A Theory of Theories[1]

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Abstract

On the basis of examples from mathematical physics, theoretical hypotheses are distinguished from generative theories. An example of the former is Green's claim that light is the vibrations of a certain type of elastic solid. An example of the later is the wave theory of light. Both hypotheses and theories are characterized in terms of theoretical principles and models, but unique to a theory is a language frame for generating its many models. The aim of theory is defined in terms of both accommodating nature and unifying nature through assimilation. The structure and use of generative theories closely resembles the structure of paradigms and their use in normal science [Kuhn 1970].

Introduction

Our conference topic begins with the statement, "what distinguishes science from all other human endeavours is that the accounts of the world that our best, mature sciences deliver are strongly supported by evidence and this evidence gives us the strongest reason to believe them." To examine this topic, we need to dive down and address the question of what sorts of accounts of the world are *intended* by science, by theories, and then discuss the extent to which these accounts are supported by evidence. To answer this question, we need to dive down further and understand theories, specifically, the aim and structure of theories. Then we can work our way back up and begin to discuss the question of how evidence supports the accounts of the world provided by our best theories. All of the examples in this paper are drawn from mathematical, physics theories, which for brevity I will refer to as *physical theories*.

We therefore start with a theory about the aim and structure of physical theory, a so-called theory of theories [see Bromberger 1992]. Just as a physical theory attempts to both fit and explain nature, so too, a theory of theories should both fit and explain theories. To fit, it must describe the phenomena, namely, theories, their structure and use. To explain, it should provide an account of why theories are as they are and this can be done by showing that the

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structure of theory is an outcome of the aims we assign them and the constraints they operate within.

The paper begins by characterizing the aim of physical theory in terms of accommodating nature and unifying nature through assimilation. A distinction between theoretical hypotheses and generative theories is then established. An example of the former is Green's claim that light is the vibrations of a certain type of elastic solid. An example of the later is the wave theory of light. Both hypotheses and theories are characterized in terms of theoretical principles and models, but unique to a theory is a language frame for generating its many models. The structure and use of generative theories closely resembles the structure of paradigms and their use in normal science [Kuhn 1970].

The Aim of Physical Theory

Theories must accommodate nature, or more precisely, our beliefs about nature. To accommodate is "to make fit, to bring into agreement". Equally important, theories must assimilate nature. The need to assimilate reflects a human bias. The world, on the surface appears complex and multifarious. Light, heat, sound and the motion of bodies appear as very different types of phenomena. The human intellect is inclined to seek sameness, commonality, at a level below appearances, showing that light, heat and sound are manifestations of the same sorts of thing. Though on the surface these phenomena seem quite different, they can be thought of as *caused* by similar mechanisms. This human inclination is sometimes described as a need to systematize or unify the world, though the concept of *assimilation* better captures the *active* nature of the intellect in making sense of the world. To 'systematize' means to arrange in accord with a definite plan or scheme. To 'assimilate' means to *make* similar—then we arrange according to some scheme.[2]

Three Examples of Theoretical Hypotheses

I will start with three brief examples of the application of physical theory to nature which illustrate the combined use of theoretical principles, models and hypotheses. Consider Green's

[2] See Nietzsche [1968] and discussion of biological epistemology. Nietzsche describes the intellect as actively making things similar in order to make them more comprehensible and likens the intellect to the biological process of assimilation by which things are made similar before they can be absorbed (comprehended) by a system.

1838 elastic solid theory of light. Green modeled the luminiferous aether as a strained "ordinary" elastic solid. His account of the reflection of light at a boundary between two optical media assumed that the rigidity **η** of the aether is the same in both media while its inertia **ρ** has different values in the two media. The general method, used by Green and others at the time, was to sufficiently detail a model of the aether so as to obtain a mathematical expression for potential energy **φ** which could then be substituted into the general variational equation:

$$\iiint \rho\{\frac{\partial^2 e_x}{\partial t^2}\delta e_x + \frac{\partial^2 e_y}{\partial t^2}\delta e_y + \frac{\partial^2 e_z}{\partial t^2}\delta e_z\}dxdydz = -\iiint \delta\phi dxdydz$$

Assuming **e** to represent the vector displacement of an aethereal particle from its equilibrium position, Green obtained a lengthy expression for potential energy **φ**, which, when substituted into the variational equation, results in the following equation of motion:

$$\rho\frac{\partial^2 \vec{e}}{\partial t^2} = -(k + \frac{1}{3}\eta)\vec{\nabla}(\vec{\nabla}\circ\vec{e}) + \eta\nabla^2\vec{e}$$

One objective of Green's theory was to derive Fresnel's sine and tangent laws of reflection. Fresnel's laws describe the ratio of the amplitude of reflected to incident light when light is incident on a planar surface separating two isotropic media. The sine law applies when the incident light is polarized in the plane of incidence and the tangent law when polarized perpendicular to the plane of incidence:[3]

$$\frac{\sin(\theta_i - \theta_r)}{\sin(\theta_i + \theta_r)} \qquad\qquad \frac{\tan(\theta_i - \theta_r)}{\tan(\theta_i + \theta_r)}$$

With the above theoretical assumptions, Green also derived the boundary conditions that must be satisfied at the interface between real elastic solids, which state that, along the interface between the two media, the three components of displacement and the three components of stress must be equal. After analyzing the reflection and refraction of an incident wave Green found that if the vibration of the aethereal molecules is at right angles to the plane of incidence, the intensity of the reflected light obeys Fresnel's sine law.

