Conditionals: Inferentialism Explicated

VINCENZO CRUPI, ANDREA IACONA

(Postprint version, forthcoming in *Erkenntnis*)

ABSTRACT

According to the view of conditionals named *inferentialism*, a conditional holds when its consequent can be inferred from its antecedent. This paper identifies some major challenges that inferentialism has to face, and uses them to assess three accounts of conditionals: one is the classical strict account, the other two have recently been proposed by Douven and Rott. As will be shown, none of the three proposals meets all challenges in a fully satisfactory way. We argue through novel formal results that a variation of the evidential account of conditionals suggested by Crupi and Iacona is the most promising candidate to develop inferentialism in a coherent formal framework.

1 HISTORY

Inferentialism, the view of conditionals investigated in this paper, rests on the idea that conditionals express inferences: a conditional holds when its consequent can be inferred from its antecendent, or equivalently when the latter provides a reason to accept the former. Although the term 'inferentialism' is a rather new label, which we take from work by Douven and colleagues, the view itself is not new.[1] In fact, an interesting thread of claims of a distinctive inferentialist flavour emerges across the history of logic.

First and foremost, inferentialism is a substantial thesis from Stoic logic. It is a key Stoic doctrine that the validity of an argument and the truth of the corresponding conditional are correlated properties.

1. K. Krzyżanowska, Wenmackers, and Douven 2013; Douven, Elqayam, and Krzyżanowska 2023. It should be pointed out that this work by Douven and colleagues characterizes inferentialism via a cluster of heterogeneous inference patterns (e.g., deductive, inductive, and abductive). This broad view is not identical to Douven's specific probabilistic theory of the acceptance of conditionals that we discuss below (section 4). Also, this use of 'inferentialism' differs from the more widespread use to denote the view by which the meaning of expressions is determined by their role in inference, although it would be in line with the latter view to adopt inferentialism in our sense. For example, Brandom 2018 seems to suggest such a convergence.

Sextus Empiricus reports the Stoic position as follows in *Against the Logicians*:

> The conclusive argument is sound, then, when after we conjoin the premises and create a conditional that begins with the conjunction of the premises and finishes with the conclusion, this conditional is itself found to be true.[2]

Scholars also agree that the Stoic notion of a valid argument was not restricted to formal validity, and might easily have included inferences that are now broadly classified as inductive rather than deductive.[3] Indeed, Iacona's phrase 'Stoic Thesis' is largely coextensive with 'inferentialism' for our purposes, and much appropriate to emphasize the historical origins of the view.[4]

Echoes of the Stoic Thesis appear sparsely but consistently throughout the late antiquity and the Middle Ages. In Boethius' *De Hypotheticis Syllogismis*, for example, it is stated that in a conditional [*in conditionali*], the reason for the inference [*consequentiae ratio*] is taken from the antecedent condition [*ex conditione*].[5] So Boethius must have assumed that conditionals somehow express inferential relations.

In Abelard's *Dialectica*, his most influential logical work, one reads that

> the meaning of a conditional [*sententia hypotheticae propositionis*] amounts to an inference [*in consecutione est*], namely to whether one thing does or does not follow from something else.[6]

The example provided is 'si est homo, est animal', where it is impossible for a man to exist without being an animal. Abelard also mentions certain authors [*quidam*] who acknowledge as true [*veras*] not only necessary inferences, but also whatever inference that is probable [*probabiles*], and accordingly take the truth of a conditional to consist either in its necessity or in its mere probability [*in sola probabilitate*]. Abelard does not elaborate any further and rejects this proposal rather firmly. The passage is nonetheless remarkable, for it confirms that lost sources must have meant the Stoic Thesis as extended to non-conclusive arguments.[7]

The Stoic Thesis surfaces again up to the latest developments of medieval logic. Ockham, in his *Summa Logicae*, explicitly says that a

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2. Sextus Empiricus, *Against the Logicians*, II, 417, in Sextus Empiricus 2005. Also see *Outlines of Scepticism*, II, 137, in Sextus Empiricus 2000.

3. See Barnes, Bobzien, and Mignucci 2008, p. 123.

4. See Iacona 2023. Taken literally, explicit statements of the inferentialist position usually only focus on the characterization of conditionals in terms of valid inference, not the other way around. Yet the converse claim that a valid inference licences the corresponding conditional is hardly ever questioned.

