Argumentations and Logic

JOHN CORCORAN

State University of New York at Buffalo
Buffalo, New York 14260, U.S.A.

ABSTRACT: Argumentations are at the heart of the deductive and the hypothetico-deductive methods, which are involved in attempts to reduce currently open problems to problems already solved. These two methods span the entire spectrum of problem-oriented reasoning from the simplest and most practical to the most complex and most theoretical, thereby uniting all objective thought whether ancient or contemporary, whether humanistic or scientific, whether normative or descriptive, whether concrete or abstract. Analysis, synthesis, evaluation, and function of argumentations are described. Perennial philosophic problems, both epistemic and ontic, related to argumentations are put in perspective. So much of what has been regarded as logic is seen to be involved in the study of argumentations that logic may be usefully defined as the systematic study of argumentations, which is virtually identical to the quest of objective understanding of objectivity.

KEY WORDS: hypothesis, theorem, argumentation, proof, deduction, premise-conclusion argument, valid, inference, implication, epistemic, ontic, cogent, fallacious, paradox, formal, validation.

Argumentation is one of the activities characteristic of rational life, in the humblest and in the most exalted senses of 'rational'. The use of reason is inseparable from argumentation. Argumentations are often involved in assenting, in dissenting and in doubting. Whether we are making up our minds or changing our minds, argumentation is often present. Argumentation is so constantly our companion that conscious effort is required even to notice it — unless there is a dysfunction. And once we have noticed it we learn to describe it and to analyse it only with great difficulty.

Some propositions are known to be true and some are known to be false. But the propositions that are important to us often include hypotheses, propositions which are neither known to be true nor known to be false. Many hypotheses have gained international attention: The Goldbach Hypothesis, The Continuum Hypothesis, The Sapir-Whorf Hypothesis, and so on. But many hypotheses concern mundane matters of limited interest. Every hypothesis is either actually true or actually false. But no hypothesis is either known to be true or known to be false — by the person for whom it is a hypothesis. Some propositions are hypotheses for some persons but not for others. For some persons the proposition that every true proposition can be known to be true is a hypothesis. For some persons this proposition had been a hypothesis but now has been settled. Some persons who now believe that they have settled the hypothesis will

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later realize that they had not settled it at all. Some propositions thought to be known to be true are not really known to be true. In fact, some of them are false. Some propositions thought to be known to be false are not really known to be false. In fact, some of them are true.

Hypotheses excite our interest and curiosity. We understand the hypothesis. We know that it is either true or false but we do not know which. How will it turn out? How can we settle the matter? Can we settle it on the basis of what we already know or do we need new information?

1. *Settling Hypotheses by Argumentations.* Argumentation is involved in settling hypotheses on the basis of what we already know. Every argumentation that deduces the hypothesis from premises already known to be true proves the hypothesis to be true. This, of course, is the *deductive method* of settling a hypothesis. Every argumentation that deduces a proposition known to be false from the hypothesis alone or from the hypothesis augmented by premises known to be true proves the hypothesis to be false. This, of course, is the *hypothetico-deductive method* of settling a hypothesis. Using the deductive method the hypothesis is proved to be true. Using the hypothetico-deductive method the hypothesis is proved to be false. But not every attempt to use one of these methods is successful. The methods themselves are reliable, but in certain situations neither can be applied and sometimes apparent applications are not successful because of mistakes made by the person attempting the application. It is important to distinguish the methods themselves from attempts to apply them.

The names Aristotle, Euclid, Archimedes, Newton, Hilbert and Tarski often come to mind in connection with the deductive method. The names Socrates, Galileo, Saccheri, Duhem and Popper often come to mind in connection with the hypothetico-deductive method. But the prestige that these two methods enjoy should not blind us to the fact that they were in use long before the Golden Age of Greek Learning and that they are in constant use today by people all over the world — many of whom have never heard of them.

The deductive method is reliable because every hypothesis deduced from propositions known to be true is itself true. Ideally speaking, every hypothesis known to be deduced from premises known to be true is itself known to be true. The hypothetico-deductive method is reliable because every hypothesis from which a false proposition has been deduced is itself false. Ideally speaking, a hypothesis is known to be false once a proposition known to be false has been deduced from it alone or from it taken together with propositions already known to be true. People have relied on these methods long before they were able to explain or, even to ask, why the methods are reliable.

In the ideal case, application of the deductive method begins after three things are available: the hypothesis itself, a set of premises known to be true and a conjecture that the hypothesis is true. The problem is to find a chain of reasoning that deduces the hypothesis from the premises. This problem is, of course, not always solvable. In the ideal case, application of the hypothetico-deductive method begins after four things are available: the hypothesis itself, a set of premises known to be true, a proposition known to be false, and a conjecture that the hypothesis is false. The problem is to find a chain of reasoning that deduces the proposition known to be false from the hypothesis augmented by the other premises. This problem, of course, is not always solvable.

In successful applications of the deductive method, the chain of reasoning is often the largest part of the argumentation; the chain of reasoning is often reported in a discourse-text several times as long as the text that expresses the premises and the hypothesis. Likewise, *mutatis mutandis*, in successful applications of the hypothetico-deductive method.

In a sense before we resort to these methods we have tacitly raised a question: is the problem of determining the truth-value of the hypothesis reducible to similar problems which have already been solved? Argumentation is involved in reducing new problems to old ones that have been solved. Argumentation connects present challenges with past successes. The deductive and hypothetico-deductive methods make it possible to reduce a new problem of a non-logical character to a new problem of a logical character, viz. the problem of whether a certain kind of argumentation exists.

In practice application of either method often starts with the hypothesis alone. Then a conjecture is made. The conjecture that the hypothesis is true leads to a search for a set of propositions known to be true and sufficient to imply the hypothesis itself. This search typically uncovers various sets of propositions. In each case we need to check whether the propositions are known to be true and whether they can be used as a premise-set from which the original hypothesis can be deduced. Thus we are led in typical cases to further hypotheses and to further applications of the deductive method — and in some cases to applications of other methods. Not every problem that we encounter is reducible to problems that we have already solved.

It happens sometimes that we find a proposition that implies the hypothesis and we actually construct a chain of reasoning establishing this implication only to discover that this implying proposition can not be used because it is not known to be true. In some cases we realize that the proposition implying the hypothesis is actually known to be false. Finding a proposition which is false and which implies the hypothesis does not by itself show that the conjecture of truth was wrong. Thinking otherwise is the *fallacy of falsified implicants*. Every true proposition is implied by infinitely many false propositions. Thus the network of implications terminating with a given hypothesis which happens to be true can be expected to involve many blind alleys.

In practice, the conjecture that the hypothesis is false, which involves us in the hypothetico-deductive method, leads us to a search for consequences of the hypothesis rather than implicants of it. The search continues for consequences of consequences, and so on, until we find a consequence known to be false. Finding a consequence that turns out to be known to be true does not by itself show that the hypothesis is true, i.e. that the conjecture of falsehood is wrong. Thinking otherwise is the *fallacy of verified consequences.* Every false proposition implies infinitely many true propositions. Thus the network of implications originating with a given hypothesis which happens to be false can be expected to involve many blind alleys.

Now, as the search for consequences of consequences of the hypothesis progresses we attempt to augment the chain (or chains) of reasoning with additional premises known to be true. This involves further hypotheses, further conjectures and further applications of the two methods and perhaps applications of other methods as well.

