Addendum to Quantum Wave Function Collapse of a System Having Three anti Commuting Elements.

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Abstract : We indicate a new way in the solution of the problem of the quantum measurement . In past papers we used the well-known formalism of the density matrix using an algebraic approach in a two states quantum spin system S, considering the particular case of three anticommuting elements. We demonstrated that, during the wave collapse, we have a transition from the standard Clifford algebra, structured in its space and metrics, to the new spatial structure of the Clifford dihedral algebra. This structured geometric transition, which occurs during the interaction of the S system with the macroscopic measurement system M, causes the destruction of the interferential factors. In the present paper we construct a detailed model of the (S+M) interaction evidencing the particular role of the Time Ordering in the (S+M) coupling since we have a time asymmetric interaction . We demonstrate that , during the measurement , the physical circumstance that the fermion creation and annihilation operators of the S system must be destroyed during such interaction has a fundamental role .

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1. Introduction

In ninety years since its beginnings quantum mechanics has had great functional and theoretical success leaving little reason to doubt its intrinsic validity. Nevertheless, we cannot ignore that some questions concerning the foundations of this theory remained unsolved, and historic debates among scientists who deeply influenced the early development of the theory remain.

The first important question concerns the problem of the wave-function collapse by measurement.

Its solution would be of relevant significance because it would provide us with a *self-consistent* formulation of the theory, which presently depends on the von Neumann postulates that have been added from the outside of the body of theory.

For a complete examination of the actual problems that are involved, we refer the reader to the several reviews that may be found in pertinent literature [1-12].

Consider the measurement of a given observable *F* on a quantum-mechanical system S that is in a normalized superposition of states

$$\psi = \sum_i c_i \varphi_i \quad ; \quad c_i = (\varphi_i, \psi) \quad ; \quad \sum_i |c_i|^2 = 1 ; \tag{1}$$

where φ_i is a normalized eigenstate of *F* , relative to an eigenvalue λ_i , $F\varphi_i = \lambda_i \varphi_i$, $(\varphi_i, \varphi_j) = \delta_{ij}$.

The probability of finding the eigenvalue λ_i during the measurement is $|c_i|^2$, the corresponding eigenstate is φ_i and during the measurement the wave function ψ is subjected to the transition $\psi \rightarrow \varphi_i$ characterizing the completed collapse .
The density matrix approach as it was initiated by von Neumann is

$$\rho_S = |\psi\rangle\langle\psi| = \sum_i \sum_j c_i c_j^* |\varphi_i\rangle\langle\varphi_j| \rightarrow \rho_{S,F_{\lambda k}} = \sum_k |\varphi_k\rangle\langle\varphi_k|. \qquad (2)$$

Usually, we consider a macroscopic measuring device M and we postulate that the states of M entangle with those of S

$$\rho = \rho_S \otimes \rho_M = \sum_i \sum_j c_i c_j^* |\varphi_i\rangle\langle\varphi_j| \otimes \rho_M \rightarrow \rho_{S,M_t} = \sum_k |c_k|^2 |\varphi_k\rangle\langle\varphi_k| \otimes \rho_{M(k)_t} \qquad (3)$$

If the system is not destroyed by the measurement, and if the interaction fits into the so called measurement of the first kind, then the quantum state after the measurement will be the eigenstate associated with the measurement outcome, or more generally (to include degenerancies), the normalized projection of the original state onto the eigensubspace associated with the outcome. This rule is known as the *projection postulate*. It originated with Dirac and von Neumann [13], and was later formalized in degenerate cases by Luders and Ludwig [14,15] .
Consider S to be a quantum two states system . The complete phase-damping by using projection postulate gives

$$D(\rho) = |0\rangle\langle 0|\rho|0\rangle\langle 0| + |1\rangle\langle 1|\rho|1\rangle\langle 1| \qquad (4)$$

Generally speaking, we have a set of mutually orthogonal projectors ($P_1, P_2, \ldots\ldots\ldots, P_N$) which complete to unity, $P_i P_j = \delta_{ij} P_j$, $\sum_i P_i = 1$, the result i is obtained with probability $p_i = \langle\psi|P_i|\psi\rangle$ and the state collapses to

$$\frac{1}{\sqrt{p_i}} P_i |\psi\rangle.$$

It is known that quantum mechanics has some peculiar features that are missing in the counterpart of classical physics. Two basic features are quantum interference and the collapse .
Starting with 2009 [16,17,18] our tentative approach was to use the Clifford algebra with the aim to construct a bare bone skeleton of quantum mechanics but giving collapse. We will deepen here some basic features evidencing in particular that in order the $\psi-$collapse to be obtained, the fermion creation and destruction operators must be realized in our considered system. .

