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Closed Timelike Curves and Time Travel: Dispelling the Myth

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Gödel's contention that closed timelike curves (CTC's) are a necessary consequence of the Einstein equations for his metric is challenged. It is seen that the imposition of periodicity in a timelike coordinate is the actual source of CTC's rather than the physics of general relativity. This conclusion is supported by the creation of Gödel-like CTC's in flat space by the correct choice of coordinate system and identifications. Thus, the indications are that the notion of a time machine remains exclusively an aspect of science fiction fantasy. The element of the identification of spacetime points is also seen to be the essential factor in the modern creation of CTC's in the Gott model of moving cosmic strings.

KEY WORDS: closed timelike curves; Gödel; Gott spacetimes.

1. INTRODUCTION

In an interesting recent paper, Ozsváth and Schucking[(1)] describe Gödel's excitement in 1949 upon learning that in his new cosmological solution of the Einstein equations, one has the ability to "travel into the past". They report on his lecturing at the IAS in Princeton on the subject with the attendance of such luminaries as Einstein, Oppenheimer and Chandrasekhar. The Gödel solution[(2)] was followed in 1956 by Kundt's calculation of its geodesics.[(3)] Evidently, Chandrasekhar was influenced by the Gödel presentation as he and Wright[(4)] independently re-calculated the geodesics. They did not find the evidence for travel into the past, what one now refers to as the presence of "closed timelike curves" (CTC's) in the geodesics they calculated, and so declared as incorrect, the Gödel claim. A CTC

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is a spacetime curve that is always future-directed, proceeding into the forward lightcone and finally reaching the point where it re-connects with the spacetime point of an earlier event in its history. In 1970, Stein[(5)] rose to Gödel's defence, noting that Gödel had never claimed that his CTC's were geodesics, suggesting that the possibility of time travel was still viable. There are interesting aspects to this geodesic-non-geodesic issue that we will discuss later.

Hawking and Ellis[(6)] ignited some considerable interest in the Gödel metric with the publication in their book of the spacelike, null and timelike Gödel curves illustrated in a diagram. Many authors subsequently entered into studies and speculations regarding CTC's, time machines and related exotica (see, e.g., Refs. 7–9). These often went beyond the Gödel metric with claims that CTC's were present in a variety of other spacetimes. Some have even taken the CTC notion so seriously as to propose experiments to search for the presence of CTC's in nature. Clearly there are serious problems associated with CTC's in that if one can travel into the past, one could presumably affect the past in some new manner leading to logical contradictions. As well, there is the issue of entropy flow as one would have to face the violation of the second law of thermodynamics, one of the holy grails of physics.

While it has long been recognized that one could trivially create CTC's in flat space by the identification of spacetime points, curiously no one to this point has ever connected this clearly artificial purely mathematical phenomenon with the supposedly physical CTC's of the Gödel spacetime and the other claimed sources. It is our contention that the connection is very direct indeed, that *the CTC's of Gödel and others simply follow from the identification of spacetime points, that they are in effect man-made rather than the consequences of exotic gravity via general relativity.*

2. CREATING CLOSED TIMELIKE CURVES

As an example of the transparently trivial variety of CTC, consider tracing the path of a circle in flat space. One begins at some specified time, say 1 PM at $\theta = 0$ and arrives back at the same spatial point $\theta = 2\pi$ (these two angles being identified) at a later time, let us say 2 PM. One can simply identify not only the spatial coordinate θ at the two extremities as one does automatically but also the time coordinate at the extremities 1 PM and 2 PM which one would normally never do. However if one does the latter as well as the former, a CTC is produced. Normally, spatial points are identified upon such a cyclical path in conformity with our physical experience. Time points are not identified because our physical experience denies it. The issue concerning the Gödel metric is the presently held belief that there is an underlying physical justification to realize a CTC in this case.