Thus an attempted application of either of these methods tends to generate an array of new hypotheses, new conjectures, new argumentations, increased awareness of the scope and limits of present knowledge, and increased awareness of the interrelatedness among propositions — whether known to be true, known to be false, or not known either way. The result of a successful application of either method is an argumentation that settles a hypothesis; in one case a proof that the hypothesis is true, i.e. a *proof* of the hypothesis; in the other case a proof that the hypothesis is false, i.e. a *disproof* of the hypothesis. Such results are clearly within the realm of justification, *apodictics*, as opposed to the realm of discovery, *heuristics*. Neither the deductive method nor the hypothetico-deductive method is a method for discovering hypotheses in the first place. Neither method is a method for discovering chains of reasoning. There are various heuristics for discovering hypotheses — perhaps the most familiar is the method of analogy. There are various heuristics for discovering chains of reasoning. Perhaps the most familiar is the so-called method of analysis which involves imagining that the desired chain of reasoning has already been constructed.

It is clear that being true is one thing and being known to be true is another. Likewise, *mutatis mutandis* for being false and being known to be false. Truth and falsehood is a matter of *ontics*. Knowledge of truth and falsehood is a matter of *epistemics*. Proof is one criterion of truth and disproof is one criterion of falsehood. Argumentations are therefore at the heart of at least some of the criteria of truth and falsehood. It is also clear that direct experience of the subject-matter is involved in other criteria of truth and falsehood, and indeed that proof and disproof presuppose such other criteria. Nevertheless, the search for an argumentation that settles a hypothesis is an attempt to apply criteria of truth and falsehood.

Sometimes the search for an argumentation that settles a hypothesis can lead to a surprising and disconcerting result. For example, sometimes we can think that we have deduced a conclusion thought to be false from a hypothesis augmented by premises thought to be true and then discover that the hypothesis itself played no role in the reasoning. This means that we have arrived at an argumentation that seems to deduce a conclusion thought to be false from premises thought to be true.

An argumentation that seems to deduce a conclusion thought to be false from premises thought to be true is called a *paradox*. The phrases 'seems to deduce', 'thought to be false' and 'thought to be true' make tacit elliptical reference to a participant. One and the same argumentation that is a paradox for one participant may seem to another participant to be a proof that its conclusion is true and it may seem to another participant to be a proof that a certain one of its premises is false and to yet another participant it may seem to involve fallacious reasoning. Perhaps the most important point here is that an argumentation that actually is a paradox for a given participant at a given time can fail to be a paradox for the same participant at a later time. It goes without saying that the converse is also true, viz. that an argumentation that is not a paradox for a given participant at one time may become a paradox for the same participant at a later time.

The process of converting a paradox into a non-paradox is called *solving* the paradox or *resolving* the paradox. People feel uncomfortable when they have a paradox. The reason for this is not hard to find: no false proposition is deducible from true propositions. In a paradox at least one of three unhappy situations occurs: either the conclusion which was thought to be false is really true, or one of the premises thought to be true is really false, or the chain of reasoning that was thought to deduce the conclusion from the premises does not really do so.

Discovery of a paradox virtually forces critical examination of beliefs and of reasoning. After the initial shock there is a period, sometimes brief and sometimes not so brief, wherein at least four propositions which had been beliefs get demoted to mere hypotheses: the proposition that the conclusion is false, the proposition that the premises are all true, the proposition that the chain of reasoning deduces the conclusion from the premises and the proposition that the conclusion is actually implied by the premises. The last two hypotheses raise two issues that concern us below. One: by what criteria do we determine a chain of reasoning to be cogent or fallacious? Two: by what criteria do we determine that a conclusion does or does not follow from given premises?

It is clear from what has been said that some argumentations are settling and that some are unsettling. Some are used to expand the scope of what we think we know. Some are used to reveal that we do not really know all that we think that we know. Some lead us to assent. Some lead us to dissent. Some lead us to doubt.

2. *Proofs: Argumentations Productive of Knowledge.* Every proof is an argumentation that proves its conclusion to be true. Every proposition proved to be true is known to be true by those persons who have proved it to be true. Every proposition known to be true is really true. There is no such thing as "a proof having a false conclusion" or "a proof whose conclusion is not known to be true". Just as "known to be true" makes tacit reference to a knower, "proof" and "proved to be true" make tacit reference to a participant or to a community of participants.

One well-known proof of the irrationality of the square-root of two uses as a premise the proposition that every number whose square is even is itself even, which of course is easy to prove. But no argumentation that uses this proposition as a premise is a proof to a person who does not know this proposition to be true. To such a person, such argumentations beg the question, they use an unwarranted premise.

Every premise of a proof is known to be true by those persons for whom that proof is conclusive. Every argumentation having a premise not known to be true by a given person begs the question to that person. The fallacy of *begging the question*, also called *unwarranted premise*, is using in an intended proof a premise not known to be true by the intended audience. Since no false proposition is known to be true, every argumentation having a false premise begs the question. Although every false proposition implies infinitely many true propositions, no false proposition "proves" even one true proposition. More precisely, no false proposition is a premise of a proof.

The terms 'proof', 'proved to be true' and even 'question-begging' make tacit reference not only to a particular audience but also to a particular time. An argumentation that begs the question for a given person at a given time may well be a proof for that person at a later time if that person gained new knowledge in the meantime. In fact, as we have already seen, one heuristic for applying the deductive method involves construction of one question-begging argumentation after another all fanning out from the original hypothesis until all of the ultimate premises are known to be true. When this happens the formerly question-begging argumentations become proofs.

Aristotle and others including Pascal and Frege seem to think that every premise of a genuine proof is either a "first principle" or else is a conclusion in an array of proofs whose ultimate premises are "first principles". A "first principle" is a proposition which is verifiable without proof by every knower. In fact, Aristotle, Pascal and Frege seem to think that it is always possible to begin with a given proposition known to be true and to work backward to find the "first principles" which form "the" ultimate premises for the given proposition. The hypothesis that such "first principles" exist has not been disproved but evidence in favor of it seems flimsy. The fact that demonstrative or apodictic knowledge presupposes prior knowledge is clear enough, but the idea that the ultimately presupposed prior knowledge is universally verifiable seems implausible. Likewise, it seems implausible to think that some verifiable propositions can not be deduced from any other verifiable propositions. The issue of the nature of the prior knowledge is not important in this essay. What is important, however, is that in order for an argumentation to be a proof to a given audience it is not necessary for its premises to be known to be true by persons not in the given audience. For example, it is common for one person to present to others an argumentation that is a proof for the presenter but which begs questions for the others.

Russell, Tarski, Popper and others seem to think that responsible and well-grounded belief is obtainable by humans but that knowledge in the strict sense is beyond our capacity. If no proposition is known to be true, then every argumentation begs the question to everyone and there is no such thing as a proof. Those who believe that knowledge is impossible describe an analogue of the deductive method as a means of increasing responsible certainty rather than as a means of gaining knowledge. It is true that we can and do increase our degree of confidence in a given proposition by deducing it from propositions that we are more confident of. This may be called *the probabilistic deductive method*. This method is rather more complicated than the deductive method and discussion of it is beyond the scope of this essay.

The hypothesis that there is no knowledge in the strict sense can not be proved and the evidence in favor of it seems flimsy. If it is true, then knowledge is an ideal to which we strive but which we never attain. If there is no knowledge, then proof is also an ideal to which we strive but which we never attain. The present essay assumes the working hypothesis that knowledge in the strict sense is difficult to obtain but is nevertheless abundant. However, not every proposition thought to be known to be true is really known to be true. There are ample grounds for caution in these matters.

