Audi, R. Ed. 1999. *Cambridge Dictionary of Philosophy*. Cambridge: Cambridge UP.

ancestral, axiomatic method, borderline case, categoricity, Church (Alonzo), conditional, convention T, converse (outer and inner), corresponding conditional, degenerate case, domain, De Morgan, ellipsis, laws of thought, limiting case, logical form, logical subject, material adequacy mathematical analysis, omega, proof by recursion, recursive function theory, scheme, scope, Tarski (Alfred), tautology, universe of discourse.

CITE AS: Corcoran, J. 1999. "Ancestral", *Cambridge Dictionary of Philosophy*. R.Audi, Ed. Cambridge: Cambridge UP. p. 65.

ancestral (of a given relation *R*), the relation (also called the transitive closure of *R*) that relates one given individual to a second if and only if the first can be "reached" from the second by repeated "applications" of the given relation *R.* The ancestor relation is the *ancestral* of the parent relation since one person is an ancestor of a second if the first is a parent of the second or the first is a parent of a parent of the second or the first is a parent of a parent of a parent of the second, and so on. Frege discovered a simple method of giving a materially adequate and formally correct definition of the ancestral of a given relation in terms of the relation itself (plus logical concepts). This method is informally illustrated as follows: in order for one person *A* to be an ancestor of a second person *B* it is necessary and sufficient for *A* to have every property that belongs to every parent of *B* and that belongs to every parent of any person to whom it belongs. This and other similar methods made possible the reduction of all numerical concepts to those of zero and successor, which Frege then attempted to reduce to concepts of pure logic. Frege's definition of the ancestral has become a paradigm in modern analytic philosophy as well as a historical benchmark of the watershed between traditional logic and modern logic. It demonstrates the exactness of modern logical analysis and, in comparison, the narrowness of traditional logic. **See also FREGE, LOGICISM, RELATION.** J.Cor.

CITE AS: Corcoran, J. 1999. "Axiomatic method", *Cambridge Dictionary of Philosophy*. R.Audi, Ed. Cambridge: Cambridge UP. p. 65.

axiomatic method, originally, a method for reorganizing the accepted propositions and concepts of an existent science in order to increase certainty in the propositions and clarity in the concepts. Application of this method was thought to require the identification of (1) the "universe of discourse" (domain, genus) of entities constituting the primary subject matter of science, (2) the "primitive concepts" that can be grasped immediately without the use of definition, (3) the "primitive propositions" (or "axioms"), whose truth is knowable immediately, without the use of deduction, (4) an immediately acceptable "primitive definition" in terms of primitive concepts for each non-primitive concept, and (5) a deduction (constructed by chaining immediate, logically cogent inferences ultimately from primitive propositions and definitions) for each non-primitive accepted proposition. Prominent proponents of more or less modernized versions of the axiomatic method, e.g., Pascal, Nicod (1893-1924), and Tarski, emphasizing the critical and regulatory function of the axiomatic method, explicitly open the possibility that axiomatization of an existent, preaxiomatic science may lead to rejection or modification of propositions, concepts, and argumentations that had previously been accepted.

In many cases attempts to realize the ideal of an axiomatic science have resulted in discovery of “smuggled premises” and other previously unnoted presuppositions, leading in turn to recognition of the need for new axioms. Modern axiomatizations of geometry are much richer in detail than those produced in ancient Greece. The earliest extant axiomatic text is based on an axiomatization of geometry due to Euclid (ll. 300 B.C.), which itself was based on earlier, no longer-extant texts. Archimedes (287-212 B.C.) was one of the earliest of a succession of post-Euclidean geometers, including Hilbert, Oswald Veblen (1880-1960), and Tarski, to propose modifications of axiomatizations of classical geometry. The traditional axiomatic method, often called the geometric method, made several presuppositions no longer widely accepted. The advent of non-Euclidean geometry was particularly important in this connection.

For some workers, the goal of reorganizing an existent science was joined to or replaced by a new goal: characterizing or giving implicit definition to the structure of the subject matter of the science. Moreover, subsequent innovations in logic and foundations of mathematics, especially development of syntactically precise formalized languages and effective systems of formal deductions, have substantially increased the degree of rigor attainable. In particular, critical axiomatic exposition of a body of scientific knowledge is now not thought to be fully adequate, however successful it may be in realizing the goals of the original axiomatic method, so long as it does not present the underlying logic (including language, semantics, and deduction system). For these and other reasons the expression ‘axiomatic method’ has undergone many “redefinitions,” some of which have only the most tenuous connection with the original meaning. See also CATEGORICITY, DEDUCTION, and FORMALIZATION. J. Cor.

CITE AS: Corcoran, J. 1999. “Borderline case”. *Cambridge Dictionary of Philosophy*. R.Audi, Ed. Cambridge: Cambridge UP. p. 96.

borderline case, in the logical sense, a case that falls within the "gray area" or "twilight zone" associated with a vague concept; in the pragmatic sense, a doubtful, disputed or arguable case. These two senses are not mutually exclusive, of course. A moment of time near sunrise or sunset may be a borderline case of daytime or nighttime in the logical sense, but not in the pragmatic sense. A sufficiently freshly fertilized ovum may be a borderline case of person in both senses. Fermat's hypothesis, or any of a large number of other disputed mathematical propositions, may be a borderline case in the pragmatic sense but not in the logical sense. A borderline case *per se* in either sense need not be a limiting case or a degenerate case. See also DEGENERATE CASE, LIMITING CASE, and VAGUENESS. J. Cor.

CITE AS: Corcoran, J. 1999. “Categoricity”. *Cambridge Dictionary of Philosophy*. R.Audi, Ed. Cambridge: Cambridge UP. p.122.

categoricity, the semantic property belonging to a set of sentences, a “postulate set,” that implicitly defines (completely describes, or characterizes up to isomorphism) the structure of its intended interpretation or standard model. The best-known categorical set of sentences is the postulate set for number theory attributed to Peano, which completely characterizes the structure of an arithmetic progression. This structure is exemplified by the system of natural numbers with zero as distinguished element and successor (addition of one) as distinguished function. Other exemplifications of this structure are obtained by taking as distinguished element an arbitrary integer, taking as distinguished function the process of adding an arbitrary positive or negative integer, and taking as universe of discourse (or domain) the result of repeated application of the distinguished function to the distinguished element. (See, e.g., Russell's *Introduction to Mathematical Philosophy*, 1919.)