Let us consider the Gödel spacetime. It describes a type of rotating universe with no expansion, and its metric is a particular example of the generic class given by Bonnor[10]

$$ds^2 = -f^{-1}[e^{\nu}(dz^2 + dr^2) + r^2 d\phi^2] + f(d\bar{t} - w d\phi)^2, \tag{1}$$

where f, ν and w are functions of r and z with the coordinates having the ranges

$$-\infty < z < \infty, \quad 0 \leqslant r, \quad 0 \leqslant \phi \leqslant 2\pi, \quad -\infty < \bar{t} < \infty \tag{2}$$

and with $\phi = 0$ and $\phi = 2\pi$ being identified as usual. The essential factor leading to the CTC is the following: the metric component

$$g_{\phi\phi} = -f^{-1}(r^2 - f^2 w^2) \tag{3}$$

changes sign at the point where $f^2 w^2 = r^2$ and hence ϕ becomes a timelike coordinate for

$$f^2 w^2 > r^2. \tag{4}$$

In this case, the spacetime curve

$$\bar{t} = \bar{t}_0, \qquad r = r_0, \qquad \phi = \phi, \qquad z = z_0 \tag{5}$$

with z_0, r_0, $\bar{t}_0$ being constants has been created as a CTC as a result of the now-timelike coordinate ϕ having $\phi = 0$ and $\phi = 2\pi$ *still being identified* as was the case when ϕ was spacelike. Clearly, there is no difficulty in deducing the nature of the curve for

$$f^2 w^2 < r^2 \tag{6}$$

as being one of a closed spacelike curve on a constant time slice. However, in case (4), the metric has *two* timelike coordinates $\bar{t}$ and ϕ. One coordinate $\bar{t}$ is held fixed while the other coordinate ϕ advances. Bizarre as it may appear, there is nothing mathematically wrong in coordinatizing a normal spacetime with more than one timelike coordinate (Synge[11] describes a situation where a normal spacetime is described with four timelike coordinates). While the mathematics allows this, it is the physical interpretation that is of concern. The interpretation becomes suspect when a timelike coordinate does not advance in the description of a timelike curve for which the physical proper time must necessarily advance. Moreover, it is essential to question the continuation of identifying the ϕ values of 0 and 2π when ϕ becomes a timelike coordinate. Clearly the identification is logical when ϕ is spacelike because this is our understanding of the azimuthal spatial symmetry that is our experience in nature. However, our experience with time is that it is non-periodic. While some might argue that continuity demands the identification when ϕ becomes timelike, there is in fact a *discontinuity* in the process of the transition, the abrupt change from spacelike to timelike. Hence this rationale is not acceptable.

It is of particular interest to consider flat spacetime in cylindrical polar coordinates

$$ds^2 = dt^2 - dr^2 - r^2 d\phi^2 - dz^2 \tag{7}$$

with the standard coordinate ranges and where $\phi = 0$ and $\phi = 2\pi$ are identified as usual, i.e.

$$(t, r, 0, z) = (t, r, 2\pi, z). \tag{8}$$

We retain the identification in ϕ for 0 and 2π as we effect the transformation

$$\bar{t} = t + a\phi, \qquad \bar{\phi} = \phi, \qquad \bar{r} = r, \qquad \bar{z} = z, \tag{9}$$

where a is a constant.

The metric becomes

$$ds^2 = d\bar{t}^2 - dr^2 - 2a\, d\bar{t}\, d\phi - (r^2 - a^2) d\phi^2 - dz^2. \tag{10}$$

This is the same form as in (1) but with constant values globally for f, w and ν. The usual approach is to consider (10) and identify in the following manner:

$$(\bar{t}, r, 0, z) = (\bar{t}, r, 2\pi, z). \tag{11}$$

The positive sign of the $g_{\phi\phi}$ component of (10) shows the character is timelike and the *imposed* closure characteristic of the ϕ coordinate given by (11) creates the closure of the curve. Note that this identification is not equivalent to (8). By the analysis of the lightcones, it is straightforward to develop the standard Fig. 1 depicting the transition from closed spacelike

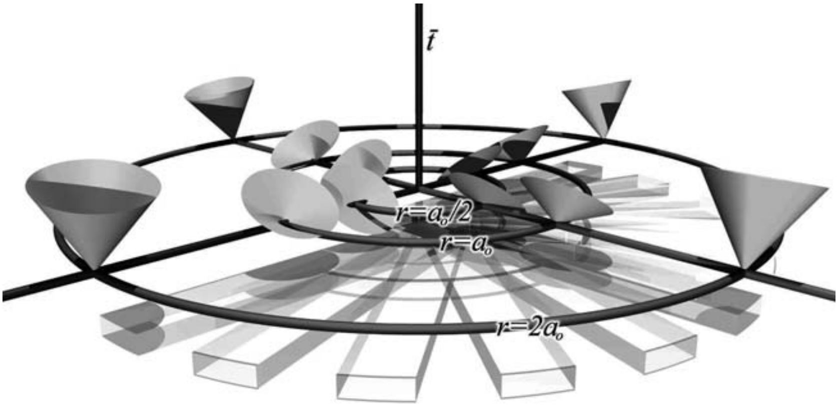

Fig. 1. Tipping light cones produce a CTC for $r < a$ in the $(\bar{t}, \phi)$ coordinates. Boxes at the bottom follow the curves for constant $\bar{t}$.

