MR2935338 (Review)   03A05 01A55
Macbeth, Danielle (1-HAV-Q)
Diagrammatic reasoning in Frege's *Begriffsschrift*. (English summary)
Synthese **186** (2012), *no. 1,* 289–314.

Summary: "In Part III of his 1879 logic Frege proves a theorem in the theory of sequences on the basis of four definitions. He claims in *Grundlagen* that this proof, despite being strictly deductive, constitutes a real extension of our knowledge, that it is ampliative rather than merely explicative. Frege furthermore connects this idea of ampliative deductive proof to what he thinks of as a fruitful definition, one that draws new lines. My aim is to show that we can make good sense of these claims if we read Frege's notation diagrammatically, in particular, if we take that notation to have been designed to enable one to exhibit the (inferentially articulated) contents of concepts in a way that allows one to reason deductively on the basis of those contents."

From the conclusion: "In Frege's concept-script, we have seen, definitions are necessary if one is to prove something about the defined concepts with which one begins. One needs the defined sign, the *definiendum*, if one is to prove something about the particular concepts one cares about; but one needs also the *definiens* if the needed logical bonds are to be forged. Only a proof that begins with definitions, and indeed with fruitful definitions, definitions that are not merely truth-functional, can be ampliative. But it is equally true that only within a proof are definitions of any interest; because definitions are stipulations, the judgments that follow from them taken one by one are, we have seen, utterly trivial. It is only definitions and proofs working together that can yield something new. Only *within* a proof can the peculiar power that resides in definitions by their nature as stipulations regarding simple and complex signs be harnessed to realize something new. In a slogan: proofs without definitions are empty, merely the aimless manipulation of signs according to rules; and definitions without proofs are, if not blind, then dumb. Only a proof can realize the potential of definitions to speak to one another, to pool their resources so as to realize something new."

Unfortunately, the paper omits definitions of key expressions including "(inferentially articulated) contents of concepts", "ampliative", "explicatory", "inference potential", "truth-condition", and "extension of our knowledge". It attributes generally to modern logicians information-containment views that were never expressed by any of the prominent logicians such as Hilbert, Gödel, Tarski, Church, and Quine. The paper distances Frege from modern logicians. On page 309 we find the following paragraph.

"In our [sc. modern] logics it is assumed that inference potential is given by truth conditions. Hence, we think, deduction can be nothing more than a matter of making explicit information that is already contained in one's premises. If the deduction is valid then the information contained in the conclusion must be contained already in the premises; if that information is not contained already in the premises [. . .], then the argument cannot be valid."

Although the paper is meticulous in citing supporting literature for less questionable points, no references are given for this. In fact, the view that deduction is the making explicit of information that is only implicit in premises has not been espoused by any current standard symbolic logic books. Between purely syntax-based proof theory and purely set-based semantics or model theory, there is no room for information-based deduction and, from the now-standard view of logic, there is no need for "information". The concept of information per se does not occur in any of the theorems of modern

logic.

Information-based views of deduction were common in earlier times before the dominance of proof theory and model theory. One of the last major figures to endorse such a view was F. P. Ramsey. In 1926 he wrote: "I can accept the account of their validity which has been given by many authorities and [...] substantially the same in Kant, De Morgan, Peirce, and Wittgenstein. All these authors agree that the conclusion of a formally valid argument is contained in its premisses; [...] that a formal deduction does not increase our knowledge, but only brings out clearly what we already know in another form. [...] The logical relation which justifies the inference is that the sense or import [sc. information] of the conclusion is contained in that of the premisses." See pp. 185–186 in Ramsey's 1926 essay "Truth and probability", published posthumously in an anthology [F. P. Ramsey, in *Foundations of mathematics*, 156–198, Routledge and Kegan Paul, London, 1931].

Information-based perspectives have only recently been revived and only by a small minority of philosophical logicians, mostly from a younger generation; for example, see the prize-winning essay by J. M. Sagüiillo [Hist. Philos. Logic **30** (2009), no. 1, 21–48; MR2488683 (2009k:03010)]. *J. Corcoran*