This essay steers a middle course between two extreme views. On the one hand, it avoids *foundationalism*, the view attributed above to Aristotle, Pascal and Frege which presupposes universally knowable "first principles" as the ultimate premises for all propositions proved to be true. On the other hand, it avoids *probabilism*, the view attributed above to Russell, Tarski and Popper which presupposes the impossibility of knowledge. In practice this essay does not conflict with much of what has been written by foundationalists or probabilists. The foundationalists rarely exhibit their "first principles" and they never prove that their alleged "first principles" are really "ultimate" or really universally knowable. The probabilists often treat responsible and well-grounded belief as if it were knowledge.

Not every argumentation whose premises are all known to be true is a proof. Every such argumentation that is not a proof involves a fallacious chain of reasoning, a chain of reasoning which does not deduce the conclusion of the argumentation from its premises. In some cases, the chain of reasoning is *fallacious per se*, for example, by virtue of logical errors. The literature of mathematics contains many instances of argumentations whose chains of reasoning are fallacious in themselves but which were later corrected. In these cases, the premises were known to be true and the conclusion was actually deducible from the premise-set but the chain of reasoning was inadequate. In some cases, the chain of reasoning is *fallacious in context* (*sc.* of the argumentation at issue), for example by virtue of using premises not among the premises of the argumentation or by virtue of reaching a conclusion other than the conclusion of the argumentation.

An argumentation is said to involve the *fallacy of premise-smuggling* (also called *hidden premise* and *suppressed premise*) if its chain of reasoning uses as premises propositions not among the premises of the argumentation. As the result of work by many thinkers, including Archimedes, Proclus, Leibniz, and Hilbert, it is now believed that many of Euclid's argumentations involve smuggled premises. Russell points out that when a "proof" seems to depend on subject-matter otherwise than in regard to the truth of the premises it is because the premises have not all been explicitly stated. Beth has observed that our modern axiom systems have been constructed by studying "proofs" of the recognized theorems to detect hidden premises.

Beth's observation illustrates the fact that the fallacy of premise-smuggling often admits of correction: an argumentation involving a given smuggled-premise can be transformed into a new argumentation wherein the given premise is not smuggled — simply by adding the formerly smuggled premise to the premise-set of the old argumentation to construct the premise-set of the new argumentation. However, if the formerly smuggled premise is not known to be true, then correction of premise-smuggling introduces question-begging. Unfortunately, it is all too common that the smuggled-premise turns out to be false, or at least not known to be true. In some cases, premise-smuggling can be corrected by adding to the chain of reasoning a subchain that deduces the smuggled-premise from the premises of the argumentation.

An argumentation is said to involve the *fallacy of wrong conclusion* (also called *ignoratio elenchi*) if its chain of reasoning reaches a conclusion other than that of the argumentation. For example, Veblen gave a long and involved argumentation with the conclusion that Euclidean geometry is reducible to affine geometry but the conclusion that he actually reached in his chain of reasoning was not this proposition, which by the way is false, but another one which is true. This fallacy is relatively common with thinkers who do not bother to check their actual results with their stated goals. The fallacy of wrong conclusion can always be repaired simply by changing the conclusion of the argumentation so that it is the same as the conclusion of the chain of reasoning. This is often useless. It would be better, when possible, to lengthen the chain of reasoning so that the conclusion of the argumentation is actually reached in the chain of reasoning.

A chain of reasoning is said to be *cogent per se* if the conclusion that it reaches is actually shown to follow from the premises that it uses. A chain of reasoning is said to be *cogent in context* of an argumentation if it is cogent *per se* and its conclusion and premises are respectively the conclusion and among the premises of the argumentation. It is possible, as an exercise, to construct from two proofs having different premise-sets and different conclusions two argumentations whose chains of reasoning are both cogent *per se* but both fallacious in context. Just interchange the two chains of reasoning.

In order to be a proof it is necessary and sufficient that an argumentation have premises all known to be true and that its chain of reasoning be cogent in context. Thus, if an argumentation is not a proof, then either it begs the question or its chain of reasoning is fallacious in context. If its chain of reasoning is fallacious in context then either it is fallacious *per se* (and hence involves a gap or logical error in the narrow sense) or else it smuggles a premise or it reaches the wrong conclusion. *Critical evaluation of an argumentation to determine whether it is a proof for a given person reduces to two basic issues: are the premises known to be true by the given person? And does the chain of reasoning deduce the conclusion from the premise-set for the given person?*

3. *Deductions: Cogent Argumentations.* Every proposition that implies a false proposition is false. We have already seen that a hypothesis can be known to be false by deducing from it alone, or from it augmented by a set of propositions known to be true, a proposition known to be false. The fact that it is possible to make deductions based on premises not known to be true is central to our intellectual life. This was already recognized by Socrates and then articulated in some detail by Aristotle. In the critical evaluation of an argumentation to determine whether it is a proof we often find that the chain of reasoning is cogent in context but that the premises are not all known to be true. We have already seen that such argumentations are nevertheless useful.

An argumentation whose chain of reasoning is cogent in context is itself said to be *cogent* and an argumentation whose chain of reasoning is not cogent in context is said to be *fallacious*. "Cogent in context", "cogent *per se*", "fallacious in context" and "fallacious *per se*" apply to discourses, or chains of reasoning, whereas "cogent" and "fallacious", *simpliciter*, apply to argumentations. An *argumentation* is a three-part system composed of a set of propositions called *the premise-set*, a single proposition called *the conclusion* and a discourse called *the chain of reasoning*.

The word 'argumentation' comes from a Latin verb meaning "to make clear". The Latin verb was itself derived from the noun for "silver". The word 'argument' would serve as well except for the fact that it is already used in logic in a sense having virtually no connection with making anything clear.

We have already seen that every proof is a cogent argumentation but that not every cogent argumentation is a proof. Every cogent argumentation that fails to be a proof for a given person begs the question for that person. The common noun '*deduction*' is often used very nearly in the sense of 'cogent argumentation'. Here we take it to be an exact synonym.

Every proof is a deduction but not every deduction is a proof. Every proof makes evident the truth of its conclusion and every deduction makes evident that its conclusion follows logically from its premise-set. Tarski and others use the terms 'proof' and 'deduction' very nearly as they are used here. Aristotle uses the terms 'demonstration' and 'perfected syllogism' to make this contrast.

Every deduction whose premises are all true has true conclusion. Every deduction whose conclusion is false has at least one false premise. No deduction has all true premises and false conclusion. As we have seen above in connection with the deductive method, not every deduction having true conclusion has all true premises. In connection with the hypothetico-deductive method we saw that not every deduction having a false premise has false conclusion. In contrast, every proof has all true premises and true conclusion.

Some probabilists, who hold that there are no proofs, still use the word 'proof' to indicate a deduction whose premises have been accepted as true on good grounds. But some probabilists use the word 'proof' as a synonym for 'deduction'. This leads to a cascade of absurd locutions such as: 'a false premise can be used to prove a true conclusion', 'a false proposition can be proved', 'some proofs prove false conclusions', etc. The distinction between proofs and deductions is more or less entrenched in ordinary learned discourse.

