142. Truth-preserving and consequence-preserving deduction rules. *Bulletin of Symbolic Logic.* 20 (2014) 130–1.

► JOHN CORCORAN, *Truth-preserving and consequence-preserving deduction rules.*
Philosophy, University at Buffalo, Buffalo, NY 14260-4150, USA
E-mail: corcoran@buffalo.edu

Following Tarski's truth-definition and consequence-definition papers [3, pp. 152–278, 409–420], we assume an interpreted formalized language: a first-order language interpreted number-theoretically. We use ordinary variable-enhanced English: for example, the English sentence schema 'every number x is such that $P(x)$' translates the first-order schema '$\forall x\ P(x)$'.

As usual, a *deduction* is a rule-governed list of sentences beginning with *premises* and ending with a *conclusion*. A system of deductions is *truth-preserving* if each of its deductions having true premises has a true conclusion [3, p. 167]—and *consequence-preserving* if, for any given set of sentences, each deduction having premises that are consequences of that set has a conclusion that is a consequence of that set [2, p.15]. Consequence-preserving amounts to: in each of its deductions the conclusion is a consequence of the premises. The same definitions apply to deduction rules considered as systems of deductions.

Every consequence-preserving system is truth-preserving. It is not as well-known that the converse fails: not every truth-preserving system is consequence-preserving [2, Appendix]. In ordinary first-order Peano-Arithmetic, the induction rule yields the conclusion 'every number x is such that: x is zero or x is a successor'—which is *not* a consequence of the null set—from two tautological premises, which *are* consequences of the null set, of course.

Truth-preserving rules not consequence-preserving are *non-logical* or *extra-logical* rules [1, §4.1]. Such rules are *unacceptable* to persons espousing traditional truth-and-consequence conceptions of demonstration [2, p.16]: a demonstration shows its conclusion is true by showing that its conclusion is a *consequence* of premises already known to be *true*.

[1] JOHN CORCORAN, *Gaps between logical theory and mathematical practice*, ***Methodological Unity of Science*** (Mario Bunge, editor), Kluwer, 1973.

[2] JOHN CORCORAN, *Founding of Logic*, ***Ancient Philosophy***, vol. 14 **(**1994), pp. 9–24.

[3] ALFRED TARSKI, ***Logic, semantics, metamathematics***, Hackett, 1983.