In 1856 Krönig developed a version of the kinetic theory of gases based on a simple model of a gas as consisting of a vast number of atoms that behave like solid, perfectly elastic

[3] When the incident wave is polarized in the plane of incidence, the vibration of the aethereal particles is at right angles to the plane of incidence. When the incident wave is polarized perpendicular to the plane of incidence, the vibration of the aethereal particles is parallel to the plane of incidence.

spheres moving with definite velocities. One objective of kinetic theory is to derive the specific heats of gases, which is defined in terms of the heat energy **E** required to raise the temperature **T** of the gas. $\mathbf{C_v}$ is the specific heat when volume is constant and is defined as:

$$C_v = \frac{\partial E}{\partial T}$$

The principle of equipartition of energy states that for a substance in thermal equilibrium its energy is equally divided between the degrees of freedom of its molecules, that is, between its translational, rotational and vibrational degrees of freedom. Previous results indicated that a mole of gas has **RT/2** units of energy per degree of molecular freedom where **R** is the universal gas constant. If we define **t**, **r** and **v** as the degrees of translational, rotational and vibrational degrees of freedom, we obtain the following expression for the energy in one mole of a gas:

$$E = (t + r + 2v)RT / 2$$

Substituting this expression for **E** into the previous equation for $\mathbf{C_v}$ and taking a simple derivative results in the following expression for $\mathbf{C_v}$:

$$C_v = (t + r + 2v)R / 2$$

The model described by Krönig has no rotational or vibrational degrees of freedom, only three degrees of translational freedom; hence, $\mathbf{C_v}$ for this model is **3R/2**.

Consider one last example, one which brings quantum theory into the discussion. Imagine that our objective is to derive the energy levels of some gas with wave mechanics. The central theoretical principle of wave mechanics is Schrödinger’s equation:

$$H\psi = E\psi$$

A general procedure for deriving the energy spectrum of a system with wave mechanics is to first construct a model, obtain the Hamiltonian **H** for the model, substitute the Hamiltonian into Schrödinger’s equation and then solve this equation to obtain the energy eigenvalues of the system.

The rigid rotator model provides a relatively simple illustration. It consists of two masses $\mathbf{m_1}$ and $\mathbf{m_2}$ connected by a mass-less rod of length **r**, with moment of inertia **I** and angular momentum operator **L**. The Hamiltonian **H** operator associated with the rigid rotator is $\mathbf{L^2/2I}$, which, when expanded in polar coordinates takes the form:

$$H = -\frac{\hbar^2}{2I}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right]$$

Substituting this expression for **H** into Schrödinger's equation, followed by a complex mathematical derivation, leads to the following expression for the quantized energy values of the system:

$$E_j = J(J+1)\hbar^2/2I$$

Theoretical Principles, Models and Hypotheses

These three examples illustrate a common pattern. In the case of Green's theory, the variational principle is applied through a specific elastic solid model. In Krönig's theory, the principle of equipartition of energy is applied through a simple gas model. In the wave mechanics example, Schrödinger's equation was applied through the rigid rotator model. Said differently, the models were designed with an eye towards applying the principles and mathematical assumptions of the theory [see Cartwright 1983]. These three examples illustrate what I will refer to as *theoretical hypotheses*, a notion which depends on an understanding of theoretical principles and theoretical models.

Examples of theoretical principles include the principles of conservation of energy and momentum, Newton's laws of motion, Schrödinger's equation, the variational principle and Maxwell's equations. First, these principles are fundamental in the sense that there is no attempt to reduce them to, or derive them from, more fundamental principles; instead, they serve as the foundation for almost all derivations of their respective theories. Second, at least in mathematical physics, they are symbolic in form and have variables as placeholders for mathematical expressions of potential energy, force, the Hamiltonian of a system and the like. And finally, theoretical principles are used by all members of the theoretical without any need for justification. Conversely, we might say that membership in the community is defined in terms of acceptance of these principles.

Because of the multiple meanings of *model*, theoretical models are two-sided, both models *of the theory* and models *of the world*. They are models of the theory in the sense that they satisfy the theoretical and mathematical statements of the theory. Cartwright [1983] and Giere [1988] have discussed this aspect of theoretical models. Giere writes, "I propose we regard the simple harmonic oscillator and the like as abstract entities having all and only the properties ascribed to them in standard textbooks" [1988, p. 78].

Properties are assigned to models with an eye towards engaging a theory's inferential mechanisms, namely, its theoretical principles and mathematical tools. Our three examples are based on models that have clearly been designed to engage *theoretical principles*. Green's elastic solid model, described in terms of particles displaced from their equilibrium position, is constructed so as to yield an expression for potential energy that can then be substituted into the variational equation. Gas models used to derive specific heats clearly indicate values for translational, rotational and vibrational degrees of freedom which are then substituted for **r**, **v** and **t** in the equation describing the energy of the system. Finally, the rigid rotator model and all of the models of wave mechanics are designed so as to yield an expression for the Hamiltonian of the system, which can then be substituted into Schrödinger's equation. At the same time, models are designed with idealizations like point masses and perfect elasticity so as to satisfy the *mathematical* needs of the theory.

Models should actually be understood as three-sided, as models of the world, of the theory's principles and mathematics, but also, as models of a generative language frame. But we will postpone a discussion of this third side of models until we turn to an analysis of theories. Up to this point, we have only been talking about theoretical hypotheses.

Models become models *of the world* through theoretical hypotheses. Our three examples are illustrations of theoretical hypotheses, although, they might also be referred to as theories, for example, Green's 1838 elastic solid theory of light. But, for reasons that will become apparent, it is better to reserve the term *theory* for the likes of the wave theory of light, the kinetic theory of gases and the quantum theory of molecular spectroscopy, and regard our the above examples as possible hypotheses of these theories. A theoretical hypothesis is therefore a statement that asserts a model as representational of some aspect of the world such that conclusions drawn from the model should reflect their parallel in the world. Examples include the statements "light is the vibrations of a Greenian elastic solid" and "hydrogen gas consists of atoms described by Bohr's model". Unlike a stand alone model, a theoretical hypothesis can be evaluated.

Illustrations of Generative Theories

The wave theory of light is consistent with conflicting theoretical hypotheses about light just as kinetic theory is consistent with many conflicting hypotheses about the same type of gas. Such options are generated from the internal structure of these theories, on, what I will eventually characterize in terms of generative language frames. But first, I will provide some historical background on the use of generative theories.