2. Theoretical Elaboration

Let us start with a proper definition of the 3-D space Clifford algebra Cl_3.
It is an associative algebra generated by three abstract algebraic elements e_1, e_2, and e_3 that satisfy the orthonormality relation

$$e_j e_k + e_k e_j = 2\delta_{jk} \quad \text{for } j,k,\lambda \in [1,2,3] \qquad (5)$$

That is

$e_\lambda^2 = 1$ and $e_j e_k = -e_k e_j$ for $j \neq k$

The algebra holds about only two postulates that are

a) it exists the scalar square for each basic element:

$$e_1 e_1 = k_1 \ , \ e_2 e_2 = k_2 \ , \ e_3 e_3 = k_3 \ \text{ with } \ k_i \in \Re \ . \tag{6}$$

In particular we have also the unit element, e_0, such that

$$e_0 e_0 = 1, \text{ and } e_0 e_i = e_i e_0 \tag{7}$$

b) The basic elements e_i are anticommuting elements

$$e_1 e_2 = -e_2 e_1 \ , \quad e_2 e_3 = -e_3 e_2 \ , \ e_3 e_1 = -e_1 e_3 \ . \tag{8}$$

Following Ilamed and Salingaros [19] we may give proof of two theorems.

Theorem n.1.

Assuming the two postulates given in (a) and (b) with $k_i = 1$, the following commutation relations hold for such algebra :

$$e_1 e_2 = -e_2 e_1 = i e_3 ; e_2 e_3 = -e_3 e_2 = i e_1 ; \ e_3 e_1 = -e_1 e_3 = i e_2 \ ; i = e_1 e_2 e_3 , (e_1^2 = e_2^2 = e_3^2 = 1) \tag{9}$$

They characterize the Clifford S_i algebra. We will call it the algebra $A(S_i)$

Theorem n.2a .

Assuming the postulates given in (a) and (b) with $k_1 = 1$, $k_2 = 1$, $k_3 = -1$, the following commutation rules hold for such new algebra:

$$e_1^2 = e_2^2 = 1 ; \ i^2 = -1 ;$$

$$e_1 e_2 = i \ , \ e_2 e_1 = -i \ , \ e_2 i = -e_1 , i e_2 = e_1 , e_1 i = e_2 \ , \ i e_1 = -e_2 \tag{10}$$

They characterize the Clifford N_i algebra. We will call it the algebra $N_{i,+1}$

Theorem n.2b .Assuming the postulates given in (a) and (b) with $k_1 = 1$, $k_2 = 1$, $k_3 = -1$, the following commutation rules hold for such new algebra

$$e_1^2 = e_2^2 = 1 ; \ i^2 = -1 ;$$

$$e_1 e_2 = -i \ , \ e_2 e_1 = i \ , \ e_2 i = e_1 , i e_2 = -e_1 , e_1 i = -e_2 , \ i e_1 = e_2 \tag{11}$$

They characterize the Clifford Ni algebra. We will call it the algebra $N_{i,-1}$

The algebra $N_{i,\pm 1}$ is the well known Clifford Dihedral algebra .

The demonstration of the two theorems as well as the construction of a bare bone skeleton of quantum mechanics were given by us in previous papers (16,17,18).

Let us evidence an important feature of Clifford algebra $A(S_i)$ **.**

In Clifford algebra $A(S_i)$ we have idempotents , two of such idempotents are

$$\psi_1 = \frac{1+e_3}{2} \quad \text{and} \quad \psi_2 = \frac{1-e_3}{2} \tag{12}$$

$\psi_1^2 = \psi_1$ and $\psi_2^2 = \psi_2$.

Let us examine now the following algebraic relations:

$$e_3\psi_1 = \psi_1 e_3 = (+1)\psi_1 \tag{13}$$

$$e_3\psi_2 = \psi_2 e_3 = (-1)\psi_2 \tag{14}$$

Similar relations hold in the case of e_1 or e_2. The inspection of (13) and (14) reveals a net analogy with a two states z-spin system constructed in their proper Hilbert space in quantum mechanics . Of course the analogy between the three basic elements e_i and quantum spin operators is trivial since we have $S_i = \frac{\hbar}{2} e_i$ and e_i relating the well known spin Pauli matrices.

Consider the previous two states system S with its proper representation in Hilbert space .

