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BETWEEN ABSTRACTION AND IDEALIZATION: SCIENTIFIC PRACTICE AND PHILOSOPHICAL AWARENESS

Abstract. The aim of this essay is to emphasize a number of important points that will provide a better understanding of the history of philosophical thought concerning scientific knowledge. The main points made are: (a) that the principal way of viewing abstraction which has dominated the history of thought and epistemology up to the present is influenced by the original Aristotelian position; (b) that with the birth of modern science a new way of conceiving abstraction came into being which is better characterized by the term idealization, the name that was later, in fact, to be used by scientists to describe their scientific activity; (c) that, however, on account of the influence of empirical and inductive philosophy, scientists have often not had sufficient methodological awareness of this new way of viewing abstraction; (d) that this new concept of abstraction has frequently been expressed in the framework of philosophies that lie outside the mainstream of contemporary epistemology or even exhibit marked anti-scientific tendencies; (e) that the theme of idealization has been taken up again in the last few decades and a great contribution in this direction has been made by the so-called Poznań school of methodology.

A commonly held view in contemporary culture is that science is essentially "abstract" knowledge; everyday experience, to which it is quite alien, is in a sense "stripped" by science in such a way as to be conceptualized with greater facility in a mathematical framework.

This universal belief would therefore lead us to expect the problem of *scientific* abstraction (i.e. as connected with the conceptualization typical of science and related themes) to be a field of investigation privileged by contemporary epistemology. This, however, is not the case: since the great debate which characterized the empirical and rationalistic tradition, interest in the problem "has been absent from the mainstream of mathematical logic and analytic philosophy" (Angelelli 2004). This can be attributed to two different reasons: on the one hand, Frege's invention of the theory of quantification is thought to have constituted a decisive turning-point which laid all previous discussions aside (Santambrogio 1992, pp. 7-14); on the other, the problem of abstraction would seem to have been considered as dealing with the context of discovery in a predominately psychological sense, that is, with the problem of the procedures which give rise to concepts. However much of 20th-century epistemology has theorized under the assumption of the irrelevance of this context, and has focused instead on the problems of accounting for and empirically verifying theories whose formulation has mostly been attributed to the creative activity of scientists, unjustified conjecture, and anticipatory intuitions capable of responding to the problems raised by experience.

This cannot, however, be explained without highlighting clearly the decisive role played by empiricism in defining the problem of abstraction. It is on the basis of this definition that 20th-century epistemology – strongly influenced by empiricism – has attempted to solve the problem by the theory of sets. The typical ways in which abstraction was conceptualized by the great figures of modern philosophy (Locke, Berkeley and Hume) constituted, in fact, a sort of "philosophical self-awareness" which was to guide thinkers such as Frege and Russell and later Carnap and Quine (to mention only the most important) towards a "solution" which, in order to be possible, is forced to neglect some of the peculiar features of abstraction (Angelelli 2004).

However, alongside the empirical way of dealing with abstraction the history of philosophy provides examples of other attempts and lines of thought, at times shared by a minority or devoid of an explicit theoretical formulation, at others immersed in philosophical contexts that did not lend themselves well to a clarification of scientific conceptualization, at others again considered to be marginal with respect to the mainstream of Western epistemology. The latter is the case of Polish philosophical thought, to which the last part of this essay will be devoted.

1. *The empiricist mainstream*

The fundamental co-ordinates within which the problem of abstraction has evolved go back to Aristotle. Having distinguished between *primary substance* (concrete individuality) and *secondary substance,* which comprises the *species* and *genus* to which individuals belong (Aristotle, *Cat.,* 2a 12-15), Aristotle conceived of the universal, the result of the process of abstraction, as that which is common to many individuals; as such - i.e. as primary substances - individuals cannot be fully captured in terms of universals. However, although real individuals elude our thought, we can conceive of the qualities, as real as the individuals themselves, which constitute what is shared by a certain number of individuals. It is just because individuals possess these common qualities that we can group them together and classify them as belonging to species and genera (Aaron 1967, pp. 7-9).

This paved the way for the classical theory of abstraction we will later find in modern empiricism, at least in its general logical lines even though the metaphysical background is different. Behind the process of abstraction there lie on the one hand the manifold forms of experience and on the other the capacity of the intellect to isolate properties common to a certain number of entities, eliminating the individual features each of them possesses. It is thus possible to proceed by isolating increasingly general properties in such a way as to achieve, at the peak of this pyramid of concepts, the most abstract representation – "being *qua* being" (Bäck 2004). This procedure of abstraction is the basis of all science, whether it be a specific one like mathematics, physics, "first philosophy", or metaphysics (Aristotle, *Metaph.,* XI, 3, 1061a 28-1061b 4; 4, 1061b, 24-27; 3, 1060b, 31-33; IV, 1-2). Obviously in Aristotle the universal concept we thus obtain is not just a subjective scheme but a *real form*, the specific type which is the active principle contained in every single concrete reality. There is a perfect correspondence between the logical process and the ontological reality (Cassirer 1910, p. 16).

A whole series of problems, however, remain unanswered. In the first place, a question which arises is that of the way in which individuals share the same qualities: are the qualities on which the process of abstraction is based identical in the various individuals who share them, or are they only similar to each other? Aristotle's answer to this question is not clear (Aaron 1967, p. 10). In addition, what is the nature of these universal concepts? Do they really exist, do they only have a conceptual existence, or are they only names? This is the famous problem of universals arising from Porphyry's introduction to Aristotle's *Categories*, which was to be one of the main disputes in medieval philosophy.

Using the terms of ordinary set theory, the question can be posed as follows. Let there be a universe of objects, U. From this let us obtain a subset of objects $A \subset U$ such that they all share a common property (e.g. whiteness). We have thus created a set

(1) $A = \{x: P(x)\}$

where $a \in A$ means that the individual a belongs to the set A and possesses the property P. By so doing we have performed an abstraction in the Aristotelian sense and we give the abstract concept thus obtained the name $[A]$ which indicates the property ("whiteness") shared by all the elements belonging to the set A. In the light of this, the fact that a given concrete individual is not definable means for Aristotle that it is the conjunction of a (possibly infinite) number n of properties, i.e. that

$$a = \sum_{i=1}^{n} P_n(a)$$

and therefore stating, for instance, that $a = P_i(a) \wedge P_j(a)$ means not capturing the substantial specificity of a as the properties with which we have defined it may also define another individual, and in addition a may continue to be such even if it is not defined by those two properties (Aristotle, *Metaph.*, VII, 15, 1040a).

We can, however, proceed in a different way. Let the following hold:

(2) $A = \{a, b, c\}$, where $a,b,c \in U$.

In this case a,b,c are the individuals that Aristotle considers as constituting *primary substance*. Still using $[A]$ to indicate the name of A, we can state that a is $[A]$, meaning that $[A]$ is the name designating the class of individuals $a,b,c,$ which have nothing in common except for the fact that they belong to A. So, in using the name $[A]$ we only use a convenient abbreviation, a conventional notation whose function is to recall the set A, i.e. the single, concrete individuals a, b, c.

In the first case we have defined a class intensively by identifying the property possessed by its elements, so identification of this property logically precedes identification of the elements belonging to the class; in the second case, on the other hand, we have given an extensive (or iterative) definition by simply listing the elements belonging to the class, without identifying any properties they may share (Lombardo-Radice 1982, p. 13); therefore (according to the axiom of extensionality) a set is completely determined by its elements, so the iterative concept of a set is different from the dichotomous concept which allows each set to be obtained by dividing all things into two categories (i.e. things which possess a certain property and things which do not). As Gödel maintained, these two concepts can be likened to a mathematical concept and a logical one (Wang 1974, pp. 200-1). This is the difference trenchantly expressed in modern times by Einstein (1936, p. 41) when he pointed out that the relationship between a scientific concept and sensorial experience "is not so much similar to that between soup and an ox as to that between a cloakroom ticket and an overcoat".

It is clear that in the first case we assume the Aristotelian point of view of properties as universals, the correspondent of which on the ontological plane is the doctrine of substantial forms: this is the thesis of realism (a universal corresponds to a really existing substance) or conceptualism (a universal is only a concept obtained by abstraction). In the second case, we opt for a world made up of individuals each of which is a substance that cannot be reduced to another and which can have nothing in common with each other. This is the thesis of extreme nominalism, according to which a universal is only a sign, a mnemonic device (*flatus vocis*) which serves to identify objects that are essentially different from each other and are only grouped together for instrumental purposes. Aristotle tried to maintain a delicate balance between the two points of view: on the one hand, a world made up of substantial individuals, of undefinable subjects, and on the other a substance that, from a metaphysical point of view, is constituted by *eidos* which possesses its own ontological reality and whose logical side is the universal conceived of by the human mind (Reale 1974, pp. 61-2).

It is obvious that each of these positions has its difficulties and these gave rise to a long controversy regarding universals. What we are interested in pointing out here, however, is that in any case in the controversy between nominalism and realism a *universal* as such is seen – with a view to either denying its existence (nominalism) or accepting it (realism and conceptualism) – as the result of an abstraction which isolates the *properties common to* several entities.

Neglecting the wealth of medieval discussion concerning this topic (cf. Spruit 2004), let us consider the modern era, in which the general context of the problem changes: the process of abstraction now loses its hold on reality and shifts to the psychological field, becoming an operation performed on the representations present in our mind. The first to observe a close link between the process of abstraction and the logical function of language was Locke, who stated that abstraction is that faculty by means of which the mind turns particular ideas into general ones. First of all we have only *particular beings* (thereby rejecting the realism of universals and thus the Aristotelian theory of substantial forms), possessing an indefinite number of qualities from which we obtain *general ideas* representing qualities of the same *sort* and indicated with a *name* or *general term*. These *ideas* which have a representative value are particular qualities which represent other qualities, as long as they belong to the same *sort*. For example, the name "white" indicates the idea of "whiteness" which represents the single white colours of certain beings (Locke 1690, I, pp. 148-9). Obviously in this case the "sort" Locke is speaking of has nothing to do with the "substantial species": although the process of abstraction is the same, the metaphysical background is profoundly different.

Significant here is Locke's stress on the fact that in the operation of abstraction nothing is added to complex ideas — the general idea is only obtained by "subtraction" of properties, that is, each time we deal with "general natures or notions", the basic operation consists "in the leaving out something that is peculiar to each individual, and retaining so much of those particular complex ideas of several particular existences as they are found to agree in" (p. 170).

In a consistently conceptualist view, names "stand for" ideas, and ideas "stand for" things. The semantic referent of language is, in the last analysis, formed by the set of individual beings, while the general or universal, referred to by general terms, does not belong to the real existence of things, but is an invention or "creature" of the intellect, since the same things "are all of them particular in their existence" (Locke 1690, I, p. 172). However, although the universal is a fictitious construction of our intellect, it would be wrong to think it is merely arbitrary. Abstract ideas have their "fundamentum in re", precisely in similarity between things, and so "the sorting of them under names is the workmanship of the understanding, taking occasion from the similitude it observes amongst them to make abstract general ideas, and set them up in the mind, with names annexed to them as patterns or forms" (p. 179).

This is precisely the thesis expressed above in (1): the name [*A*] designates the general idea that expresses the property *P* which in turn defines the set *A*, which is thus constructed by our intellect on the basis of the similarities between the properties of generic elements *x* belonging to *U*. Hence Locke's anti-essentialism: for him only nominal essence exists, not "real" essence, which "comes to be nothing but that abstract idea which the general or sortal … name stands for" (p. 182). In short, in Locke Aristotle's dual articulation between *primary* and *secondary* substances disappears. He only maintains Aristotle's *tode ti*, the concrete individual with its irreducible particularity, while the *eidos* is discarded.

What is, however, unclear in Locke is the meaning of the concept of sort that he uses when he states that general ideas represent all those of the same "sort". What does this "sort" consist of? Obviously, it has been stated, all things are particular and consequently so are our mental representations of them. If abstraction consists of isolating the qualities they have in common, and adds nothing to them, and these qualities are always different from each other, how are we to view them as belonging to the same "sort"? The problem can only be solved by hypothesizing a creative mental activity whereby qualities perceived by the senses are used as a basis for constructing a "type" defined by its meaning rather than its representative nature. In effect, Locke sometimes takes the universal to signify a fixed and unchanging meaning, and not the selection of a nucleus of common properties taken on account of its connotations, the only objectivity in which is the fact that it has a precise, unchanging definition. This approach, however, is only hinted at: it is not a fully developed theory, and so the dominant approach to abstraction remains the one outlined previously (Aaron 1967, pp. 32-6).

It is precisely these difficulties which gave rise to criticism, and it is significant that even those who rejected Locke's ideas operated in the same conceptual universe, albeit in a negative sense. A case in point is Berkeley and his critique of the very idea of formulating abstract ideas. Borrowing Locke's assumption that every idea is always a particular idea, he maintained that "a word becomes general by being made the sign, not of an abstract general idea but, of several particular ideas, any one of which it indifferently suggests to the mind" (Berkeley 1710, p. 79).

This denial of the existence of abstract ideas does not mean that "general ideas" do not exist – they are just particular ideas used "to represent or stand for all other particular ideas of the same sort" (Berkeley 1710, p. 80). This ineliminable function of general ideas leads to the erroneous conclusion that there exist "abstract general ideas" which constitute a sort of intermediary between words and ideas.