5. Boethius, *De Hypotheticis Syllogismis*, in Boethius 1847, p. 832.

6. Abelard, *Dialectica*, in Abelard 1956, p. 271 (our translation).

7. Abelard, *Dialectica*, in Abelard 1956, p. 271-2.

conditional [*condicionalis*] is equivalent to an inference, so that it is true when the antecedent entails the consequent [*quando antecedens infert consequens*].[8] An important anonymous treatise of the same period, the *Liber Consequentiarum*, provides a sharp and unequivocal phrasing of this equivalence:

> Every inference [*consequentia*] is equivalent to a conditional [*aequivalet condicionali*] composed of the antecedent and consequent [*ex antecedente et consequente*] of the given inference with 'if' put in front of the antecedent, and conversely every conditional is also equivalent to an inference composed of the antecedent and consequent of the given conditional with 'therefore' put in front of the consequent.[9]

The idea that conditionals express inferences, along with the use of the term 'consequens', which is closely related to 'consequentia', survived for long time and remained largely undisputed until the end of the XIX century. A clear illustration of its pervasiveness is provided by the following description of hypothetical judgments in Kant's *Blomberg Logic*, which is based on his logic lectures in the early 1770s:

> one always finds the relation of ground to consequences. Now in conditioned judgments, that which contains the ground is called *antecedens* or also *prius*. That which contains the consequences, however, is called in these judgments *consequens* or *posterius*.[10]

Another telling example in the same spirit, drawn from Mill's *System of Logic*, is the following:

> When we say "If the Koran comes from God, Mohammed is the prophet of God", we do not intend to affirm either that the Koran does come from God, or that Mohammed is really his prophet. Neither of these simple propositions may be true, and yet the truth of the hypothetical proposition may be indisputable. What is asserted is not the truth of either of the propositions, but the inferribility of the one from the other.[11]

Between the end of the XIX century and the beginning of the XX century, the inferentialist ideas expressed in the quotations above started losing momentum, as a new paradigm of logical analysis, which emerged in the works of Frege, Russell, and Wittgenstein, privileged the material account of conditionals originally defended by

8. Ockham, *Summa Logicae*, II.31, in Ockham 1998, p. 186.
9. *Liber consequentiarum*, 123.199-203, edited in Schupp 1988 (p. 109), and quoted from King 2001, p. 132 (translation slightly modified).
10. Kant 1992 9, pp. 222-223.
11. Mill 1882, p. 102.

Philo, the view that a conditional is true when it is not the case that its antecedent is true and its consequent is false.[12]

Interestingly, this crucial historical turn also marks the origins of the symbol $\supset$, which nowadays occurs in logic textbooks. When Peano published his *Formulaire de Mathématique* in 1894, he used a symbol $\subset$, which looked like a 'C', with the intention to represent a relation of consequence between two sentences. That is, he wrote $\alpha \subset \beta$ to mean that α is a consequence of β.[13] A few years later, Russell took this symbol from Peano and reversed it, that is, he wrote $\beta \supset \alpha$ to mean that β implies α, where implication was now understood as plain material implication. This is the use of the symbol that we find in Russell and Whitehead's *Principia Mathematica*.[14] Subsequently, the latter use became standard, and the original inferentialist meaning of Peano's notation faded into oblivion.

As will emerge in the next sections, inferentialism has eluded a canonical formal develoment so far, which explains at least in part its present lack of popularity among logicians. In comparison, the striking technical and theoretical success of the material account has granted it a central place in the orthodoxy of the XX century, prompting the impression that, "much confusion has been produced in logic by the attempt to identify conditional statements with expressions of entailment", as Kneale and Kneale once put it.[15] However, in spite of the widespread dominance of the material account, the idea that conditionals express inferential relations has never really disappeared. Several contemporary authors, including C.I. Lewis, Ramsey, Goodman, Ryle, Mackie, and Strawson, have described the behaviour of conditionals in natural language along inferentialist lines.[16] This resilience suggests that the notion of inference is deeply rooted in our pretheoretic understanding of conditionals, or so we are inclined to believe.

2 THREE MAJOR CHALLENGES

In section 1 we presented a brief historical overview of inferentialism. Now we will describe three major challenges that an inferentialist account of conditionals has to face.

12. Sextus Empiricus, in *Outlines of Scepticism*, II, 110-11, ascribes this reading to Philo, see Sextus Empiricus 2000, p. 96.