The complex coefficients $c_i \, (i = 1,2)$ are the well known probability amplitudes for the considered quantum state

$$\psi = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \quad \text{and} \quad |c_1|^2 + |c_2|^2 = 1 \tag{15}$$

For a pure state in quantum mechanics it is $\rho^2 = \rho$. We have a corresponding algebraic member that in $A(S_i)$ is given in the following manner

$$\rho_S = a + be_1 + ce_2 + de_3 \tag{16}$$

with

$$a = \frac{|c_1|^2}{2} + \frac{|c_2|^2}{2}, \; b = \frac{c_1^* c_2 + c_1 c_2^*}{2}, \; c = \frac{i(c_1 c_2^* - c_1^* c_2)}{2}, \quad d = \frac{|c_1|^2 - |c_2|^2}{2}$$

In our scheme a theorem may be demonstrated in Clifford algebra [20,21] . It is that

$$\rho_S^2 = \rho_S \leftrightarrow a = \frac{1}{2} \;\; \text{and} \;\; a^2 = b^2 + c^2 + d^2 \quad \text{and} \;\; \mathrm{Tr}(\rho) = 1 \tag{17}$$

Let us write the state of the two state quantum system S with connected quantum observable S_3 relating e_3 of $A(S_i)$. We have

$$\left|\psi\right\rangle = c_1\left|\varphi_1\right\rangle + c_2\left|\varphi_2\right\rangle \ , \ \varphi_1 = \begin{pmatrix}1\\0\end{pmatrix} \ , \ \varphi_2 = \begin{pmatrix}0\\1\end{pmatrix} \tag{18}$$

and

$$|c_1|^2 + |c_2|^2 = 1$$

As we know, the density matrix of such system is easily written

$$\rho = a + be_1 + ce_2 + de_3 \tag{19}$$

with

$$a = \frac{|c_1|^2 + |c_2|^2}{2} \ , \ b = \frac{c_1^* c_2 + c_1 c_2^*}{2} \ , \ c = \frac{i(c_1 c_2^* - c_1^* c_2)}{2} , \ d = \frac{|c_1|^2 - |c_2|^2}{2} \tag{20}$$

where in matrix notation, e_1 , e_2, and e_3 are the well known Pauli matrices

$$e_1 = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix} , \ e_2 = \begin{pmatrix}0 & -i\\i & 0\end{pmatrix} , \ e_3 = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix} \tag{21}$$

The (17) and (19) coincide .

To examine the consequences, starting with the algebraic element (16), write it in the two equivalent algebraic forms that are obviously still in the algebra A(Si).

$$\rho_S = \frac{1}{2}(|c_1|^2 + |c_2|^2) + \frac{1}{2}(c_1 c_2^*)(e_1 + e_2 i) + \frac{1}{2}(c_1^* c_2)(e_1 - ie_2) + \frac{1}{2}(|c_1|^2 - |c_2|^2)e_3 \tag{22}$$

and

$$\rho_S = \frac{1}{2}(|c_1|^2 + |c_2|^2) + \frac{1}{2}(c_1 c_2^*)(e_1 + ie_2) + \frac{1}{2}(c_1^* c_2)(e_1 - e_2 i) + \frac{1}{2}(|c_1|^2 - |c_2|^2)e_3 \tag{23}$$

The (22) and (23) coincide , and both such expressions contain the following interference terms.

$$\frac{1}{2}(c_1 c_2^*)(e_1 + e_2 i) + \frac{1}{2}(c_1^* c_2)(e_1 - ie_2) \tag{24}$$

and

$$\frac{1}{2}(c_1 c_2^*)(e_1 + ie_2) + \frac{1}{2}(c_1^* c_2)(e_1 - e_2 i) \tag{25}$$

We know that they represent the hard problem in a theory of quantum collapse .

We may write (23) in the following terms

$$\rho_S = \rho_{1S} + \rho_{S,\mathrm{int}.} \tag{26}$$

where

$$\rho_{1S} = \rho_S = \frac{1}{2}(|c_1|^2 + |c_2|^2) + \frac{1}{2}(|c_1|^2 - |c_2|^2)e_3 \tag{27}$$

and

$$\rho_{S,\text{int}} = \frac{1}{2}(c_1c_2^*)(e_1 + e_2 i) + \frac{1}{2}(c_1^* c_2)(e_1 - ie_2) \tag{28}$$

or equivalently

$$\rho_{S,\text{int}} = \frac{1}{2}(c_1c_2^*)(e_1 + ie_2) + \frac{1}{2}(c_1^* c_2)(e_1 - e_2 i) \tag{29}$$

The mechanism that induces the collapse of the wave function is now evident. During the interaction of the system S with the macroscopic apparatus M the previous interference terms are destroyed. It never can happen until we assume in algebraic terms that the $A(Si)$ algebra is acting in the ($S+M$) interaction. and that ,during such coupling ($S+M$), the system undergoes a transition from the Clifford algebra $A(S_i)$ to the dihedral algebra $N_{i,\pm1}$. If , probabilistically speaking, the macroscopic instrument reads $S_3 = +\frac{\hbar}{2}$, it means that the algebra $N_{i,+1}$ has prevailed . If instead the macroscopic instrument reads $S_3 = -\frac{\hbar}{2}$, it means that the algebra $N_{i,-1}$ has prevailed.