This criticism of Locke's views is not inconsistent: if abstraction is seen as a progressive "subtraction" of properties (which for Berkeley is the only way of conceiving it), one could fall into the absurd situation of supposing that it is possible to have, for instance, an idea of a triangle, that is "*neither oblique, nor rectangle, equilateral, equiecrural, nor scalenon, but all and none of these at once*" (p. 81). This of course is in contrast with the work of the geometer: although his demonstrations do refer to "universal notions", they are not universal in the sense that they are formed by abstraction; hence, for example, any particular property of a triangle is not universal because it has been proven for a general abstract triangle which is neither equilateral nor scalene, etc. and is thus considered to be valid for every particular triangle. For Berkeley, universality lies in the relation between a particular idea and the particulars it represents; thus whatever kind of triangle a geometer is dealing with and providing a demonstration of, it "equally stands for and represents all rectilinear triangles whatsoever, and is in that sense *universal*" (p. 82). So we do not have a name "standing for" an abstract idea which "stands for" things but simply a particular idea which "stands for" other particular ideas.

It is precisely with reference to the way in which mathematical demonstrations can be of universal validity that Berkeley outlines his alternative to Locke's solution: demonstrations are of a universal nature not because their referents are abstract general ideas but because in the course of the demonstration no use is made of any of the particular features which distinguish individual triangles from each other. As has been rightly pointed out, Berkeley's alternative "delineates with great accuracy what was to become the modern concept of mathematical demonstration by universal generalization" (Santambrogio 1992, p. 44). Berkeley's procedure would, in fact, seem to correspond to what is known nowadays, in systems of natural deduction, as the rule of the introduction of a universal quantifier: if the property P holds for any x, then it holds for all x.

It is clear that we are dealing here with the position previously described as (2): $[A]$ is only a name designating the class of individuals a,b,c. And when we utter or use the name $[A]$ we are not thinking of a general idea but only the particular idea of an element of A whose function is to represent all the others. A question which arises, however, is: what makes it possible for the various different *particular* ideas to be represented by a single *particular* idea? That is, what mechanism causes a given name (that of a triangle) referring to a particular idea (a particular triangle) to bring to mind other particular ideas (other particular triangles)? If all ideas are equally different, then one idea could recall *any* other idea *whatsoever*. The fact that it does not, i.e. that it recalls certain ideas but not others, means that these ideas are not equally different, that they do have something in common, that they belong to the same "sort", that *in fine*, they are *less different* from each other than they are from other ideas. Here we come back to Locke's problem in another guise: it is not an abstract idea that represents qualities of the same sort, but a particular idea which recalls other particular ideas in some way connected with it.

The need to answer this question led Berkeley, in the second edition of his work (published 24 years after the 1710 edition), to take up a more conciliatory stand and admit the possibility of considering a particular individual as having only one particular property, e.g. triangularity (Berkeley 1710, p. 82); in short he rejected the existence of what the previous tradition regarding universals termed "concrete universal", while he accepted "abstract universals": "it is not triangularity that Berkeley finds unacceptable, but the idea of a general triangle" (Santambrogio 1992, p. 41). Hence the connection between particular ideas, represented by another particular idea, lies in the fact that they all share the same property ("triangularity"), and so are of the same "sort". Once again, therefore, Berkeley conceives of abstraction as an operation that captures what several entities have in common, as he deems it possible to think of "triangularity" as a property common to several particular triangles, but considers it absurd and contradictory to believe, as Locke did, in the existence of the idea of a general abstract triangle which possesses no properties other than that of triangularity.

Once this is accepted, Berkeley's place in the framework of (2) changes significantly. It is, in fact, true that the general name [A] indicates a set of particular individuals – and not an abstract general idea as Locke believed – but Berkeley's acceptance of the existence of abstract universals allows the set to be defined by means of the corresponding predicate. It would therefore be possible for him to admit a definition of the type $A = \{x\colon P(x)\}$. The only difference with respect to Locke at this point is his refusal to admit a general object as a reference of the general term [A], i.e. a concrete universal conceived of as an independent object, a separate entity, the obvious consequence being that he maintained that it was impossible for a mode or a quality to exist separately from the substance of which it is a mode or quality. In other words, Berkeley saw the function of the common property as making predication possible, which is fundamental in order to define the set or "sort" of entities each of which is represented by a particular idea which "stands for" one of its particular elements.

This possibility of predication must, however, be based on the recognition that entities resemble each other to a certain extent on account of that particular common property which, conceived of as an abstract universal, constitutes a predicate that can be applied to all other similar entities. The procedure is a dual one: in the multitude of particulars we recognize entities that resemble each other because they have a property in common; we then predicate this property of both the entities already recognized and others that possess it. It was, however, this concept of similarity that defied all empirical attempts to clarify definitively the capacity of the human intellect to group together entities which are irremediably different. On the one hand Berkeley, as was typical of his line of thought, was led to solve the problem of membership in the same sort – which Locke achieved by means of abstract universal ideas – by recourse to the doctrine of the regularity with which God produced ideas, which would ensure that they were recognisably similar (cf. Mugnai 1979, pp. 50-7); on the other hand Hume, who consciously took up the same stand as Berkeley, attempted to understand the operations of the human intellect on a psychological basis, and thus regarded the notions of similarity and *habit* as "*original* qualities of human nature" (Hume 1739, I, p. 321), since "to explain the ultimate causes of our mental actions is impossible" (p. 330); the original capacity of the human intellect to recognize similar things, a capacity which is not susceptible to further analysis, thus becomes the foundation of our associational operations, and the new notion of "habit" takes on the function of recalling to the imagination simple ideas that are similar to each other.

The inevitable consequence of such a solution, however, is that it creates doubts as to whether an objective natural science can exist unless it is also based on habit, and the science of Newton is a problem it is unable to solve. In effect, however the major representatives of empiricism looked at the matter, they failed to resolve the problem of accounting for the transition from experience (essentially individual), to science (with its universal and abstract concepts). The problem of "similarity" was the stumbling-block for all attempts to explain the nature of concepts, no matter whether the process of abstraction was accepted (as it was by Locke) or rejected (as it was by Berkeley and Hume). Unless the human intellect was to be denied possession of a generalising, abstracting faculty, the only way out was to view this power differently from the way in which it had been conceived of up to then. Was it not possible that there was something fundamentally wrong in the way the process of abstraction, and thus the formation of concepts, especially scientific ones, had been viewed? This was the question many German thinkers were asking themselves in the second half of the eighteenth century, including Kant (cf. Tagliagambe 1980, pp. 106-69).

Kant's positive answer to the question is well known. He tended to view that original productive power of the intellect on a transcendental plane: it was capable of adding that "something extra" to empirical data which would provide the abstracting, universalizing function of scientific concepts. To follow his line of reasoning here would take us far from the main theme we are dealing with. Suffice it to say that we can still see the attitude to the problem of abstraction typical of classical empiricism in John Stuart Mill. It is clear that he also considered the abstraction through which general concepts are formed to be mere generalisation: "when we form a set of phenomena into a class, that is, when we compare them with one another to ascertain in what they agree, some general conception is implied in this mental operation" (Mill 1843, p. 650). The general concept thus reached is the result of such a comparison and is, therefore, obtained by abstraction from single things. It is important to note that according to Mill "the conception is a conception *of* something; and that which it is a conception of, is in the facts" (p. 651); it is, in effect, a *realistic* concept, which confines itself to grasping what already exists in experience: we are therefore in perfect harmony, at least as far as the process of abstraction is concerned, with the empirical tradition.

What, however, is important to point out in connection with this is that Berkeley's solution was later taken as a point of no return in the debate about general objects, and as a basis for building the standard contemporary theory of universals and predication. This was the work of Frege (Santambrogio 1992, pp. 63-88): resuming Berkeley's thesis that only individual objects exist, he based the logic of generality on quantifier-variable notation. Due to his refusal to accept the existence of indefinite (or general) objects, a statement like "a whale is a mammal" is for Frege equivalent to saying "all whales are mammals". So he also shares the view that concrete universals (the idea of a "whale in general") do not exist, and the function of the abstract universal ("mammality") is to act as predicate in the corresponding statement.

This stand finds its fullest expression in the concept of "definition by abstraction" which, introduced by Peano, was then to be taken up by Bertrand Russell (1903, pp. 219-20) although he partially altered its original meaning (Angelelli 2004). It consists of identifying a set by means of a "common property" its members possess (Berkeley's "triangularity"), which is none other than a symmetrical, transitive (and reflexive) relation, i.e. a relation of equivalence. As Russell states, "this principle amounts, in common language, to the assertion that transitive symmetrical relations arise from a common property, with the addition that this property stands, to the terms which have it, in a relation in which nothing else stands to those terms. It gives the precise statement of the principle, often applied by philosophers, that symmetrical transitive relations always spring from identity of content" (Russell 1903, p. 220).

Thanks to this translation of the abstract universal into the language of set theory, thus solving the question of "similarity" by the theory of relations, a definite solution to the problem of abstraction and the nature of abstract entities was thought to have been found. According to the standard position in modern logic, as exemplified by Quine (1961), the referents of predicates are attributes, or preferably classes, so universals are simply the same as sets. The possibility of a general abstract object or idea is denied, but there remains an uninvestigated capacity to capture one or more common properties in a given universe of objects and then to predicate them of all the elements which are part of that universe. As Carnap puts it, the sign of a class in a sense represents what is common to these objects, i.e. to the elements of the class (cf. Carnap 1928, p. 130); or again: "Such a definition of a family of properties by way of the equivalence class of an equivalence relation is often called *definition by abstraction*" (1954, p. 137). This procedure is seen as the perfection and conclusion of the empirical debate concerning the formation of concepts and the nature of abstraction: "*The passage from the set A to the quotient set* A/ε (where ε is an equivalence) *summarises and specifies the process whereby concepts are formed starting from objects*, and in more general terms, the ordinary *process of abstraction*, consisting of identifying elements that are, it is true, different, but that all enjoy a common 'property'" (Lombardo-Radice 1982, p. 27).

In contemporary logic and epistemology the debate about the processes of abstraction thus dies a natural death: the attention is focused exclusively on the nature of theoretical entities, predicability, and the problem of justifying the axiomatic-deductive systems in which concepts are implicitly defined; but there remains a weight hanging about its neck, a conviction naturalistically assumed to be obvious: that the theoretical terms of science can either be reduced to observable properties by procedures featuring varying degrees of directness or, where the procedure of generalization is applied, that they are no more than a grouping together of common properties (cf. for example Mach 1883, pp. 156, 471; 1896, pp. 189-90; 1905, p. 132; Braithwaite 1953, p. 14; Hempel 1952, p. 105; Nagel 1961, pp. 17-8; Carnap 1966, p. 285). The technical tools of first-order predicate logic and the language of set theory harmoniously embrace the empirical or even phenomenological – if not nominalistic – viewpoint of a great part of contemporary epistemology. Solidarity between the classical concept of abstraction and contemporary logic is thus a feature of much of contemporary epistemology and thanks to it epistemology and mathematics have visibly flourished.

Nevertheless, the solution to the problem of general objects and universals as we have described it has left certain grey areas which have had negative repercussions on our understanding of the structure of scientific theories and the peculiar nature of conceptualisation applied in them. Whereas, in fact, the concept of abstraction based on set theory is quite efficient at tackling the concepts of mathematics, or those of formal or formalised sciences in general, it encounters considerable limits when it attempts to account for the nature of scientific theories whose structure comprises theoretical concepts, whose purely extensional interpretation would be an arduous task.

2. *The abstraction of the scientists*

If we take a look at how working scientists have interpreted what they do, it is apparent that when their thought manages to escape the snare of an empiricist philosophical self-awareness, they often put forward theories of scientific conceptualization and abstraction which diverge from the theory developed by the empirical tradition, as described previously. In many cases there is a sort of co-existence of two attitudes: on the one hand, we have a methodological awareness that takes its epistemological and methodological models from the philosophy moulded by the Baconian revolution against the abstract scholasticism of the Middle Ages in the name of experience and of an inductive procedure that tries to remain faithful to it; on the other, we have actual scientific practice that cannot reconcile itself with this philosophical framework and often puts forward modes of theorization and conceptualization that do not lend themselves well to the conceptual apparatus available to the gnoseological awareness of the times.

It is thus not by chance that modern science, in its earliest stages, felt the need to turn to philosophers who were at the time relatively unknown, perhaps even looking for them in classical antiquity, to find concepts and ideas that could support them against the dominant Aristotelianism. And what could be more natural than to turn to the Stagyrite's great antagonist, i.e. Plato?

It was, in fact, Plato, as well as Archimedes, who inspired Galileo to lay the foundations of modern science, rejecting the qualitative orientation of medieval peripatetic philosophy and basing his view of science on the concept of abstraction as the identification of common qualities. His trust in "geometrical reasons", his faith in the analysis of merely quantitative relationships, subverted the scholastic-Aristotelian edifice and were corroborated by a Plato freed from any esoteric, numerological or magic contamination. The pages of the *Saggiatore* devoted to the mathematical language of nature are the most evident sign of assimilation of a geometrical and methodical Plato rather than a Plato steeped in mystery and soteriology, a Plato mediated by the teaching of Archimedes, which is the sole explanation of Galileo's Platonic inheritance (cf. Dollo 1989; Minazzi 1994, pp. 257-71). The *communis opinio* of the times drew a clear line between Aristotelians and Platonists:

> If you claim that mathematics enjoys a superior status, if you ascribe it a real value and a dominant position in physics, then you are a Platonist. If, on the other hand, you see mathematics as an abstract science which therefore is of less value than those sciences – physics and metaphysics – that deal with real being, if in particular you state that physics needs no other basis than experience and is to be constructed directly on perception, that mathematics should be content with a secondary,

> subsidiary role, then you are an Aristotelian (Koyré 1943, p. 160).

It was the impossibility of applying mathematical tools to the concepts of Aristotelian physics and those elaborated during the Middle Ages to overcome its most obvious incongruities (as, for example, with the theory of *impetus*) that led to a need for a new way of conceiving the relation between nature and reason, i.e. between mathematics and reality, and the introduction of experimentation as an indispensable intermediary to bridge the gap between matter and geometrical figures.