to null to timelike curves. The result is a diagram similar to that which displays the curves of the Gödel universe as shown in the standard texts (see e.g., Ref. 6). Figure 1 would indicate that for $r_0 < a$, there are closed timelike curves.

It is indeed seen that the ϕ-curve is a closed timelike curve for a fixed $r_0 < a$. The curve is always timelike, and hence the proper time flows monotonically and never becomes imaginary, i.e., the curve does not reverse and proceed into the past lightcone. If we transform the "cylindrical coordinates" $(\bar{t}, r, \phi, z)$ into the more familiar "cartesian coordinates" $(\bar{t}, \bar{x}, \bar{y}, \bar{z})$, we find that the ϕ curve follows the trajectory

$$\bar{t} = \bar{t}_0, \tag{12}$$

$$\bar{x} = r_0 \cos\phi, \tag{13}$$

$$\bar{y} = r_0 \sin\phi, \tag{14}$$

$$\bar{z} = z_0, \tag{15}$$

$$ds^2 > 0 \qquad \text{(time-like) } \forall\phi \in [0, 2\pi] \tag{16}$$

and this timelike curve returns to the original location in spacetime as a CTC.

However, we recall that the original spacetime, with metric (7) and standard coordinate ranges and identifications, is simply ordinary flat spacetime. The metric (10) was derived simply from a coordinate transformation. The essential element that led to the CTC in this flat space was the continued demand that ϕ exhibit closure even when it became a timelike coordinate.

3. ALTERNATIVE IDENTIFICATIONS AND THE GÖDEL SPACETIME

At this point, we present a more natural choice of identification for these curves. Consider the transformation of the lightcones of Fig. 1 back into the original fiducial (t, r, ϕ, z) coordinates. This is illustrated in Fig. 2. The reason for the apparent tilting of these lightcones in Fig. 1 with respect to the $(\bar{t}, \phi)$ coordinates as r varies is clear: The curves $t + a\phi = \bar{t}_0, r = r_0, z = z_0$ being helices, are inside/outside the lightcone for $r_0 < a$ and $r_0 > a$, respectively. However, removing the imposed closure in ϕ when it becomes a timelike coordinate eliminates the CTC characteristic.

In Gödel's[(2)] spacetime, the metric

$$ds^2 = a^2 \left(d\bar{t}^2 - d\bar{r}^2 + \frac{1}{2} e^{2\bar{r}} d\phi^{-2} + 2e^{\bar{r}} d\bar{t}\, d\bar{\phi} - d\bar{z}^2 \right) \tag{17}$$

is expressed with timelike cordinates $\bar{t}$,$\bar{\phi}$ globally. The underlying 3 + 1 character of the spacetime is hidden. However, it is advantageous to have the metric expressed in a form that displays the $3 + 1$ character explicitly. This is achieved with the transformation

$$\bar{t} = t + \frac{r\phi}{2}(1 - \ln r) + \frac{1}{2}\ln r, \tag{18}$$

$$\bar{r} = r\phi, \tag{19}$$

$$\bar{\phi} = -\frac{1}{2} e^{-r\phi} \ln r, \tag{20}$$

$$\bar{z} = z. \tag{21}$$

The metric becomes

$$\frac{ds^2}{a^2} = dt^2 - \left[\phi^2 + \frac{1}{8r^2}(r\phi \ln r - 1)^2\right] dr^2 - \left[\frac{3}{4}r^2 + \frac{1}{8}(r\ln r)^2\right] d\phi^2 - dz^2$$
$$-\frac{1}{4}\left(8r\phi + r\phi(\ln r)^2 - \ln r\right) dr d\phi + r dt d\phi. \tag{22}$$

It is to be noted that in the process, ϕ dependence in the metric appears.[1] Similar situations occur in other cases in general relativity such as with the Schwarzschild metric. The identification

[1] Because of the ϕ dependence in the metric, ϕ is not a suitable coordinate for periodic identification. The main advantage of this form over (17) is its explicit $3 + 1$ nature. In fact, if we were to force identification, it would result in a discontinuity of the metric which is inadmissable.