In the first case the basic commutation rules that hold are those given in theorem 2a ,

$$e_1e_2 = i \text{ , } e_2e_1 = -i \text{ ,} \tag{30}$$

$$e_2 i = -e_1 \text{, } ie_2 = e_1 \text{, } e_1 i = e_2 \text{, } ie_1 = -e_2 \tag{31}$$

The density matrix becomes

$$\rho_{S,+1} = \rho_{1S} + \rho_{S,\text{int.}} \tag{32}$$

with

$$\rho_{S,\text{int}} = \frac{1}{2}(c_1c_2^*)(e_1 + e_2 i) + \frac{1}{2}(c_1^* c_2)(e_1 - ie_2) = 0 \tag{33}$$

In the second case the basic commutation rules that hold are those given in theorem 2b,

$$e_1e_2 = -i \text{ , } e_2e_1 = i \text{ , } e_2 i = e_1 \text{ , } ie_2 = -e_1 \text{ , } e_1 i = -e_2 \text{ , } ie_1 = e_2 \tag{34}$$

The density matrix becomes

$$\rho_{S,-1} = \rho_{1S} + \rho_{S,\text{int}}. \tag{35}$$

with

$$\rho_{S,\text{int}} = \frac{1}{2}(c_1 c_2^*)(e_1 + ie_2) + \frac{1}{2}(c_1^* c_2)(e_1 - e_2 i) = 0 \tag{36}$$

The macroscopic apparatus has the task to differentiate $\rho_{S,+1}$ from $\rho_{S,-1}$ on the basis of its dihedral algebra, destroying interference .

There is another important feature in such mechanism . The basic matrix density expression , written previously in equivalent manner in the (22) and (23) and valid only in the *A*(S_i) algebra, (this is to say before *S* interacts with *M*) contains two algebraic elements that in quantum mechanics relate the Fermion annihilation and creation operators . In fact they are explicitly expressed in such basic matrix density expression

$$\rho_S = \frac{1}{2}(|c_1|^2 + |c_2|^2) + \frac{1}{2}(c_1 c_2^*)(e_1 + e_2 i) + \frac{1}{2}(c_1^* c_2)(e_1 - ie_2) + \frac{1}{2}(|c_1|^2 - |c_2|^2)e_3 \tag{37}$$

They act in $A(S_i)$ before of the interaction of S with M . The new key is here : when the system S interacts with M , the new commutation relations , given previously for the dihedral algebra in theorems 2a and 2b, or the (31) or the (34) , act and they completely cancel the presence of the algebraic terms corresponding to the two fermion creation and annihilation operators. Quantum collapse requires the cancellation of such two operators and it happens during the transition from an $A(S_i)$ to $N_{i,\pm 1}$ dihedral Clifford algebra in its algebraic , geometric and metric structure . This represents the basic mechanism of the ($S+M$) interaction.

We have the counterpart using the Hamiltonian and the evolution operator in quantum mechanics .

Consider the quantum system S and indicate by ψ_0 the state at the initial time in Hilbert space . The state at any time t will be given by

$$\psi(t) = U(t)\psi_0 \quad \text{and} \quad \psi_0 = \psi(t=0) \tag{38}$$

An Hamiltonian *H* must be constructed such that the evolution operator *U(t),* that must be unitary, gives $U(t) = e^{-iHt}$.