Aristotle and Plato had based their contrasting solutions on the evaluation of this very "gap". Aristotle had sanctioned the inapplicability of a perfect instrument like geometry to the essentially imperfect entities of nature, both physical and celestial; Plato had attributed this application not to the physical nature in which man lives but to the perfect world of Ideas. For Aristotle,

> the minute accuracy of mathematics is not to be demanded in all cases, but only in the case of things which have no matter. Therefore its method is not that of natural science; for presumably all nature has matter. Hence we must inquire first what nature is: for thus we shall also see what natural science treats of [and whether it belongs to one science or to more to investigate the causes and the principles of things] (Aristotele, *Metaph.* II (a), 3, 995a, 15-20).

So he also excluded the possibility of applying geometry to the study of the stars:

> And astronomy also cannot be dealing with perceptible magnitudes nor with this heaven above us. For neither are perceptible lines such lines as the geometer speaks of (for no perceptible thing is straight or curved in this way; for a hoop touches a straight edge not at a point, but as Protagoras said it did, in his refutation of the geometers), nor are the movements and complex orbits in the heavens like those of which astronomy treats, nor have geometrical points the same nature as the actual stars (Aristotele, *Metaph.* III(b), 997b, 34-36/998a, 1-6).

Plato also started from the view that astronomy could not be studied by considering the real movement of celestial bodies, insofar as it was imperfect, and reserved this specific science for things which "are not accessible to sight, but only to reason and thought" as "we should use the heavenly decorations merely as illustrations to help us study the other realm" (Plato, *Rep.*, VII, 529d). Geometry is therefore a science of ideal models; indeed, it is the science which allows us to introduce ourselves to knowledge of those Ideas on the model of which sensible entities are forged. Geometers

> in the course of their discussions make use of visible forms, despite the fact that they're not interested in visible forms as such, but in the things of which the visible forms are likenesses: that is, their discussions are concerned with what it is to be a square, and with what it is to be a diagonal (and so on), rather than with the diagonal (and so on) which occurs in their diagrams (Plato, *Rep.*, VI, 510d-e),

so the figures they model are just examples, "images" that help us to see things that can only be perceived by the intellect. In this way Plato sees mathematical objects as reduced to the grasp of the intellect and transcending the sensory realm. That is, the contradictory and deceiving nature of sense experience can only be overcome by mathematization. So thinking of unity and plurality at the same time, with the consequent perplexity it causes for the human mind, leads to investigation: unity leads to numbers and hence mathematics comprises a series of problems that lead to investigation, thus making it suitable for philosophers. In short, it

> forces the mind to turn towards the realm where the most blessed part of reality is to be found ... it is particularly good at guiding the mind upwards and forcing one to discuss numbers in themselves. It excludes the slightest hint, in a discussion, of numbers which have attendant visible or tangible material objects ... it forces the mind to rely purely on intellectual processes and to aim for truth in itself (Plato, *Rep.*, VII 525c-526b).

Plato's celebration of the "polemical" function of mathematics regarding sensibility is accompanied by his critique of sensation as the source of knowledge and the basis of science. When he criticises the empirical concept that knowledge is a "true opinion" (*doxa*) his target is not only Protagoras, the milieu of the Sophists and Cynics, or the rhetoric of Gorgias and Isocrates, but also Socrates, who saw the discovery of universals as the result of an inductive process. Socratic concepts are therefore *koinà*, obtained from comparison of sense objects, which are very different from ideas, hence its aporetic nature. Plato's refusal to view scientific concepts, science itself, as the result of the generalization, based on experience, of common properties (thus totally departing from Socrates) leads him to value geometry as a means of access to ideal entities that are not "abstracted" from their intelligible forms but are "models" of them. This is in clear contrast with Aristotle's concept of abstraction, which is quite consistent with his adhesion to the plane of common sense and rejection of Plato's world of ideas, corresponding to the attempt to capture the essence expressed by means of a subject-predicate definition whose terms are obtained by abstraction. As we have seen, in this case the essence (*eidos*) is the translation to the plane of the intelligible and permanent of what is obtained from common sense by qualitative abstraction.

Galileo's great revolution consisted of the capacity to combine what had previously been kept separate, that is, to unite the perfection of mathematical apparatus with the discreteness of sense data, avoiding not only the Scholastic-Aristotelian reliance on empirical data and its qualitative wealth but also the flight towards the "region beyond the sky" in search of the perfection that cannot be found in the world of men, which was typical of the Platonic tradition (with all its soteriological and symbolic consequences). In this way Galileo was inspired by the Platonic view but he modified it radically, replacing the ontology of ideal perfect forms with a new ontology of mathematical entities used in theories formulated to describe the world of phenomena; Galileo opposed philosophical knowledge, the acquisition of which was for Plato the aim of mathematics, with knowledge of the great book of nature, written in mathematical characters (McMullin 1985).

To do so it was, however, necessary to free nature from all its accidental features, to simplify it, to make it increasingly ideal, to render it susceptible to the application of geometrical calculations. That is, it was no longer necessary to reason about spheres or the imperfect, rough or yielding surfaces that present themselves to circumspect empirical investigation, but to examine ideal spheres, perfectly smooth bodies, perfectly uniform movement. It was, in short, necessary to elaborate concepts which could not be simple abstractions from nature, from its common properties, but which constituted a counter-factual creation, a polemic against experience. For Galileo, science could no longer consist of the simple recording and generalization of phenomena and all the details of their evolution; its aim was rather to capture the process in its pure form, free from random influences. It is no coincidence that a philosopher who contested modern science and its "unrealistic" image of the world like Feyerabend was to criticize Galileo's method for this very reason, claiming that Aristotle's science had a greater capacity to remain faithful to the empirical, to common sense (Feyerabend 1978). As Amos Funkenstein (1986, p. 89) claimed in his illuminating book, from the theoretical and experimental point of view, the strength and novelty of 17th-century science lay in its capacity to extract things from their context and analyse the relationships between them in ideal isolation. This was recognised by many of those who used it as a new form of abstraction or generalization and they viewed it as an element of superiority over traditional natural science.

Nothing is more symptomatic of the difference between these two alternative ways of conceiving science than the controversy between the Aristotelian Simplicius and the mathematician Salviati (who represents Galileo in the *Dialogue Concerning the Two Chief World Systems: Ptolemaic and Copernican*). When the latter states that two spheres, on coming into contact, touch each other at one point, Simplicius immediately objects in the name of faithfulness to the phenomenal manifestations of empiria, stating that the demonstration referred to abstract spheres, not material ones. The "mathematician" Galileo explains, however, that just as a book-keeper has to subtract the weight of crates and other forms of packaging in order to obtain the net weight of sugar, silk and wool, when the geometer-philosopher wishes to apply to concrete phenomena effects that have been demonstrated in the abstract, he has to get rid of all material impediments; if he is able to do so, Galileo states, things will turn out to be as exact as arithmetical calculations. Any errors will not be due to reference to the concrete or the abstract, to geometry or physics, but to the person making the calculations (Galilei 1632, pp. 251-2). As Koyré has written: “the real object of the *Dialogues* is not the opposition between two astronomical systems, but the validity of mathematical science, of the mathematical explanation of Nature, as opposed to the non-mathematical explanation given by common sense and Aristotelian physics” (Koyré 1943, p. 158).

To counter Simplicius’ stubborn attachment to the concreteness of the intelligible world – which Feyerabend would certainly have shared – the mathematician-geometer Galileo performed what was to be defined, three hundred years later, as “starving the reality of the phenomena under investigation”. Only by creating fictitious, ideal entities and then descending from them by means of experiment and approximation to the “roughness of experience” is it possible to combine mathematics and reality. This is the *ex suppositione* argument that Galileo defends in his letters to P. Carcay and G.B. Baliani (Galilei 1980, I, pp. 944, 962) and expounds when he responds to objections similar to those of Simplicius in his later work *Discorsi*. In discussing movement, he admits that conclusions demonstrated in the realm of the abstract are “altered” in the concrete sphere; but he appeals to the authority of Archimedes to justify the unrealistic assumptions he makes *ex suppositione*, both on account of the limited incidence they have on calculations and because it is possible to introduce corrections which will ensure that the calculations approximate the behaviour of real bodies (Wallace 1974). Only by proceeding with this form of idealization and the creation of fictitious entities is it possible to operate scientifically; in fact science cannot accurately account for accidents of gravity, velocity and so on, as they are infinitely variable. However, to deal with matter scientifically, it is necessary to abstract from them and, once the conclusions thus rid of impediments have been drawn and demonstrated, they must be used with the limitations that experience gradually proves to exist (Galilei 1638, p. 779). So *ex suppositione* argumentation consists of creating non-realistic physical models by assuming values and properties which cannot be ascertained empirically and therefore cannot be the result of abstraction from common properties. Galileo reasoned *ex suppositione* by imagining movement towards a point: starting at a slow speed it gradually accelerates, its speed increasing in proportion to the length of time that has passed; and he conclusively demonstrated several accidents of this movement (Galilei, Letter to P. Carcay, 5 June 1637, in 1980, p. 944). It is thanks to this idealizing operation, to its capacity to create possible worlds of clear, perfect conceptual figures, that modern science is born; only by virtue of the replacement of everyday experience with scientific experience, of common objects with physical objects, can mathematics unite with the “discreteness” of sense data - thus overcoming the prohibition formulated by Aristotle.

It is clear from this viewpoint that ideal experiments played a central role in Galileo’s methodology and the birth of modern science (Such 1977). Only by means of experiments was it possible to establish the principle of inertia, as the conditions in which a body continues to move indefinitely not only cannot be observed but are almost counter-factual. They represent extreme cases in which one or more variables take a zero value. But this transition from what is real to what is imaginary is useful "only if we abandon the hope of obtaining valid generalizations from the so-called immediate sense data" (Funkenstein 1986, p. 154). 17th-century scientists were proud of this new concept of abstraction, which they called the "method of resolution". The capacity to consider things "in themselves and for themselves", isolating phenomena from their context, was something that the Scholastics and Aristotle had not been able to do.

Galileo's methodological revolution therefore comprised two fundamental stages. First he carried out a preliminary *simplification* of the world, in the sense that it was deprived of some of its features, which are therefore completely absent in the universe of scientific discourse. Referring once more to the formula illustrated above, if the universe of all entities U possesses a (possibly infinite) number n of properties, the classical operation of abstraction consists of the procedure indicated by (1), i.e. of obtaining from U a subset A of entities sharing a given property P:

$$A = \{x: P(x)\}$$

It is obvious that the complementary set is $A' = \{x: \neg P(x)\}$ so that $U = A \cup A'$. This means that if the set A comprises all white things (i.e. things which share the property of "whiteness"), its complement A' is given by all things which are not white, so the universe is a set of white and non-white things. If, on the other hand, we consider Galileo's way of proceeding, we will see that his simplification does not consist of this type of abstraction: he does not take "whiteness" into any consideration whatsoever, so the entities that he investigates are neither white nor non-white; in other words, no colour can be predicated of them so it is useless to ask what colour they are. Whereas in the former case (abstraction) the space of the properties an abstract object possesses is the same as the object from which it has been abstracted, in the latter case (Galilean simplification) this space of properties decreases, so the simplified universe to which the entity belongs is different from the previous one. That is, it is not possible to return to the original universe U by simply uniting the complementary sets, because the set of objects obtained by simplification has no complement. Examples of this way of proceeding can be found throughout Galileo's works, as for example when he maintains that he does not want to "try the essence" of things in his investigations, but only to investigate some of their features, or when, in his famous distinction between primary and secondary qualities, he confines science to knowledge of the former alone, and so on.

The second fundamental stage is that of idealization, which consists of making the counter-factual assumption that a certain property (or magnitude, to use a mathematical term) has a value of zero. In this case the formula would be of the following type:

$$A = \{x: P(x) = 0\}$$

in which the set A comprises all objects which have a certain property annulled; for example, mass is punctiform, i.e. equal to zero, or there is no air resistance. So a generic object would always be composed by a number of properties with a zero value and other properties with a value other than zero. In this way, if we wished to reconstruct Galileo's law of free fall (Nowak 1994, pp. 118-9), we would have:

$$\text{if } fb(x,e) \wedge v_0(x) = 0 \;\&\; r(x) = 0 \wedge g(e) = const, \text{ then } s(x) = 1/2gt^2(x)$$

which reads: if x is a body in free fall in the direction of the earth ($fb(x,e)$) and the initial speed v_o is equal to zero and the forces of resistance r are equal to zero and the gravitational force g remains constant at the various heights, then the distance s covered by the body is given by the formula $1/2gt^2$. As can be seen, in this case Galileo was well aware that he had made at least one idealizing assumption, i.e. that the body was moving in a vacuum (he was not at that time aware that a constant gravitational force was also an idealizing assumption). From this awareness he got his idea that when experiments in the "abstract" are compared with the actual behaviour of bodies, it is necessary to introduce corrections which will allow the calculation to "approximate" what has actually been observed. This can be done, for example, by taking into account the resistance of the air and thus modifying the formula as a consequence of the fact that $r(x) = 0$ no longer holds. Hence the great difference between simplification and idealization: the former does not admit procedures of approximation or concretization, i.e. simplifications are not part of the formulation of the law but are just implicitly assumed; they are, so to speak, the ontology of the world on which operations of idealization are performed. They are a preliminary definition of the ontology governing the work of a scientist and used by him to investigate certain types of object about which it is senseless to ask certain questions (as makes no sense, for instance, to ask about the influence of the colour of a body on the law of fall). It is only thanks to this preliminary operation that idealizing procedures can be applied.

