smith (2008).

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