More precisely, a postulate set is defined to be *categorical* if every two of its models (satisfying interpretations or realizations) are isomorphic (to each other), where of course, two interpretations are *isomorphic* if between their respective universes of discourse there exists a one-to-one correspondence by which the distinguished elements, functions, relations, etc., of the one are mapped exactly onto those of the other. The importance of the analytic geometry of Descartes involves the fact that the system of points of a geometrical line with the "left-of relation" distinguished is isomorphic to the system of real numbers with the "less-than" relation distinguished. Categoricity, the ideal limit of success for the axiomatic method considered as a method for characterizing subject matter rather than for reorganizing a science, is known to be impossible with respect to certain subject matters using certain formal languages. The concept of categoricity can be traced back at least as far as Dedekind; the word is due to Dewey.

See also AXIOMATIC METHOD, LÖWENHEIM-SKOLEM THEOREM, MATHEMATICAL ANALYSIS, and MODEL THEORY. J. Cor.

CITE AS: Corcoran, J. 1999. "Church, Alonzo". *Cambridge Dictionary of Philosophy*. R.Audi, Ed. Cambridge: Cambridge UP. p. 140.

Church, Alonzo (1903-1995), American logician, mathematician, and philosopher, known in pure logic for his discovery and application of the Church lambda operator, one of the central ideas of the Church lambda calculus, and for his rigorous formalizations of the theory of types, a higher-order underlying logic originally formulated in a flawed form by Whitehead and Russell. The lambda operator enables direct, unambiguous, symbolic representation of a range of philosophically and mathematically important expressions previously representable only ambiguously or only after elaborate paraphrasing. In philosophy, Church advocated rigorous analytic methods based on symbolic logic. His philosophy was characterized by his own version of logicism, the view that mathematics is reducible to logic, and by his unhesitating acceptance of higher-order logics. Higher-order logics, including second-order, are ontologically rich systems that involve quantification of higher-order variables, variables that range over properties, relations, and so on. Higher-order logics were routinely used in foundational work by Frege, Peano, Hilbert, Gödel, Tarski and others until around World War II, when they suddenly went out of favor. In regard to both his logicism and his acceptance of higher-order logics, Church countered trends, increasingly dominant in the third quarter of this century, against reduction of mathematics to logic and against the so-called "ontological excesses" of higher-order logic. In the 1970s, although admired for his high standards of rigor and for his achievements, Church was regarded as conservative or perhaps even reactionary. Opinions have softened in recent years.

On the computational and epistemological sides of logic Church made two major contributions. He was the first to articulate the now widely accepted principle known as Church's Thesis, that every effectively calculable arithmetic function is recursive. At first highly controversial, this principle connects intuitive, epistemic, extrinsic and operational aspects of arithmetic with its formal, ontic, intrinsic and abstract aspects. Church's Thesis sets a purely arithmetic outer limit on what can be achieved by computational manipulations. Church's further work on Hilbert's "decision problem" led to the discovery and proof of Church's Theorem—basically that there is no computational procedure for determining of a finite-premised first-order argument whether it is valid or invalid. This result contrasts sharply with the previously known result that the computational truth-table method is sufficient for determining of a finite-premised truth-functional argument whether it is truth-functionally valid or invalid. Church's Thesis at once highlights the vast difference between propositional logic and first-order logic and sets an outer limit on what can be achieved by "automated reasoning".

Church's mathematical and philosophical writings are influenced by Frege's thought, especially by Frege's semantic distinction between sense and reference, Frege's emphasis on purely syntactical treatment of proof, and Frege's doctrine that sentences denote (are names of) their truth-values. **See also CHURCH'S THESIS, COMPUTABILITY, FORMALIZATION, HILBERT, HILBERT'S PROGRAM, LOGICISM, RECURSIVE FUNCTION THEORY, SECOND-ORDER LOGIC, TRUTH-TABLE, TYPE THEORY**. J. Cor.

CITE AS: Corcoran, J. 1999. "Conditional". *Cambridge Dictionary of Philosophy*. R.Audi, Ed. Cambridge: Cambridge UP. p. 140.

conditional, a compound sentence such as 'if Abe calls, then Ben answers' in which one sentence, the antecedent, is connected to a second, the consequent, by the connective 'if...then'. Propositions (statements, etc.) expressed by conditionals are called 'conditional' propositions (statements, etc.) and, by ellipsis, simply 'conditionals'. The ambiguity of the expression 'if...then' gives rise to a semantic classification of conditionals into material conditionals, causal conditionals, counterfactual conditionals, and so on. In traditional logic, conditionals are called hypotheticals and in some areas of mathematical logic conditionals are called implications. Faithful analysis of the meanings of conditionals continues to be investigated and intensely disputed. **See also PROPOSITIONS, COUNTERFACTUAL, IMPLICATION, CORRESPONDING CONDITIONAL, TRUTH TABLE**. J. Cor.

CITE AS: Corcoran, J. 1999. "Convention T". *Cambridge Dictionary of Philosophy*. R.Audi, Ed. Cambridge: Cambridge UP. p.185.

convention T, a criterion of material adequacy (of proposed truth definitions) discovered, formally articulated, adopted and so-named by Alfred Tarski in connection with his 1929 definition of the concept of truth in a formalized language. Convention T is one of the most important of several independent proposals Tarski made concerning philosophically sound and logically precise treatment of the concept of truth. Various of these proposals have been criticized, but convention T has remained virtually unchallenged and has come to be regarded almost as an axiom of analytic philosophy. To say that a proposed definition of an established concept is materially adequate is to say that it is "neither too broad nor too narrow", i.e. that the concept it characterizes is coextensive with the established concept. Since, as Tarski emphasized, for many formalized languages there are no criteria of truth, it would seem that there can be no general criterion of material adequacy of truth definitions. But Tarski brilliantly finessed this obstacle by discovering a specification which is fulfilled by the established correspondence concept of truth and which has the further property that any two concepts fulfilling it are necessarily coextensive. Basically, convention T requires that in order for a proposed truth definition to be considered materially adequate it must imply all of the infinitely many relevant Tarskian biconditionals: e.g. the sentence 'Some perfect number is odd' is true if and only if some perfect number is odd. Loosely speaking, a Tarskian biconditional for English is a sentence obtained from the form

the sentence -------is true if and only if -----------

by filling the right blank with a sentence and filling the left blank with a name of the sentence. Tarski called these biconditionals 'equivalences of the form T' and he referred to the form as a 'scheme'. Later writers refer to the form as 'scheme T' or 'schema T'. **See also FORMAL SEMANTICS, GÖDEL'S INCOMPLETENESS THEORY, MATERIAL ADEQUACY, SATISFACTION, TRUTH, TARSKI.** J. Cor.