These two methodological steps – simplification and idealization (with the consequent concretization) – are completely missing in Aristotelian science. In their place there is adherence to common sense, to everyday experience; and the only adequate way to conceptualize this is abstraction, as we have shown previously. Further confirmation of this is given by the fact that, to be precise, Aristotle himself also used ideal experiments on certain occasions; however, unlike Galileo, these ideal experiments do not attempt to formulate a general law, valid for both factual and extreme cases, but to reduce *ad impossibile* (to the impossible) a false universal characteristic. No mediation is possible between factual propositions or generalizations relating to our world and counter-factual hypotheses (with all their implications). They are incompatible because they describe incommensurable conditions (Funkenstein 1986, pp. 156-7). So, in analyzing the movement of a body in a *vacuum*, Aristotle aimed to demonstrate the inadmissibility of a vacuum on the basis of the consideration that whereas velocities in the plenum are commensurable in proportion to the medium the body is moving through (i.e.: $v_1/v_2 = m_1/m_2$), this relation would lose all meaning if m_2 were equal to zero (i.e. if the movement occurred in a *vacuum*) as there is no proportion between zero and a finite magnitude: "The movements of two equal bodies moved by equal forces in the void and in the plenum have no common measure" (Funkenstein 1986, p. 159).

Although medieval physicists, who were used to dealing with the concept of *de potentia Dei absoluta* (according to which "any thing that does not imply contradiction is possible to God", as St Thomas Aquinas, Duns Scotus, Ockham and various other philosophers of the time maintained), were less reluctant to hypothesize extreme or even counter-factual conditions, and therefore viewed the function of ideal experiments in a more constructive way, Galileo's kinematic definitions and theorems were not merely an exercise in systematically imagining a "rational physics" but a tool to reconstruct reality, and they had to be susceptible to experimental verification, albeit obliquely. Galileo and the other great scientists of the modern era who followed in his footsteps – such as Huygens, Descartes, Pascal and Newton – used imaginary experiments in a way that differs *toto coelo* from those of their medieval predecessors, not as regards discipline and scientific rigour but in their physical interpretation. Counter-factual states had never been conceived in the Middle Ages as commensurable with any of the factual states from which they had been extrapolated, while for Galileo an extreme case, even when it does not describe reality, was the main element of his explanation, even though he was aware of how absurd the procedure looked to his adversaries (Funkenstein 1986, pp. 174-7).

Idealizing abstraction was therefore a fundamental stage in the construction of Galilean dynamics and the new science of nature. This was made possible by the convergence of two fundamental events: the re-discovery of the scientific work of Archimedes, who had also applied the procedure, even though he had not given an explicit methodological formulation of it, and the philosophical backing and general perspective of Plato regarding the function of ideal models: stripped of its ontological dimensions and the numerological and mystical meaning it had been adorned with in the Middle Ages, this perspective had represented an antidote to the Aristotelian methodological framework, an alternative *auctoritas* on which argumentation could be based; in short, a sort of embryo "philosophical self-awareness" which could amply justify the new way of relating to the world of phenomena.

Whereas we find in Galileo a balance between scientific practice and philosophical-methodological awareness, this has not always happened in the history of philosophy and science. Although some scientists remained faithful in their scientific activity to the method devised by Galileo – which we can consider as characterizing the whole of modern science and indeed marking its epistemological maturity – they then clothed their procedures with philosophical concepts that did not grasp their most authentic meaning or at times even clashed with them, often due to reasons that had nothing to do with science but were part of the broader cultural climate in which all scientists are inevitably involved. Galileo's balance between scientific practice and methodological awareness was shattered. The result of this destruction was either scientific practice devoid of a theoretical dimension or a theory in singular contrast with what a scientist actually does in his laboratory once he has shed the cloak of the philosopher. Or again, it may happen that the philosopher or scientist, armed with sound empirical principles and wanting to adhere to common sense – possibly as a reaction against some kind of particularly harmful metaphysics dominant at the time – criticizes the idealizing procedures of physics and science, and rejects any concept not obtained by means of "sound abstraction" from the solid ground of phenomenal "facts" (thus often deliberately taking the position of philosophers like Protagoras or the typical objection of the sceptics to the abstractness of scientific laws); with a view to objecting to scientific knowledge, they may propose debatable viewpoints like the superiority of philosophical rationality over that of science, capable of grasping that essence of reality that escapes the "abstract" concepts of excessively mathematized physics, or they may hypothesize new forms of rationality to replace the obsolete scientific rationality propounded by classical science, which they consider inadequate to grasp the manifold nuances and imperfections of the world of the senses.

There is no doubt that a major concept which developed in the 17th century was one according to which natural laws often referred to ideal abstract conditions and to entities which, although they did not exhaustively describe reality, were thought to be essential in any description of reality (Funkenstein 1986, p. 192). But this "new mode of abstraction" which underlay the formulation of the laws, rules and principles governing physical entities that we find in Boyle, Huyghens and Newton was at times steeped in the empirical philosophy that opposed the rationalistic speculation of the period. Newton, who is universally considered as having continued and perfected Galilean science on both a physical and a methodological level, thus took as the banner of his natural philosophy that *hypotheses non fingo* which is unanimously recognised as the banner of the new Baconian empirical and inductive framework. And yet Duhem, at the beginning of this century, criticized this inductivist theoretical superstructure (Duhem 1906, pp. 214-25) and it has been sufficiently well documented that Newton made use of ideal concepts and a type of abstraction that can by no means be identified with that proposed in the same period by the great masters of empiricism we examined previously (Such 1977, 1990; Boscarino 1990). Newton's justified struggle against the ontology and metaphysics of substantial causes and the possibility of admitting occult qualities ended up by clouding the authentic meaning of the concept of "hypothesis", making it adhere to an empirical philosophy that clashes singularly with the way in which he constructed his dynamics.

The same applies to Descartes. If we read his scientific works, we are in no doubt that he applied the Galilean method; he too proceeded *ex suppositione*, hypothesizing perfectly flat hard ground, balls that always roll at the same speed and encounter no resistance, bodies with no weight, size or shape, and so on. In this way he made use, like Galileo, of idealizations (Shea 1991, pp. 236-7), even though his excessive love for mathematics caused him to construct a physics that was totally hypothetical, and prevented him from returning to that solid ground of experience that Galileo considered indispensable to give hypotheses experimental concreteness. And yet when Descartes theorized as a philosopher he could not help thinking of abstraction and the formulation of concepts in a way similar to that followed by empirical philosophy (Descartes 1644, p. 626). Strange as it may seem, the same can be said of the other main representatives of rationalism, the two schools (rationalism and empiricism) finding it impossible to reach an agreement as to the possibility that another kind or source of knowledge other than experience could exist (cf. for example Spinoza 1677, pp. 110-1; Leibniz 1704, pp. 223-30).

Subsequent science did not depart very far from the Galilean view of the concepts it employed, even though its methodological awareness was not always adequate. The great scientists of the 19th century, for instance, had no doubts as to the way in which their theoretical work was to be interpreted; Boltzmann, in reaction to the orientation typical of Mach, thought that no equation was a simple transcription of experience, but rather an idealization of it. In knowledge, therefore, thought does not reproduce or merely abstract from experience, but creates a mental image (*Bild*) of it, with which it can represent a multitude of phenomena. Hertz and even Einstein followed the same orientation (Barone 1983, pp. 176 ff.). One of the best examples is Werner Heisenberg (1959, pp. 61-3), who clearly understood the significance of the work of Newton and thus the overall sense of the way in which science proceeds, throwing light on the new way in which it forges its theoretical tools by the procedure of idealization, which is quite distinct from normal empiricist abstraction and can be more readily identified with the way in which art proceeds (1959b, pp. 128-30).

This awareness is clearly present in modern science, even in research areas which would seem more tied to descriptive ideals: even metallurgy, as John Archibald Wheeler points out, never shows us perfect iron or a perfect crystal. There are usually various defects and imperfections to be observed separating the structures of an almost ideal – but never completely ideal – crystalline structure. This does not, however, make us abandon the concept of an ideal crystal, even though we know that nature will never show us one (Wheeler 1983, p. 398). The same awareness has matured even further in the last few decades, a period in which science has abandoned a whole series of idealizing assumptions that characterised previous 20th-century research, recognizing their counter-factual nature and declaring the necessity for a new scientific rationality. This is what happened in Ilya Prigogine's nonequlibrium systems (Prigogine 1962), René Thom's catastrophe theory (Thom 1962), Benoit Mandelbrot's theory of fractal objects (Mandelbrot 1977) and finally the new theories of deterministic chaos that have fascinated physicists and philosophers in the last few years (Nicolis 1995).

Although there is great awareness in all these cases that scientific concepts are constructs of the creativity of the scientist, not obtained from experience by means of the abstraction typical of the empirical tradition, that they are of a counter-factual nature and thus describe models or ideal situations which are far from any descriptive or generalizing orientation, this emerges more from considerations that are marginal to scientific practice than from explicit epistemological theorization. In general (with the exception of Prigogine, perhaps the most explicit and methodologically accurate of them all) this research activity remains unexpressed and has not been given adequate attention in contemporary epistemology, either within the framework of the Standard Conception of Scientific Theories or in the New Philosophy of Science. As we have seen, this has been almost completely dominated by the view of abstraction outlined in the previous section of this essay. Philosophy has thus unconsciously won a great victory over a science that wished to liquidate it and epistemologists, whose intention it was to become the spokesmen for this science, have tacitly borrowed from philosophy a conceptual apparatus that is quite inadequate for the methodological innovations that have been introduced.

3. *The philosophical outsiders*

Not all philosophy, however, is embraced by the logical-analytical trend we have outlined so far. There have been other traditions of thought that have tried to elaborate a different way of viewing abstraction, at times coming into open contrast with the empirical tradition and at others trying to renew it. The thinkers and theoretical trends in question are "outsiders" to contemporary epistemological thought both because they are in principle polemical towards scientific rationality and because they belong to philosophical schools that are more projected towards developing their own theoretical approach than to interacting fruitfully with contemporary scientific thought. And even when they have tried to do so, their attempt has been ignored by the philosophy of the analytical tradition, which is diffident to any attempt at a "philosophical" approach to scientific thought. Only in the last few years has there been a fresh attempt to develop and promote the ideas expressed by some of the thinkers belonging to these trends.

To remain in a period closer to our own, it is in Hegel that we find the first objection to the traditional empiricist way of considering the relationship between the abstract and the concrete. In the introductory remarks to the section devoted to the "doctrine of the concept", of the *Science of Logic*, Hegel criticizes the ordinary idea of the relationship between concept and empirical matter (or "the manifold of the intuition and representation") that considers the intellect as an empty container which, on the one hand, in contact with the empirical world, acquires reality by obtaining contents and on the other works on reality through abstraction, elevating it to universality. This abstraction is an operation which "neglects" the content acquired as being useless for the concept. So the abstract is considered as having less worth than the empirical matter the intellect works on. This happens because

> In this conception, abstraction means that from the concrete, one or another feature is extracted only for our subjective advantage, in such a way that with omitting numerous other *properties* or *qualities* of the object nothing of their value and of their *merit* should be lost. But, as the Real, they are always left as something fully valid although over there, on the other side, it is only an *impotence* of the intellect that it does not embrace such richness and must limit itself to poor abstraction (Hegel 1816, pp. 258-9).

Hegel opposes this erroneous concept of the process of abstraction with what he thinks is the right one:

> Abstractive thought is not to be regarded as a simple putting aside of sense data whose reality would not be thereby put in question, but rather as the taking away (*Aufheben*) and the reduction of the material as a phenomenon to the *essence* which manifests itself only in the concept (p. 259).

In so doing, Hegel refused to view abstraction as a simple elimination of sense data in the search for common qualities; according to this approach, he claimed, "intelligible matter" undergoes "no prejudice", that is, it remains unaltered in the concept. In this, Hegel differed from Mill, who considered the intellect as not adding anything to the representation. On the contrary, he saw abstraction as a reduction of the material to the essential by active transformation on the part of the intellect, with the aim of grasping what lies beyond its phenomenal manifestations.

In effect for Hegel it would be a great mistake to believe that the natural principle on which conceptual reflection is based is the "truth". Of course, the sensitive view (*Anschauung*) or the singular being (*Sein*)

> … are … the condition of the concept, but they are not therefore the unconditioned as such (*das und für sich Unbedingte*); rather in the concept their existence is removed and thereby the appearance which we considered as reality conditioned (p. 260).

In other words, empirical reality is the real starting point for scientific inquiry, the premise without which theorization would not even be possible, but it only leads to science when it is *abandoned*, that is, when we use it as a base on which to build ideal models of physical systems that are in themselves "unconditioned" or free from disturbing particulars. This is a clear criticism of the "uncritical positivism" of some of his contemporaries, especially Comte. Hegel opposes the "residual concept of truth" – to be found in Comte and all forms of positivism including neopositivism – consisting of the thesis that truth is what is left once the cognitive process has been cleared of any perturbation (as Bacon classically argued), with the need for a "treatment" of the datum that cannot be grasped in all its immediateness and therefore the necessity at each stage of the cognitive process of mediation between subject and object (Negt 1975, p. 29). This is the only approach that can give knowledge which is not confined to the description or generalization of particular phenomena but rather grasps their objective essential structure - which it is possible to achieve only in theory and which does not coincide with the intuitive sense-datum. As Ilienkov states, "for Hegel, a concept expresses the essence of the phenomena being contemplated, an essence that is by no means an element abstractly common to single phenomena" (Ilienkov 1960, p. 17). This gives a sense to the distinction, typical of Hegelian idealism, between *essence* and *phenomenon* viewed as a manifestation of the former.