One objective of kinetic theory, as applied to some gas, is the derivation of its specific heat. Historically, a physicist may have considered two types of options when confronted with this problem, one based on vortex models, the other on "billiard ball" type models. Rankine and others championed vortex models which portrayed atoms as a nucleus, enveloped by a rotating elastic atmosphere which is retained in place by attractive forces of the nucleus. As the heat of a gas is increased the rotational kinetic energy and centrifugal force increase, thus increasing pressure. There were several types of vortex models just as there were many variations of "billiard ball" models, Krönig's model being the simplest of this type of model.

The particles of Krönig's simple model have only three degrees of motion, namely, translation in three directions. But in 1857, Clausius, in reflecting upon Krönig's model, imagined more complex models that would exhibit both rotational and translational degrees of freedom. In modern terms, he imagined rigid rotators, and particles attached by "springs", which exhibit various degrees of freedom, and result in a variety of values for specific heat using the following equation:

$$C_v = (t + r + 2v)R / 2$$

Further variations of the "billiard ball" type of model were developed in the process of fitting the kinetic theory to experiment. Jeans, for example, considered the implications of molecules that can aggregate with one another and assumed that in raising the temperature of a gas, work is not only done in increasing the energy of its individual molecules but also in separating them, which, if the case, would increase predicted values of $\mathbf{C_v}$. Likewise, Maxwell considered particle shape and modeled gases with different ratios of spherical and non-spherical particles, and depending on the assumed ratio, computed different values for the specific heat of a gas.

The above options provide these theories with many models and theorists with many possible hypotheses when confronting a problem. If a specific hypothesis fails to accommodate experimental results, then an alternative can be developed, without in the process calling into question the theory or its basic principles. But, foreshadowing a question we will return to, how does the community reach consensus about what are, and what are not, *legitimate* hypotheses of the theory. After all, we can not understand theories or their evaluation, without answering this question.

An illustration of the question of legitimacy, of standards, is found in the context of the classical theory of light, our second example of a generative physical theory. Green's elastic solid model of light, though successful in deriving Fresnel's sine law of reflection, was inconsistent with Fresnel's tangent law of reflection. Eventually MacCullagh [1839] developed an elastic solid model of light consistent with both laws of reflection as well as the absence of longitudinal waves. But to illustrate the issue of hypothesis legitimacy, let us begin with MacCullagh's paper of 1837. Mathematically, this paper achieved similar results to his paper of 1839, but it was "without dynamical" foundation. In particular, MacCullagh's 1837 paper was based on assumptions about the boundary conditions between two optical media that are violated in the case of an ordinary elastic solid (such as Green's). This would not have been a problem if MacCullagh had backed his mathematical assumption with a model that provided dynamical foundation for the alleged boundary conditions. But he did not, and recognizing this deficiency himself, concluded that his 1837 paper amounted to nothing more than fortunate conjecture without any theoretical foundation.

MacCullagh's 1839 model, however, provided a dynamical foundation for the assumptions of his 1837 conjecture. He assumed a new type of elastic solid, one whose potential energy depends only on the *rotation* of its volume elements. Substituting the expression for potential energy into the variational equation led to an elegant expression for the equation of motion that describes purely transverse waves and is consistent with both of Fresnel's laws of reflection:

$$\rho \frac{\partial^2 \vec{e}}{\partial t^2} = \mu \nabla^2 \vec{e}$$

About MacCullagh's work, Whittaker concludes, "there can be no doubt that MacCullagh really solved the problem of devising a medium whose vibrations, calculated in accordance with the correct laws of dynamics, should have the same properties as the vibrations of light" [1989, p. 144]. Nonetheless, MacCullagh's solution was not immediately embraced because of questions concerning its legitimacy and whether it adhered to required standards. Although, on the surface, the notion of a rotationally elastic solid offered some dynamical foundation for his mathematical assumptions, MacCullagh failed to describe a mechanical model that illustrated the dependence of potential energy on rotation. In 1889, Thomson (Lord Kelvin), addressed this concern, and constructed, not simply in thought, a mechanical system consisting of spheres, rigid bars, and spherical caps, which illustrated the concept of rotational elasticity.

While it is not actually necessary to actually build a physical model in order to legitimate a theoretical model, *something* is needed in order to justify an equation; after all, one can not simply write down any equation, derive a desired result, and declare success in the sense of having solved a problem with a theory. But we have started our discussion of the classical theory of light down in the details; let us bring the discussion up a few levels and paint a broader picture of this generative theory and then return to the question of standards.

One objective of the classical theory of light was to fit various experimental results, including those relating to reflection, refraction and polarization. But the first order of business was to simply account for the fact that light travels from one place to another in a finite duration. Broadly conceived, there were two basic responses to this aim which divided the theory into two major versions, the corpuscular and wave theories of light. From the perspective of classical mechanics, one could think of light as like a dart, or a projectile, that travels from one place to another. Or one could think of light as traveling as a progressive disturbance that initiates with the agitation of some particles, which communicate the disturbance to neighboring particles, which in turn agitate their neighboring particles, and so forth. Newton and Laplace were advocates of corpuscular hypotheses, while Young, Fresnel, Stokes, Green, MacCullagh, Thomson and others developed versions of the wave theory of light.