It is well known that, given a finite N-level quantum system described by the state $\psi(t)$, its evolution is regulated according to the time dependent Schrödinger equation

$$i\hbar \frac{d\psi(t)}{dt} = H(t)\psi(t) \quad \text{with } \psi(0) = \psi_0 . \tag{39}$$

Let us introduce a model for the hamiltonian *H(t).* We indicate by H_0 the hamiltonian of the system S, and we add to H_0 an external time varying hamiltonian, $H_1(t)$, representing the coupling to which the system S is subjected by action of the measuring apparatus. We write the total hamiltonian as

$$H(t) = H_0 + H_1(t) \tag{40}$$

so that the time evolution will be given by the following Schrödinger equation

$$i\hbar \frac{d\psi(t)}{dt} = [H_0 + H_1(t)]\psi(t) \tag{40a}$$

We have that

$$i\hbar \frac{dU(t)}{dt} = H(t)U(t) = [H_0 + H_1(t)]U(t) \quad \text{and } U(0)=I \tag{41}$$

where *U(t)* pertains to the special group *SU(N).*

Let $A_1, A_2, \ldots\ldots, A_n$, ($n=N^2-1$), are skew-hermitean matrices forming a basis of Lie algebra *SU(N).* In this manner one arrives to write the explicit expression of the hamiltonian *H(t).* It is given in the following manner

$$-iH(t) = -i[H_0 + H_1(t)] = \sum_{j=1}^{n} a_j A_j + \sum_{j=1}^{n} b_j A_j = Hamiltonian\ System\ S + Hamiltonian\ Apparatus\ M \tag{42}$$

where a_j and $b_j = b_j(t)$ are respectively the constant components of the hamiltonian of *S* and the time-varying control parameters characterizing the action of the measuring apparatus *M* .

In order to continue such discussion we have to introduce the operator *T,* the time ordering parameter (for details see reff. [9,10,11]), in order to correctly describe the time (*S+M*) interaction , being in this case *M* a macroscopic apparatus marked from strong irreversibility . We have

$$U(t) = T\exp(-i\int_0^t H(\tau)d\tau) = T\exp(-i\int_0^t (a_j + b_j(\tau))A_j d\tau) \tag{43}$$

that is the well known Magnus expansion. Consequently, *U(t)* may be expressed by exponential terms as it follows

$$U(t) = \exp(\gamma_1 A_1 + \gamma_2 A_2 + \ldots\ldots + \gamma_n A_n) \tag{44}$$

on the basis of the Wein-Norman formula

$$\Xi(\gamma_1, \gamma_2, \ldots\ldots, \gamma_n)\begin{pmatrix} \dot{\gamma}_1 \\ \dot{\gamma}_2 \\ \ldots \\ \dot{\gamma}_n \end{pmatrix} = \begin{pmatrix} a_1 + b_1 \\ a_2 + b_2 \\ \ldots \\ a_n + b_n \end{pmatrix} \tag{45}$$

with Ξ n x n matrix, analytic in the variables γ_i. We have $\gamma_i(0)=0$ and $\Xi(0)=I$, and thus it is invertible. We obtain

$$\begin{pmatrix} \dot{\gamma}_1 \\ \dot{\gamma}_2 \\ \dots \\ \dot{\gamma}_n \end{pmatrix} = \Xi^{-1} \begin{pmatrix} a_1+b_1 \\ a_2+b_2 \\ \dots \\ a_n+b_n \end{pmatrix} \quad (46)$$

Consider a simple case based on the superposition of only two states. We have

$$\psi = [y_1, y_2]^T \quad \text{and} \quad |y_1|^2 + |y_2|^2 = 1 \quad (47)$$

We have here an *SU(2)* unitary transformation, selecting the skew symmetric basis for *SU(2).* We will have that

$$e_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \;,\; e_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \;,\; e_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (48)$$

The following matrices are given

$$A_j = \frac{i}{2} e_j \;,\; \text{j = 1,2,3} \quad (49)$$

The reader may now ascertain that the previously developed formalism is moving in direct correspondence with our Clifford algebra A(Si).

We are now in the condition to express *H(t)* and *U(t)* in our case of interest. The most simple situation we may examine is that one of fixed and constant control parameters b_j. In this condition the hamiltonian *H* will become fully linear time invariant and its exponential solution will take the following form

$$e^{t(\sum_{j=1}^{3}(a_j+b_j)A_j)} = \cos(\frac{k}{2}t)I + \frac{2}{k} sen(\frac{k}{2}t)\left(\sum_{j=1}^{3}(a_j+b_j)A_j\right) \quad (50)$$

with $k=\sqrt{(a_1+b_1)^2+(a_2+b_2)^2+(a_3+b_3)^2}$. In matrix form it will result

$$U(t) = \begin{pmatrix} \cos\frac{k}{2}t + \frac{i}{k} sen\frac{k}{2}t(a_3+b_3) & \frac{1}{k} sen\frac{k}{2}t[a_2+b_2+i(a_1+b_1)] \\ \frac{1}{k} sen\frac{k}{2}t[-a_2-b_2+i(a_1+b_1)] & \cos\frac{k}{2}t - \frac{i}{k} sen\frac{k}{2}t(a_3+b_3) \end{pmatrix} \quad (51)$$

and, obviously, it will result to be unimodular as required.