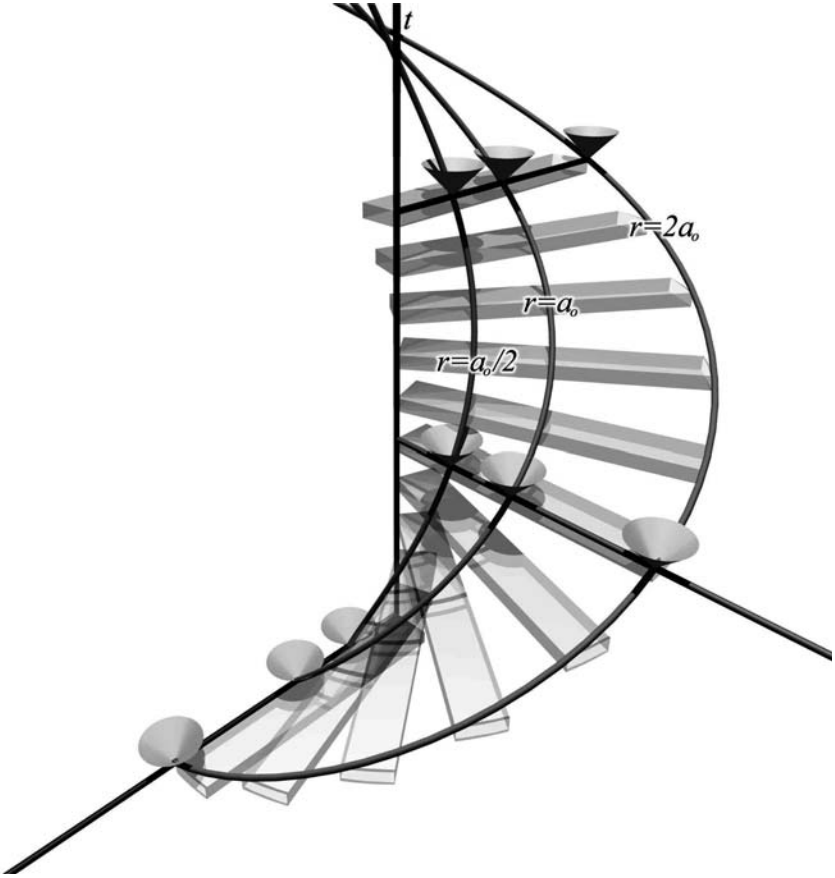

Fig. 2. The boxes in the figure are now at constant t. In the (t, ϕ) coordinate system, the spacelike, null and timelike curve are seen as a unified family of curves advancing monotonically in time t. Evolving curves never close in terms of t and so there are no CTC's with the periodic time restriction removed. The fixed $\bar{t} = \bar{t}_0$ surface is actually helicoidal in this case.

$$(\bar{t}, \bar{r}, 0, \bar{z}) = (\bar{t}, \bar{r}, 2\pi, \bar{z}) \tag{23}$$

is transformed to

$$(t, 1, \phi, z) = (t + 2\pi(1 - \phi)e^{\phi}, e^{-4\pi e^{\phi}}, \phi\, e^{4\pi e^{\phi}}, z) \tag{24}$$

and in this form, there is no suggestion of any identification of spacetime points that would yield closure in this explicitly $3 + 1$ coordinate system.