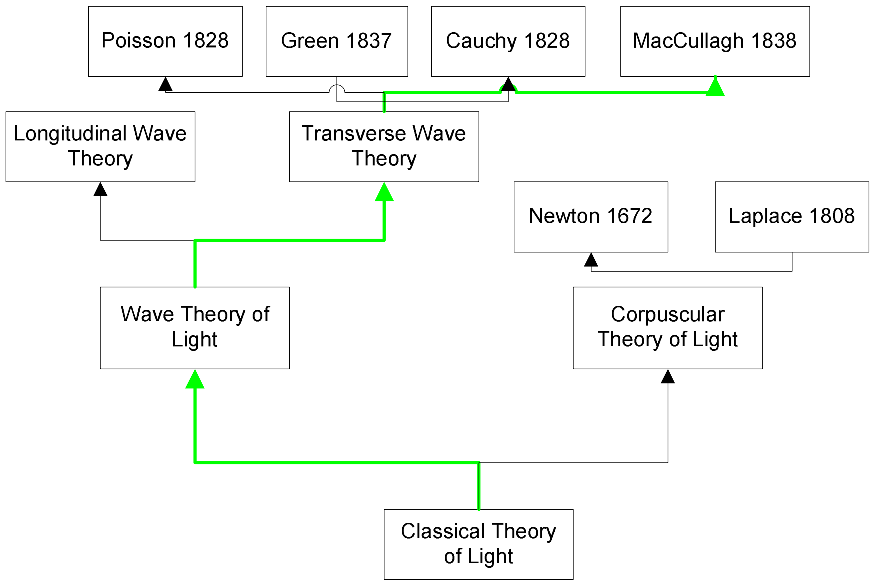


Figure 1 Tree diagram, evolution path for classical theory of light

Through a process similar to evolution and natural selection (figure 1), alternative hypotheses of the classical theory of light were developed and tested for fitness with respect to experimental results. Combined requirements relating to reflection, refraction, polarization and other experimental results, eventually gave preference to the wave theory of light, more specifically, to the transverse wave theory of light. The assumption of a transverse wave theory of light was quickly translated into the need for elastic solid models since it was known that such a medium could support transverse waves. Finally, many hypotheses of the elastic solid theory of light were developed, based on various options, such as those relating to the rigidity and inertia of the medium.

Expectations of a Theory of Theories

I will argue that the kinetic theory of heat and the classical theory of light are generative theories and that such theories consist of theoretical principles, theoretical hypotheses and a mechanism for generating (and legitimating) the models of the theory. But before detailing this response, we need to understand the criteria for judging a theory of theories. After all, just like a physical theory, a theory of theories should be evaluated according to some recognized standards, and following the example of physical theory, it is appropriate that a theory of theories fit and explain theories.

In the case of a theory of theories, what we are fitting is a practice, known to us through historical documents and textbooks. Any such account of physical theory had better take account of the use of theoretical principles, theoretical models and mathematical derivation. But it also needs to capture *standards* of practice, and the fact that members of the community, in understanding a theory, have criteria for agreeing on what is, and what is not, a legitimate expression of the theory. Kuhn emphasizes this point in writing, “No puzzle-solving enterprise can exist unless its practitioners share criteria which, for that group and for that time, determine when a particular puzzle has been solved” [293]. This does not mean that practitioners must agree on which model to use for a given problem. But there must be criteria, shared by the community, for deciding what counts as solving a problem. Although these standards may not be clear cut rules, as evidenced by the debate about the legitimacy of MacCullagh’s rotationally elastic solid theory, without something in this role, a sort of anarchy would ensue.

Likewise, a theory about physical theories will ideally explain specific instances of physical theory, just as kinetic theory aims to explain the specific heat of a gas by showing it as the outcome of some underlying mechanism. So too, a theory of theories should explain the structure and use of theories as a consequence of their aims and the constraints they operate within.

The Structure of Generative Theories

Earlier I described theoretical hypotheses in terms of theoretical models, governed by theoretical principles and mathematics. These concepts are also essential to understanding generative theories, but something in addition is needed to capture a theory’s mechanism for generating and legitimating its models.

According to Kuhn, models supply the group with “preferred or permissible analogies and metaphors” which “help to determine what will be accepted as an explanation and as a puzzle-solution” [1970, 184]. Directionally this seems correct, but I believe the concept of *generative language frames*, as developed by Barsalou [1992] and others might provide a better framework for what Kuhn was after here. More work needs to be done in this area, but I will sketch a role for language frames in the context of generative physical theories.

Figure 2 illustrates a modified version of a language frame given by Barsalou for the category *car*. Barsalou describes several features and uses for language frames but I will only focus on those features which seem particularly suited to an analysis of the structure of physical theory, namely, attributes and values, the concept of a model of a language frame, exemplars, the enablement relation, and the idea that the choice of a specific model of a language frame is goal driven.

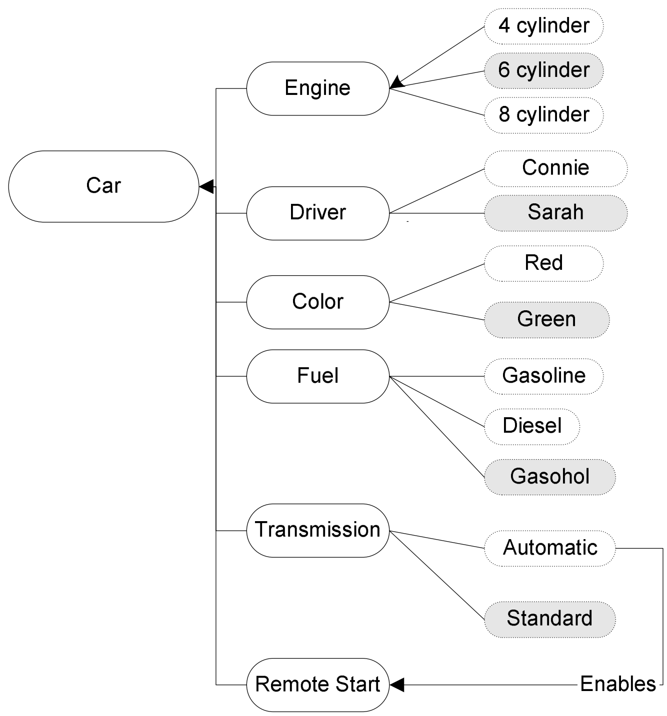


Figure 2 Generative language frame for car category

Figure 2 illustrates various attributes of *car* and their possible values. The concept of enablement is also illustrated and in this case reflects the fact that *automatic transmission* is an enabler for *remote start* but *standard transmission* is not. Shading is used to identify a specific instantiation of the language frame, in this case, a *green, six cylinder*, *standard transmission*, *gasohol* fueled car that *Sarah* will drive. Barsalou defines an exemplar as an instantiated frame, in which case, the car defined about would be an exemplar. I would prefer to use the term *model* for any instantiation of a frame, and reserve the term *exemplar* for special instantiations, special models, those that have particular significance. A model of a language frame is therefore a model in the sense in which an ideal spring is a model of certain theoretical equations. The car model selected above has all, and only, those attributes and values assigned to it.