CITE AS: Corcoran, J. 1999. "Converse, outer and inner". *Cambridge Dictionary of Philosophy*. R.Audi, Ed. Cambridge: Cambridge UP. p. 186.

converse, outer and inner, respectively, the result of "converting" the two "terms" or the relation verb of a relational sentence. The outer converse of 'Abe helps Ben' is 'Ben helps Abe' and the inner converse is 'Abe is helped by Ben'. In simple, or atomic, sentences the outer and inner converses express logically equivalent propositions, and thus in these cases no informational ambiguity arises from the adjunction of 'and conversely' or of 'but not conversely', despite the fact that such adjunction does not indicate which, if either, of the two converses is intended. However, in complex, or quantified, relational sentences such as 'Every integer precedes some integer' genuine informational ambiguity is produced. Under one set of normal interpretations of the respective sentences, the outer converse expresses the false proposition that some integer precedes every integer, the inner converse expresses the true proposition that every integer is preceded by some integer. More complicated considerations apply in cases of quantified doubly relational sentences such as 'Every integer precedes every integer exceeding it'. The concept of scope explains such structural ambiguity: in the sentence 'Every integer precedes some integer and conversely' in the outer sense 'conversely' has wide scope, whereas taken in the inner sense 'conversely' has narrow scope. See also ambiguity, converse, relation, scope, structural ambiguity. J. Cor.

CITE AS: Corcoran, J. 1999. "Corresponding conditional". *Cambridge Dictionary of Philosophy*. R.Audi, Ed. Cambridge: Cambridge UP. p. 187.

corresponding conditional (of a given argument), any conditional whose antecedent is a (logical) conjunction of all of the premises of the argument and whose consequent is the conclusion. The two conditionals, 'if Abe is Ben and Ben is wise, then Abe is wise' and 'if Ben is wise and Abe is Ben, then Abe is wise', are the two corresponding conditionals of the argument whose premises are 'Abe is Ben' and 'Ben is wise' and whose conclusion is 'Abe is wise'. In the case of a one-premise argument, the corresponding conditional is the conditional whose antecedent is the premise and whose consequent is the conclusion. The limiting cases of the empty premise set and the infinite premise sets are treated in different ways by different logicians; one simple treatment considers such arguments as lacking corresponding conditionals. The principle of corresponding conditionals is that in order for an argument to be valid it is necessary and sufficient for its corresponding conditionals to all be tautological. The commonly used expression 'the corresponding conditional of an

argument' is also used when two further stipulations are in force: first, that an argument is construed as having an (ordered) sequence of premises rather than an (unordered) set of premises; second, that conjunction is construed as a polyadic operation that produces in a unique way a single premise from a sequence of premises rather than as a dyadic operation that combines premises two by two. Under these stipulations the principle of the corresponding conditional is that in order for an argument to be valid it is necessary and sufficient for its corresponding conditional to be tautological. These principles are closely related to modus ponens, to conditional proof and to the so-called deduction theorem. **See also ARGUMENT, CONDITIONAL, CONDITIONAL PROOF, LIMITING CASE, MODUS PONENS, NULL CLASS, PROPOSITION, TAUTOLOGY**.
J. Cor.

CITE AS: Corcoran, J. 1999. "Degenerate case". *Cambridge Dictionary of Philosophy*. R.Audi, Ed. Cambridge: Cambridge UP. p. 215.

degenerate case: an expression used more or less loosely to indicate an individual or class that falls outside of a given background class to which it is otherwise very closely related, often in virtue of an ordering of a more comprehensive class. A degenerate case of one class is often a limiting case of a more comprehensive class. Rest (zero velocity) is a degenerate case of motion (positive velocity) while being a limiting case of velocity. The circle is a degenerate case of an equilateral and equiangular polygon. In technical or scientific contexts, the conventional term for the background class is often "stretched" to cover otherwise degenerate cases. A figure composed of two intersecting lines is a degenerate case of hyperbola in the sense of synthetic geometry, but it is a limiting case of hyperbola in the sense of analytic geometry. The null set is a degenerate case of set in an older sense but a limiting case of set in a modern sense. A line segment is a degenerate case of rectangle when rectangles are ordered by ratio of length to width, but it is not a limiting case under these conditions. **See also BORDERLINE CASE, LIMITING CASE**. J. Cor.

CITE AS: Corcoran, J. 1999. "De Morgan, Augustus". *Cambridge Dictionary of Philosophy*. R.Audi, Ed. Cambridge: Cambridge UP. p. 218.

De Morgan, Augustus (1806-1871), prolific British mathematician, logician, philosopher of mathematics, philosopher of logic, remembered chiefly for several lasting contributions to logic and philosophy of logic including discovery and deployment of the concept of universe of discourse, co-founding of relational logic, adaptation of what are now known as De Morgan's Laws, and several terminological innovations including coining the expression 'mathematical induction'. His main logical works, the monograph *Formal Logic* (1847) and the series of articles "On the syllogism" (1846-1862) demonstrate wide historical and philosophical learning, synoptic vision, penetrating originality, and disarming objectivity. His relational logic treated a wide variety of inferences involving propositions whose logical forms were significantly more complex than those treated in the traditional framework stemming from Aristotle, e.g., "Every ancestor of a parent of a person is a parent of an ancestor of the person", "If every doctor is a teacher then every ancestor of a doctor is an ancestor of a teacher”. De Morgan’s conception of the infinite variety of logical forms of propositions vastly widens that of his predecessors and even that of his able contemporaries such as Boole, Hamilton, Mill, and Whately. De Morgan did as much as any of his contemporaries toward the creation of modern mathematical logic. **See also DE MORGAN’S LAWS, LOGICAL FORM, RELATIONAL LOGIC, UNIVERSE OF DISCOURSE**. J. Cor.