Returning to (1), consider the transformation for the curve where r, z (and hence f, ν and w) are held constant as

$$dt = d\bar{t} - w\,d\varphi,$$
$$d\Phi = \frac{w^2 f - r^2 f^{-1}}{2fw}d\varphi - d\bar{t}. \tag{25}$$

The line element for this curve has a particularly useful form:

$$ds^2 = \frac{f}{(w^2f^2 + r^2)^2}\Big((w^2f^2 - r^2)^2 dt^2 - 8f^2w^2r^2 d\Phi dt - 4f^2w^2r^2 d\Phi^2\Big). \tag{26}$$

Now, t is a timelike coordinate and Φ is a spacelike coordinate regardless of whether (4) or (6) holds. It is seen that with these coordinates, the azimuthal coordinate Φ is maintained explicitly as a truly angular coordinate throughout, unlike the case with the Gödel-like approach. There are no ambiguities of interpretation. Thus, these coordinates are particularly valuable. Holding $\bar{t}$ constant, say $\bar{t} = 0$ for simplicity, the curve is seen to have the equation in parametric form

$$t = -w\varphi,$$
$$\Phi = \frac{(w^2 f - r^2 f^{-1})\varphi}{2fw} \tag{27}$$

with parameter φ. Eliminating φ between the two equations, it is seen that Φ is simply a linear function of t with proportionality factor dependent upon the particular (r, z) chosen. At this point, we are on familiar ground. We naturally ascribe standard geometric (in this case cylindrical polar-like) character by identifying the spatial points for Φ (rather than for φ). However, as we are familiar from other analogous situations, we have no reason to ascribe periodicity to t. Time flows monotonically without repitition as it does in conventional flat space, as is our experience in nature, i.e., while the spatial points are retraced *ad infinitum*, they do so at successively later times. They do so here as in the previous examples in Fig. 2.

It is essential not to lose sight of the fact in the study of CTC's, our experience in nature has already been imposed prior to any analysis. We demand that physical curves always evolve into the *forward* lightcone, that time for a physical observer is monotonic in its evolution. We recognize that once the forward direction of time is set, it must always continue to flow in that vein, as is our experience. If we were to allow a reversal in the flow, the achievement of CTC's would be trivial. Similarly, it is equally natural to deny periodicity to the time coordinate as our experience with the real world is that while we readily re-visit spatial locations, we do not re-visit points in time. The essential confusion with Gödel is the notion that an angular coordinate, once set logically to have a certain periodicity when spacelike should by necessity maintain that periodicity when it becomes timelike. It is certainly a *choice* that can be made but there is nothing that makes it a necessity. Indeed, to adopt that choice is to force the realization of a CTC in a particular spacetime. However, the choice we would argue as more natural, is *not* to force the periodicity and this does not yield a CTC in this different spacetime. There is nothing in the field equations to guide us in one direction as opposed to the other. We would argue that the choice of a system of coordinates in which there are two timelike coordinates with one held fixed for a timelike curve is an unfortunate choice from the point of view of clarity. Our choice is to choose coordinates that maintain their timelike or spacelike character. In so doing, we recognize that the imposition of periodicity is a choice rather than a necessity.

Let us return to the issue of whether or not the CTC's of Gödel are geodesic. We would argue that the authors in Ref. 4 were correct in raising this point. To be a geodesic curve is to be traceable without any extraneous elements, to "fall freely". Had it been the case that the Gödel CTC's were geodesic, then one might have argued that the deviation from our normal experience would not be so radical. However, once it is seen that they are not geodesics, there is the immediate requirement for an agency to force the particular spacetime trajectory, i.e. a "time machine". This is more serious in that to recreate the conditions for true closure, the elements of the time machine must also follow closure in time. In so doing, there would be a reversal in entropy flow that further compounds the demands upon one's credulity.

4. GOTT'S CTC CREATION FROM MOVING COSMIC STRINGS

It is our contention that the essence of CTC creation stems from the process of identification of spacetime points, that it is a mathematical man-made choice rather than the result of general relativity and the solutions of the field equations. This applies to even some of the modern versions such as the example put forward by Gott.[9] In the Gott system, a CTC is constructed using two moving cosmic strings and the mathematics of Lorentz boosts. A pair of simple examples will illustrate its main feature and how the existence of such CTC's arises.

First, consider a flat 1+1 spacetime in which a strip $-1 < \bar{x} < 1$ is removed, i.e. the points identified are $(\bar{t}, -1)$ and $(\bar{t}, 1)$. For this example,