Specific models of a frame result from a series of choices. These choices should be seen as goal driven. The selected car model reflects a compromise between a desire for quickness, environmentalism and comfort. The *standard transmission* is optimal for quickness, and good for gas consumption, but it implies that the car can not have *remote start*. The choice of a *six cylinder* engine reflects a compromise between miles per gallon and quickness, and *gasohol* was selected because of environmental commitment.

Language frames provide a useful medium for representing model generation in the context of physical theories. Choices within the language frame of a physical theory, as well as the specification of the language frame itself, are goal driven. First, models of the language frame must be suited to application of the theory's principles and mathematics. Second, the selection of a specific model of the frame will be guided by the goal of solving a given problem, for example, deriving the energy spectrum of some gas. And finally, and we will discuss this more later, the goal is to not overly inflate the frame and allow too many model types.

Consider a language frame for the kinetic theory of heat. Within the frame, alternative models result from a series of choices, and, as we will see, a very similar frame is used by various quantum theories, such as quantum theory of molecular spectra.

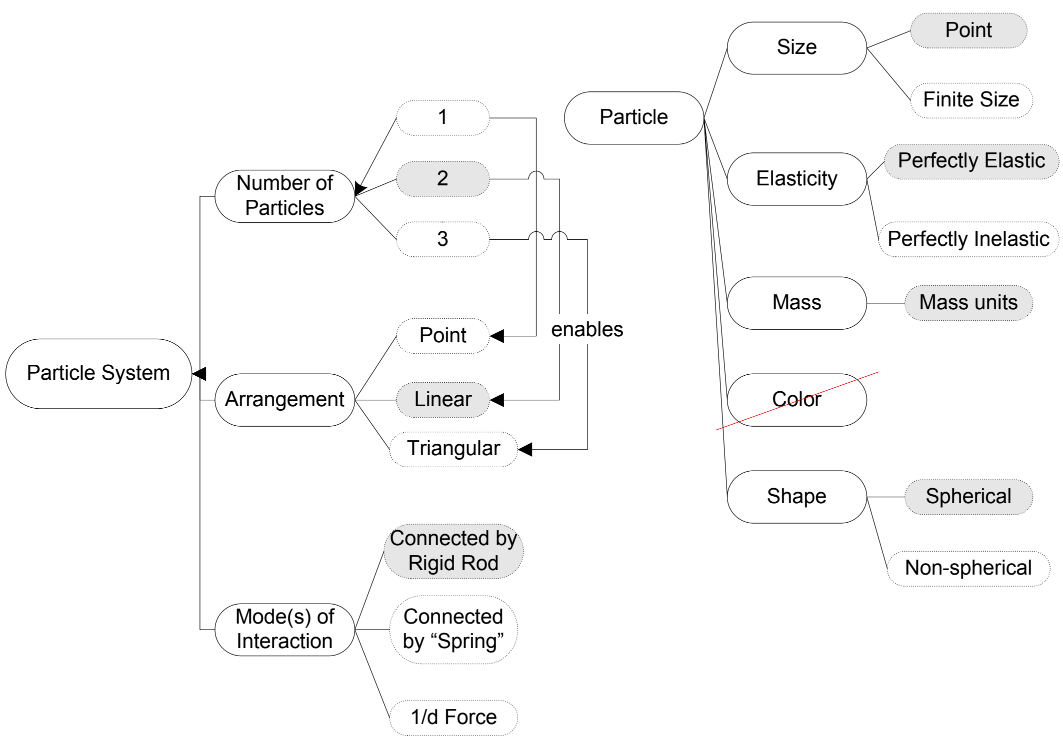


Figure 3 Partial language frames for particle system and particle

Figure 3 illustrates a partial frame for *particle system* and *particle*. The attributes of particle system include *number* of particles, *arrangement* and *modes of interaction* between particles. The attributes of particle include *mass*, *elasticity*, *size* and *shape*. Attribute and value decisions are constrained by the requirements of applying the theory's principles and mathematics as well as the goal of accommodating nature. For the time being we will assume a fixed frame and hold off on discussion of expanding the frame so as to accommodate anomalies or additional types of phenomena.

In the *particle* frame, *color* has been crossed out because, unlike the other attributes, it is irrelevant with respect to the assumed principles of kinetic theory. These principles require models defined in terms of mass, elasticity, size, shape and the like. Likewise, the mathematical needs of the theory are captured by selection of attributes and values that lend themselves to mathematical derivation, such as point masses, and perfect elasticity. The concept of enablement is essential here because some attribute values are only permitted if other attributes have certain values. If, for example, the *number* of particles in the system is two, then a *triangular arrangement* of particles is not permitted. If, on the other hand, a three particle system is selected, then there are three choices as to *arrangement*— *point*, *linear*, and *triangular*. A specific model of the combined language frames, in this case, a rigid rotator model, is identified by shading specific attribute values—the key attribute values are *two* particles, *rigid rod* connection and *linear arrangement*.

Figure 4 illustrates some of the models that can be generated with these simple language frames as well as the values of specific heat obtained with these models. Although the frame is simple, permutations of its attribute values allow for many models and values of $\mathbf{C_v}$, even more if other attributes are exploited. Maxwell, through consideration of the attribute of particle shape, imagined different ratios of *spherical* and *non-spherical* particles and was able to predict a continuous range for $\mathbf{C_v}$ between some interval.

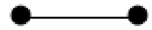
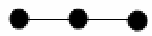
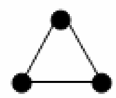
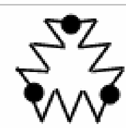
Model Description	Model Picture	T	R	V	Cv
Monatomic		3	0	0	3R/2
Rigid-Diatomic		3	2	0	5R/2
Vibrational-Diatomic		3	2	1	7R/2
Rigid-Linear-Triatomic		3	2	0	5R/2
Vibrational-Linear-Triatomic		3	3	0	6R/2
Rigid-Nonlinear-Triatomic		3	2	4	13R/2
Vibrational-Nonlinear-Triatomic		3	3	3	12R/2

Figure 4 Models, degrees of freedom and specific heats

Not all frames are so easily deciphered and constructed. This frame happens to lend itself to a sort of "tinker-toy" interpretation, such that the models of the frame are objects one could build with rigid rods, springs and spheres. But the critical point here is that there must be *something* that plays this role, the role of identifying legitimate expressions of the theory.