CITE AS: Corcoran, J. 1999. “Domain”. *Cambridge Dictionary of Philosophy*. R.Audi, Ed. Cambridge: Cambridge UP. p. 218.

domain, of a science, the class of individuals that constitute the subject-matter of the science. Zoology, number theory and plane geometry have as their respective domains the class of animals, the class of natural numbers and the class of plane figures. In *Posterior Analytics* 76b10, Aristotle observes that each science presupposes its domain, its basic concepts and its basic principles. In modern formalizations of a science using a standard first-order formal language, the domain of the science is often, but not always, taken as the universe of the intended interpretation or intended model, i.e., as the range of values of the individual variables. **See also AXIOMATIC METHOD, FORMAL LOGIC, FORMALIZATION, MODEL THEORY, ONTOLOGICAL COMMITMENT, UNIVERSE OF DISCOURSE, VARIABLE**. J. Cor.

CITE AS: Corcoran, J. 1999. “Ellipsis”. *Cambridge Dictionary of Philosophy*. R.Audi, Ed. Cambridge: Cambridge UP. p. 257-8.

ellipsis, also known as elliptical expression, an expression (spoken or written) from which semantically or syntactically essential material has been deleted, usually for conciseness. Elliptical sentences are often used to answer questions without repeating material occurring in the questions. For example, the word ‘Lincoln’ may be an answer to the question of the authorship of the Gettysburg Address or to the question of the birthplace of George Boole. The single word ‘Lincoln’ can be seen as an elliptical name when used as an ellipsis of ‘Abraham Lincoln’ or ‘the City of Lincoln’, and it can be seen as an elliptical sentence when used as an ellipsis for ‘Abraham Lincoln wrote the Gettysburg Address’ or for ‘George Boole was born in the City of Lincoln’. Other typical elliptical sentences are: Abe is a father of two [children], Ben arrives at twelve [*sc*. noon], Carl is two years old and Dan is three [*scilicet* years old]. A typical ellipsis that occurs in discussion of ellipses involves citing the elliptical sentences with the deleted material added in brackets (often with ‘*sc*.’ or ‘*scilicet*’) instead of also presenting the complete sentence. Ellipsis also occurs above the sentential level, e.g., where commonly assumed premises are omitted in the course of argumentation. The word *enthymeme* designates an elliptical argument expression from which one or more premise-expressions have been deleted. The expression *elliptic ambiguity* designates ambiguity arising from ellipsis. **See also AMBIGUITY, ARGUMENT, LOGICAL FORM.** J. Cor.

MATERIAL SUBMITTED BUT DELETED BY THE EDITOR: "as in the following two examples: Russell read Frege more than Peano [did]; By now the Goldbach Conjecture should be known to be true or [known to be] false. The expression *the ellipsis* also denotes the string of three dots used to indicate an infinite continuation or a deletion as in: The natural numbers are zero, one, two, ...; The double-digit numbers are ten, eleven, twelve, ..., ninety-nine. Ambiguity and incompleteness of expression resulting from the use of the ellipsis in arithmetical sentences has been discussed by philosophers including Frege and Wittgenstein.

CITE AS: Corcoran, J. 1999. "Laws of thought". *Cambridge Dictionary of Philosophy*. R.Audi, Ed. Cambridge: Cambridge UP. p. 489.

laws of thought, laws by which or in accordance with which valid thought proceeds, or that justify valid inference, or to which all valid deduction is reducible. Laws of thought are rules that apply without exception to any subject matter of thought, etc.; sometimes they are said to be the object of logic. The term, rarely used in exactly the same sense by different authors, has long been associated with three equally ambiguous expressions: the law of identity (ID), the law of contradiction (or non-contradiction; NC), and the law of excluded middle (EM).

Sometimes, these three expressions are taken as propositions of formal ontology having the widest possible subject matter, propositions that apply to entities *per se:* (ID), everything is (i.e., is identical to) itself; (NC) no thing having a given quality also has the negative of that quality (e.g., no even number is non-even); (EM) every thing either has a given quality or has the negative of that quality (e.g., every number is either even or non-even). Equally common in older works is use of these expressions for principles of metalogic about propositions: (ID) every proposition implies itself; (NC) no proposition is both true and false; (EM) every proposition is either true or false. Beginning in the middle to late 1800s these expressions have been used to denote propositions of Boolean Algebra about classes: (ID) every class includes itself; (NC) every class is such that its intersection ("product") with its own complement is the null class; (EM) every class is such that its union ("sum") with its own complement is the universal class. More recently the last two of the three expressions have been used in connection with the classical propositional logic and with the so-called *protothetic* or quantified propositional logic; in both cases the law of non-contradiction involves the negation of the conjunction ('and') of something with its own negation and the law of excluded middle involves the disjunction ('or') of something with its own negation. In the case of propositional logic the "something" is a schematic letter serving as a place-holder, whereas in the case of protothetic logic the "something" is a genuine variable. The expressions 'law of non-contradiction' and 'law of excluded middle' are also used for semantic principles of model theory concerning sentences and interpretations: (NC) under no interpretation is a given sentence both true and false, (EM) under any interpretation, a given sentence is either true or false.

The expressions mentioned above all have been used in many other ways. Many other propositions have also been mentioned as laws of thought, including the *dictum de omni et nullo* attributed to Aristotle, the substitutivity of identicals (or equals) attributed to Euclid, the so-called identity of indiscernibles attributed to Leibniz, and other "logical truths." The expression "laws of thought" gained added prominence through its use by Boole (1815-64) to denote theorems of his "algebra of logic"; in fact, he named his second logic book *An Investigation of the Laws of Thought* (1854). Modern logicians, in almost unanimous disagreement with Boole, take this expression to be a misnomer; none of the above propositions classed under 'laws of thought' are explicitly about thought per se, a mental phenomenon studied by psychology, nor do they involve explicit reference to a thinker or knower as would be the case in pragmatics or in epistemology. The distinction between psychology (as a study of mental phenomena) and logic (as a study of valid inference) is widely accepted.

See also CONVENTIONALISM, DICTUM DE OMNI ET NULLO, PHILOSOPHY OF LOGIC, SET THEORY. J. Cor.