Every deduction is an argumentation whose conclusion is implied by its premise-set, but not every argumentation whose conclusion is implied by its premise-set is a deduction. In ideal applications of the deductive method we begin with a premise-set and a conclusion that follows but is not known to follow. The problem is to construct a chain of reasoning that shows the conclusion to be a consequence of the premise-set. It is obvious that not every attempt to construct such a chain of reasoning is successful. It often happens that a fallacious chain of reasoning is found in an argumentation whose conclusion actually follows from its premise set. The idea that an argumentation is cogent if its conclusion follows from its premise-set is a species of the *process/product fallacy*; thinking that a process must be correct if it results in a correct product. The point is familiar: it is possible to obtain correct results using incorrect procedures — either by making compensating errors or by some other means.

It is obvious that not every argumentation in the same form as a proof is itself a proof. For example, if we have a proof using mathematical induction as a premise and we uniformly replace *number* (*sc.* natural number) by *integer*, we have transformed the proof into an argumentation having a false premise. "Every set of integers containing zero and closed under successor contains every integer" is false. The set of natural numbers is a counterexample. This is the familiar point, alluded to in the Russell quote above, that proof depends on matter as well as on form. Some argumentations in the same form as a proof have false premises and thus beg the question. It is at the level of deduction that the principle of form enters logic.

Every two argumentations in the same logical form are both cogent or both fallacious. Every argumentation in the same form as a deduction is itself a deduction. Every argumentation in the same form as a fallacious argumentation is itself fallacious. No fallacious argumentation has the same form as a deduction. These are *principles of form for argumentations.* Hindsight enables us to find suggestions of these principles in Aristotle's writings but Aristotle did not attempt to articulate them.

The principle of form has been heralded as the basis for an important economy of thought in the deductive method. According to this principle each deduction may be regarded as a template for constructing an endless sequence of other deductions. The creative energy expended in the construction of one deduction serves as well for all others in the same form.

In order to use an already constructed deduction as a template for generating new deductions it is necessary to know how to transform a given argumentation into another argumentation having the same form. The simplest form-preserving transformation is the operation of substituting one new non-logical term for every occurrence of a given non-logical term. By 'new' here is meant "not already occurring in the argumentation operated on" and, of course, the semantic category of the new term must be the same as the one it replaces. For example, "number" can replace "integer" but it cannot replace "one", "even", "divides", "square-root", "plus", etc. The operation just described is called *one-new-term substitution.*

Every argumentation obtained from a given argumentation by a finite sequence of one-new-term substitutions is in the same logical form as the given argumentation and every argumentation in the same logical form as a given argumentation involving only finitely many non-logical terms is obtained from the given argumentation by a finite sequence of one-new-term substitutions. Extending this result to the case of argumentations involving infinitely many non-logical terms is a mere technicality.

The importance of the principle of form for argumentations in effecting an economy of thought is apt to be exaggerated. When we need to find a chain of reasoning to deduce a given conclusion from given premises, it may very well be easier to construct the chain of reasoning *ab initio* than to search through files of already constructed deductions. Be this as it may be, the principle of form has another use much more important than effecting economy of thought, viz. testing for fallacious argumentations.

Every argumentation whose premises are true and whose conclusion is false is fallacious and every argumentation in the same form as a fallacious argumentation is fallacious. Therefore, we can determine that a given argumentation is fallacious by transforming it into another argumentation whose premises are known to be true and whose conclusion is known to be false. This method can be used to determine conclusively the fallaciousness of a given argumentation but it does not, by itself, determine exactly where the chain of reasoning breaks down. In practice, it is often easy to find the mistake once it is known for certain that there is one.

4. *Arguments*: "*Hollow Argumentations*". The "core" of an argumentation is its chain of reasoning. In fact, the word *core* is a convenient acronym for 'Chain of Reasoning'. The "bounds" of an argumentation are its premise-set and its conclusion. The metaphor that an argumentation "begins" with its premise-set and "ends" with its conclusion is reflected in the etymology of the words 'premise' and 'conclusion'. It often happens in practice, however, that construction of an argumentation ends with the construction of the chain of reasoning and begins, not with the premise-set alone, but with the premise-set taken together with the conclusion. It is also common for a chain of reasoning to be generated before the premise-set and conclusion have been chosen. It even happens, as we have seen, that the process of generating an argumentation begins with the conclusion and works backward alternatively generating intermediate tentative "premises" and subchains of reasoning only reaching the premise-set at the end.

The expression *argument* (more clearly *premise-conclusion argument*) indicates the two-part system that "bounds" an argumentation. An argument can be constructed from an argumentation by deleting the chain of reasoning. In a sense, therefore, an argument is a "hollow argumentation". More explicitly, an *argument* is a two-part system composed of a set of propositions called its *premise-set* and a single proposition called its *conclusion*. The word 'argument' is widely used in technical and semi-technical works on logic but it is rarely used in this sense in ordinary discourse. Every argument "bounds" infinitely many argumentations but no argument *is* an argumentation. The expressions 'premise-set' and 'conclusion' are role words which have no meaning except in connection with an argument. Every proposition is a premise of infinitely many arguments and it is the conclusion of infinitely many arguments. An argument can be constructed by arbitrarily choosing a set of propositions to serve as premise-set and a single proposition to serve as conclusion.

In some cases a premise used in a chain of reasoning is not a premise of an argument that bounds it. In this case, the argumentation smuggles a premise. In some cases the conclusion of the chain of reasoning is not the conclusion of an argument that bounds it. Here we have *ignoratio elenchi*, or wrong conclusion. In some cases the bounding argument of an argumentation has premises not among the premises used in the chain of reasoning. In fact every argumentation having infinitely many premises has infinitely many premises that are not used, i.e. that are not premises of its chain of reasoning. The reason for this is that every chain of reasoning, whether cogent or fallacious, is finite and therefore uses only a finite number of premises. This may seem to be partly a matter of terminology. What is not merely terminological is the fact that every cogent chain of reasoning is finite. This is closely related to the fact that deduction is a temporal activity performed by thinkers. Every cogent chain of reasoning makes evident (to those thinkers to whom it is cogent) the fact that its conclusion is implied by the premises it uses. Some mathematicians who work with infinite sets and infinite sequences seem to have a tendency to overlook the inherent finiteness of chains of reasoning.

Infinite arguments, i.e. arguments having an infinite number of premises, had already been considered by Aristotle and they have become important in modern logic. Consider the sequence of propositions generated from "zero is exceeded by zero plus one" by repeated replacement of both occurrences of "zero" by "zero plus one". The set of these propositions does not imply "Every number is exceeded by itself-plus-one" but if a suitable proposition of mathematical induction is added the resulting set does imply the generalization. Here are two infinite arguments, in one the conclusion does not follow from the premise-set and in the other it does follow.

There is no condition on a proposition-set or proposition and no relationship between a proposition-set and a proposition necessary for them to be the premise-set and conclusion of an argument. If the conclusion is a logical consequence of the premise-set, the argument is said to be *valid*. If the conclusion is not a logical consequence of the premise-set, the argument is said to be *invalid*. Every argument is either valid or invalid and no argument is both valid and invalid. Every argument obtained by adding premises to a valid argument is valid. Every argument obtained by deleting premises from an invalid argument is invalid.

The fact that Tarski and other mathematical logicians lack the concepts "argument", "valid" and "invalid" does not entail that they cannot express certain facts. To say that an argument is valid is to say that certain propositions imply a certain proposition or that a certain proposition follows from or is a consequence of certain propositions. Aristotle, it should be said, lacked a relational verb for "implies" and lacked a relational noun for "consequence".