Structure of Generative Theories Revisited

To summarize up to this point, a generative theory consists of theoretical principles, a language frame for generating the theory's models, and a set of theoretical hypotheses that, somewhat artificially, reflect the intent of the theory at some point in time. This statement requires two points of clarification. First, theoretical models were previously described as two sided, as models of the world and as models of theoretical equations. But theoretical models are also models of the generative language frame assumed by a theory. Second, a generative theory is not simply a tool for hypothesizing about the world, but, as a theory about the world, it must also assume theoretical hypotheses. We have seen that generative theories are consistent with many hypotheses about the same phenomenon, for example, about the mechanisms underlying the propagation of light and the specific heat of a gas. Hence, to associate a generative theory with any given set of hypotheses is somewhat arbitrary, but aimed at capturing the preferred, perhaps exemplary, hypotheses of the theory at some point in time. Figure 5 summarizes the structure and use of generative theories. The activity described characterizes normal science and the current analysis of generative theories is proposed as a further detailing of Kuhn's concept of a paradigm.

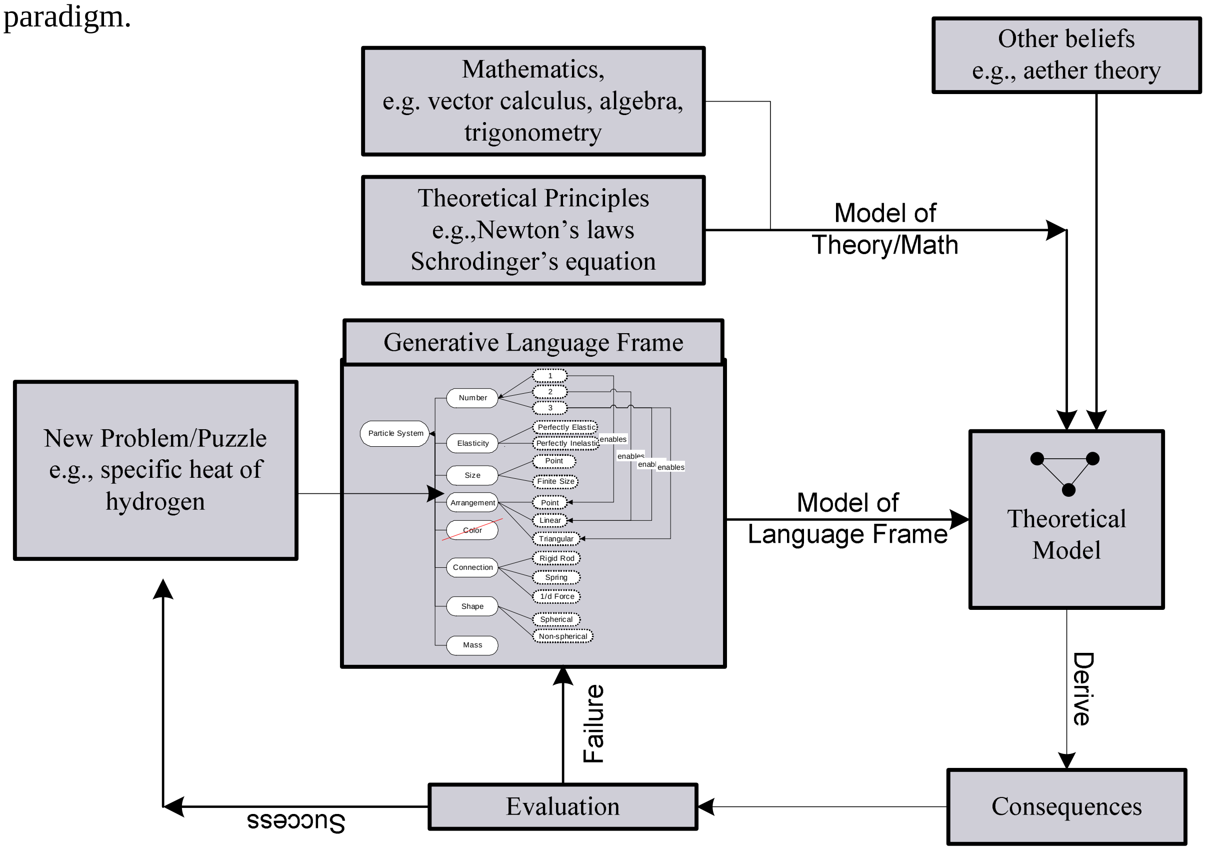


Figure 5 Feedback diagram for generative theory testing (paradigms and normal science)

We start with a specific problem (or puzzle), for example, to derive the specific heat of hydrogen or the reflection of polarized light. Next, a specific model of the language frame is selected, which *then* becomes a model of the theory's principles and mathematics. A theoretical hypothesis can then be formulated, for example, light is the vibrations of a certain type of elastic solid. The theoretical model is then set in motion by the theory's principles and mathematics. The first step is typically writing down a mathematical version of a key theoretical principle, such as the variational equation, f=ma or Schrödinger's equation. The result is a specific version of one of these equations. Mathematical, theoretical derivation proceeds and implications of the hypothesis are drawn which are compared with experimental results. Success implies that the community can move onto another problem in an effort to assimilate more of nature to the generative theory (or paradigm). In addition, the theoretical hypothesis might be given exemplary status in the theory's literature just as the stand alone model might be dubbed an exemplar and given a section in standard textbooks. Failure typically results in returning to the language frame to select an alternative model, a selection that will be informed by previous failures. A critical experiment that appears to implicate the entire language frame may imply that there is no point in returning to the language frame in search of a better model.

