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Tarski, Alfred

Collected papers. Vol. 1.

1921--1934. Edited by Steven R. Givant and Ralph N. McKenzie.

Contemporary Mathematicians.

Birkhäuser Verlag, Basel, 1986. xiv+659 pp. *ISBN* 3-7643-3280-8

Alfred Tarski (1901--1983) is widely regarded as one of the two giants of twentieth-century logic and also as one of the four greatest logicians of all time (Aristotle, Frege and Gödel being the other three). Of the four, Tarski was the most prolific as a logician. The four volumes of his collected papers, which exclude most of his 19 monographs, span over 2500 pages. Aristotle's writings are comparable in volume, but most of the Aristotelian corpus is not about logic, whereas virtually everything written by Tarski concerns logic more or less directly. There is no doubt that Tarski wrote more on logic than any other author; he started publishing on logic in 1921 at the age of 20 and continued until his death at the age of 82. Two of his works appeared posthumously [Hist. Philos. Logic **7** (1986), no. 2, 143--154; MR0868748 (88b:03010); Tarski and Givant, *A formalization of set theory without variables*, Amer. Math. Soc., Providence, RI, 1987; MR0920815 (89g:03012)]. Tarski's voluminous writings were widely scattered in numerous journals, some quite rare. It has been extremely difficult to study the development of Tarski's thought and to trace the interconnections and interdependence of his various papers. Thanks to the present collection all this has changed, and it is likely that the increased accessibility of Tarski's papers will have the effect of increasing Tarski's already enormous influence.

The present collection contains all of Tarski's "papers" (in their original languages), except the three assembled in Tarski, A. Mostowski, and R. M. Robinson's book *Undecidable theories* North-Holland, Amsterdam, 1953; MR0058532 (15,384h)], the above-mentioned posthumous article, and the historically important 1933 Polish original of the famous truth-definition article, which the reader would have expected to find in Volume 1, covering 1921--1934, and whose 1935 German translation appears in Volume 2, covering 1935--1944. The included German translation is actually longer than the excluded Polish original by virtue of several pages of added postscript.

The fact that almost all of the papers in the collection are photographic copies of the originals is very convenient, and in some cases essential, for historical research. The original pagination has been deleted, but it can readily be recovered because each paper is supplied with a fresh title page giving the original starting and ending pagination.

In Volume 1 \{see the next three reviews for Volumes 2--4\} there are articles in Polish, French and German, together with English translations of two Polish articles on the "degree of equivalence" of (plane, Euclidean) polygons. Nine of the papers in this volume (four French, five German) have

already been translated by J. H. Woodger in Tarski's well-known earlier collection, *Logic, semantics, metamathematics. Papers from 1923 to 1938*, [Oxford Univ. Press, Oxford, 1956; MR0078296 (17,1171a); second edition, Hackett, Indianapolis, IN, 1983; MR0736686 (85e:01065)], cited below as LSM. Since Tarski himself saw neither the translations nor the printer's proofs of the translations in the first edition of LSM before they were printed, it is not surprising that he saw fit to make various changes in the second edition of LSM. Moreover, it was not until after publication that Tarski was able to see the article "Les opérations logiques et les ensembles projectifs" coauthored with C. Kuratowski [in this collection, Vol. 1, see heading, 549--560]; Tarski and Kuratowski obtained the results together but Kuratowski wrote the paper on his own. Unfortunately, the paper does not meet Tarski's standards.

The collection lacks a preface that could have given the reader the above and other useful information. Moreover, there are no indices. However, each paper is preceded by a page giving bibliographic information in addition to the title page already mentioned. Volume 4 ends with a complete Tarski bibliography prepared by Givant and previously published with the so-called Tarski Survey [J. Symbolic Logic **51** (1986), no. 4, 913--941; MR0865919 (88e:01057)].

This excellent collection will be valued by logicians, philosophers of logic and historians of logic for generations to come. Its four tasteful and sturdy volumes required an immense expenditure of energy, time and thought. A credit to its editors and its publisher, it is a fitting memorial to its author.