In this and in most works on logic, to say that an argument is (logically) valid is simply to say something about the relationship of its conclusion to its premise-set, viz. that the conclusion follows (logically) from, is (logically) implied by, is a (logical) consequence of, its premise-set. The words 'logically' and 'logical' are *redundant rhetoric* which are added or omitted according to taste or other inconsequential consideration. The words 'necessarily' and 'necessary' are likewise merely rhetorical. An argument that is valid is necessarily valid and conversely. Other words that are used in this way are 'formal', 'formally', 'deductive' and 'deductively'.

There are many useful ways to characterize the relation "logical consequence". In order for a conclusion to be a logical consequence of a premise-set it is necessary and sufficient for the information of the premise-set to include that of the conclusion, in other words, for there to be no information in the conclusion beyond that already in the premise-set. In order for a conclusion to be a logical consequence of a premise-set it is necessary and sufficient that it be logically impossible for the premises to all be true with the conclusion false. In order for a conclusion to be a logical consequence of a premise-set it is necessary and sufficient that were the premises all true then necessarily the conclusion would be true, in other words, that were the conclusion false then necessarily at least one premise would be false.

It would be an illusion to think that any one of the above characterizations by itself or even in combination with the others is sufficient to uniquely identify "logical consequence" for every reader. Those who have grasped the concept do not need any characterization. Indeed these readers will easily find objections to the above. Those who have not yet grasped the concept will need to experience examples of instances and examples of non-instances and they will need hints as well. The problem of characterizing "logical consequence", despite insightful attempts by Carnap, Tarski, and Quine, is still open.

No true proposition implies even one false proposition. Every true proposition is implied by infinitely many false propositions. Every false proposition implies infinitely many true propositions. Every proposition implying its own negation is false. Every proposition implied by its own negation is true. Every proposition implying a certain proposition and also implying the negation of that certain proposition is false. Every proposition implied by a certain proposition and also by the negation of that certain proposition is true. Every proposition implies itself.

It is of course not the case that every true proposition implies every other true proposition. Nor is it the case that every false proposition implies every true proposition. Nor is it the case that every false proposition implies every other false proposition. Except in the case of all true premises and false conclusion, the validity or invalidity of an argument is not determined by the truth-values of its propositions. Validity, or implication, is not "truth-functional".

The *principles of form for arguments*, which are not to be found either in Aristotle or in Tarski, are the following. Every argument in the same form as a valid argument is valid. Every argument in the same form as an invalid argument is invalid. Every two arguments in the same form are both valid or both invalid.

Despite the absence of the principles of form in Aristotle and in Tarski each uses a principle of form as the basis for proofs of invalidity. Not every invalid argument is known to be invalid. Ideally speaking, every argument whose premises are known to be true and whose conclusion is known to be false is known to be invalid. In a sense, such arguments may be said to be "obviously" invalid. This method of establishing invalidity, which is called the *method of fact*, is applicable only to a small fraction of the invalid arguments. The principle of form makes it possible to reduce invalidity of arguments not obviously invalid to the invalidity of obviously invalid ones.

In order to establish that "No proposition is both true and false" does not imply "Every proposition is either true or false" it is sufficient to notice that "No number is both positive and negative" is true whereas "Every number is either positive or negative" is false. In order to establish the invalidity of a given argument it is sufficient to exhibit an argument known to be in the same form whose premises are known to be true and whose conclusion is known to be false. The proposition "Every consequence of a consequence of a proposition is again a consequence of that proposition" is not implied by the proposition "Every consequence of a consequence of a consequence of a proposition is again a consequence of that proposition". To see this take 'is an opposite of' to mean "is logically equivalent to the negation of" and substitute "opposite" for "consequence".

A *counterargument* for a given argument is an argument having all true premises and false conclusion and in the same form as the given argument. The *method of counterarguments* for establishing invalidity amounts to exhibiting a known counterargument, i.e. an argument known to be a counterargument. The method of counterarguments is applied with meticulous precision throughout Aristotle's logical works. It is employed throughout the history of logic, e.g. to show that the parallel postulate is not implied by the other basic premises of geometry and to show that the continuum hypothesis is not implied by the axioms of set theory. It is the only method for establishing invalidity mentioned in Tarski's works. Whether it is the only method possible is a question that seems not to have been discussed systematically.

Not every valid argument is known to be valid. Much of current research in philosophy, mathematics, theoretical physics and other fields would be pointless were this not the case. In many cases an argument under investigation is actually valid and the goal of the investigation will only be achieved once the argument is known to be valid. It is surprising that some writers seem to think that a proof is a valid argument whose premises are true. This double fallacy involves conflation of the ontic and the epistemic: conflating "true" with "known to be true", and conflating "valid" with "known to be valid". Ideally speaking every proposition known to be implied by propositions known to be true is itself proved to be true. But a proposition that is implied by propositions known to be true is not necessarily proved to be true unless it is also known to be implied. Either Goldbach's Hypothesis or its negation is implied by propositions known to be true, viz. by the axioms of arithmetic. And yet neither Goldbach's Hypothesis nor its negation has been proved to be true. Likewise, a proposition known to be implied by propositions which are true is not necessarily proved to be true unless the implying propositions are known to be true. Every proposition is implied by itself. *A fortiori* every true proposition is implied by a true proposition. But not every true proposition has been proved to be true.

In some cases thinking that an argument shows or proves may be confusing the technical meaning of the word 'argument' with its etymological connotation. This type of confusion is often risked when stipulative definitions are used.

It is clear that a valid argument *per se* "proves" nothing, not even that its conclusion follows from its premise-set. In order to know that a valid argument is valid it is sufficient to deduce the conclusion from the premises, to construct a chain of reasoning that is cogent in the context of the argument. Thinking that a valid argument "shows" its own validity is an *ontic/epistemic fallacy*. Moreover, it involves confusion of the ontic relation-verb 'implies' with the epistemic action-verb 'deduce' (or 'infer'). Implication is a static atemporal relation from sets of propositions to single propositions. Deduction is an epistemic action performed by thinker-agents in "drawing forth" particular information from other information in which it is already contained. Deduction takes place in time. Deduction is the process of coming to know implication.

It is clear that in order to know whether a given argument is valid or invalid it is never necessary to know the truth-values of the premises and conclusion (of the given argument). A deduction is sufficient for knowledge of validity. A counterargument is sufficient for knowledge of invalidity. Failure to achieve a deduction does not establish invalidity. Failure to achieve a counterargument does not establish validity. The absence of positive evidence by itself is never conclusive negative evidence and the absence of negative evidence by itself is never conclusive positive evidence. In the absence both of a deduction and of a counterargument the validity/invalidity of the argument is often unknown.

Validating an argument is determining that it is valid, gaining knowledge of its validity. *Invalidating* an argument is determining that it is invalid. It is convenient to refer to the process of constructing a deduction for an argument as *deducing* the argument and to say that an argument admitting of validation by deduction is *deducible*. Likewise it is convenient to refer to the process of constructing a counterargument for an argument as *refuting* the argument and to say that an argument admitting of invalidation by counterargument is *refutable*. It is clear that deducibility and refutability are participant-relative.

The above terminology applying to arguments is paralleled by a commonly used terminology applying to propositions.

Verifying a proposition is determining that it is true, gaining knowledge of its truth. *Falsifying* a proposition is determining that it is false. The process of constructing a proof for a proposition is *proving* the proposition, and a proposition admitting of verification by proof is *provable*. The process of constructing a disproof of a proposition is *disproving* the proposition and a proposition admitting of falsification by disproof is *disprovable*. It is clear that provability and disprovability are participant-relative.