Hegel's basic intention would seem to have been to stress the fundamental difference between descriptive and theoretical science – whereas the former collects and orders facts to describe their morphology or becoming, the latter constructs theories that do not confine themselves to describing reality but aim at giving an explanation of it by constructing ideal models that can only progressively be approached to it: thus, scientific theory (which, for Hegel, was speculative philosophy and not the empirical science of the age) grasps the "truth" of what at first sight is a "simple happening".

Marx was to borrow this concept of theoretical science and he tried to apply it to his construction of economic science, in explicit reaction to empiricism. The "abstract" concepts which are part of theory are quite different from that "empty identity or abstract universality" (Hegel 1816, p. 665) obtained by following the procedure of abstraction typical of empiricism. Stripping Hegelian philosophy of its mystical shell, Marx viewed scientific theory as the construction of ideal models making use of concepts whose nature does not consist of simply setting aside differences in order to grasp what several empirical phenomena have in common, but rather of postulating ideal entities whose features contrast with the empirical reality they aim to account for (Coniglione 1990a).

Hegel and the philosophical currents he inspired (such as some aspects of Marxism) were not, however, the only ones to oppose the abstraction of the empiricists. We cannot ignore the attempt within the Kantian tradition to develop a transcendental approach, trying to overcome the rigid categorial limits that had made Neo-Positivism so diffident towards the philosophy of Kant and his "synthetic a priori". Undoubtedly the most interesting thinker in this connection was Ernst Cassirer, a reading of whose works would have induced many of the representatives of the Vienna Circle to modify their judgement regarding transcendental philosophy.

In his epistemological masterpiece *Substanzbegriff und Funktionbegriff* (1910) (Chapter I), Cassirer clearly theorized the difference between abstraction as conceived by empiricism, whose origins he rightly traced back to Aristotle, and the specific form of conceptualization of modern science when dealing with the "theory of the formation of concepts". The Aristotelian doctrine of concepts is based on the capacity of the spirit to isolate, from the vast multiplicity of features present in things, those characteristics that several of them share by virtue of their similarity. In this way a concept is defined by means of its proximate genus and specific differentia, and the specific differentia "does not *exist* save as a part of a concrete presentation and burdened with all the attributes of presentation" (Cassirer 1910, p. 10). For the "psychology of abstraction", which Cassirer viewed as being typical of both conceptualism and nominalism from ancient to modern times, the logical meaning of conceptual form boils down to the "simple capacity of reproducing any given content of presentation" (p. 10). Recognizing in a presentation that the perceived object has equal (or at least similar) characteristics, the intellect makes a "progressive solidification of these features that agree, their fusion into a unitary, indivisible whole, [which] constitutes the psychological nature of concept, which is consequently in origin as in function merely a totality of memory-residues, which have been left in us by perceptions of real things and processes" (p. 11). The concepts arrived at in this way are thus the same as the genera-concepts of Aristotle, which are typical of descriptive, classifying natural science. Consistently with his Kantianism, Cassirer opposes this way of viewing abstraction with the active function of the intellect, which comes into play in particular in mathematical science and theoretical physics:

> "Abstraction", as it has hitherto been understood, does not *change* the constitution of consciousness and of objective reality, but merely institutes certain limits and divisions in it; it merely divides the parts of the sense-impression but adds to it no new datum. In the definition of pure mathematics, however … the world of sensible things and presentations is not so much reproduced as transformed and supplanted by an order of another sort. If we trace the method of this transformation, certain forms of relation, or rather an ordered system of strictly differentiated intellectual functions, are revealed, such as cannot even be characterized, much less justified, by the simple schema of "abstraction'. And this result is also confirmed if we turn from the purely mathematical concepts to those of theoretical physics. For in their origin the same process is shown, and can be followed in detail, of the transformation of the concrete sensuous reality, – a process which the traditional doctrine cannot justify (Cassirer 1910, p. 14).

Analyzing, for instance, the modern concept of "energy", Cassirer shows that it is unexplainable if the traditional way of considering abstraction is applied, as it would lead to substantialization and thus conceal its functional nature. This means that the fundamental concepts of natural science continuously transcend actual data and the scientific image of nature "arises first through a process of idealization, in which the indefinite data of sensation are supplanted by their strict, conceptual limits" (pp. 127-8). Even laws which appear to be close to experience can be reasonably expressed only with reference to ideal extreme figures with which we replace the empirical data of sensorial perception by means of a concept. This construction of "extreme structures" is performed by means of "an independent and constructive activity", without which "the world of perception would not be merely a mosaic but a true chaos" (p. 128). Direct contact between scientific concepts and perceived facts is lost:

> No scientific theory is directly related to these facts, but is related to ideal limits, which we substitute for them intellectually. We investigate the impact of bodies by regarding the masses, which affect each other, as perfectly elastic or inelastic; we establish the law of the propagation of pressure in fluids by grasping the concept of a condition of perfect fluidity; we investigate the relations between the pressure, temperature and volume of gas by proceeding from an "ideal" gas and comparing a hypothetically evolved model to the direct data of sensation. … If the procedure of natural science only consisted in substituting the ideal limiting cases for the directly observable phenomena, then we could attempt to do justice to this method by a simple extension of the positivistic schema. For the objects with which the theoretical consideration of nature is concerned, although they fall beyond the real field of empirical perception, seem to lie on the same line with the members of this field; and the laws, that we assert, do not seem to represent a transformation so much as a mere extension of certain perceptible relations. Yet, in sooth, the relation between the theoretical and factual elements at the basis of physics cannot be described in this simple way (p. 130).

There is a complete detachment, a total separation between concept and reality. None of the concepts of science is part of sensorial perception because we cannot do without limiting concepts. Taking as examples Lagrange's principle of virtual velocities and Galileo's principle of inertia, Cassirer points out that

> The fundamental theoretical laws of physics throughout speak of cases that are never given in experience nor can be given in it; for in the formula of the law the real object of perception is replaced by its ideal limit. The insight gained through them never issues from consideration of the real alone, but from the possible conditions and circumstances; it includes not only the actual, but also the "virtual" process (Cassirer 1910, p. 174).

This line of thought was later to be incorporated within a more general view of nature as eminently symbolic of human culture, by which Cassirer stressed the non-speculative character of scientific concepts: they are freely created intellectual symbols so, as Hertz had pointed out in his "theory of symbols", the value of physical concepts does not lie in their capacity to reflect but in their logical value and the fact that they "obey the general requirements" of the logic of scientific knowledge, the most important of which is the need, a priori, for clarity, absence of contradiction and descriptive unambiguity (Cassirer 1923, pp. 5-7). The exact sciences have, in fact, progressed because they have been built up as "symbolic systems", in which a symbol is not just an accessory, a tool to convey certain contents, but that which constitutes the contents themselves (pp. 19-20). It is no coincidence that he sees Galileo as the first to have understood this truth regarding knowledge when he had to imagine a completely isolated body that is not a real object and could never be found in nature: "without introducing these quite unreal premises Galileo would not have been able to formulate his theory of movement" (Cassirer 1948, p. 129). This applies to all great scientific theories: at first they were always paradoxical, and no common intellectual courage was required to propose and defend them (pp. 129-30).

> This could be expressed, somewhat paradoxically, by saying that Galileo's equations claim to be correct not because they apply always and everywhere, and because this "always" and "everywhere" has been experimentally demonstrated by him, but rather because strictly speaking they never apply anywhere. They are based on ideal cases, not on immediately given, empirically real ones. And all those laws which have been established by classical physics, following the example of Galileo, are of the same nature (Cassirer 1937, p. 83).

Cassirer had an extremely modern view of scientific knowledge and to a certain extent anticipated the criticism of classical neo-positivistic theorization that was to be put forward by the New Philosophy of Science: the inexistence of empirical data not mediated by theory, his critical attitude towards Mach's reductionism and positivistic descriptivism and the holistic nature of science and his criticism of the Baconian concept of *experimentum crucis,* the multiplicity of conceptual images both in individual sciences and among different sciences, whereby he was led to the problem of incommensurability and "meaning variance, the incompleteness of verification and the settling function of falsification, the non-representative and constructivistic character of the concept of truth, the influence of the philosophical perspective in deciding between two empirical theories, and so on (cf. 1910, *passim*).

Cassirer belonged to the great central European philosophical tradition that played a crucial role among the outsiders to the empirical-analytical mainstream. On the same post-Kantian wavelength we can also mention the fictionalistic theory of Hans Vaihinger, which was taken into even less consideration due to the apparently paradoxical nature of his idea of an "as if" philosophy, the radicalism of which ended up by attributing a counter-factual, fictional nature to any concept or representative human activity. And yet no other philosopher was as aware as Vaihinger of the constructive and symbolic-artificial character of human thought, especially scientific thought (Vaihinger 1922).

Within the same tradition we find two lines of thought inspired by Franz Brentano and his school: although to a different extent and with different degrees of epistemological awareness, these two lines eventually converged to propose an alternative way of conceiving scientific conceptualization: on the one hand we have the teaching of Husserl before he became involved in phenomenology, and on the other the great, although little known, tradition of the Polish analytical school founded by Kazimierz Twardowski.

As regards the first line of thought, it is worthwhile recalling here that Husserl devoted a large amount of his *Logical Investigations* to the analysis of the "modern theories of abstraction" (especially in the *Second Investigation*), highlighting their contradictions and difficulties and countering empirical abstraction - which views general objects as mere abbreviations dictated by economy of thought and therefore reduces them to singular individual experiences, representations of and judgements about single facts - with the sound arguments of the "idealist". Accordingly, when he states that any attempt to reduce these ideal units to single real facts is absurd and it is therefore impossible to split a concept up into any set of single units without a concept that will give them unity of thought. Empiricism, in fact, does not grasp the "ideal" nature of scientific concepts, which can be arrived at only by what Husserl calls "idealizing abstraction": it is this that allows us to grasp those ideal meanings expressed in utterances, which cannot be reduced to a collection of empirical objects nor to recognition of the common properties shared by the objects our senses perceive. Although a natural scientist performs various kinds of subjective actions, he knows well that the expression is the accidental element; what is essential is the concept, the ideally identical meaning. All science, in relation to its objective statute, is an ideal set of meanings. To make this difference clearer, Husserl more than once describes the difference between the colour "red" we see with our eyes, which is something individual, which is here and now, which appears and disappears, and red as a species, which is an ideal unity and of which it would be absurd to say that it appears and disappears:

> We do something wholly different if, looking at an intuited concretum, we refer to its sensed redness, the individual feature it has here and now, and if, on the other hand, we refer to the Species Redness, as when we say that Redness is a Colour. And just as, while regarding some concrete case, we refer, not to it, but to its universal, its Idea, so, while regarding several acts of such Ideation, we rise to the inwardly evident recognition of the identity of these ideal unities which are meant in our single acts (Husserl 1900, p. 149).

Or again, criticizing Berkeley's concept of abstraction, he uses the example of the triangle to reproach him with confusing "the basis of abstraction with what is abstracted, the concrete instance, from which our consciousness of the universals draws intuitive fulness, with the object our thought intends" (p. 378). Berkeley thinks that a demonstration is performed on the concrete triangle drawn in ink on the paper, but "no geometrical proposition holds for the drawn figure as a physical object, since the latter is not really rectilinear, nor a geometrical figure at all. We can find no ideal geometrical properties in it, as colour is found in an intuited coloured object" (p. 378; see also p. 398). Although the mathematician is looking at the drawing, in his thought acts he makes no reference to it. In such cases, in fact, our attention does not focus on the concrete object of our intuition, or on the partial content which is the result of empiricistic abstraction, but on the Idea in the sense of a Specific Unity, an abstractum in the logical sense:

> In logic and epistemology, therefore, abstraction must not be said to be a mere stress on a partial content, but a peculiar consciousness which, on an intuitive basis, directly apprehends a Specific Unity (p. 379).

We reach this idea through an intuitive act; in short:

> we directly apprehend the Specific Unity *Redness* on the basis of a singular intuition of something red. We look to its moment of red, but we perform a peculiar act, whose intention is directed to the "Idea', the "universal'. Abstraction in the sense of this act is wholly different from the mere attention to, or emphasis on, the moment of red; to indicate this difference we have repeatedly spoken of *ideational or generalizing abstraction* (p. 432).

Given this stand, Husserl's rejection of a purely extensionalistic concept of scientific concepts is quite natural.

His critical attitude towards the classic empirical concept of abstraction can be found throughout his works. In the last of these it is applied to history when he discusses the idealizing character of modern science as founded by Galileo (cf. Husserl 1954, pp. 51-128). Now, however, he is no longer interested in evaluating this typical mode of acquiring scientific knowledge so much as proposing as an alternative – on account of its artificial nature – re-discovery of the "world of life" as the only resource that can help European science to get over its "crisis". Husserl's thought thus belongs to a broad current of contemporary philosophy that sees scientific rationality as an enemy of philosophical speculation and therefore wishes to confine it within narrower limits, depriving it of any authentic cognitive value. This also accounts for the fact that his phenomenology did not give rise to any serious epistemological thought concerning science and was more or less ignored by contemporary analytical philosophy, thus losing its elements of originality, which included a new way of conceiving scientific conceptualization once it was freed of any deprecatory intentions. In this respect, Husserl remains a misunderstood philosopher.