Starting with this matrix representation of time evolution operator *U(t),* we may deduce promptly the dynamic time evolution of quantum state at any time t writing

$$\psi(t) = U(t)\psi_0 \tag{52}$$

assuming that we have for ψ_0 the following expression

$$\psi_0 = \begin{pmatrix} c_{true} \\ c_{false} \end{pmatrix} \tag{53}$$

having adopted for the true and false states (or yes-not states, +1 and –1 corresponding eigenvalues of such states) the following matrix expressions

$\varphi_{true} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\varphi_{false} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Finally, one obtains the expression of the state $\psi(t)$ at any time

$$\begin{aligned}\psi(t) = &\left[c_{true}\left[cos\frac{k}{2}t + \frac{i}{k}sen\frac{k}{2}t(a_3+b_3)\right] + c_{false}\left[\frac{1}{k}sen\frac{k}{2}t\left[(a_2+b_2)+i(a_1+b_1)\right]\right]\right]\varphi_{true} + \\ &\left[c_{true}\left[\frac{1}{k}sen\frac{k}{2}t\left[i(a_1+b_1)-(a_2+b_2)\right]\right] + c_{false}\left[cos\frac{k}{2}t - \frac{i}{k}sen\frac{k}{2}t(a_3+b_3)\right]\right]\varphi_{false}\end{aligned} \tag{54}$$

As consequence, the two probabilities *$P_{true}(t)$* and *$P_{false}(t)$,* will be given at any time t by the following expressions

$$P_{true}(t) = (A^2+B^2)\cos^2\frac{k}{2}t + \frac{1}{k^2}sen^2\frac{k}{2}t(P^2+Q^2) + \frac{senkt}{k}(AP+BQ) \tag{55}$$

and

$$P_{false}(t) = (C^2+D^2)\cos^2\frac{k}{2}t + \frac{1}{k^2}sen^2\frac{k}{2}t(S^2+R^2) + \frac{senkt}{k}(RC+DS) \tag{56}$$

where

$A= Re\ c_{true}$, $B=Im\ c_{true}$, $C=Re\ c_{false}$, $D=Im\ c_{false}$,

$P=-D(a_1+b_1)+C(a_2+b_2)-B(a_3+b_3)$,

$Q=C(a_1+b_1)+D(a_2+b_2)+A(a_3+b_3)$,

$R=-B(a_1+b_1)-A(a_2+b_2)+D(a_3+b_3)$,

$S=A(a_1+b_1)-B(a_2+b_2)-C(a_3+b_3)$

Until here we have developed only standard quantum mechanics. The reason to have developed here such formalism has been to evidence that at each step it has its corresponding counterpart in Clifford algebraic framework *A(Si),* and thus we may apply to it the two theorems previously demonstrated, passing from the algebra $A(Si)$ to $N_{i,\pm1}$. In fact, to this purpose, it is sufficient to multiply the (50) by the (53) to obtain the final forms of $c_{true}(t)$ and $c_{false}(t)$

In the final state we have that

$$\psi_t = \begin{pmatrix} c_{true}(t) \\ c_{false}(t) \end{pmatrix} \tag{57}$$

We may now write the density matrix that will result to have the same structure of the previously case given in the (22-23) . In the Clifford algebraic framework it will pertain still to the Clifford algebra *A(Si).* In order to describe the wave-function collapse we have to repeat the same procedure that we developed previously from the (22) to the (37), considering that, in accord to our criterium, we have to pass from the algebra A(Si) to $N_{i,\pm 1}$, and obtaining

$$\rho \to \rho_M = \left| c_{true}(t) \right|^2 \tag{58}$$

in the case $N_{i,+1}$

and

$$\rho \to \rho_M = \left| c_{false}(t) \right|^2 \tag{59}$$

in the case $N_{i,-1}$, as required in the collapse.

Using Clifford algebra, we have given now a complete theoretical elaboration of the problem of wave function reduction in quantum mechanics also considering the process under the profile of the time dynamics. A time value of the collapse may be also obtained by a_i and b_j.

To avoid difficulties that could arise to have considered only an n=2 dimensional situation , we may also consider now the explicit case of the ($S+M$) interaction in their corresponding tensor product .