Provability is a criterion of truth and disprovability is a criterion of falsity. Deducibility is a criterion of validity and refutability is a criterion of invalidity.

Once these distinctions are clear it is easy to restate some of the perennial problems concerning the scope and limits of human knowledge. The hypothesis that every true proposition is verifiable, which has been called the *hypothesis of sufficient reason*, is analogous to each of the following hypotheses: every false proposition is falsifiable, every valid argument is deducible, every invalid argument is refutable. The converses of all four of these hypotheses are, of course, true. Each hypothesis, then, amounts to a hypothesis to the effect that an epistemic property is coextensive with a corresponding ontic property. There are two further hypotheses deserving of mention: every valid argument is validatable, every invalid argument is invalidatable. As with all perennial problems in philosophy, insight and maturity can be achieved through careful discussion of them.

5. *Derivations: Cogent Chains of Reasoning.* A proof solves the problem of whether its conclusion is true by reducing it to a problem that has already been solved, viz., the problem of whether its premises are all true. A deduction reduces the problem of whether its conclusion is true to a problem that has not necessarily been solved, viz. the problem of whether its premises are all true. But it is equally accurate to make a figure-ground shift and to observe that a deduction reduces the problem of whether at least one of its premises is false to the problem of whether its conclusion is false. But it is again equally accurate to say that a deduction actually solves a problem, viz. the problem of whether its premise-set implies its conclusion, or as we will say, the problem of whether its bounding argument is valid.

How does a deduction solve this problem? How does a deduction make evident that the answer is affirmative? How does a deduction make evident the fact that its bounding argument is valid? What makes a chain of reasoning cogent? This is the *cogency question.* Discussion of this question requires that we examine some types of cogent chains of reasoning. Let us use the word *derivation* to indicate a chain of reasoning that is cogent *per se*, i.e. that establishes, shows, makes clear, makes evident the fact that its final conclusion is a logical consequence of the propositions it uses as premises.

A very simple type of derivation perhaps the simplest, is the class of linear derivations. Roughly speaking, a chain of reasoning is *linear* if it is a sequence of propositions beginning with one of its premises, ending with the conclusion, and each subsequent member of which either is a premise or else is the conclusion of a *component argument* whose premises have already occurred. It is clear that the cogency of a linear derivation presupposes knowledge of the validity of the component arguments. A deduction whose derivation is linear seems to reduce the problem of the validity of its bounding argument to problems already solved, viz. to the problems of the validity of the component arguments.

In order for a linear deduction to be cogent for a given person, that person must have knowledge of the validity of the component arguments. A linear chain of reasoning that is cogent for one person need not, and normally will not, be cogent for all other persons. An argumentation using a component argument not known to be valid by the intended audience involves the *fallacy of begging the argument* which, of course, is analogous to the fallacy of begging the question. A component argument not known to be valid is commonly referred to as "a gap in the chain of reasoning". In some cases a begged argument is later deduced to the satisfaction of the intended audience and in these cases we may say that the gap has been filled by interpolation of additional steps of reasoning. In some cases, however, a begged argument is actually invalid and, thus, the gap cannot be filled. The term *non sequitur* is used in either case.

As Poincaré and others have pointed out, knowledge of the correctness of each step in a sequence is not sufficient for knowledge of the correctness of the sequence. It is possible to validate each step in a derivation without gaining knowledge that the conclusion is implied by its premises. It is clear then that the cogency of a linear derivation requires, besides knowledge of the validity of the component arguments, knowledge that the conclusion of a "sequential chain" of valid arguments is implied by its "ultimate premises".

There are several "chaining principles" which were already known to Alexander and to the Stoics. Perhaps the simplest such principle applies to the case of a three-line derivation constructed by chaining two one-premise arguments: every given argument whose conclusion is the conclusion of a valid one-premise argument the premise of which is the conclusion of a second valid one-premise argument the premise of which is a premise of the given argument is valid. This amounts to the so-called *principle of transitivity of consequence*: every consequence of a consequence of a given proposition is again a consequence of that proposition. Another example is the principle that applies to a chaining of three one-premise arguments: every consequence of a consequence of a consequence of a given proposition is again a consequence of the given proposition.

Then we have the case of the "cascading" of three two-premise arguments: every given argument whose conclusion is the conclusion of a valid two-premise argument the premises of which are each conclusions of valid two-premise arguments the premises of which comprise the premises of the given argument is valid. It is clear that for each method of chaining arguments to make a linear derivation there is a chaining principle to the effect that whenever the component arguments are valid the conclusion is implied by the premises.

There seems to be no independent evidence to support the hypothesis that every person for whom a linear deduction is cogent knows an appropriate chaining principle. An alternative hypothesis is that every person who follows or constructs linear derivations knows of a few principles from which the others can be deduced. However, even were it true there is little hope that it could be used to explain the cogency of linear derivations.

The point here is that we should not be tempted to believe that the person to whom a derivation is cogent proved the proposition that the argument is valid by deducing it from metalogical propositions already known to be true. In the first place, this would involve us in an infinite regress: the objectlanguage deduction presupposing a metalanguage deduction presupposing a metametalanguage deduction and so on. In the second place, it may involve us in a self-contradiction. A cogent derivation makes evident (to anyone for whom it is cogent) that its conclusion is implied by its premises; by referring to a metalogical proof we ran the risk of having assumed that the derivation did not make the implication evident.

Explaining a derivation is like explaining a "joke". If the explanation is needed to get the participant to laugh then it was not a joke. If the "joke" plus the explanation elicits laughter then it is the combination which is the joke, not just the "joke" *simpliciter*. Likewise, if the explanation is needed to get the participant to validate the argument then the chain of reasoning was not cogent to the participant. If the added explanation succeeds then the original chain of reasoning with the explanation added is cogent, not the original chain of reasoning *simpliciter*. To paraphrase Church, it is never necessary to prove that a proof is a proof.

The cogency question should never be construed as the problem of explaining why a derivation is not really cogent as it stands, nor as the problem of explaining why a chain of reasoning that is not cogent really is cogent. Just as the humor question, let us say, is the problem of explaining why a funny joke is funny and is not the problem of rescuing an unsuccessful "joke".

The discussion thus far shows that a deduction does not reduce the validity of its bounding argument to the validity of its component arguments *alone* even if it is acceptable to say that a deduction reduces the validity of its bounding argument to the validity of its component arguments.

The problem of chaining, let us say, is not the obvious problem in explaining cogency. The obvious problem, rather, is the problem of how the validity of the component arguments is known. To say that they are known by deduction is true but evasive and perhaps even question-begging. It is clear that if any argument is known to be valid by reduction to arguments already known to be valid then some arguments must have been known to be valid without being so reduced. Such ultimate arguments may be said to be *immediately validated.* This does not mean that it takes no time to validate them. It only means that they have been validated without "mediating reasoning". Aristotle calls these "perfect syllogisms". They correspond to trivial derivations i.e. to linear derivations having but one component argument. Of these Aristotle says that nothing need be added in order for it to be evident that the conclusion follows. Of such arguments we say that the conclusion is deduced *immediately* from the premises. Quine uses the phrase "visibly sound" in a parallel situation. Quine by the way is one of the few mathematically-oriented logicians who mention this topic at all. Tarski has not written a word on it.