4. *The Polish connection*

Husserl therefore indicated a radical alternative to the direction taken by empiricism, which was then rejected by contemporary epistemology and logic. The same path, starting from the same Brentanian roots, was taken by K. Twardowski in Poland and subsequently (not, however, without ambiguities) by his disciples. As early as 1894, by his triple distinction between the *act*, *content* and *object* of representations, Twardowski distinguished between the psychological image of an object (the "content" of a representation) and the object to which the image refers, thus abandoning any kind of empirical concept, according to which an object is a combination variously obtained from subjective impressions, something which represents a sort of synthesis, a vague memory or mnemonic trace. Of course he still viewed the object of a general representation as "a group of components that are common to several objects" (Twardowski 1894, p. 85). But the fact that he stressed that the content of a representation belonged to a different domain from the one to which the object of the representation belonged was to lead him to render the object of psychology (a "psychological fact") independent of that of epistemology (a "scientific concept") (cf. Twardowski 1912). His recognition of the autonomy of the object of a representation paves the way towards "de-psychologising" science and hence an epistemology with an anti-empirical basis.

In connection with this Twardowski makes an important distinction between "sense-images" (or "presentations", *wyobrażenia spostrzegawcze*), "memory-images" (or "representations", *wyobrażenia odtwórcze*) and productive images (*wyobrażenia wytwórcze*, conceived as a free creation of "fantasy", i.e. imagination); the sources of the latter are sense-images but, unlike memory-images, they perform a metamorphosis, thanks to which "the synthesis [of impressions] is presented as something new, an involuntary or arbitrary product of the fantasy" (Twardowski 1898, p. 127). The originality of this operation of synthesis performed by the intellect immediately presents the images ("ideas", in the sense of Hume) we possess (whether they be ideas of physical or spiritual objects) as a whole whose component parts we can only distinguish *a posteriori*, by means of psychological analysis. They are thus essentially *concrete*, that is, it is impossible to distinguish the factors composing single images; any analysis to distinguish their parts is always an *a posteriori* operation. This analysis is, of course, abstraction (p. 137). But this type of abstraction is exactly the opposite of the process of abstraction typical of empiricism: first, in this approach to images (ideas), Twardowski is clearly far from the associative psychology derived from empiricism, according to which they are the fruit of a combination of impressions on the basis of their similarity: an idea is grasped immediately, by means of an act similar to Husserl's idealizing abstraction; secondly, the role of abstraction is of great importance: it is not the instrument by means of which from particular impressions we reach general or complex ideas, but the means by which, given an original whole autonomously created by the intellect through a creative act that cannot be further analyzed, it is possible to distinguish its component parts. That this view is similar to *Gestalt* psychology would seem to be confirmed by the fact that *Gestalt* theory had a certain influence in the Vienna environment in which Twardowski received his formation, especially on Brentano (Smith 1989).

Images or ideas are distinguished from concepts which are characterized by a lack of concreteness and visibility; it derives from the impossibility of imagining or, in other words, is a failed or impossible productive image (cf. Twardowski 1898, pp. 147-8). Thus, for example, a mathematical point or a square circle cannot be imagined but they can be conceptually represented: what Twardowski was aiming at was to guarantee the autonomy of an object from single psychological contents and throw light on the creative, productive role of human thought in elaborating scientific concepts. The object all knowledge revolves around is therefore quite different from the content of our consciousness because, in the case of an imaginable object, it is a connected whole that is given to us through a primitive act and not by a combination of simple impressions; with a concept, we are dealing with a completely counter-factual activity whereby a representative judgement is formed which attributes an imagined object with a certain number of properties it does not in fact possess (pp. 153-4; 190). Twardowski's explicit acceptance (1924, p. 292) of Vaihinger's fictionalistic theory of concepts (Vaihinger 1922), which was also clearly anti-empirical in inspiration, confirms his basic intentions: Twardowski judges his theory of abstraction to be in harmony with his own theory of concepts which, at least in certain cases, can be identified with Vaihinger's fictions.

It seems, however, clear that the fundamental role played by representative judgement in Twardowski's approach gives all concepts, not only some of them, elements of artificiality that make them very similar to what Vaihinger means by the terms fiction and semi-fiction: "if Vaihinger's fiction is a conscious deformation of reality, then Twardowski's representative judgement – which attributes the object of the supporting image with characteristics that it does not in reality possess – is also a fiction" (Paczkowska-Łagowska 1980, p. 188). By fiction, Vaihinger means "those particular forms of representation that not only contradict reality but are contradictory in themselves" (Vaihinger 1922, p. 30). These include mathematical concepts, like that of the point Twardowski used as an example of a concept, and which Vaihinger defines in the same way – as something to which is attributed a characteristic that is not only inexistent but contradictory (p. 62). Semi-fictions, on the other hand, are those representations which contradict given reality but are not in themselves contradictory (p. 30) – well-known examples are a golden mountain or a red chalkboard.

Despite these insights, Twardowski did not develop an organic theory of science. The seeds he sowed bore fruit, however, in the work of his disciples, especially in the philosophical concepts of the great logician Jan Łukasiewicz, in those of the lesser known Tadeusz Czeżowski, and in part also in those of the creator of radical conventionalism, Kazimierz Ajdukiewicz.

In Łukasiewicz's essay on creativity in science (Łukasiewicz 1912), which was to have a great influence on Polish culture for several decades (Giedymin 1986, p. 193), we can find an extremely modern approach to scientific theories which not only foreshadows the development of contemporary epistemology after the crisis of the neo-positivist paradigm and has a number of analogies with the conjecturalistic stand of Popper, but also shows that Poland provided fertile ground for acceptance of Popper's ideas, which were grafted in the post-war years onto the trunk of Marxism. Łukasiewicz energetically stresses the creative character of science: it does not aim at mere reproduction of reality, as in a photograph or a gramophone record, but is closer to the way an artist paints a picture. The non-reproductive nature of science can be grasped from simple awareness that it is not "omniscience", i.e. it does not aim to know or collect detailed facts but only to synthesise their general features. Otherwise the most banal truism would be part of science, which would be absurd. Facts, however, have to be ordered, they have to be given shape and so they need to be provided with something they did not originally possess, something which we draw from reason. The "general nature" of physical laws is not given by experience, as experience only shows us a vast number of individual facts (Łukasiewicz 1912, p. 17).

The abstract, creative nature of science is also evident from Łukasiewicz's analysis of Galileo's law concerning the fall of bodies. Its creative character lies in the relationship it establishes (given by the formula $v = gt$):

> No measurement is exact. Hence it is impossible to state that the velocity is *exactly* proportional to the time of fall. Thus *neither* does the form of the relationship *reproduce* facts that are empirically given: the *entire* relationship is a product of the creative activity of the human mind.
> Indeed, we know that the law governing the fall of heavy bodies can be true only in approximation, since it supposes such non-existent conditions as a constant gravitational acceleration or a lack of resistance offered by air. Thus it does not reproduce reality, but only refers to a *fiction*.
> That is why history tells us that the law did not emerge from the observation of phenomena, but was born *a priori* in Galileo's creative mind. It was only *after formulating* his law that Galileo verified its consequences with facts. Such is the role of experience in every theory of natural sciences: *to be a stimulus for creative ideas and to provide subjects for their verification* (p. 9).

Thus Łukasiewicz stresses the unreal character of the situation described by a physical law: the law includes counter-factual statements, so rather than referring to reality it describes an ideal model (which is viewed as a "fiction") (Zamecki 1977, p. 93). Łukasiewicz insists on the authentically hypothetical nature of science: hypotheses, the free creations of the human mind, are "*permanent* elements of knowledge and not temporary ideas that by verification can be changed into established truths" (1912, p. 10). Science is, in short, an open system undergoing constant improvement in which ultimate truths are never achieved and whose statements are essentially always hypothetical.

As far as Czeżowski is concerned, an interesting thesis is put forward in his essay *Pozytywizm a idealizm w pojmowaniu nauki* (1936) (Positivism and Idealism in Understanding Science) where he discusses the concepts of science typical of idealism and positivism. By the term "idealism" (but it would be more accurate to say "Platonism"), Czeżowski does not mean classic German idealism so much as the Brentanian school, later continued in Poland by Twardowski and Łukasiewicz. This difference emerges above all in their way of viewing scientific judgement:

> Scientific sentences are abstract products as compared with concrete thoughts and spoken or written statements; they are ideal objects completely different from the empirical, individual, concrete objects of the positive world. For positivists science is a system of empirical objects connected by natural links; for idealists, science is a system of ideal objects connected by logical links (Czeżowski 1936, pp. 6-7).

The difference between positivism and idealism, Czeżowski continues, is also to be seen in their way of viewing the aim of science. For positivism it consists of description of the real world by means of scientific laws which obey the principle of economy of thought. So scientific laws are just an abbreviation or description, and progress in science consists of progressive refining of this description, thus perfecting the principle of economy of thought. For the idealist, on the other hand, "general concepts and scientific laws are not descriptive formulae but ideal construction of thought" (Czeżowski 1936, p. 10).

The view of science that Czeżowski defines is a continuation of the position of Twardowski and Łukasiewicz, stressing their idea of the creative role of the scientist. This continuation of the approach of Twardowski and Łukasiewicz is also explicitly confirmed when he comes to discuss the so-called "method of analytical definition", which is both the method he uses in all his discussion and philosophical-scientific analysis (Czeżowski 1963, p. 161) and that used by modern science from Galileo onwards (Czeżowski 1956, pp. 209-213). In effect, Czeżowski traces the method of analytical description back to the philosophical trend which took the name of analytical philosophy and was represented in Poland by Twardowski, who applied it in his "O istocie pojęć" (Twardowski 1924) and which in turn had its origins in Brentano's descriptive psychology, where the method of analytical description was applied to the world of psychological phenomena, classifying them by means of a process of abstraction not based on induction but conceived as the result of intuition (Dąmbska 1979, p. 21). Unlike experimental description, which is applied for instance by a psychologist who accumulates data to obtain statistical generalizations, analytical description leads to general apodictic statements obtained by referring to additional assumptions and eliminating the complications existing in the phenomenon described. It does not lead to the formation of gender and species so much as to the construction of *types*:

> The difference between gender or species and type can be characterised in general by saying that gender or species is an abstraction which includes the particular examples described, whereas type is chosen (or hypothetically constructed – the so-called ideal type) to describe the model to which the others, which it helps describe, more or less resemble (Czeżowski 1953, p. 199).

In the case of analytical description, inductive generalization is obviously not used; the act of generalization present in it is a particular cognitive act based on the analysis of the meaning of the name of the object described. This defining nature of analytical description relates it to the *a priori* statements of mathematics. Just as these are valid by definition for all the objects they comprise and exclude *a priori* dissimilar cases or introduce additional clauses to explain anomalies, so, in the description of free fall made by Galileo, for instance, friction and resistance on the part of medium are excluded, only to be introduced subsequently to explain cases of departure from the law of free fall. So once the concept of type is fixed, it is not falsified by cases which do not agree with the statements describing its properties; they are treated as atypical cases, whose deviance from type is accounted for by means of subsidiary circumstances (pp. 200-1).

Analytical description as it is seen by Czeżowski differs both from inductive generalisation and from the method of abstraction (which consists of extracting what is common to a set of objects), as well as from the analysis of individual objects aiming at specifying their peculiar characteristics in various circumstances (as in historical and philological science). Analytical description is the fundamental method of all empirical research from physics to biology, and also of all non-empirical research, such as mathematics, geometry and logic. This idealising approach is explained more explicitly and with greater precision in a more recent essay, where Czeżowski distinguishes between the concepts of abstraction, generalization and idealization and, referring to Galileo's analysis of the free fall of heavy bodies, states:

> The Galilean model of free fall is an idealized one: a resistance of friction is omitted in the case of a body rolling down, and an air resistance. Idealization results from omitting the factors which may become sufficiently small not to cause any disturbance in the investigated phenomena. ... Idealization must not be equated with generalization ... The difference is, however, that generalization widens the scope of the term generalized, whereas idealization modifies it in a different way: generalizing one steps over to the superior term; whereas idealization is shifting the element in a sequence to which it was put until it arrives at the border of this sequence which is designed by an ideal type (Czeżowski 1975, pp. 13-4).

This way of proceeding is what Czeżowski had previously called the "method of analytical description", consisting of choosing those characteristics of investigated objects that are considered to be definitional and thus provide an analytical definition, as we saw above. Unlike description, this "loses a direct link with the world of individuals, which is the object of description, to create species (or sets) as abstract objects" (p. 10). So analytical definitions only refer to reality through the creation of a *model* (or *type*), and scientific theory is not descriptive of the phenomenon but of this model:

> ... the result of the construction of a theory with the help of a method of analytical description is the model for an investigated domain of the phenomena. Such a model is obtained by assuming definitions, the basic definitions in axiomatized theories being axioms themselves. The definitions in question and their consequences were called aprioristic or analytical sentences, and these names referred jointly to different ways of conceiving these principal sentences. According to the view upheld in the present paper these principal definitions are – to make it simple – means towards the end, namely towards constructing a theory and making models which, in turn, lead to the cognitive grasp and linguistic fixing of the richness of empirical data providing knowledge of the world (p. 17).

This method of analytical description thus refers to a creative moment which as such cannot be subjected to any a priori regulation with which the scientist and also the philosopher, as it is valid both for empirical and mathematical sciences and for philosophy, creatively defines ideal objects, as representatives of the models of the phenomena being investigated. The type "presents the phenomenon under investigation in somehow pure form, without minor disturbances, and it is also a point of reference for characterizing the elements of a sequence according to their distance from it" (Czeżowski 1975, p. 13).

Lastly, during his radical conventionalism phase Ajdukiewicz was clearly aware that physics makes use of ideal concepts which refer to events that can never occur in reality:

> ... almost all the sciences exhibit this "tendency toward idealization". Physics, e.g., sets up theses about ideal gases, although it is well known that no such gases exist. Again, in mechanics, the concern is with motions which run their course under conditions never actually realized. Physics does this, we submit, because it is the only possible way for knowledge to approach the reality. First of all, one sets up laws which hold rigorously only for ideal gases; for real gases, they hold only within a rather larger error of approximation. Only after the first step are we in a position to transform these laws with a view toward reducing the initial error of approximation. To begin by instantly demanding an absolute agreement between the laws and reality is to set much too difficult a problem for ourselves (Ajdukiewicz 1934, p. 88).