CITE AS: Corcoran, J. 1999. "Limiting case". *Cambridge Dictionary of Philosophy*. R.Audi, Ed. Cambridge: Cambridge UP. p. 504-5.

limiting case, an expression used more or less loosely to indicate that an individual or subclass of a given background class is maximally remote from "typical" or "paradigm" members of the class with respect to some ordering which is not always explicitly mentioned. The number zero is a limiting case of cardinal number. A triangle is a limiting case of polygon. A square is a limiting case of rectangle when rectangles are ordered by ratio of length to width. Certainty is a limiting case of belief when beliefs are ordered according to "strength of subjective conviction"; Knowledge is a limiting case of belief when beliefs are ordered according "adequacy of objective grounds". A limiting case is necessarily a known case (member) of the background class; in contrast a BORDERLINE CASE need not be a case and a DEGENERATE CASE may clearly be not a case at all. **See also BORDERLINE CASE and DEGENERATE CASE**. J. Cor.

CITE AS: Corcoran, J. 1999. "Logical form". *Cambridge Dictionary of Philosophy*. R.Audi, Ed. Cambridge: Cambridge UP. p. 511-12.

logical form, the form obtained from a proposition, a set of propositions or an argument by abstracting from the subject-matter of its content terms or by regarding the content terms as mere place-holders or blanks in a form. In a logically perfect language the logical form of a proposition, a set of propositions or an argument is determined by the grammatical form of the sentence, the set of sentences or the argument-text expressing it. Two sentences, sets of sentences or argument-texts are said to have the same grammatical form, in this sense, if a uniform one-one substitution of content words transforms the one exactly into the other. The sentence 'Abe properly respects every agent who respects himself' may be regarded as having the same grammatical form as the sentence 'Ben generously assists every patient who assists himself'. Substitutions used to determine sameness of grammatical form cannot involve change of form words such as 'every', 'no', 'some', 'is', etc. and they must be category-preserving i.e. they must put a proper name for a proper name, an adverb for an adverb, a transitive verb for a transitive verb, and so on. Two sentences having the same grammatical form have exactly the same form words distributed in exactly the same pattern and, although they of course need not have, and usually do not have, the same content words, they do have exactly the same number of different content words. The most distinctive feature of form words, which are also call syncategorematic terms or logical terms, is their topic neutrality; the form words in a sentence are entirely independent of and are in no way indicative of its content or topic.

Modern formal languages used in formal axiomatizations of mathematical sciences are often taken as examples of logically perfect languages. Pioneering work on logically perfect languages was done by George Boole (1815-64), Gottlob Frege (1848-1925), Giuseppe Peano (1858-1952), Bertrand Russell (1872-1970) and Alonzo Church (1903-1995). According to the principle of logical form, an argument is [logically or formally] valid or invalid in virtue of logical form. More explicitly, every two arguments in the same form are both valid or both invalid. Thus, every argument in the same form as a valid argument is valid and every argument in the same form as an invalid argument is invalid. The argument form that a given argument fits (or has) is not determined solely by the logical forms of its constituent propositions; the arrangement of those propositions is critical because the process of interchanging a premise with the conclusion of a valid argument can result in an invalid argument. Of course, the principle of logical form, from which formal logic gets its name does not apply to non-formal conceptions such as material, enthymematic, or analytic validity.

The principle of logical form is commonly used in establishing invalidity of arguments and consistency of sets of propositions. In order to show that a given argument is invalid it is sufficient to exhibit another argument as being in the same logical form and as having all true premises and false conclusion. In order to show that a given set of propositions is consistent it is sufficient to exhibit another set of propositions as being in the same logical form and as being composed exclusively of true propositions. The history of these methods traces back through noncantorian set theory, noneuclidian geometry and medieval logicians (especially Saint Anselm) to Aristotle. These methods must be used with extreme caution in languages such as English which fail to be logically perfect as a result of ellipsis, amphiboly, ambiguity, etc. For example, “This is a male dog” implies “This is a dog” but “This is a brass monkey” does not imply “This is a monkey”, as would be required in a logically perfect language. Likewise, of two propositions commonly expressed by the ambiguous sentence ‘Ann and Ben are married’ one does and one does not imply the proposition that Ann is married to Ben.

Quine and other logicians are careful to distinguish, in effect, the (unique) logical form of a proposition from its (many) schematic forms. The proposition (A) “if Abe is Ben, then if Ben is wise Abe is wise” has exactly one logical form, which it shares with (B) “if Carl is Dan then if Dan is kind Carl is kind” whereas it has all of the following schematic forms: (1) if P then if Q then R; (2) if P then Q; (3) P. The principle of form for propositions is that every two propositions in the same logical form are both tautological (logically necessary) or both non-tautological. Thus, although propositions A and B are tautological, there are non-tautological propositions that fit the three schematic forms just mentioned.

Failure to distinguish logical form from schematic form has led to fallacies. According to the principle of logical form quoted above, every argument in the same logical form as an invalid argument is invalid, but it is not the case that every argument sharing a schematic form with an invalid argument is invalid. Contrary to what would be fallaciously thought, the conclusion “Abe is Ben” is logically implied by the following two propositions taken together, “if Abe is Ben then Ben is Abe” and “Ben is Abe” even though the argument shares a schematic form with invalid arguments “committing” the fallacy of affirming the consequent. **See also AMBIGUITY, FORMAL LOGIC, LAWS OF THOUGHT, LOGICAL SYNTAX, TAUTOLOGY**. J. Cor.

CITE AS: Corcoran, J. 1999. “Logical subject”. *Cambridge Dictionary of Philosophy*. R.Audi, Ed. Cambridge: Cambridge UP. p. 511-12.

logical subject, in Aristotelian and traditional logic, the common noun, or sometimes the intension or the extension of the common noun, that follows the initial quantifier word (‘every’, ‘some’, ‘no’, etc.) of a sentence as opposed to the grammatical subject which is the entire noun phrase including the quantifier and the noun, and in some usages, any modifiers that may apply. The grammatical subject of ‘Every number exceeding zero is positive’ is ‘every number’, or in some usages, ‘every number exceeding zero’, whereas the logical subject is ‘number’, or the intension or the extension of ‘number’. Similar distinctions are made between the logical predicate and the grammatical predicate: in the above example, ‘is positive’ is the grammatical predicate whereas the logical predicate is the adjective ‘positive’, or sometimes the property of being positive or even the extension of the word ‘positive’. In standard first-order logic the logical subject of a sentence under a given interpretation is the entire universe of discourse of the interpretation. See also **LOGICAL FORM, GRAMMAR, SUBJECT, UNIVERSE OF DISCOURSE.** J. Cor.