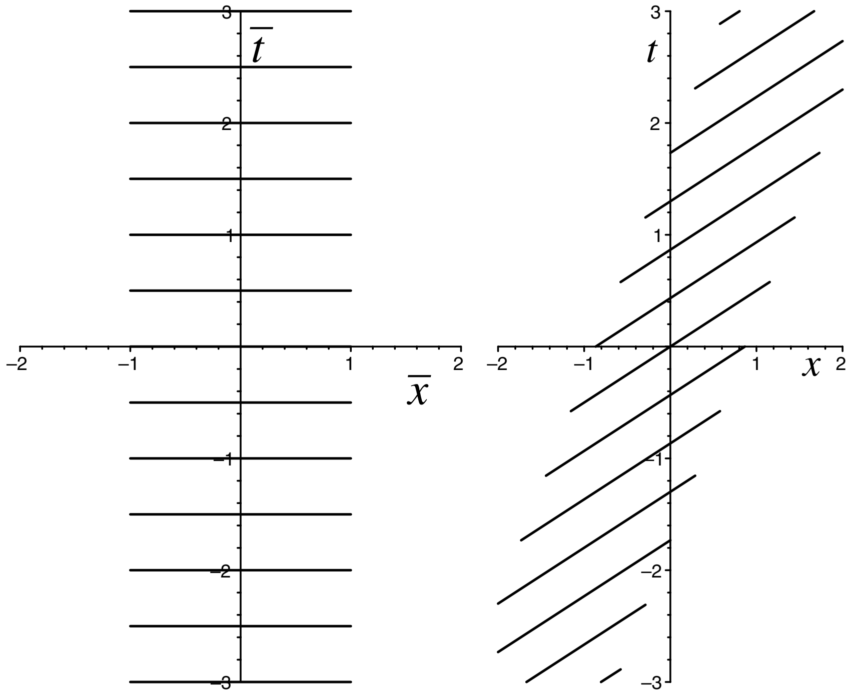

Fig. 3. In the left figure, the identifications after the removal of the strip $-1 < \bar{x} < 1$ are shown using horizontal line-segments. The right figure illustrates the same identification of points after the Lorentz boost.

we shall call the region $\bar{x} < -1$ the negative side and the $\bar{x} > 1$ the positive side. If one applies a Lorentz boost from $(\bar{t}, \bar{x})$ to (t, x) *before* identifying the points, the two edges of the cut as seen in the new coordinate system will "slip" as shown in Fig. 3.

It is seen in Fig. 4 that any object from the positive side, crossing the identified strip to the negative side will be displaced back in t value. In a sense, making the transition over the identified points e_1 and e_2 in this direction allows travel into the past. In spite of this, causality is not violated because any attempts to close the object's world line would require another crossing through the identified strip. However, traveling through this strip in the opposite direction would have the t value *increased* by the same amount. Thus, no events in the future of e_2 coincide with e_1, i.e. it is impossible to return to the initial event via a timelike trajectory.

On the other hand, if the identification were made before the Lorentz boost was applied, the spacetime would be continuous. All events would

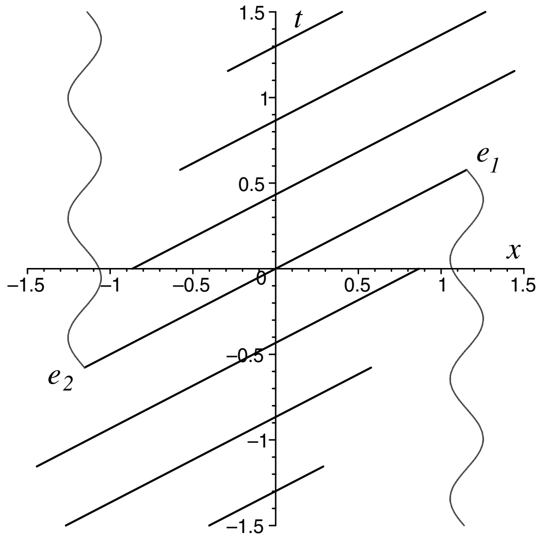

Fig. 4. This shows a possible worldline of a massive object as it crosses the identified strip. Events e_1 and e_2 are identified. In this coordinate, the t-value of e_2 is less than that of e_1 so one can say that e_2 occurred before e_1.

be mapped smoothly from one coordinate to another without any jump in "time". This will be the key feature to be employed in the next example. The choice of appropriate order of identification will be discussed later.