13. C. Peano 1894, pp. 10-11.

14. Russell and Whitehead 1910. Peano himself used the reversed symbol in the following editions of his *Formulaire*, but without associating it to Russell's reading.

15. Kneale and Kneale 1962, p. 134.

16. C. I. Lewis 1912, p. 529, Ramsey 1990, p. 156, Goodman 1947, p. 117, Ryle 1950, Mackie 1973, p. 83, Strawson 1950, p. 233.

Challenge 1: Key test cases. Consider the following sentences, which concern a series of tosses of a fair coin and Real Madrid's football season:

(1) If the first 5 tosses are all heads, there will be at least 5 heads in the first 1.000 tosses.

(2) If Real Madrid loses the first 10 matches, its coach will be fired.

(3) If Real Madrid loses the first 10 matches, there will be at least 5 heads in the first 1.000 tosses.

(4) If Real Madrid loses the first 10 matches, 5+5=10.

Intuitively, (1)-(4) are not all equally compelling: while (1) and (2) seem perfectly reasonable, (3) and (4) strike as odd. The first impression one has with (3) and (4) is that something is missing, some sort of connection between the antecedent and the consequent. After all, one may be tempted to say, the result of a football match has nothing to do with the outcome of a coin toss, or with a mathematical truth.

The intuitive difference just illustrated causes explanatory troubles to most extant accounts of conditionals. On the material account, (1)-(4) are all true as long as their antecedent is false, which is very likely.[17] The suppositional views based on the Ramsey Test do not fare better. On the probabilistic account suggested by Adams, which equates the acceptability of a conditional with the conditional probability of its consequent given its antecedent, (1)-(4) are all highly acceptable, due to the high probability of their consequent given their antecedent.[18] On the modal account developed by Stalnaker and Lewis, according to which a conditional is true when its consequent is true in the closest world, or worlds, in which its antecedent is true, (1)-(4) are all true, given that they satisfy the condition required.[19] A similar result is obtained on the belief revision account due to Gärdenfors and others, according to which a conditional is acceptable just in case its consequent belongs to the belief state obtained by adding the antecedent to one's set of beliefs.[20] The intuitive difference just illustrated causes explanatory troubles to most extant accounts of conditionals. On the material account, (1)-(4) are all true as long as their antecedent is false, which is very likely. The suppositional views based on the Ramsey Test do not fare better. On the probabilistic

17. Much like the material account, trivalent truth-functional treatments, such as that offered in Égré, Rossi, and Sprenger 2020, also fails to discriminate the cases (1)-(4).

18. Adams 1965.

19. Stalnaker 1991, D. Lewis 1973.

20. Gärdenfors 1978.

account suggested by Adams, which equates the acceptability of a conditional with the conditional probability of its consequent given its antecedent, (1)-(4) are all highly acceptable, due to the high probability of their consequent given their antecedent.[21] On the modal account developed by Stalnaker and Lewis, according to which a conditional is true when its consequent is true in the closest world, or worlds, in which its antecedent is true, (1)-(4) are all true, given that they satisfy the condition required.[22] A similar result is obtained on the belief revision account due to Gärdenfors and others, according to which a conditional is acceptable just in case its consequent belongs to the belief state obtained by adding the antecedent to one's set of beliefs.[23]

Admittedly, there may be diverse stories, including pragmatic stories, of why (1) and (2) seem perfectly reasonable while (3) and (4) strike as odd.[24] However, if one could explain this intuitive difference in purely semantic terms, it would be a worthwile achievement. This is precisely the project pursued by inferentialists. In their perspective, the correct explanation must be that (1) and (2) correspond to justified inferences, whereas the same does not hold for (3) and (4). A formal account of conditionals where a symbol $\triangleright$ is suitably defined in terms of the inferential relation postulated should be able to explain the intuitions about (1)-(4) precisely on this ground. That is, once (1)-(4) are represented as sentences of the form $p \triangleright q$, the account should imply that (1) and (2) hold while (3) and (4) do not hold.

Challenge 2: Probabilistic relevance. One way to draw the line between cases like (2), in which some relation of support plausibly ties the antecedent to the consequent, and cases like (3), in which no such relation seems to obtain, is to resort to probabilistic considerations. While in the case of (2) there is a clear probabilistic correlation between antecedent and consequent, a key damning feature of (3) is that the credibility of its antecedent and the credibility of its consequent are unrelated, that is, the two sentences are probabilistically independent. Arguably, an inferentialist account of conditionals should be able to make sense of this remark, allowing for an explicit connection with the language of probability. Ideally, the account should imply that $p \triangleright q$ holds only if there is some degree of positive relevance between p and q in a suitable probabilistic analysis.