CITE AS: Corcoran, J. 1999. “Material adequacy” in *Cambridge Dictionary of Philosophy*. R.Audi, Ed. Cambridge: Cambridge UP.

material adequacy, the property that belongs to a formal definition of a concept when that definition characterizes or “captures” the extension (or material) of the concept. Intuitively, a formal definition of a concept is materially adequate if and only if it is neither “too broad” nor “too narrow”. Alfred Tarski advanced the state of philosophical semantics by discovering the criterion of material adequacy of truth definitions contained in his convention T. Material adequacy contrasts with analytic adequacy, which belongs to definitions that provide a faithful analysis. Defining an integer to be even if and only if it is not the sum of two consecutive integers would be materially adequate but not analytically adequate, whereas defining an integer to be even if and only if it is a multiple of two would be both materially and analytically adequate. Material adequacy also contrasts with formal correctness, which belongs to definitions that meet certain grammatical requirements designed to prohibit circularity and other similar defects. **See also CONVENTION T, DEFINITION, FORMAL SEMANTICS, TARSKI, TRUTH.** J. Cor.

CITE AS: Corcoran, J. 1999. “Mathematical analysis”. *Cambridge Dictionary of Philosophy*. R.Audi, Ed. Cambridge: Cambridge UP. p. 540-41.

mathematical analysis, also called standard analysis, the area of mathematics pertaining to the so-called real number system, i.e. the area that can be based on an axiom set whose intended interpretation (standard model) has the set of real numbers as its domain (universe of discourse). Thus, analysis includes, among its many subbranches, elementary algebra, differential and integral calculus, differential equations, the calculus of variations, and measure theory. Analytic geometry involves the application of analysis to geometry. Analysis contains a large part of the mathematics used in mathematical physics. The real numbers, which are representable by the ending and unending decimals, are usefully construed as (or as corresponding to) distances measured, relative to an arbitrary unit length, positively to the right and negatively to the left of an arbitrarily fixed zero point along a geometrical straight line. In particular, the class of *real numbers* includes, as increasingly comprehensive proper subclasses, the natural numbers, the integers (positive, negative, and zero), the rational numbers (or fractions), and the algebraic numbers (such as the square root of two). Especially important is the presence in the class of real numbers of non-algebraic (or transcendental) irrational numbers such as pi. The set of real numbers includes arbitrarily small and arbitrarily large, finite quantities, while excluding infinitesimal and infinite quantities.

Analysis, often conceived as the mathematics of continuous magnitude, contrasts with arithmetic (natural number theory), which is regarded as the mathematics of discrete magnitude. Analysis is often construed as involving not just the real numbers but also the imaginary (complex) numbers. Traditionally, analysis is expressed in a second-order or higher-order language wherein its axiom set has categoricity; each of its models is isomorphic to (has the same structure as) the standard model. When analysis is carried out in a first-order language, as has been increasingly the case since the 1950s, categoricity is impossible and it has nonstandard models in addition to its standard model. A *non-standard model* of analysis is an interpretation not isomorphic to the standard model but nevertheless satisfying the axiom set. Some of the non-standard models involve objects reminiscent of the much-despised "infinitesimals" that were essential to the Leibniz approach to calculus and that were subject to intense criticism by Berkeley and other philosophers and philosophically sensitive mathematicians. These non-standard models gave rise to a new area of mathematics, non-standard analysis, within which the fallacious arguments used by Leibniz and other early analysts form the heuristic basis of new and entirely rigorous proofs.

See also CALCULUS, CATEGORICITY, PHILOSOPHY OF MATHEMATICS.J. Cor.

CITE AS: Corcoran, J. 1999. "Omega". *Cambridge Dictionary of Philosophy*. R.Audi, Ed. Cambridge: Cambridge UP. p. 529-30.

omega, the last letter of the Greek alphabet (ω). Following Cantor (1845-1911), it is used in lowercase as a proper name for the first infinite ordinal number, which is the ordinal of the natural ordering of the set of finite ordinals. By extension it is also used as a proper name for the set of finite ordinals itself or even for the set of natural numbers. Following Gödel (1906-78), it is used as a prefix in names of various logical properties of sets of sentences, most notably omega-completeness and omega-consistency.

Omega-completeness, in the original sense due to Tarski, is a syntactical property of sets of sentences in a formal arithmetic language involving a symbol '0' for the number zero and a symbol 's' for the so-called *successor function*, resulting in each natural number being named by an expression, called a numeral, in the following series: '0', 's0', 's00', and so on. For example, five is denoted by 'sssss0'. A set of sentences is said to be omega-complete if it (deductively) yields every universal sentence all of whose singular instances it yields. In this framework, as usual, every universal sentence, 'for every *n*, *n* has *P*', yields each and every one of its singular instances, '0 has P', 's0 has P', 'ss0 has P', etc. However, as had been known by logicians at least since the Middle Ages, the converse is not true, i.e., it is not in general the case that a universal sentence is deducible from the set of its singular instances. Thus, one should not expect to find omega-completeness except in exceptional sets. The set of all true sentences of arithmetic is such an exceptional set; the reason is the semantic fact that every universal sentence (whether or not in arithmetic) is materially equivalent to the set of all its singular instances. A set of sentences that is not omega-complete is said to be omega-incomplete. The existence of omega-incomplete sets of sentences is a phenomenon at the core of the 1931 Gödel incompleteness result, which shows that every “effective” axiom set for arithmetic is omega-incomplete and thus has as theorems all singular instances of a universal sentence that is not one of its theorems. Although this is a remarkable fact, the existence of omega-incomplete sets *per se* is far from remarkable, as suggested above, in fact, the empty set and, equivalently, the set of all tautologies are omega-incomplete because each yields all singular instances of the non-tautological formal sentence, here called *FS*, that expresses the proposition that every number is either zero or a successor.

Omega-consistency belongs to a set that does not yield the negation of any universal sentence all of whose singular instances it yields. A set that is not omega-consistent is said to be omega-inconsistent. Omega-consistency, of course, implies consistency in the ordinary sense; but it is easy to find consistent sets that are not omega-consistent, e.g., the set whose only member is the negation of the formal sentence *FS* mentioned above. Corresponding to the syntactical properties just mentioned, there are analogous semantic properties whose definitions are obtained by substituting ‘(semantically) implies’ for ‘(deductively) yields’.