Next, we consider a 2+1 system $(\bar{t}, \bar{x}, \bar{y})$, as opposed to the 1+1 dimensional system in the previous example. Instead of having one strip $-1 < \bar{x} < 1, -\infty < \bar{t} < \infty$ removed for $\bar{y}=0$, we will consider two strips removed: the "front" strip being $-1 < \bar{x} < 1, -\infty < \bar{t} < \infty, \bar{y}=y_1$, where y_1 is a positive constant and the "back" strip being $-1 < \bar{x} < 1, -\infty < \bar{t} < \infty, \bar{y}=-y_1$. Consider a Lorentz boost with velocity $+\beta_s$ in the positive $\bar{x}$-direction for half of the space $\bar{y}\equiv y \geq 0$ and another boost in the negative $\bar{x}$-direction for the other half, $y < 0$. This is possible because the two half-spaces are flat and hence, they can be stitched together.[9]

That there is a violation of causality can be seen as follows: a traveller starts at an event E_1 on the right side of the front strip (with the front being $y = y_1$) and crosses over the $+\beta_s$ Lorentz-boosted strip to travel "back in time" to event E_2 as seen in Fig. 5. With y_1 sufficiently small, he could proceed via a timelike path to the back strip ($y = -y_1$) at event E_3. At this point, he crosses over the $-\beta_s$ Lorentz-boosted strip to event E_4. From E_4, he could follow a timelike trajectory to return to his original

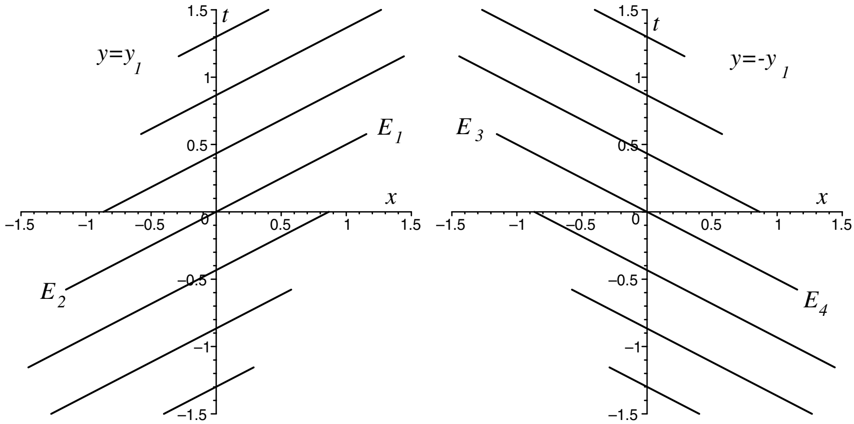

Fig. 5. These are two Lorentz-boosted strips in opposite directions: the front being $y = y_1$ and the back being $y = -y_1$. Points E_1 and E_2 are identified as with points E_3 and E_4.

position in space and time at event E_1. Thus, his worldline is a CTC. This illustration captures the essential mechanism of the Gott-produced CTC.[(9)]

The primary difference between this scenerio and that of Gott is in the choice of coordinate system. Gott chose a coordinate system where each cosmic string is at rest (in the barred coordinates) and thus E_1, E_2, E_3, and E_4 and all intermediate events are simultaneous in that system. Either system of coordinates is acceptable.

The presence or absence of a CTC rests in identifying respectively the points *after* or *before* the Lorentz boost is applied. Gott chose the first form of identification and hence he realized a CTC. The question arises as to which approach is the more natural one in dealing with such a system. Regardless of the viability or non-viability of cosmic strings, one would expect continuity and axial symmetry of the spacetime around one cosmic string at rest, i.e. the "wedge" that is removed in the construction of a cosmic string from non-singular flat spacetime should not be detectable. This is just as in the case of an axially symmetric cone which does not display the wedge that one would create to paste it together into a cone from the original plane. As a string is Lorentz-boosted, this continuity should be maintained even though the axial symmetry is lost. On the other hand, if there *were* a discontinuity,[2] there would be no preferred location for it to

[2] One might suspect that the discontinuity is only associated with the coordinate system (a mathematical labelling) and have no physical implications. However, consider the physics of fluid-flow over a plane transformed into a cone via identifications: one either applies the physics *before* or *after* identifying. The former will generally result in a physical discontinuity of the flow.

appear because of the ultimate axial symmetry. Thus, the more reasonable scenerio from the point of view of physics (at least to the extent that one is inclined to regard these constructs as physical) is in the identification of points *before* the Lorentz boost. In so doing, one rules out the closed timelike curve as envisaged by Gott.

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