{Volumes 2--4 are reviewed below.}

Reviewed by *J. Corcoran*

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Collected papers. Vol. 2.
1935--1944. Edited by Steven R. Givant and Ralph N. McKenzie.
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{For Volume 1 see the preceding review.}

Of the one hundred-odd papers written by Alfred Tarski in his lifetime, some of the best known are reprinted in this, the second volume of his *Collected papers.* This volume is sure to be consulted more often than any of the other three. Here there are thirty-one papers photographically reproduced in their original languages (German, French and English). Eight of the German papers have already appeared in English translation by J. H. Woodger in LSM (see the reference in the preceding review). Tarski took the opportunity afforded by the second edition of LSM to make various typographical,

translational and technical emendations, to do further cross-referencing of other work both earlier and later, and to expand the historical notes (see Tarski's postscript, LSM, second edition, p. xiv). For these reasons and because changes had been introduced in the course of some of the Woodger translations, it has become even more important, particularly from the perspective of a historian of mathematical logic, to have the original papers. The editors and the publisher are to be especially commended for providing photographic copies and not mere reprintings or translations.

Almost one hundred fifty of the seven hundred pages of Volume 2 are devoted to the famous "Wahrheitsbegriff", the German translation of Tarski's 1933 Polish paper on the concept of truth in formalized languages, which is among the most important papers ever written on mathematical logic. In many respects it marks a watershed between pre-model-theoretic and model-theoretic mathematical logic. Not only does this paper provide a mathematically rigorous articulation of several ideas that had been developing in earlier mathematical logic, it also presents foundations on which later logic could be built. For example, it gives a categorical axiomatization of string theory, something that had never been attempted before in the history of logic. The fact that an axiomatic treatment of formal syntax was needed by the logic community of the time is further confirmed by the appearance at about the same time of another axiomatization independently constructed on a different primitive basis by the distinguished German mathematical logician Hans Hermes, "Semiotik, eine Theorie der Zeichengestalten als Grundlage für Untersuchungen von formalisierten Sprachen", which is the entire no. 5 of *Forschungen zur Logik und zur Grundlegung der exakten Wissenschaften* [Leipzig, 1938; Zbl **20**, 98]. In addition to the philosophical, expositional, and foundational contents of the "Wahrheitsbegriff", it also contains the first statement and proof of what is now known as Tarski's theorem, viz. that the property of being a Gödel number of a true sentence of number theory is not definable in terms of the standard primitives of number theory or, very loosely, that number-theoretical truth is not number-theoretically definable.

Other especially important papers in this volume include the 1936 German paper on logical consequence (pp. 271--281), the 1937 French paper on the deductive method (pp. 325--333), and the 1941 English paper on the calculus of relations (pp. 571--587). The first of these three is widely accepted as a definitive explication of the semantic notion of logical implication (as opposed to material implication and to deducibility) as it occurs in the mathematical literature. The second is a paper that Tarski came to reaffirm in the 1970s as a succinct articulation of his views on the nature and purpose of the deductive method and the role of logic in scientific thought. This paper had been regarded as supplanted by Chapter 6 of Tarski's *Introduction to logic and to the methodology of deductive sciences* [Oxford Univ. Press, New York, 1941; MR0003375 (2,209a); fourth edition, edited by J. Tarski, forthcoming]. A. Church wrote in a review [J. Symbolic Logic **6** (1941), 30--32] that this Chapter 6 "should altogether supersede

older and more familiar, but less accurate accounts" of the axiomatic method. The French article, which is written entirely in natural language without logical or mathematical symbols and which can be read by itself, was regarded by Tarski as superior to the above-mentioned Chapter 6, which involves symbols, presupposes previous chapters, and is much longer. The French article makes it clear to historically and philosophically informed readers that Tarski was much more sympathetic to the tradition of (going back through Pascal) taking logical consequence as the central notion of logic than to the tradition of going back through Leibniz, absorbed as it was with the notion of logical truth (analytic truth, tautology).

\{Volumes 3 and 4 are reviewed below.\}

Reviewed by *J. Corcoran*

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Collected papers. Vol. 3.
1945--1957. Edited by Steven R. Givant and Ralph N. McKenzie. Contemporary Mathematicians.
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01A75 (03-03 03-06 03A05 03B15)
For Volumes 1 and 2 see the preceding two reviews.\}

Volume 3 contains twenty-eight mathematical papers published between 1945 and 1957; one is in German, the rest are in English. Fifteen were written by Tarski alone. Three are coauthored with B. Jónsson, two are coauthored with J. C. C. McKinsey, and each of the following is a coauthor with Tarski of one paper: \n A. Horn, L. H. Chin, W. Szmielew, J. M. C. Fell, E. W. Beth, E. C. Smith, Jr., and R. L. Vaught\en. The mathematical papers concern various areas, including abstract algebra, definability, decidability, model theory, foundations of geometry and set theory.