The Greek letter omega and its English name have many other uses in modern logic. Carnap introduced a non-effective, non-logical rule, called the *omega rule*, for “inferring” a universal sentence from its singular instances; adding the omega rule to a standard axiomatization of arithmetic produces a complete but non-effective axiomatization. An omega-valued logic is a many-valued logic whose set of truth-values is or is the same size as the set of natural numbers.

See also COMPLETENESS, CONSISTENCY, GÖDEL’S INCOMPLETENESS THEOREMS. J. Cor.

ERRATA CORRECTED: ‘Omega-inconsistency, of course, implies consistency’ changed to ‘Omega-consistency, of course, implies consistency’.

CITE AS: Corcoran, J. 1999. “Proof by recursion”. *Cambridge Dictionary of Philosophy*. R.Audi, Ed. Cambridge: Cambridge UP. p. 749-50.

proof by recursion, also called proof by mathematical induction, a method for conclusively demonstrating the truth of universal propositions about the natural numbers. The system of (natural) numbers is construed as an infinite sequence of elements beginning with the number 1 and such that each subsequent element is the (immediate) successor of the preceding element. The (immediate) successor of a number is the sum of that number with 1. In order to apply this method to show that every number has a certain chosen property it is necessary to demonstrate two subsidiary propositions often called respectively the basis step and the inductive step. The *basis step* is that the number 1 has the chosen property; the *inductive step* is that the successor of any number having the chosen property is also a number having the chosen property (in other words, for every number n, if n has the chosen property then the successor of n also has the chosen property). The inductive step is itself a universal proposition that may have been proven by recursion.

The most commonly used example of a theorem proved by recursion is the remarkable fact, known before the time of Plato, that the sum of the first n odd numbers is the square of n. This proposition, mentioned prominently by Leibniz as requiring and having demonstrative proof, is expressed in universal form as follows: for every number n, the sum of the first n odd numbers is n^2.

$1 = 1^2$, $(1 + 3) = 2^2$, $(1 + 3 + 5) = 3^2$, and so on.

Rigorous formulation of a proof by recursion often uses as a premise the proposition called, since the time of De Morgan, the principle of mathematical induction: every property belonging to 1 and belonging to the successor of every number to which it belongs is a property that belongs without exception to every number. Peano (1858-1932) took the principle of mathematical induction as an axiom in his 1889 axiomatization of arithmetic (or the theory of natural numbers). The first acceptable formulation of this principle is attributed to Pascal.
See also DE MORGAN, OMEGA, PHILOSOPHY OF MATHEMATICS. J. Cor.

CITE AS: Corcoran, J. 1999. "Scheme". *Cambridge Dictionary of Philosophy*. R.Audi, Ed. Cambridge: Cambridge UP. p. 818.

Scheme, also schema (plural schemata), a metalinguistic frame or template used to specify an infinite set of sentences, its instances, by finite means, often taken with a side condition on how its blanks or place-holders are to be filled. The sentence 'Either Abe argues or it is not the case that Abe argues' is an instance of the excluded middle scheme for English: 'Either...or it is not the case that...' where the two blanks are to be filled with one and the same (well-formed declarative) English sentence. Since first-order number theory can not be finitely axiomatized the mathematical induction scheme is used to effectively specify an infinite set of axioms: 'If zero is such that...and the successor of every number such that...is also such that..., then every number is such that...' where the four blanks are to be filled with one and the same arithmetic open sentence such as 'it precedes its own successor' or 'it is finite'. Among the best known is Tarski's scheme T: '...is a true sentence if and only if...' where the second blank is filled with a sentence and the first blank is filled by a name of the sentence. **See also CONVENTION T, LOGICAL FORM, METALANGUAGE OPEN SENTENCE, PHILOSOPHY OF MATHEMATICS, TARSKI.** J. Cor.

CITE AS: Corcoran, J. 1999. "Scope". *Cambridge Dictionary of Philosophy*. R.Audi, Ed. Cambridge: Cambridge UP. p. 822.

scope, the "part" of the sentence (or proposition) to which a given term occurrence "applies" under a given interpretation of the sentence. If the sentence 'Abe does not believe Ben died' is interpreted as expressing the proposition that Abe believes that it is not the case that Ben died, the scope of the occurrence of 'not' is 'Ben died'; interpreted as "It is not the case that Abe believes that Ben died", the scope is the rest of the sentence, i.e. 'Abe believes Ben died'. In the first case we have "narrow scope", in the second "wide scope". If 'Every number is not even' is interpreted with narrow scope, it expresses the false proposition "Every number is non-even", which is logically equivalent to "No number is even". Taken with wide scope it expresses the true proposition "Not every number is even", which is equivalent to "Some number is non-even". Under normal interpretations of the sentences, the term 'hardened' has narrow scope in 'Carl is a hardened recidivist' whereas 'alleged' has wide scope in 'Dan is an alleged criminal'. Accordingly, "Carl is a hardened recidivist" logically implies "Carl is a recidivist", whereas "Dan is an alleged criminal", being equivalent to "Allegedly, Dan is a criminal", does not imply "Dan is a criminal". Insertion of the word 'only' in various places in a sentence can be used as an experiment to demonstrate its array of "potential scopes", as in 'Ed only says what is acceptable to Fran'. Scope considerations are useful in analyzing structural ambiguity and in understanding the difference between the grammatical form of a sentence and the logical form of a proposition it expresses. In a logically perfect language grammatical form mirrors logical form, there is no scope ambiguity, the scope of a given term occurrence is uniquely determined by its context. **See also AMBIGUITY; CONVERSE; CONVERSE, OUTER AND INNER; RELATION, STRUCTURAL AMBIGUITY**. J. Cor.

CITE AS: Corcoran, J. 1999. "Tarski, Alfred". *Cambridge Dictionary of Philosophy*. R.Audi, Ed. Cambridge: Cambridge UP. p. 902

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Tarski, Alfred (1901- 1983) Polish-born American mathematician, logician, and philosopher of logic famous for his investigations of the concepts of truth and consequence conducted in the 1930s. His analysis of the concept of truth in syntactically precise, fully interpreted languages resulted in a definition of truth and an articulate defense of the correspondence theory of truth. Sentences of the following kind are now known as Tarskian biconditionals: 'The sentence "Every perfect number is even" is true if and only if every perfect number is even.' One of Tarski's major philosophical insights is that each Tarskian biconditional is, in his words, a partial definition of truth and, consequently, all Tarskian biconditionals whose right-hand sides exhaust the sentences of a given formal language together constitute an implicit definition of 'true' as applicable to sentences of that given formal language. This insight, because of its penetrating depth and disarming simplicity, has become a staple of modern analytic philosophy. Moreover, it in effect reduced the philosophical problem of defining truth to the logical problem of constructing a single sentence having the form of a definition and having as consequences each of the Tarskian biconditionals. Tarski's solution to this problem is the famous Tarski truth definition, versions of which appear in virtually every mathematical logic text.