Later on, however, influenced by neo-positivism, Ajdukiewicz was to incline towards an increasingly radical form of positivism, thus abandoning the ideas he had previously expressed. Ajdukiewicz is a typical example of a philosopher of science who correctly identifies the idealizational method as applied to science, but does not succeed in giving it an adequate place in his own epistemology, which remains largely a result of the inductivistic framework on which the classical way of viewing abstraction is based. The same could be said of Tadeusz Kotarbiński: his recognition of the idealising method, connected with rejection of the empirical view of abstraction, is combined with acceptance of typically empirical inductive procedures (cf. Coniglione 1999).

The line we have briefly described here – which follows the path leading from Meinong and Brentano's theory of objects to the elaborations developed in different degrees and with different degrees of awareness by Twardowski's school – is an element of originality in the Polish analytical tradition. Together with Husserl on the one hand and Hegel on the other, it put forward a concept of science that was radically different from the positivistic models existing both before and after the war and indicated a way of conceiving scientific conceptualisation that broke with the standard view of abstraction as proposed by classical empiricism, later reshaped in contemporary epistemology by the sophisticated technical tools of contemporary logic.

There exist, however, other lines of thought in Polish philosophy and other philosophers who have made important contributions in this direction, thinkers whose cultural background was quite different from that provided by the analytical tradition. We must not, in fact, forget the importance of the methodological thought of Leon Petrażycki (a sociologist, philosopher and theoretician of law who lived in St. Petersburg and Warsaw at the turn of the century). Attacking positivism, he maintained that the basic cause of the lack of understanding of social laws lay in an inductivistic and descriptivistic interpretation of them. Stressing the role of theory, viewed not as a mere registration of empirical facts but rather as a tool whereby empirical data could be ordered and explained, he maintained that laws do not aim to describe reality but to explain it, so their existence cannot be ascertained by observing phenomena, events and facts. Although the various "laws" concerning social phenomena describe something that never occurs in reality, they are still useful to explain what happens and to predict future events. The significance of these formulae, Petrażycki points out, is that they indicate what would happen if there were no perturbing circumstances - what our reality would approximate and in effect does approximate. This approximation grows when the role of the perturbing circumstances as compared to that of the active agent according to the given formula is reduced (Petrażycki 1907, pp. 181-2).

The contents of social laws are ideal dependences which capture the basic structure of society once any disturbing factors have been eliminated; hence they do not describe deterministic events but only objective trends. This is also expressed by his stating that the task of a scientific theory is to grasp the essential features of the objects that fall into its theoretical domain, i.e. those properties that allow us to understand the fundamental nomic relationships that hold between properties. Petrażycki therefore criticized the method of abstraction by induction, that is, selecting features common to several objects and then generalizing them. This was a sterile issue for him as, in pure Popperian style, it is not the way that we get our ideas that is important but the method we use to control them: therein lies the scientific nature of our hypotheses.

This viewpoint was shared, again in the field of sociology, by the Marxist thinker Ludwik Krzywicki (1923) with his "structural method". In describing social systems, especially the simplest ones such as primitive and tribal societies, he built regular theoretical models. He was, in fact, convinced that only such models could enable one to go beyond mere description and grasp their regular features. To construct theoretical models of this kind it is necessary to conceptualize the societies under investigation by abstracting from the particular temporal and spatial circumstances as well as the particular features of the peoples by whom these societies were constituted. The systematic application of this particular kind of "abstraction" and rejection of the inductivism typical of positivistic methodology were characteristic elements of the Marxist method. The aim of science was to build ideal models that could explain the laws governing reality. Only in this way could Marxism become an empirically testable science, sharing the same methodological statute as the natural sciences (Klawiter 1975).

Another interesting but almost unknown example of the valuing of idealizational procedures is that of the greatest contemporary Polish sociologist Florian Znaniecki. In the early stage of his thought, when he still intended to be a philosopher and was interested above all in the problem of the statute of values, he more than once stressed the essentially mediating function of thought and criticized the concept of immediate or "pure" experience as proposed by Bergson, Le Roy and Mach (Znaniecki 1911, pp. 41-5). This led him to counter material reality, individual consciousness and social life with an ideal world which is the result of human cognitive activity. Recalling Plato, Znaniecki stated that philosophy has from the start been aware that it is knowledge and that it takes on this character when its concepts and truths belong to an ideal world. The knowledge of experience is thus idealization, whereby empirical objects are replaced by ideal objects, i.e. empirical concepts and relationships are replaced by ideal relationships (judgements and deductions). Our knowledge of the material, psychological and social world is therefore acquired through the mediation of an ideal world (Znaniecki 1913, p. 174).

Znaniecki dealt with the problem in a more systematic fashion in what can be considered his most important philosophical work, in which he devotes a whole chapter to the "problem of idealization" (Znaniecki 1912, pp. 386-99). With an undoubtedly Kantian accent, he confirmed the shaping function of scientific thought, which lies in its idealizing capacity, the only means by which knowledge of reality can be achieved. It is significant that the only philosopher whom he mentions in this context is Cassirer (p. 397).

It was, however, necessary to pass from the plane of general statements to one on which it was possible to construct an articulate epistemological proposal that could compete with the contemporary philosophy of science as regards both clarity and the capacity to analyse existing scientific theories. And we can by no means claim that the philosophers mentioned so far proposed an articulate theory of science, either because this was not their main intention (as in the case of those who were more interested in sociological enquiry) or because the influence and prestige of the great Western analytical tradition soon suffocated those elements of originality that Polish scientific philosophy had inherited from its cultural background, which was different from that of the Vienna Circle.

Only towards the end of the 60s was there any movement in this direction. The new philosophical trend arose from the convergence of two factors: on the one hand the influence of the Polish analytical school, enriched by knowledge of Popperian philosophy; on the other the importance of reflection on the thought of Marx which, as we have seen, supported a decidedly anti-empirical concept of science, taking what in Hegel was "upside down" and setting it on its feet again (Coniglione 1990a).

This new approach to science took shape in the so-called "Poznań School of Methodology" whose thought centred round the concept of "science as idealization" (Coniglione 1990b). The philosophical reflection of the School was based on a non-positivistic approach to natural sciences that rejected the model of science proposed by the Vienna Circle, Carnap in particular (Nowak and Kmita 1968). It arose from the need to re-interpret Marxism in such a way as to throw light on its scientific character, and was thus related to the Polish Marxist tradition (mainly represented by Kelles-Krauz, Krzywicki and Chwistek) that has privileged its cognitive content and seen it as a continuation of the rationalistic scientific tradition in Western culture. Explicit reference was made to the rigorous, crystalline intellectual style typical of Neo-Positivism (whose Polish counterpart was the Lvov-Warsaw School), as opposed to the literary style in which Marx's thought was usually presented. The School, however, rejected the basic philosophical theses of Neo-Positivism, as was consistent with the anti-positivistic methodological naturalism proposed in the 1968 volume.

This concept of science, which constitutes the original matrix of the School, was initially formulated by Leszek Nowak, Jerzy Kmita and Jerzy Topolski, although with differing degrees of emphasis. The rigorous intellectual style of the Lvov-Warsaw School was applied to the analysis of Marx's thought, favouring his later works and stressing their "scientific" character, thus rejecting the humanistic reading of Marx inspired by Hegel, which was widespread at the time following knowledge of his earlier works. The basic idea was that the main theoretical assumptions of Marx's methodology and positivistic epistemology agreed that there was a methodological unity between natural sciences and human sciences, even though they differed in many other basic philosophical assumptions. It was respecting the latter that Marxism was to develop its own idea of science: rejecting all combination, it was to compete in rigour and clarity with the most advanced epistemologies and at the same time overcome the limitations of Neo-Positivism (Kmita, Nowak and Topolski 1975).

It is in Marx's *Capital* that a new way of considering abstraction is found, a way typical of his "modelling" method that is called "idealization" in order to distinguish it from the classical, empirical way of viewing it: the term abstraction – Nowak claims – is in a way ambiguous. On the one hand we use it to mean, for example, that the content of a given term has been constructed by abstracting from certain properties of specific objects and considering other characteristics. In this sense, an abstract theory is simply a theory about sets of objects, characterized by the fact that some characteristics of the objects in question are taken into consideration and others not. In the other sense, an abstract theory is identified with an idealisational theory which deals with certain ideal types (Nowak 1971, p. 21).

This means that the constructs Marx uses in his *Capital*, which are those of mature science, are not empirically given, nor are they inductive generalizations based on experience (i.e. "abstracted" from it in the prime sense of the term); they are constructs which correspond to nothing that exists in reality, for instance a "perfect gas" or "two-class capitalism", a "rational individual", etc. So "in the effort to become acquainted with the reality that surrounds us, science apparently distances itself from it" (Nowak 1970, p. 127) and scientific theory "is not the mere description of the phenomena that surround us, but is formulation by means of idealising assumptions, i.e. assumptions that set aside certain features of the phenomena being investigated in order to stress other, more essential, ones" (p. 139).

To make this concept of idealization clearer, Nowak compares it to caricature:

> Let us see what a cartoonist does: he leaves out some details of the person presented, thus stressing what he considers important. That is, he employs the method of exaggeration: he does not present everything but distorts a person or a situation by neglecting some features he thinks minor ones. Science … in fact does the same. When a physicist constructs the concept of a material point, he does not present physical objects but distorts them – he assumes that they have zero dimensions and focuses on other properties of these bodies (e.g. mass) which he considers more essential for the physical magnitudes he investigates. In short: science consists in the same method we find in caricature (Nowak 1980, p. 134).

So an ideal entity like a "perfect gas" has no reference in fact and therefore cannot be extensionally reduced to any class of individuals (nor is it an element belonging to a class of abstraction), but only designates conceptual constructs intensionally defined by means of properties, whose extension is null. A conceptual construct that is more *abstract* than another will have a wider field of denotation and a narrower one of connotation, whereas a more *idealized* conceptual construct will have the same denotation or class of factual reference as another, less idealised one (it is in any case empty) but its connotation will be broader. The denotation of the term "perfect liquid", for example, is no broader than that of the term "incompressible liquid", as the class of factual reference is in both cases empty (to our knowledge, liquids that are perfect or incompressible do not exist), but the connotation of the former, or its conceptual content, is broader (as a perfect liquid is at the same time incompressible and has no viscosity) (Nowak 1975, pp. 23-5).

The essential part of every scientific law, at least as regards science that has gone beyond the "threshold of maturity" physics crossed with Galileo, is to be found in ideal concepts of the kind described. However, a scientific law is never elaborated in a vacuum of knowledge, by means of a disembodied comparison between the intelligence of the researcher and pure nature: it is formulated against the background of a preliminary ontological structuring of reality (deriving from the scientific tradition to which the researcher belongs, from shared philosophical concepts which in turn may depend on a broader view of science that a certain community or even civilization possesses at any one time) that specifies the kinds of magnitude to be taken into consideration to account for the behaviour of a given phenomenon and how they are to be connected with each other. Between these magnitudes, or parameters, which are thought to affect the phenomenon F being studied (and which are part of the so-called "space of factors essential for F") a hierarchy of priority is established, going from the most influential (the "main factors") to those that affect it least ("secondary factors"). This hierarchy of factors is called an "essential structure"; it is a hypothesis the researcher makes about the phenomenon being investigated, a hypothesis that will have to be checked to ascertain its capacity to produce explanatory scientific laws. Its aim is to allow the (provisional) omission, in the formulation of the law, of factors considered to be secondary, so as to take only the "main" factors into account, which are thought to be capable of describing the phenomenon under investigation, albeit with a certain degree of approximation. To this aim "idealizing assumptions" of the form $p(x)=0$ are introduced, thanks to which the researcher eliminates the secondary factors (in this case it is assumed that the factor p, concerning the generic element x belonging to the universe being investigated, has a value of zero) and tries to establish a nomic connection only between the magnitude being studied and the main factors. The result is a conditional statement, the premise of which contains both realistic conditions and idealizing assumptions (and is thus *counterfactual*):

$$\mathrm{L}^{k}: \quad U(x) \wedge q_1(x)=0 \wedge q_2(x)=0 \wedge, \ldots, \wedge q_k(x)=0 \rightarrow F(x)=g_k[G(x)]$$

where U is naturally the realistic condition determining the reality to which the statement refers, so it is met by any element belonging to our universe of discourse U; that is, if $x \in U$ then $U(x)$ is true for any x. In practice, the universe of discourse U indicates the set of objects our investigation refers to and the propositional function $U(x)$ means that a generic object x is an element in this universe; for example, "x is a commodity", "x is a body", "x is a molecule", etc. In addition, $q_i(x)=0$ is the idealizing assumption, as there does not exist in U any object a such that $q(a)=0$; for example, there is no body that is perfectly *rigid* or any gas that possesses properties such as to allow it to be defined as *ideal*. Finally, g_k is the dependence proposed by the researcher according to the kind of relation between the phenomenon F and the factor G considered to comply with the ontic relational principles of the ontological perspective the researcher has assumed to be valid.

Once it has been empirically checked, the statement can aspire to the title of *scientific law*. It is checked either by direct comparison through an appropriate procedure of approximation or, more frequently, by means of progressive "concretization"; the latter, which is fundamental to this concept of science, consists of progressively removing the idealizing assumptions and replacing them with realistic conditions, so as to bring the statement as close as possible to the phenomenal plane, to make it increasingly "realistic". We thus have an initial concretization L^{k-1}, with $q_k(x) \neq 0$, a second concretization L^{k-2} again with $q_{k-1}(x) \neq 0$, etc., until we reach the final concretization with which we may obtain a factual statement L^0.