In addition, there is a philosophical paper, "The fundamental ideas of pansomatism", written by the philosopher Tadeusz Kotarbinski in Polish and translated here into English by Tarski with the assistance of D. Rynin. Pansomatism (from "pan" = all + "soma" = body) is a materialistic philosophy diametrically opposed not only to the Platonistic realism of Frege, Gödel, and many other classically oriented mathematical logicians but also to the mentalistic views often associated with intuitionism and other constructivistic foundational philosophies. According to pansomatism, all existent objects are "bodies" (i.e. material, spatio-temporal, physical objects) some of which are also "souls" (i.e. sentient, experiencing objects). It is evident already from his 1933 truth-definition paper, which mentions Kotarbinski several times, that Tarski thought of the syntactic aspects of metamathematics from a physicalistic point of view. The fact that Tarski decided to include the Kotarbinski translation in his *Collected papers* is further confirmation of his materialistic inclinations, however incongruous they may be with the strong set-theoretic premises that were the hallmark of his approach to metamathematics and that contrast

with the finitistically and syntactically oriented approach of the Hilbert school. Since Tarski wrote virtually nothing about his personal philosophy, the inclusion of the Kotarbinski paper might be taken as a kind of substitute for a philosophical essay.

Although it is unusual to include in one person's collected papers an article written by another, the mathematics and logic communities are fortunate that the Kotarbinski paper has been included in Tarski's collected papers. Kotarbinski was one of the three most important of Tarski's teachers, the other two being the mathematician \n S. Lesniewski\en and the historian of logic J. Lukasiewicz. Tarski's 1956 collection is dedicated to Kotarbinski, who, by the way, was President of the Polish Academy of Sciences from 1957 to 1962. A. Mostowski, the distinguished Polish mathematical logician, who was a friend, collaborator and former student of Tarski, saw fit to mention Tarski's sympathies with Kotarbinski's philosophy in his article on Tarski [in *Encyclopedia of philosophy, Vol. 8*, 77--81, Macmillan, New York, 1967].

The most famous work in Volume 3 is probably *A decision method for elementary algebra and geometry*, prepared for publication with the assistance of McKinsey [Rand Corporation, Santa Monica, CA, 1948; MR0028796 (10,499f); second edition, Univ. California Press, Berkeley, CA, 1951; MR0044472 (13,423a)]. Here Tarski shows, among other things, that the first-order theory of the real numbers under addition and multiplication is decidable. This contrasts with the Gödel-Church result that the first-order theory of the natural numbers under addition and multiplication is undecidable. Nonlogicians often find Tarski's result paradoxical when juxtaposed with the Gödel-Church result. The basic ideas of this work and its influence on subsequent mathematical research in algebra and logic have been discussed by L. van den Dries [J. Symbolic Logic **53** (1988), no. 1, 7--19; MR0929371 (89h:01040)].

Other papers in Volume 3 were discussed in the paper just cited and in the other contributions by various authors to the Tarski Survey [ibid. **51** (1986), no. 4, 865--941; ibid. **52** (1987), no. 4, vii; ibid. **53** (1988), no. 1, 1--91].

{Volume 4 is reviewed below.}

Reviewed by *J. Corcoran*

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1958--1979. Edited by Steven R. Givant and Ralph N. McKenzie.
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{For Volumes 1--3 see the preceding three reviews.}

The fourth and final volume of the set contains eighteen papers by Tarski published in English between 1958 and 1979. Seven are by Tarski alone. One has as coauthors J. Doner and A. Mostowski. One has as coauthors C. C. Chang and B. Jónsson. Two are coauthored with L. W. Szczerba. The

remaining are each coauthored with one of the following: D. Scott, Jónsson, L. Henkin, P. Erdös, H. J. Keisler, Doner.

One of the papers, "The completeness of elementary algebra and geometry", written in the late 1930s, had been typeset in Paris but not published due to wartime conditions. Finally in 1967, one hundred copies were printed from the original set of printer's proofs dated 1940 as corrected and prepared for publication in 1966 by J. Ng. This paper might prove to be of historical importance because, as indicated in a note added by Tarski, it is the prewar precursor of the now famous monograph *A decision method for elementary algebra and geometry* [RAND Corporation, Santa Monica, CA, 1948; MR0028796 (10,499f); second edition, Univ. California Press, Berkeley, CA, 1951; MR0044472 (13,423a)]. This paper can be expected to be useful in determining changes that took place in Tarski's thinking about model theory after he came to the United States.