Clifford $A(Si)$ algebra at order n=4 [16,17,18] is

$$E_{0i} = I^1 \otimes e_i ; \qquad E_{i0} = e_i \otimes I^2 \tag{60}$$

The notation ⊗ denotes direct product of matrices, and I^i is the *i*th 2x2 unit matrix. We have two distinct sets of Clifford basic unities, E_{0i} and E_{i0}, with

$$E_{0i}^2 = 1 ; \qquad E_{i0}^2 = 1, \qquad i = 1, 2, 3; \tag{61}$$

$$E_{0i} E_{0j} = i E_{0k} ; \qquad E_{i0} E_{j0} = i E_{k0} , \quad j = 1, 2, 3; \qquad i \neq j$$

and

$$E_{i0} E_{0j} = E_{0j} E_{i0} \tag{62}$$

with (*i, j, k*) cyclic permutation of (*1, 2, 3*).

Let us examine now the following result

$(I^1 \otimes e_i)(e_j \otimes I^2) = E_{0i} E_{j0} = E_{ji}$ (63)

We have $E_{0i} E_{j0} = E_{ji}$ with $i = 1, 2, 3$ and $j=1, 2, 3$, $E^2_{ji} = 1$, $E_{ij} E_{km} \neq E_{km} E_{ij}$, and $E_{ij} E_{km} = E_{pq}$ where p results from the cyclic permutation (i, k, p) of ($1, 2, 3$) and q results from the cyclic permutation (j, m, q) of ($1, 2, 3$).

In the case $n = 4$ we have two distinct basic set of unities E_{0i}, E_{i0} and, in addition, basic sets of unities

(E_{ij}, E_{ip}, E_{0m}) with (j, p, m) basic permutation of ($1, 2, 3$). We may now give the explicit expressions of E_{0i}, E_{i0}, and E_{ij} . E_{0i} refers to the measured system S while E_{j0} refers to the measuring apparatus . E_{ij} characterizes instead the coupling .

$$E_{01} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}; \; E_{02} = \begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix}; \; E_{03} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad (64)$$

$$E_{10} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}; \; E_{20} = \begin{pmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \\ i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix}; E_{30} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix};$$

$$E_{11} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}; \; E_{22} = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}; \; E_{33} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix};$$

$$E_{12} = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}; E_{13} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}; E_{21} = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix};$$

$$E_{31} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}; E_{23} = \begin{pmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & i \\ i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{pmatrix}; E_{32} = \begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \end{pmatrix}.$$

We have now some different sets of Clifford algebras $A(Si)$

(E_{01},E_{12},E_{13}), (E_{01},E_{22},E_{23}) ,(E_{01},E_{32},E_{33}), (E_{02},E_{11},E_{13}), (E_{02},E_{21},E_{23}) ,(E_{02},E_{31},E_{33}),
(E_{03},E_{11},E_{12}) ,(E_{03},E_{21},E_{22}), (E_{03},E_{31},E_{32}) ,(E_{10},E_{23},E_{33}), (E_{10},E_{22},E_{32}),(E_{10},E_{21},E_{31})
(E_{20},E_{13},E_{33}), (E_{20},E_{12},E_{32}) ,(E_{20},E_{11},E_{31}) ,(E_{30},E_{13},E_{23}) ,(E_{30},E_{12},E_{22}) ,(E_{30},E_{11},E_{21}) (65)

We may apply the theorems n.1 and n.2 to each of such sets and consider the $A(S_i)$ and $N_{1,\pm1}$ algebras that we used in the previous case of application.

Fixed such algebraic premises, we have to extend the previous elaboration considering explicitly the presence of the measurement apparatus M obtaining

$$\rho=\rho_S\otimes\rho_M=\sum_i\sum_j c_i c_j^*\left|\varphi_i\right.><\left.\varphi_j\right|\otimes\rho_M\rightarrow\rho_{S,M,t}=\sum_k\left|c_k\right|^2\left|\varphi_k\right.><\left.\varphi_k\right|_t\otimes\rho_{M(k),t}\qquad(66)$$

We have connected the set E_{0i} to the quantum system S to be measured, and the set E_{i0} to the measuring apparatus M . The basic set E_{ij} couples S with M . The resulting density matrix ρ is

$$\rho=\begin{pmatrix} a & b_1+ib_2 & c_1+ic_2 & d_1+id_2\\ b_1-ib_2 & e & f_1+if_2 & q_1+iq_2\\ c_1-ic_2 & f_1-if_2 & h & t_1+it_2\\ d_1-id_2 & q_1-iq_2 & t_1-it_2 & s\end{pmatrix}\qquad(67)$$

ρ of the (67) is still a member of the Clifford algebra $A(S_i)$.