Challenge 3: Logical profile. In an inferentialist perspective, $p \triangleright q$ is meant to say that p is a reason for q. Which principles of conditional

21. Adams 1965.

22. Stalnaker 1991, D. Lewis 1973.

23. Gärdenfors 1978.

24. Douven, Elqayam, and Krzyżanowska 2023 discuss a range of alternative approaches, and find them all eventually defective. On the other hand, Lassiter 2022 and Bourlier et al. 2023 defend a pragmatic analysis on both theoretical and empirical grounds.

logic should be validated or violated by statements of this kind? To serve the purposes of inferentialism, the logic of $\triangleright$ should be plausible as a logic of reasons. Logical principles that reasons arguably fulfil should thus be retained, whereas logical principles that reasons arguably contradict should be rejected. A valuable guideline here is the thought that an adequate theory of reasons should be able to model both conclusive reasons and non-conclusive or defeasible reasons, thus presenting a general pattern of logical results that applies to reasons in general. In the extension from conclusive to defeasible reasons, the loss of logical strength would ideally remain within limits: the theory should definitely reject defective principles, but not drop ones that seem to plausibly survive for defeasible reasons.

In the next three sections we will present three inferentialist accounts of conditionals, and we will show how Challenges 1-3 can be used to assess the plausibility of each of these accounts.

3 THE STRICT ACCOUNT

The first account to be discussed is the classical strict account, which defines $p \triangleright q$ as $\Box(p \supset q)$, that is, $p \triangleright q$ is true just in case it is impossibile that p is true and q is false. This account, which goes back to Diodorus, was revived by C.I. Lewis at the beginning of the twentieth century, and is still a widely discussed option in the contemporary debate on conditionals.[25]

The strict account is inferentialist in a straightforward sense. As long as a valid argument is understood as an argument in which it is impossible that the premises are true and the conclusion is false, the strict account implies that a conditional is true just in case the corresponding argument is valid. This is the sense that some logicians of the past had in mind when they made inferentialist claims. A clear example is Abelard. As we have seen, Abelard defines a true conditional as an inference in which something follows from something else, and rejects the hypothesis that the truth of a conditional consists in its mere *probabilitas*. The same equation between true conditionals and deductively valid arguments is postulated by C.I. Lewis, Ramsey, Goodman, and others.[26]

Despite its venerable tradition, however, the strict account seems clearly unable to meet Challenges 1-3 in a satisfactory way, because the criterion of truth it imposes on conditionals is too strong in a crucial

25. Sextus Empiricus, in *Outlines of Scepticism*, II, 110-11, ascribes this reading, or at least a temporal version of it, to Diodorus, see Sextus Empiricus 2000, p. 96. C. I. Lewis 1914 is a seminal paper on the topic. More recently, the strict account has been developed in different ways in Lycan 2001, Gillies 2009, Kratzer 2012, among other works.

26. C. I. Lewis 1912, p. 529, Ramsey 1990, p. 156, Goodman 1947, p. 117.

sense as well as arguably too weak in other respects. Let us consider these challenges one by one.

Challenge 1 causes serious troubles to the strict account. On this account, (1) is true because it is impossible that its antecedent is true and its consequent is false. However, (2) and (3) turn out to be both false, given that the truth of their antecedent does not rule out the falsity of their consequent. Moreover, (4) turns out to be vacuously true, because its consequent expresses a necessary truth.

As to (2), an advocate of the strict account might appeal to contextual restrictions on the domain of quantification and claim that, once we consider the set of worlds that are relevant in the context of utterance, there is no world in that set in which the antecedent is true and the consequent is false. The fact, however, is that if some such story can be provided to accommodate (2), one may wonder why a similar story shouldn't apply to (3), due to the extremely high probability of its consequent, thus again losing the intuitive difference between (2) and (3). After all, the possibility of not having 5 heads in 1000 tosses of a fair coin is negligeable for most purposes.