Being immediately validated is inherently participant-relative, i.e. to say that an argument is immediately validated is to make tacit reference to a thinker who has immediately validated it. To such a thinker the linear derivation consisting solely of premises and conclusion had been cogent at a time when no extended deduction for it had been done. This is not to say that no extended deduction could have been done. Moreover, saying that an argument is immediately validated by a given person neither implies nor excludes that it is immediately validated by many other persons, perhaps even by all thinkers. Saying that an argument is immediately validated is saying something about the epistemic history of a thinker.

The combinations "immediately valid", "immediately implies", "immediately follows from" and the like are incoherent. They mix an epistemic concept, "immediately", with an ontic concept, "valid", "implies", etc., in a way that gibberish results. If an argument is immediately validated we may say that the premise-set *leads immediately* to the conclusion or that the conclusion is *immediately deduced* from or *immediately derived* from the premise-set.

Despite the fact that many responsible thinkers have concluded that no proposition is known to be true, virtually no one has concluded that no argument is known to be valid. Despite the fact that many responsible thinkers have concluded that no proposition is immediately verified, virtually no one has challenged the view that some arguments are immediately validated. Nevertheless, understanding of deduction, the linchpin of rationality, requires understanding of how immediately validated argument are known to be valid. The *fundamental problem of deduction* is to explain how immediately validated arguments are known to be valid.

The above description of linear derivations is an oversimplification. The crucial deficiency is that it describes the derivation as a sequence of objectlanguage propositions. The most casual observation of the facts reveals that this is not so. Every derivation involves instructions for carrying out a train of thought and there is no way that a sequence of objectlanguage propositions can contain instructions for deducing one of them from others. Many *propositionals*, let us say, in a derivation are imperative, not declarative. They instruct us to carry out various epistemic acts most prominent of which are assuming and inferring: "assume such and such", "from so and so infer such and such". Besides the assume-instructions that introduce premises of the derivation there are assume-instructions that introduce *auxiliary* assumptions, which are introduced for purposes of reasoning and which are later discharged. There are others as well.

An auxiliary assumption in a derivation initiates a subchain of reasoning the completion of which serves as the mediating basis for an inference from the previous assumptions. Perhaps the simplest case is the *simple indirect derivation*. Here after assuming the premises we assume as an auxiliary the negation of the conclusion thereby initiating a subchain. We then proceed to make a sequence of immediate inferences until we arrive at a contradiction, i.e. until we have a proposition and its negation. Then we note the contradiction and end the subchain. On the mediating basis of the subchain we infer the conclusion itself. The last line of an indirect deduction is something to the following effect: "Since we have deduced a contradiction from the initial premises augmented by the negation of the conclusion, we infer the conclusion itself from the initial premises alone".

Incidentally, when a person knows to be true the negation of any proposition the person knows to be false, a disproof of a hypothesis amounts to an indirect proof of (the truth of) the negation of the hypothesis. Thus, the hypothetico-deductive method often admits of being transformed into a submethod of the deductive method. It may well be the case that indirect deduction, which is rather intricate, is an evolutionary descendant of the hypothetico-deductive method, which is elemental.

Many sensible things have been said about indirect derivations, e.g. that an indirect derivation shows that the conclusion follows from the premises by showing that the negation of the conclusion is inconsistent with the premises, or that an indirect derivation shows that the conclusion follows from the premises by showing that it is logically impossible for the premises to be true with the conclusion false.

According to standard accounts linear derivations are not constructed by chaining arguments but rather by applying rules of inference (to premises and then to results of previous applications). Of course, the resulting derivations could be described either way. In fact, each such rule amounts to a set of arguments, viz. the set of instances of the rule. For example, in order for an argument to be an instance of *modus ponens* it is necessary and sufficient that it be a two-premise argument one of whose premises is the conditional whose antecedent is the other premise and whose consequent is the conclusion.

The observation that derivations are constructed by chaining immediately validated arguments that can be collected under rules is of course due to Aristotle. Moreover, the hypothesis that deduction is in some sense a rule-governed activity seems well-confirmed by practice. Nevertheless, this hypothesis does not seem to contribute to the solutions of any of the problems already mentioned and, in fact, it suggests further problems.

As has been pointed out by others, there are three species of knowledge: *objectual knowledge* or knowledge *of* objects (entities, concepts, propositions, argumentations), *operational knowledge* or known-*how* to perform various tasks, and *propositional knowledge* or knowledge *that* propositions are true or false. It is already clear that cogency of a derivation requires all three and that it pivots on operational knowledge. In order to follow a derivation it is necessary to be able to perform various operations most prominently assuming and immediate inferring but also operations involved in "chaining".

6. *Expressions and Meanings*. Some sentences express propositions and some do not. The sentence 'Two exceeds one' expresses the true proposition "Two exceeds one". The sentence 'One exceeds two' expresses the false proposition "One exceeds two". The properties "true" and "false" have as their range of applicability the class of propositions. Any attempt to affirm or to deny "true" or "false" of a non-proposition results in gibberish, incoherence, category error, nonsense. The sentences 'One is true' and 'One is false' do not express propositions at all.

It is likewise incoherent to attempt to affirm or to deny "true" or "false" of an argument, an argumentation, or a chain of reasoning. The class of arguments is the range of applicability of "valid" and of "invalid". It is incoherent to say that a proposition is valid or is invalid. The class of argumentations is the range of applicability of "conclusive" (*sc.* "apodictic") and of "inconclusive" (*sc.* "nonapodictic"). It is incoherent to say that a proposition or an argument is conclusive or is inconclusive.

"Cogent *per se*", "cogent in context", "fallacious *per se*", and "fallacious in context" all apply exclusively to chains of reasoning. "Cogent" and "fallacous" *simpliciter* apply exclusively to argumentations. An argumentation is cogent or fallacious according as its chain of reasoning is cogent in context or fallacious in context.

The relation-verb 'implies' is tenseless. Its subject is a set of propositions and its object is a proposition. The sentence 'One implies two' is incoherent, as is the sentence 'Tarski implies two'. In other English writing the sentence 'Tarski implied that logic is a science' is coherent. In fact it expresses a true proposition but only when 'implies' is taken in another sense. The other sense is called *agent implication.*

The action-verb 'deduce' is tensed. Its subject is a thinker, its direct object is a proposition and its indirect object is a set of propositions. It is often the case that the distinction between a set of propositions and a single proposition is unmarked but it must always be understood in order to avoid incoherence. 'Euclid deduced the Pythagorean Theorem from the Basic Premises of Geometry' expresses an (arguably) true proposition. 'Russell deduced his whole body from his left foot' is incoherent (but humorous praise of Russell's powers of reasoning).

Tragic confusion in logic has resulted from failure to heed the above and other selection restrictions, or semantic-category constraints. The importance of observing them was first pointed out by Plato in the *Cratylus.* The theory of semantic categories was advanced by Aristotle and more recently by Husserl, Russell, Tarski, Quine and others. It was Quine, for example who made it clear that 'If two exceeds three then three exceeds two' is coherent and expresses a true proposition whereas 'Two exceeds three implies three exceeds two' is incoherent and '"Two exceeds three" implies "Three exceeds two"' is coherent but expresses a false proposition.

There is no way to develop a coherent philosophy of logic without careful attention to coherent discourse.

Many logicians, whether mathematically oriented or not, believe that "true" and "false" are properties of sentences rather than of propositions as is presupposed in this essay. This is a fundamental disagreement that should not be glossed over. It is easy to read remarks to the effect that a sentence is true (or false) as elliptical for remarks that the sentence expresses a proposition which is true (or false). But there are many places where this becomes awkward. For example, the sentence 'Every even number is not prime' admits of "scope ambiguity" in regard to 'not' and thus can be used to express the true proposition "Not every even number is prime" as well as to express the false proposition "Every even number is non-prime". To avoid this and other conflicts, Quine and Tarski restrict the class of sentences under consideration to sentences of formalized (or regimented) languages from which ambiguity has been eliminated.