Several papers in this volume are already well known. Among the best known are the 1959 paper "What is elementary geometry?" [in *Proceedings of an International Symposium held at the University of California* (Berkeley, CA), 16--29, North-Holland, Amsterdam, 1959; MR0106185 (21 \#4919)], the 1968 paper "Equational logic and equational theories of algebras" [in *Contributions to mathematical logic* (Hannover, 1966), 275--288, North-Holland, Amsterdam, 1968; MR0237410 (38 \#5692)] and the 1969 paper "Truth and proof" [Sci. Amer. **220** (1969), no. 6, 63--77; per revr.]. All three of these papers are masterpieces of exposition; the first two also include mathematical results not previously reported.

By the 1930s finite, categorical axiom sets were known for various nonelementary (higher-order) geometrical theories, in particular for the second-order theory of the Euclidean plane under betweenness and segment-congruence (equidistance); see, e.g., LSM (*Logic, semantics, metamathematics*, see the reference in the review of Volume 1), pp. 306 ff. As certain logicians, including Tarski, came to doubt the foundational significance of higher-order logic, these results seemed to lose some of their importance and to be seen more as challenges to attempt a construction of adequate elementary (first-order) foundations for geometry. In the 1959 paper, Tarski considered the elementary subtheory of the above second-order theory of betweenness and congruence; he chose an infinite, recursive subset as an axiom set and he showed, among other things, that his axiom set is complete and that the theory is not finitely axiomatizable.

The 1968 paper is an accessible expository survey of the metamathematics of equational logic as it existed at the time. In it Tarski presented in a clear and original way the framework of basic concepts and results in the field, which originated in 1935 with the Birkhoff completeness theorem [see G. Birkhoff, Proc. Cambridge Philos. Soc. **31** (1935), 433--454; Zbl **13**, 1]. In addition, Tarski announced some new results and some open problems. The 1969 paper "Truth and proof" is one of the finest pieces of expository writing

in all of mathematical logic. In many ways it is Tarski the mentor at his best. This paper is an excellent source of "quotable quotes" characteristic of Tarski's sensitive, sober, and careful approach to logic and metamathematics. In this paper Tarski discussed the interconnections between the Gödel incompletability theorem and his own undefinability theorem in the context of an introductory overview of the modern metamathematical reconstruction of the axiomatic method. The logician can detect the technical virtuoso in the background of this article but the nonlogician reading it sees only the patient, effective and helpful teacher.

In addition to the papers, this volume also contains over ninety abstracts published between 1924 and 1977. Some of the "abstracts" are rather substantial summaries of papers. Reading the abstracts in order gives a fascinating chronology of Tarski's interests. This volume also contains about forty pages of "Problems, reviews, [and] contributions to discussions" many of which give welcome insight into Tarski's attitudes concerning various controversial issues, e.g., implicit definition and the significance of syntactical consistency proofs. Unfortunately the editors did not see fit to include indexes. The volume ends with an excellent bibliography by S. Givant, previously published with the Tarski Survey [J. Symbolic Logic **51** (1986), no. 4, 913--941; MR0865919 (88e:01057)] and here reprinted without change. Intended to include all of Tarski's published "works", where the latter term is construed broadly, it has nine sections listing, respectively, 110 papers, 94 abstracts, 19 monographs, 23 exercises and problems, 21 contributions to discussions, 4 reviews, 4 publications edited, 7 project reports, and a single letter that happened to have been published, posthumously. The letter, which is interesting from philosophical and historical viewpoints, was published after this volume and thus is not included. See "A philosophical letter of Alfred Tarski" with prefatory note by M. White [J. Philos. **84** (1987), no. 1, 28--32; MR0876004 (88d:03008)]. In addition to the nine sections, there is an appendix listing items published under other authorship but containing results attributable in whole or in part to Tarski. Besides giving full bibliographic data on each item, Givant has provided other valuable information including, among other things, reviews in Mathematical Reviews and the Journal of Symbolic Logic, relevant abstracts, partial and total reprintings, translations, and revised versions. To the best knowledge of the reviewer this bibliography is free of error (typographical or other) and it was comprehensive at the time of its initial publication, although some of the extensive unpublished writings of Tarski, now housed in the Mathematics Department at the University of California at Berkeley, are certain to be published in the future, and some of Tarski's writings are certain to receive further translation.

The mathematical community owes a debt of gratitude to Givant and McKenzie for their efforts in producing this invaluable collection of Alfred Tarski's works. It is only when we see Tarski's papers collected in one place that we can begin to appreciate the scope and profundity of his influence on modern mathematical thought and, in particular, on modern mathematical

logic. Mathematical logic as we know it today is almost inconceivable without Tarski's contributions.

Reviewed by *J. Corcoran*