Challenge 2 raises a related problem. As long as the strict account is unable to draw a principled distinction between (2) and (3), it fails to capture the apparent difference of probabilistic relevance between (2) and (3). The fact is that the probabilistic counterpart of a strict conditional would be a conditional that gets value 1 when the conditional probability of its consequent given its antecedent is 1, and 0 otherwise, which is too coarse-grained a criterion to make sense of cases like (2) and also not fine-grained enough to retain (1) without retaining (4).

Now consider Challenge 3. The strict account definitely has some virtues when it comes to the logical profile of $\triangleright$. Here are two principles that reasons arguably fulfill and that hold for strict conditionals. The first is *AND*, the principle according to which $p \triangleright q$ and $p \triangleright r$ entail $p \triangleright (q \wedge r)$. This inferential rule arguably holds for reasons in general: if p is a reason for q, and p is a reason for r, it seems to follow that p is a reason for $q \wedge r$. The second is *OR*, the principle according to which $p \triangleright r$ and $q \triangleright r$ entail $(p \vee q) \triangleright r$. If each of p and q is a reason for r, it seems to follow that $p \vee q$ is a reason for r. The strict account also invalidates inference rules that reasons arguably do not fulfill, such as *Conjunctive Sufficiency*, the principle according to which $p \wedge q$ entails $p \triangleright q$. Clearly, it may happen that p and q both hold but are totally unrelated, so that p is not a reason for q.[27]

27. Recently, whether people comply with Conjunctive Sufficiency has also been a matter of empirical investigation in the psychology of reasoning. Cruz et al. 2016 and Douven, Elqayam, and Hasshim, ms, have drawn quite divergent conclusions from the available evidence.

The main shortcoming of the strict account, however, is that it validates *Monotonicity*, the principle according to which $p \triangleright q$ entails $(p \wedge r) \triangleright q$ for any r. This principle is at odds with the very idea that $\triangleright$ represents defeasible inference: on a widespread understanding of defeasibility, to say that p is a defeasible reason for q is to say precisely that, for some r, $p \wedge r$ is not a reason for q.

A related worry concerns *Right Weakening*, the principle according to which $p \triangleright q$ entails $p \triangleright r$ whenever $q \models r$, where $\models$ is classical logical consequence. Although Right Weakening is very reasonable when one restricts consideration to conclusive reasons, it becomes more problematic as a rule for reasons in general. Arguably, it may be the case that p is a reason for q without thereby being a reason for r, in spite of the fact that $q \models r$, because by weakening the conclusion the positive relevance of the premise can decrease or get lost.

More generally, it is arguable that in order to represent defeasible inference as distinct from conclusive inference, some principle that holds for the strict conditional must fail. As we will explain, the two accounts discussed in the next two sections weaken the logic of the strict conditional in different ways.

4 DOUVEN'S THRESHOLD/INCREMENT ACCOUNT

According to Douven, $p \triangleright q$ is acceptable when (i) the conditional probability of q given p is high enough — relative to a threshold greater than 0.5 — and (ii) p gives some amount of evidential support to q. Following a standard probabilistic construal of evidential support, (ii) means that the conditional probability of q given p is higher than the *un*conditional probability of q itself. This requirement is intended to capture the intuition that the antecedent of an acceptable conditional must be relevant to its consequent.[28]

The threshold/increment account suggested by Douven is fully satisfactory in addressing Challenges 1 and 2, but not as much in addressing Challenge 3. Challenge 1 is fully met because (1) and (2) turn out to be clearly acceptable while (3) and (4) turn out to be clearly unacceptable. In (1) and (2), the conditional probability of the consequent given the antecedent is high enough, and higher than the unconditional probability of the consequent. In (3) and (4), by constrast, despite the high conditional probability of the consequent given the antecedent, (ii) is violated: that conditional probability is just as high as the unconditional probability of the consequent. In particular, Douven calls (3) and similar sentences "missing link

28. See Douven 2016, p. 108.

conditionals". Another label, adopted by Cruz and Over, is "Walrus conditionals".[29]

In fact Douven uses precisely examples such as (2) and (3) to make an important point against Adams' probabilistic account. Experiments have shown that by and large people judge conditionals such as (3) as significantly less plausibile than conditionals such as (2), even if the corresponding conditional probabilities are matched.[30] This fact, known as *relevance effect*, shows that it would be wrong to assume, as McGee once put it, that Adams' probabistic account "describes what English speakers assert and accept with unfailing accuracy".[31] The same point applies, as we have seen, to other suppositional views such as the Stalnaker-Lewis account and the belief revision account.