However, in eliminating ambiguity such writers make it awkward or impossible to discuss phenomena traditionally and properly discussed in logic. For example, discussion of the *fallacy of ambiguity* requires the distinction between expression and meaning. For example, a person might believe that "No even number is prime" on the strength of an alleged proof whose premises are "2 is an even number" and "2 is not prime". The person first (correctly) deduces "Not every even number is prime" but writes the result 'Every even number is not prime'. Then this is read "Every even number is non-prime" from which "No even number is prime" is (correctly) deduced. The fallacy consists in reading a sentence one way when the proposition intended is being inferred and reading it another way when the other proposition intended is being used as the basis of an inference.

Just as we have distinguished propositions from sentences we likewise distinguish discourses from discourse-texts, arguments from argument-texts and argumentations from argumentation-texts. The property "ambiguous" applies exclusively to expressions. In order for an expression to be ambiguous it is necessary and sufficient for it to express two or more meanings. It is clear that ambiguity is participant-relative even though a given expression is often ambiguous to every member of a given community of participants. It is incoherent to say that a proposition (or any other non-expression) is ambiguous.

Just as some sentences are elliptical, likewise other expressions, e.g. argument-texts are elliptical. In some cases, perhaps most, ellipsis involves ambiguity. For example, the sentence '6 has more divisors of 30 than 15' expresses a true proposition if the ellipsis is 'of' but it expresses a false proposition if the ellipsis is 'does'. The two propositions in question are expressed:

6 has more divisors of 30 than of 15.
6 has more divisors of 30 than does 15.

In an elliptical discourse-text, words are omitted from sentences and sentences themselves are omitted. Making the distinction of argument-text from argument makes it possible to make sense of the ancient doctrine of the *enthymeme*, incoherently described as an argument with a suppressed premise. An enthymeme is an elliptical argument-text. It is incoherent to say that an enthymeme is valid or invalid as these properties apply exclusively to arguments.

Many proof-texts are elliptical. Some problems in the history of mathematics involve supplying missing parts of elliptical discourses so that the reasoning of ancient thinkers can be reconstructed and appreciated. Is it a miracle that Euclid's geometry is pervaded by fallacious reasoning and yet contains no false "theorems" or is Euclid's reasoning elliptically expressed? Although modern logicians are meticulous about full expression of objectlanguage proofs, virtually every argumentation-text intended to express a proof of a metatheorem either is enthymematic or else expresses argumentation that smuggles premises. Historians of modern logic might find it ironic that modern logic books are pervaded by the same deficiencies (viz. ellipsis and premise-smuggling) that the books themselves charge Euclid with.

7. *Conclusion.* Some people say that logic is about "argument forms" and not about "concrete arguments", i.e. arguments whose non-logical terms are actual concepts as opposed to schematic letters, variables, or what have you. In this essay the expression 'argument' is used in such a way that every argument is concrete, there is no such thing as "an abstract argument", "an argument devoid of concrete terms", "a formal argument", etc. Every argument is composed of propositions and every proposition is either true as it stands or false as it stands.

Aristotle says that the subject-matter of logic is proof, that logic is the systematic study of proofs. Quine says that logic is the systematic study of tautologies. A *tautology*, of course, is a proposition that is implied by its own negation, or, as Quine says, "is true no matter what". What Quine has in mind is that since a tautology is implied by itself and by its own negation, it is true whether it is true or not, and thus is "true no matter what". It is also clear, as Quine has also emphasized, that in order to be a tautology it is necessary and sufficient for a proposition to be implied by absolutely every and any proposition. Since a consequence of each of two propositions has no information not contained in both, if a proposition and its negation share no information, a tautology is empty, i.e. devoid of information. This should be no surprise to those who take as a paradigm of tautology any of the following: "Every number is identical to itself". "Every even number is even", "Every number is either even or not even". So Quine thinks that logic is the systematic study of empty propositions, whether he would put it this way is beside the point.

The negation of a tautology is a *contradiction*, a proposition that implies its own negation. A contradiction may be said to be "false no matter what" since it implies itself and it implies its own negation, and thus is false whether it is false or not. In order to be a contradiction it is necessary and sufficient that a proposition imply absolutely every and any proposition. Thus a contradiction contains any information contained in any proposition. In other words, a contradiction is absolutely comprehensive, it contains all information.

Now Aristotle says that opposites are studied by the same science. This might imply that any science of tautologies is a science of contradictions as well, and thus that Quine's view implies that logic is also the systematic study of contradictions. Quine should not object to this. Quine says that the only reason that tautologies are important is that implication can be explained by means of "tautology". But it is as correct to say, in the same sense, that implication can be explained by means of "contradiction". By the way, we have just seen that implication can be used to explain "tautology" and "contradiction".

Quine also says that the reason implication is important is that, since every proposition implied by a true proposition is true, we use knowledge of implication to justify or to form new beliefs on the basis of old beliefs. Thus, by tracking down the explanations that Quine gives for his apparently strange view, we find that it is not so far from Aristotle's view after all. This conclusion is all the more plausible once we take account of the fact that in pronouncing on the nature of logic the masters were probably not intending to give precise "theorems" but merely to point the student in the right direction, to give the beginner an idea of what lies ahead.

Now if logic is the study of proofs and if opposites are studied in the same science, then logic is the study of proofs and of non-proofs, of conclusive argumentations and of inconclusive argumentations, in short, of argumentations. Thus Aristotelian principles lead to the view that logic is the study of argumentations. Taking logic to be the study of argumentations, in the fullest sense, takes logic to include a vast array of phenomena.

If the goal of a pronouncement on the nature of logic is to point the student in the right direction, then it seems to me that logic should be pronounced to be the systematic study of argumentations. As we have seen, this will bring the student to see the relevance of logic to many fields and to see the relevance of many fields to logic. Philosophy, linguistics, mathematics and cognitive psychology are perhaps the fields with the greatest commonality with logic. If "rational animal" is even a fair approximation to a definition of "human", then logic should be prominent among the humanities. But perhaps logic is more of a quest than a subject. Perhaps logic should be defined as the quest for an objective understanding of objectivity. This would be in keeping with the spirit of Aristotle, Ockham, Boole, Frege, Russell, Tarski, and Quine, to mention only a few of the tireless workers who have created this magnificent edifice.

The fact that this essay has revealed difficult problems about the nature of logic and about the possibility of objectivity may lead some readers to infer that the goal of the essay is more destructive, or deconstructive, than constructive. Such an inference would be a distortion of my intentions and it would be contrary to the spirit of logic. Objectivity requires that objective standards of criticism be applied to themselves. Otherwise dedication to objectivity becomes a mockery of itself. Logicians cannot aspire to objective evaluation of methodology in other fields unless they apply the same standards to logic itself. Dogmatism and skepticism are twin enemies of objectivity. Self-criticism of logic, and in particular the attempt in this century to give a rational reconstruction of logic, has led both to revolutionary advances in logic and, more importantly, to a richer understanding of the human condition. Knowing that a proposition is true does not require knowing that it is known to be true nor does it require a feeling of certainty. It is the denial of this point, not the admission of it, that leads to dogmatism and to skepticism.

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