But success, or the criterion for success, needs clarification. In upper right corner of figure 5 there is reference to evaluation of a hypothesis in terms of consistency with the hypotheses of other theories. For example, do we need to worry about the interaction of the molecules of kinetic theory with an elastic solid that pervades all space? Likewise, astronomy tells us that the planets are not appreciably slowing down as they move through space, but if an aether fills all of space, then classical mechanics implies that the planets should slow down over time. One might object that this sort of reasoning implies that we are taking models *too literally* and are not reflecting the theory's *intended* account of the world. One might argue that what we were really after here was the equations for an elastic solid and not the hypothesis that there *really is* an elastic solid occupying all space. I will counter this objection by arguing that by disassociating theoretical models from the language frames they originated within, a theory fails to assimilate and unify nature. But first, let us consider how theories do in fact assimilate and unify nature.

Generative Theories and Assimilation

While Newton's laws and Schrödinger's equation are critical to any account of how classical and quantum theories unify nature, by themselves, these principles are impotent when it comes to unifying nature. For the sake of argument, imagine theories without recourse to language frames and their models. Different applications of f=ma and Schrödinger's equation require different versions of these equations, in the end, in order to solve a variety of problems, many mathematical versions of these equations are required. Figure 6 shows just a few of the many versions of f=ma and Schrödinger's equation.

$\vec{F} = m\vec{a}$	$H\psi = E\psi$
$\rho \frac{\partial^2 \vec{e}}{\partial t^2} = -3\eta\vec{\nabla}(\vec{\nabla} \cdot \vec{e}) - \eta(\vec{\nabla} \times (\vec{\nabla} \times \vec{e}))$	$\left(\frac{h^2}{8\pi^2 m}\nabla^2 + E + \frac{e^2 Z}{r}\right)\psi = 0$
$\rho \frac{\partial^2 (\vec{b} + \vec{c})}{\partial t^2} = \eta\vec{\nabla}^2\vec{c} + (\kappa + \frac{4}{3}\eta)\vec{\nabla}^2\vec{b}$	$\frac{d^2\psi}{dx^2} + \frac{8\pi^2\mu}{h^2}(E - 2\pi^2 \upsilon_0^2 \mu x^2)\psi = 0$
$\rho \frac{\partial^2 \vec{e}}{\partial t^2} = -(\kappa + \frac{1}{3}\eta)\vec{\nabla}(\vec{\nabla} \cdot \vec{e}) + \eta\vec{\nabla}^2\vec{e}$	$\left(\frac{1}{2m}\sum_i p_i^2 + \frac{1}{2}\sum_k \frac{p_k^2}{M_k} + V\right)\psi = E\psi$
$\rho \frac{\partial^2 \vec{e}}{\partial t^2} = \mu\nabla^2\vec{e}$	$\frac{L^2}{2I}\psi = E\psi$

Figure 6 Mathematical versions of f=ma and Schrödinger's equation

Without recourse to the semantic content of models, we have two options for capturing the systematic unification of nature by theories. One option is to derive these equations from some smaller set of more fundamental principles. As an example of this exercise, consider the four mathematical versions of Schrödinger's equation shown in figure 6, and try to derive them from a smaller number of equations. A second option is to list all of these equations as fundamental, but then the theory has too many "fundamental principles" and fails to somehow reduce the many to the few.

Assimilation is not achieved not through mathematic derivation of many equations from a few equations; rather, it is achieved through the semantic content of models. The equations of a theory should be seen as *versions* of a handful of fundamental principles, and though each version is based on a unique model, the models are similar in terms of semantic content, that is, are instantiations of a common language frame. The result is a small number of fundamental principles that govern a large number of models that are all similar in the sense that they stem from a common language frame. In this way, theories assimilate the multifarious surface phenomena to deeper, but similar, causal mechanisms.

To summarize the dependence of theories on models, models are indispensable to theory and should not be thought of as scaffolding that can be removed once a theory is constructed. Hertz wrote, "Maxwell's theory is Maxwell's equations". The aether models are superfluous. In the end, that appears to be true, but for that reason, Maxwell did not assimilate electromagnetic phenomena to classical mechanics. He introduced a different language frame, one based on fields, as well a new set of theoretical principles. Metaphorically, models are critical to assimilation and provide the organic matter that unites the symbolic expressions of a theory. Poincare offers a lovely metaphor which, though used with different intent, can be seen as describing the mathematical equations of a theory after their models are removed:

> . . . You have doubtless seen those delicate assemblages of silicious needles which form the skeleton of certain sponges. When the organic matter has disappeared, there remains only a frail and delicate lacework, True, nothing is there but the silica, but what is interesting is the form this silica has taken, and we could not understand it if we did not know the living sponge which has given it precisely this form.

Accommodation versus Assimilation Trade-offs

The aims of accommodating and assimilating nature pull theories in opposite directions. They do so for two reasons, one of which is obvious, the other more subtle.

Assimilation is optimized if the theory's language frame and cast of models is limited. At one extreme is a language frame similar to that of Descartes' physics, limited to particles of different sizes and shapes, where the only type of force admitted is that due to collision. The metaphor is very restrictive and judging from history unable to fit diverse phenomena. To better accommodate the world, the language is given more freedom, perhaps initially allowing for different degrees of elasticity or types of interactions beyond collision. 1/r forces might be added to the language frame as possible values for the particle to particle *interaction* attribute. The acceptance of new attributes, or new attribute values, may require a change in standards. Kuhn makes this point in writing, "Must a theory of motion explain the cause of the attractive forces between particles of matter or may it simply note the existence of such forces? Newton's dynamics was widely rejected because, unlike both Aristotle's and Descartes' theories, it implied the latter answer to the question. When Newton's theory had been accepted, a question was therefore banished from science" [1970, p. 148]. But in loosening standards in this manner, assimilation is compromised in order to better accommodate nature.

Accommodation and assimilation, however, can interact in a more subtle way. While the semantic content of models provides the grounds for assimilation, it also endows models with the means for interacting with other beliefs and this can get hypotheses into trouble. If the aether is an elastic solid then that *means* it is a resistive medium that should interact with planetary motion and the molecules of kinetic theory. Stokes addressed this concern by likening the aether to an extreme example of substances like "shoemaker's wax" which is rigid enough to permit elastic vibration, but plastic enough to allow others bodies to pass slowly through it. Stokes' proposal *may* provide a solution to this problem, but MacCullagh and others did not endow their models with these properties and then show that the resulting hypothesis was consistent with the observed motion of planets as well as Fresnel's laws of reflection and the absence of longitudinal light waves.