Challenge 2 is also fully met. It is a straightforward consequence of the threshold/increment account that in an acceptable conditional the antecedent is relevant for the consequent precisely in the sense that, assuming the former, the probability of the latter is higher than it would be otherwise. So the connection with probabilistic relevance is direct and general.

Now let us turn to Challenge 3. In Douven's framework, a logic for $\triangleright$ is developed from the idea of acceptability preservation for all probability distributions and all thresholds. The threshold/increment account is surely effective in avoiding principles of conditional logic that are dubious for inferentialists. At least three cases deserve attention: Conjunctive Sufficiency, Monotonicity, and Right Weakening. These three principles are invalid according to Douven's theory, which we take to be a desirable result. However, the threshold/increment account is not equally effective in preserving principles of conditional logic that seem plausible for inferentialists. Here two key examples are AND and OR: Douven's theory does not validate these two principles.[32]

5 Rott's difference-making account

The third account, due to Rott, hinges on the notion of *difference-making*: for a conditional to hold, its antecedent has to make a difference as concerns the credibility of its consequent. More precisely, $p \triangleright q$ holds if and only if (i) q holds in all closest worlds in which p holds, and (ii) it is not the case that q holds in all closest worlds in which $\neg p$ holds. While (i) expresses the Ramsey Test, (ii) is an additional clause

29. Cruz and Over, forthcoming. The two labels are meant to overlap only partially. In particular, according to Cruz and Over 2023, some pragmatically acceptable conditionals would qualify as "missing-link" but are not Walrus conditionals.

30. See Skovgaard-Olsen, Singmann, and Klauer 2016.

31. McGee 1986, p. 485.

32. Douven 2016, pp. 129-130.

devised to capture the intuition that q holds in virtue of p. Rott labels "Relevant Ramsey Test" the combination of (i) and (ii).[33]

The difference-making account has good prospects for Challenge 2, but faces rather serious difficulties relative to Challenges 1 and 3. Consider Challenge 1. On this account, the intuitive difference between (2) and (3) is explained by saying that (3), unlike (2), does not satisfy (ii): plausibly, at least 5 heads in 1.000 tosses will arise in the closest worlds in which Real Madrid does not lose the first 10 matches. Moreover, (4) turns out to be unacceptable as well, given that again (ii) is not satisfied. The problem is, however, that the difference-making account does not make sense of the intuitive plausibility of (1). In fact, (1) is predicted to be equally unacceptable, and for the same reason: even in the closest worlds in which there is some tails in the first 5 coin tosses, there will still be at least 5 heads in the first 1.000 tosses.

Challenge 2 also raises a non-trivial question. Rott's theory is spelled out deliberately in a qualitative framework to represent doxastic states of acceptance and non-acceptance. While this is a totally legitimate move, soliciting a connection with probability can be motivated by analogy with the Ramsey Test. The modal interpretation of the Ramsey Test has a counterpart in Adams' probabilistic semantics as a requirement that the conditional probability of the consequent given the antecedent be high. Since Rott's theory implies a strengthening of the Ramsey Test through an additional clause, one is led to figure out what a probabilistic condition corresponding to this clause would look like. The most natural idea is that the probability of the consequent given the negated antecedent be low. A high value of $P(q|p)$ and a low value of $P(q|\neg p)$ are surely enough to represent the positive probabilistic relevance of p for q. However, no project has been thoroughly pursued so far to establish how such relevance may arise from the fulfilment of the Relevant Ramsey Test. To this extent, bridging the gap between the qualitative framework of possible worlds and the quantitative structure of probabilities remains an open issue for this account. In general, a systematic study of how modal and probabilistic characterizations are coupled seems an interesting research project of its own, regardless of whether a theory effectively achieves all its philosophical goals. Earlier attempts in this direction have provided significant insight in the case of the suppositional conditional.[34]

33. See Rott 1986 and Rott 2022. Rott relies on the AGM formalism as his favourite technical machinery. However, framing the theory in a possible world semantics will be immaterial for our purposes and will make subsequent comparisons easier. See Raidl 2021 for a discussion.