$$\rho=a(\frac{E_{00}+E_{03}+E_{30}+E_{33}}{4})+e(\frac{E_{00}+E_{30}-E_{03}-E_{33}}{4})+h(\frac{E_{00}+E_{03}-E_{30}-E_{33}}{4})+$$
$$s(\frac{E_{00}-E_{03}-E_{30}+E_{33}}{4})+\left[b_1(\frac{E_{01}+E_{31}}{2})-b_2(\frac{E_{02}+E_{32}}{2})\right]+\left[c_1(\frac{E_{10}+E_{13}}{2})-c_2(\frac{E_{23}+E_{20}}{2}\right]+$$
$$\left[d_1(\frac{E_{11}-E_{22}}{2})-d_2(\frac{E_{12}+E_{21}}{2})\right]+\left[f_1(\frac{E_{11}+E_{22}}{2})+f_2(\frac{E_{12}-E_{21}}{2})\right]+$$
$$\left[q_1(\frac{E_{10}-E_{13}}{2})+q_2\left(\frac{E_{23}-E_{20}}{2}\right)\right]+\left[t_1(\frac{E_{01}-E_{31}}{2})+t_2(\frac{E_{32}-E_{02}}{2})\right]\qquad(68)$$

We must now pass from *A(S$_i$)* to $N_{i,\pm1}$. Consider that , during $(S+M)$ interaction, the $A(S_i)$ algebra is vanishing, leaving the place to $N_{i,\pm1}$. In such transition E_{33} is now assuming numerical value +1 and this is to say that E_{03}, E_{30} during the transition (measurement in $N_{i,\pm1}$) are assuming or $E_{03}=E_{30}=+1$ or $E_{03}=E_{30}=-1$. By inspection of the (68), it is seen that terms with e and h go to zero. It remains the term with a for $E_{03}=E_{30}=+1$ and the term with s for $E_{03}=E_{30}=-1$. All the terms containing b_i, c_i, d_i, f_i, q_i, t_i ($i=1,2$) go to zero and the wave function collapse has happened.

Let us explain as example as the term

$$\frac{E_{02}+E_{32}}{2}\qquad(69)$$

pertaining to b_2, goes to zero.

Owing $(S+M)$ interaction, transition $A(S_i)$ -> $N_{i,\pm1}$ is happening , E_{33} is becoming +1. By inspection of the (65), it is seen that the basic algebraic set $A(Si)$ in which E_{33} enters is (E_{01},E_{32},E_{33}). Passing from the algebra $A(Si)$ to the algebra $N_{i,+1}$ (in fact we have attributed to E_{33} the numerical value +1) we obtain the new commutation rule that

$$E_{01}E_{32} = i\,. \quad (70)$$

On the other hand, considering in $A(S_i)$ the set (E_{01}, E_{02}, E_{03}) of the (65) with attribution to E_{03} the numerical value -1, we have the new commutation rule in N_i that

$$E_{01}E_{02} = -i \quad (71)$$

In conclusion we have that

$$E_{32} = E_{01}i \quad (72)$$

and

$$\frac{E_{02} + E_{32}}{2} = \frac{E_{02} + E_{01}i}{2} = \frac{-E_{01}i + E_{01}i}{2} = 0 \quad (73)$$

Following the same procedure, one obtains that also the other interference terms are erased and in conclusion, passing from the algebra $A(S_i)$ to $N_{i,\pm 1}$, the density matrix ρ, given in (68), is reduced to be

$$\rho = a\left(\frac{E_{00} + E_{03} + E_{30} + E_{33}}{4}\right) + s\left(\frac{E_{00} - E_{03} - E_{30} + E_{33}}{4}\right) \quad (74)$$

where in the new application of the $N_{i,\pm 1}$ algebra, we may have

or

$$E_{03} = E_{30} = +1 \;(E_{33} = +1) \quad (75)$$

and thus

$$\rho \rightarrow \rho_M = a \quad (76)$$

or

$$E_{03} = E_{30} = -1 \;(E_{33} = +1) \quad (77)$$

and thus

$$\rho \rightarrow \rho_M = s \quad (78)$$

and the collapse has happened.

3. Conclusion

We have given a first solution to the problem of quantum collapse in quantum mechanics but using a quantum system having only three anticommuting elements. The central approach is that , during the interaction of the given quantum system with the macroscopic apparatus, we have a transition from the standard Cl_3 Clifford algebra $A(S_i)$, having its proper algebraic , geometric and metric signatures to a new dihedral, $N_{i,\pm 1}$ Clifford algebra having new algebraic, geometric and metric signatures. This is the basic feature of quantum collapse . It enables us to destruction of fermion creation and destruction operators in the given quantum system S composed by three anticommuting elements .

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