Likewise, in the end, advocates of kinetic theory overlooked an apparent conflict between their hypotheses and the assumption of an aether. In the early part of the nineteenth century many physicists believed in a luminiferous aether and were therefore reluctant to advance a kinetic theory of heat because they suspected an inconsistency between the two theories. Clausius was one of the first physicists to overcome this reluctance and as a consequence he was critical to the early development of kinetic theory. In general, physicists often ignore conflicts between the hypotheses of different theories and one can imagine several reasons why this is the case. One reason is that the task of fitting nature with theories is hard enough already.

But the question remains as to whether these conflicts *should* be overlooked, or expressed differently, how should we understand the intended accounts of our best theories? Earlier I mentioned that one might object that concerns about, for instance, the consistency between the kinetic theory of gases and the assumption of an aether, suggest that models are being taken too literally. In response, I argue that we can not have it both ways—rely on the semantic content of models in order to make a case for assimilation, and ignore semantic content when it interacts with other hypotheses and makes fitting nature more difficult.[4]

Explaining Generative Theories

Finally, I turn to the questions of *why* and *must* physical theories look like this. Can we imagine other alternatives, theories constructed in different ways, which achieve similar goals?

[4] It is interesting that while philosophers worry about underdetermination, physicists neglect to consider interactions between their theories that could "further determine them".

While not prepared to address this question in its entirety, I will at least attempt an argument for the necessity of models and language frames.

It is important to acknowledge that language frames and their models have a *primary* and *secondary* role in the context of physical theory. Their primary role relates to assimilation and the goal of showing many phenomena as consequences of similar sorts of mechanisms that are governed by a small number of basic principles. And *because* of this role, language frames and their models establish standards that are used to identify what are legitimate expressions of a theory.

It is not necessary for theories to assume models in the sense of familiar pictures, for example, cogs, wheels, pulleys and springs. The theoretical models of quantum theory certainly challenge the imagination. But this is *after* the principles and mathematics of the theory have imposed their requirements on these models. Models *start* as models of a familiar language frame. After they are *then* further characterized by theoretical principles like Schrödinger's equation and the mathematics of quantum theory, we can certainly lose sight of the model. But we should not lose sight of their origin as models of a familiar language frame. There is a tendency to think of *modern* physics as free of models. This point of view is echoed by Dirac's proclamation that quantum theory has no dependence on what can be perceived, talked about, and that one should simply stick with the equations, the mathematics. Even in quantum theory there is a familiar language that serves as the origin for its models, without which, the theory would not assimilate any more than a long list of equations.

The language frames of classical and quantum theory have much in common. In fact, the language frame previously illustrated for classical kinetic theory is remarkable similar to that used in the quantum theory of specific heats and the quantum theory of molecular spectroscopy. A cursory study of Herzberg's "The Spectra and Structure of Simple Free Radicals" reveals a language frame built on particle systems consisting of varying numbers of particles, arranged in different ways and connected by the likes of abstract rigid rotators and ideal springs. The models of the language frame exhibit different rotational, vibrational and electronic states. When the models *then* become models of the principles and equations of quantum theory so as to derive transition probabilities and line spectra, indeed, the models become rather peculiar. But we should not lose sight of their origin in a familiar language frame.

Atavism

In order to fit nature, theories require many variations of their theoretical principles. Unification of nature, through assimilation, is achieved by recognizing these equations as *versions* of a handful of fundamental principles, and though each version is based on a unique model, the models are similar in terms of semantic content, that is, they are instantiations of a common language frame. Quine [1953], in *Two Dogmas of Empiricism* remarks that "language is social and so depends for its development upon intersubjective reference". The reliance of theories on language, and the dependence of language on intersubjective reference, implies that generative physical theories are necessarily atavistic and dependent on something more primitive than themselves. Models, originating in everyday experiences with the mid-sized objects of intersubjective reference, keep reappearing-- in classical mechanics, in the classical theories of light, heat and sound, and finally in quantum mechanics. On the topic of philosophical atavism, Nietzsche wrote:

> …. the most diverse philosophers unfailingly fill out again and again a certain basic scheme of *possible* philosophies. Under an invisible spell they always trace once more the identical orbit: however independent of one another they may feel [1990, p. 20].

Whether or not this is true of philosophy, something like this occurs in physics. Modern theories are more dependent on ancient ideas than we might prefer to recognize. But the orbit theories trace is certainly not identical. What reappears are not theoretical models but only models in the more limited sense of models as models of a language frame. Theoretical models are both models of a language frame *and* models of a theory's principles and mathematics and progress requires that they change from one theory to the next.

Conclusions

What distinguishes science from all other human endeavours is that the accounts of the world that our best, mature sciences deliver are strongly supported by evidence and this evidence gives us the strongest reason to believe them.

I have barely reached the point at which the topic of theory confirmation can be addressed, but hopefully I have provided a framework that might be useful in this context. In approaching the issue of theory evaluation and confirmation, it is important to contrast the testing of theoretical hypotheses with the testing of a generative theory.

Theoretical hypotheses, such as Green's elastic solid hypothesis about light, can easily be reconstructed as derivations where consequents are mechanically derived from antecedents; hence, if consequents do not agree with experiment, the hypothesis is in trouble. For pragmatic reasons the principles and mathematics are typically exempt from questioning, and the models are the preferred victim [Quine 1953]. But, given that these hypotheses are the hypotheses of a broader theory, which aims at assimilating nature, the semantic content of their models is essential and interaction with other beliefs, other theories, must be reconciled—but only if theories are to both accommodate and unify nature through assimilation.

On the other hand, failure of generative theories is less obvious and the traditional contexts of discovery and justification become intertwined as discovering, creating new models, is integral to the evaluation of these theories. Finally, it is important to recognize *everything* that is tested when we evaluate a generative theory. We are testing theoretical principles and mathematics, but we are also testing the suitability of a familiar language to simultaneously accommodate and assimilate nature.

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