Tarski's second most widely recognized philosophical achievement was his analysis and explication of the concept of consequence. Consequence is interdefineable with validity as applied to arguments: a given conclusion is a consequence of a given premise-set if and only if the argument composed of the given conclusion and the given premise-set is valid; conversely, a given argument is valid if and only if its conclusion is a consequence of its premise-set. Shortly after discovering the truth definition, Tarski presented his "no-countermodels" definition of consequence: a given sentence is a consequence of a given set of sentences if and only if every model of the set is a model of the sentence (in other words, if and only if there is no way to reinterpret the non-logical terms in such a way as to render the sentence false while rendering all sentences in the set true). As Quine has emphasized, this definition reduces the modal notion of logical necessity to a combination of syntactic and semantic concepts, thus avoiding reference to modalities and/or to "possible worlds."

After Tarski's definitive work on truth and on consequence, he devoted his energies largely to more purely mathematical work. For example, in answer to Gödel's proof that arithmetic is incomplete and undecidable, Tarski showed that algebra and geometry are both complete and decidable. Tarski's truth definition and his consequence definition are found in his 1956 collection *Logic, Semantics, Metamathematics* (2d ed., 1983): article VIII, pp. 152-278, contains the truth definition; article XVI, pp. 409-20, contains the consequence definition. His published articles, nearly 3,000 pages in all, have been available together since 1986 in the four-volume *Alfred Tarski, Collected Papers*, edited by S. Givant and R. McKenzie.

See also GÖDEL'S INCOMPLETENESS THEOREMS, LOGICAL CONSEQUENCE, TRUTH. J. Cor.

CITE AS: Corcoran, J. 1999. "Tautology". *Cambridge Dictionary of Philosophy*. R.Audi, Ed. Cambridge: Cambridge UP. p. 902-3.

tautology, a proposition whose negation is inconsistent, or (self-) contradictory, e.g., 'Socrates is Socrates', 'Every human is either male or non-male', 'No human is both male and non-male', 'Every human is identical to itself', 'If Socrates is human then Socrates is human'. A proposition that is (or is logically equivalent to) the negation of a tautology is called a (self-)contradiction. According to classical logic, the property of being implied by its own negation is a necessary and sufficient condition for being a tautology and the property of implying its own negation is a necessary and sufficient condition for being a contradiction. Tautologies are logically necessary and contradictions are logically impossible.

Epistemically, every proposition that can be known to be true by purely logical reasoning is a tautology, and every proposition that can be known to be false by purely logical reasoning is a contradiction. The converses of these two statements are both controversial among classical logicians. Every proposition in the same logical form as a tautology is a tautology and every proposition in the same logical form as a contradiction is a contradiction. For this reason sometimes a tautology is said to be *true in virtue of form* and a contradiction is said to be *false in virtue of form*; being a tautology and being a contradiction (tautologousness and contradictoriness) are formal properties. Since the logical form of a proposition is determined by its logical terms ('every', 'some', 'is', etc.), a tautology is sometimes said to be true in virtue of its logical terms and likewise *mutais mutandis* for a contradiction.

Since tautologies do not exclude any logical possibilities, they are sometimes said to be "empty" or "uninformative"; and there is a tendency even to deny that they are genuine propositions and that knowledge of them is genuine knowledge. Since each contradiction "includes" (implies) all logical possibilities (which, of course, are jointly inconsistent), contreadtictions are sometimes said to be "overinformative." Tautologies and contradictions are sometimes said to be "useless," but for opposite reason. Most precisely, according to classical logic, being implied by each and every proposition is necessary and sufficient for being a tautology and, coordinately, implying each and every proposition is necessary and sufficient for being a contradiction.

Certain developments in mathematical logic, especially model theory and modal logic, seem to support use of Leibniz's expression 'true in all possible worlds' in connection with tautologies. There is a special subclass of tautologies called *truth-functional tautologies* that are true in virtue of a special subclass of logical terms called *truth-functional connectives* ('and', 'or', 'not', 'if', etc.). Some logical writings use 'tautology' exclusively for truth-functional tautologies and thus replace "tautology" in its broad sense by another expression, e.g., 'logical truth'. Tarski, Gödel, Russell, and many other logicians have used the word in its broad sense, but use of it in its narrow sense is widespread and entirely acceptable.

Propositions known to be tautologies are often given as examples of *a priori* knowledge. In philosophy of mathematics, the logistic hypothesis of logicism is the proposition that every true proposition of pure mathematics is a tautology. Some writers make a sharp distinction between the formal property of being a tautology and the non-formal metalogical property of being a law of logic. For example, ‘One is one’ is not metalogical but is a tautology, whereas ‘No tautology is a contradiction is metalogical but is not a tautology.

See also LAWS OF THOUGHT, LOGICAL FORM, LOGICISM. J. Cor.

CITE AS: Corcoran, J. 1999. “Universe of discourse”. *Cambridge Dictionary of Philosophy*. R.Audi, Ed. Cambridge: Cambridge UP. p. 941.

Universe of discourse, of a discussion, treatise, or discourse, the usually limited class of individuals under discussion, whose existence is presupposed by the discussants and which in some sense constitute the ultimate subject-matter of the discussion. Once the universe of a discourse has been established, expressions such as ‘every object’ and ‘some object’ refer respectively to every object in the universe of discourse and to some object in the universe of discourse. The concept of universe of discourse is due to De Morgan in 1846, but the expression was coined by Boole eight years later. When a discussion is formalized in an interpreted standard first-order language, the universe of discourse is taken as the “universe” of the interpretation, i.e. as the range of values of the variables. Quine and others have emphasized that the universe of discourse represents an ontological commitment of the discussants. In a discussion in a particular science, the universe of discourse is often wider than the domain of the science, although economies of expression can be achieved by limiting the universe of discourse to the domain. **See also DOMAIN, FORMAL LOGIC, MODEL THEORY, ONTOLOGICAL COMMITMENT, VARIABLE**. J. Cor.

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