This means conceiving of science as a succession of increasingly concrete statements (i.e., a succession of models), the last of which may be realistic, that is, its antecedent includes no idealizing assumption.

> The structure of a scientific theory *t* is thus given by a sequence of models M^k, M^{k-i}, ..., M^i, A^{mi}, where M^k is the most abstract model with *k* idealising conditions, and M^{k-1}…M^i are its successive concretisations. Lastly, A^{mi} is an approximation of the least abstract of these models, M^i , to empirical reality (Nowak 1994b, p. 20).

However, this stage of ultimate "concretization" is never reached in science, so recourse to procedures of approximation is always necessary.

To elaborate the concept of science we have outlined Nowak. His disciples also used a range of logical and formal tools, and they presented it explicitly as an alternative to the Standard Concept of scientific theory that dominated Western thought at the time. First of all, they attached great importance to the modelling nature of science: theories are seen as a succession of increasingly realistic models, linked to each other by "concretization"; this led them to a considerable sophistication of the relationship between theoretical apparatus and empirical data, unlike the line taken by Popper, whose rigid view of falsification they opposed (along with Kuhn and Lakatos, who were to point out the "resistance" to falsification put up by theories); for this to happen, in fact, first of all it is necessary to apply a procedure of adjustment and refinement in which empirical refutation does not lead *sic et simpliciter* to rejection of the theory but to procedures of "concretization" and modification of its essential structure to distinguish between authentic counter-examples and *prima facie* counter-examples:

> Now, in actual scientific practice the *prima facie* counter-examples are taken as confirmations, and not disconfirmations, of the idealizational law. And when a fact regarding such a statement is found, then the main effort of theoreticians is to prove that it is merely a *prima facie* counter-example, i.e. that it suffices to concretize that law in order to explain the discrepancy and take what seems to negate the law as a confirming case (Nowak 1992, p. 15).

The "scientific revolution" is therefore the final stage in a complex procedure developed over a period of time, not the result of an "instantaneous rationality" deriving from an immediate and unequivocal conflict between basic statements and consequences that can be deduced from a theory. However, the concept of science as idealization does not lead to positions that deny the possibility of establishing a rational link between successive theories by attaching excessive importance to theoretical apparatus and the "resistance" of scientific theories; adopting the principle of correspondence proposed by Bohr (and timidly also appreciated by Popper who, however, never developed all its implications), Nowak made it the fulcrum around which the concept of the historical evolution of science rotates, also establishing a typology of the various relations linking successive theories; this is the direction taken in particular by W. Krajewski (1977), I. Nowakowa (1994) and J. Such (1977).

We should not, however, neglect the fact that the crisis of the Standard Concept of Scientific Theory and the dissolution of the Popperian perspective also led a number of Western epistemologists to search for alternative ways, thus giving rise to a debate concerning the theoretical foundations on which the traditional image of science was constructed, which had after all been accepted by both Popper and his most radical opponents. As this re-orientation followed the re-evaluation of the role of models in science, it has been pointed out that adequate attention to the importance of the role of idealization and the consequent use of models in science would cause a real epistemological revolution (Harré 1970, p. 15). Although this thesis may not be acceptable, it must be recognized that well-known Western epistemologists have followed this orientation towards idealization or at least shown an interest in it (Cartwright 1983, 1989; Niiniluoto 1986; Dilworth 1990; Ludwig 1981; Suppe 1974, 1989; Cohen 1990, 1991; Harré 1990; etc.).

Another important point stressed by the School is the realistic stand. It derives from having introduced the notions of "essential structure" and "main" and "secondary" factors: science, in fact, does not aim to describe and organize the phenomenal data provided by immediate experience, but to grasp the nomological relations that are the essence of reality and are only manifest when we operate by means of idealization. Consistently with the teaching of Marx, reality is not "flat", but hierarchically structured into different levels, only the most fundamental of which are required to obtain a law governing the phenomenon being investigated. Galileo's law of free fall, for example, only takes gravity and time into consideration, neglecting all other factors because they are secondary (e.g. air resistance, wind, etc.). This way of viewing science is obviously very close to Popper's "modified essentialism", according to which although our scientific laws can never describe the ultimate essence of the world, they can "investigate in ever-greater depth its structure, or … increasingly essential or ever deeper properties" (Popper 1984, p. 156).

Finally, the counter-factual nature of scientific laws requires a different way of viewing the concept of truth, which can never be seen as a simple correspondence (or reflection), as the latter presupposes a phenomenistic vision of reality, whereas the approach of the Poznań School presupposes ontological essentialism. The classical concept of truth, if applied to laws as they are viewed in this concept of science, "leads to an absurd conclusion that all idealizational statements are true in the classic sense, since they are emptily satisfied by actual objects" (Nowak 1980, p. 134). This depends on the fact that an idealizational law, having a counter-factual assumption in its antecedent (i.e. one that is not true in the light of available knowledge), would still be true from a logical viewpoint, regardless of the functional connection established in its consequent. The essentialist concept of truth therefore opposes the idea of correspondence between between scientific statements and empirical facts with the thesis that an idealisational statement is true inasmuch as the essential structure hypothesized and the nomological links between the factors established in it are similar to the essential structure effectively governing the investigated phenomenon; that is, in the sense that "the internal structure of scientific constructs is isomorphic to the structure of the basic ontological forms of the reality described" (p. 125). And this similarity is confirmed by the explanatory fecundity and empirical robustness which are found in the law.

There is no point in dwelling any further here on concepts that are easily accessible (cf. Nowak 1992) and widely discussed both in general (cf. the essays in Brzeziński et al. 1990a-b; Brzeziński and Nowak 1992; Dilworth 1992; Kuokkanen 1994; Nowakowa 1994; Shanks 1998) and as regards their particular applications (cf. Hamminga and De Marchi 1994; Brzeziński et al. 1997).

This is, of course, not the first time a modelling approach to Marx has been proposed; but never before has scientific re-appraisal of his works been made in an epistemological context that creatively exploits both the Polish analytical tradition and the liveliest elements of Western analytical thought. In doing so the merit of the Poznań school lay in both the formal precision with which the distinction between abstraction and idealization was made, and the systematic way in which they developed an organic vision of science based on re-interpretation and original re-elaboration of many of the traditional categories of Marx's thought. The fecundity of this reconstruction is confirmed by the multitude of contributions its supporters made in almost all fields of knowledge, from psychology with J. Brzeziński, to biology with K. Łastowski, history with J. Topolski, pedagogy, the re-interpretation of historical materialism, and so on.

However, methodological awareness of the distinction between abstraction and idealization was not associated with an equally adequate knowledge of its historical roots, so at times the reflections of the Poznań methodologists almost seem to ignore the philosophical depth of the concepts on which their works focus. Except for due recognition of the work of Galilei, Darwin, Marx and a few other thinkers, the historical dimension occupies a negligible part of the thousands of pages the School produced, thus confirming the analytical and methodological trend that has characterized most of 20th-century Polish philosophy.

5. *Some metaphilosophical conclusions*

In the course of this lengthy analysis, we have considered the positions taken by scientists and philosophers towards the problem of abstraction, distinguishing it from idealization. We have also seen the diversity of the stands taken: on the one hand those who have theorized and applied procedures of abstraction, viewing them as the correct scientific method and the main way to acquire knowledge, and on the other those who have applied idealizing procedures and were also aware of the methodological novelty they represented, formulating in varying degrees of detail the theoretical grounds for their use in scientific research. In between, there are a wide variety of positions featuring different forms of methodological and practical awareness.

We are dealing here with a typical case in which the tension between scientific practice and methodological awareness takes on a multitude of forms. These range from the adoption on the part of scientists of an ingenuous philosophy of science borrowed from the dominant philosophical climate or a primitive metaphysics, dangerous in that science is held to be "pure", alien from any kind of philosophical consideration (as pointed out by Louis Althusser and, earlier, Friedrich Engels), to full methodological awareness of their scientific practice (which is much less common in the history of science).

The position of working scientists, who seldom devote much effort to methodological considerations, must be distinguished from that of philosophers (especially 20th-century philosophers of science) who see their task as explicit reflection on science, in the attempt either to theoretically "found" modern science or, more modestly, to understand its procedures, in the hope that they can then be applied to other fields of human research, bringing with them the benefits of a method whose validity has already been established. In this way, philosophers (insofar as they are not scientists, i.e. they perform a "second degree" reflection on scientific research, their thought thus becoming "metascience") may recognize the work of a scientist as applying the idealizational method but they are not necessarily also able to elaborate a complete scientific methodology in which it plays a central role, thus being distinguished from the inductive procedures behind empirical abstraction. Or again, they may recognise this idealizing procedure and ascribe to it the proper methodological importance, distinguishing it from abstraction and elaborating an epistemology oriented in this characteristic of scientific research.

On the basis of these considerations, we can hypothesise the following types of attitudes on the part of scientists and philosophers:

$\mathbf{a}_1$) the scientist who does not adopt the idealizational method in practice or theory;

$\mathbf{a}_2$) the scientist who does adopt the idealizational method in both practice and theory;

$\mathbf{a}_3$) the scientist who adopts the idealizational method in practice but not in theory (i.e. has no epistemological awareness of it and even interprets it incorrectly);

$\mathbf{a}_4$) the scientist who does not adopt the idealizational method in practice but does so in theory (perhaps with reference to some other discipline or field of knowledge rather than science).

As far as philosophers of science are concerned, we can have the following cases:

$\mathbf{b}_1$) the philosopher who does not recognise the practice of the idealizational method in mature scientific research, or develop a theory about it;

$\mathbf{b}_2$) the philosopher who recognizes the practice of the idealizational method in mature scientific research, but does not develop a theory about it (thus confining himself to the recognition of its existence and explicitly embraces a scientific methodology that has nothing to do with idealization);

$\mathbf{b}_3$) the philosopher who recognises the practice of the idealisational method in mature scientific research and develops a theory about it (i.e. he elaborates a complete scientific methodology based on this method, thus distinguishing it from other methodologies of an inductivistic nature);

$\mathbf{b}_4$) the philosopher who does not recognize the practice of the idealizational method in mature scientific research, but does develop a theory about it (i.e. he elaborates a philosophical theory which contemplates it as being typical of some other discipline, often as an alternative to or even polemical towards scientific procedure interpreted according to the concepts typical of inductivism).

The historical reconstruction we have made so far clearly allows us to locate the various thinkers we have mentioned in one of these various categories. There is therefore no doubt - if the historical reconstruction is correct - that Aristotle can be placed in position $\mathbf{b}_1$, whereas $\mathbf{a}_2$ is more suitable for Galileo. Newton, who applied the idealisational procedure but placed in the framework of an inductivistic perspective, interpreting it as classical abstraction, is a case which comes under $\mathbf{b}_3$. At the same time, the whole empiricist tradition which goes from Locke to the Circle of Vienna can be interpreted as a case of $\mathbf{b}_1$, while the philosophical outsiders belong either to $\mathbf{b}_2$ (if they did not explicitly develop a theory about the idealizational method but encapsulated it in a generically inductivistic methodology, as in the case of Twardowski, Łukasiewicz, Kotarbiński and Ajdukiewicz), or to $\mathbf{b}_3$ (as in the case of philosophers such as Marx, Cassirer, Vaihinger, Husserl, Czeżowski, the Poznań School and the more recent developments in contemporary epistemology: Niiniluoto, Cartwright, Suppe, Dilworth, Ludwig etc.). Finally we can also find in $\mathbf{b}_4$ a place for thinkers like Znaniecki and Andrzej Malewski (1951), who contrasted the inductivistic approach to natural sciences typical of the epistemological mainstream of this century; or Plato and Hegel, as they did not recognize the idealizational nature of scientific research, ascribing it a higher form of philosophical speculation and thus rendering it superior to the simply "doxical" forms of scientific knowledge. This explains why both felt it was necessary to re-interpret science: eliminating the "mystic shell" of their "re-interpretation" it would be possibile to extract its rational kernel, i.e. its methodological contents, in order to apply it to the procedures typical of scientific knowledge: this is what was done by both Galileo and Marx (Coniglione 1990).

It is only with mature science, whose threshold was crossed by the work of Galileo, that a new scientific methodology based on idealizational procedures and the systematic elaboration of ideal models was introduced; and it is only in the last few decades that epistemology has devised suitable conceptual tools for this change in scientific practice. As Hegel would have said, Athena's owl always takes flight as twilight descends. And twilight, at the end of a millennium, seems to be the destiny of contemporary epistemology, whose death has been announced by many (Williams 1992).

This, however, is for posterity to decide. The purpose of this essay was only to emphasize and clarify certain points that are important for a better understanding of the historical events that have accompanied philosophical thought on scientific knowledge, that is: (a) that the prevalent way in which abstraction has been viewed up to the present in the history of thought and epistemology can be traced back to Aristotle; (b) that with the birth of modern science a new way of conceiving abstraction came to the fore; better characterized by the name of idealization, it was what scientists actually did in their scientific practice; (c) that, however, on account of the influence of empirical and inductivistic philosophy, scientists have often not possessed sufficient methodological awareness of this new way of viewing abstraction; (d) that this new concept has often been expressed by philosophers working outside the mainstream of contemporary epistemology or even with a marked anti-scientific intent; and (e) that renewed interest in the theme of idealization can be said to have arisen in the last few decades and that a great contribution in this direction was made by the so-called Poznań School of methodology, the genesis of which in the context of Polish philosophy we have attempted to outline.

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