34. See Adams, E. W. 1977 and Leitgeb 2017, 6.2. If one defines the acceptability of $\alpha \triangleright \beta$ as the difference between $P(\beta|\alpha)$ and $P(\beta|\neg\alpha)$ in case $P(\beta|\alpha) \geq P(\beta|\neg\alpha)$ (and zero otherwise, or in case $P(\alpha) = 0$), distinctive logical features of Rott's difference-

Let us turn to Challenge 3. In Rott's approach, the logic is developed from the idea of acceptability preservation for all rankings of possible worlds, where rankings of possible worlds can be understood as belief states. The difference-making account aptly avoids some crucial principles that are dubious for inferentialists, such as Monotonicity, Conjunctive Sufficiency, and Right Weakening. However, other principles also fail while being plausible instead, such as OR. Moreover, some principles validated by this account appear devoid of sound justification in an inferentialist perspective. A rather striking example is *Affirming the Consequent*, the inference from $p \triangleright q$ and q to p. Suppose that q holds in all closest worlds (which means that q is acceptable). Then p must also hold is those worlds, for otherwise (ii) would be violated, against the assumption that $p \triangleright q$ holds. Here is an example:

(5) If a meteorite hits Carol's house, her favourite mug will be broken.

It makes good sense to endorse (5) as compelling in an inferentialist perspective. The difference-making interpretation of $\triangleright$ then implies that, given (5) and the additional assumption that Carol's favourite cup of coffee has broken, one can conclude that the space rock strike occurred. This is quite odd. After all, $p \triangleright q$ is meant to convey that q can be inferred from p, not the other way around. So it seems that Challenge 3 is not convincingly met, as the logic generated by the difference-making account is both unduly weak in certain respects and unduly strong in others.[35]

6 THE CHRYSIPPUS TEST

So far we have presented three main challenges that inferentialism has to face, and we have discussed three inferentialist accounts of conditionals, showing how each of them is affected by serious difficulties. The aim of the rest of the paper is to develop an account of conditionals that yields better results with respect to our three challenges. This account, as we will explain, is an amended version of the evidential account suggested by Crupi and Iacona.[36]

The core idea of the evidential account is that $p \triangleright q$ holds just in case p and $\neg q$ are *incompatible*, where the incompatibility between p and $\neg q$ is taken to define the relation of support that obtains between p and q.

making conditional are recovered. Several interesting results of this kind have now been proved in Calderisi 2023, and Rott 2023a. The difference between acceptability so defined and the probabilistic version of the evidential account reflects well-known alternatives in measures of evidential support. See Brössel 2013 for an overview.

35. The relevant results for the logical profile of Rott's difference-making conditional are in Rott 2022

36. Crupi and Iacona 2022a, Crupi and Iacona 2022b

Crupi and Iacona call *Chrysippus Test* this incompatibility condition, because the first clear formulation of it goes back to Chrysippus, at least according to the secondary sources.[37]

The Chrysippus Test implies a direct connection between conditionals and arguments, to the extent that a valid argument is understood as an argument in which the premises are jointly incompatible with the negation of the conclusion. This is surely a widely accepted view as concerns conclusive arguments, and it can work as a fruitful guideline to think about defeasible arguments too by suitably extending the notion of validity.[38]

In fact, Crupi and Iacona identify two distinct forms of incompatibility, which yield two distinct senses in which the corresponding inference can be valid. One is *absolute incompatibility*, which rules out the possibility of holding together and so qualifies as the strongest form of incompatibility. When p and $\neg q$ are absolutely incompatible, the inference from p to q is concusively valid. The other is *relative incompatibility*, which implies that the possibility of holding together is remote, although it exists. When p and $\neg q$ are relatively incompatible, the inference from p to q is defeasibly valid.

The next two sections outline two independent ways to formally specify the Chrysippus Test: one relies on a probabilistic semantics, the other relies on a modal semantics. So we will provide a probabilistic version and a modal version of the evidential account. Both versions are based on Crupi and Iacona's work, although they contain two crucial amendments. The first amendment is common to both versions, while the second only concerns the modal version.

The two versions of the evidential account will be phrased by using a single language L defined on the basis of a propositional language L_p constituted by a finite set L_{pa} of atomic formulas $p, q, r, \ldots$, the connectives $\neg, \supset$, and the brackets $(,)$. The alphabet of L extends the alphabet of L_p by adding the connectives $\Box, >, \triangleright$. The formulas of L are defined as follows: if $\alpha \in L_p$, then $\alpha \in \mathsf{L}$; if $\alpha \in L_p$, then $\Box\alpha \in \mathsf{L}$; if $\alpha, \beta \in L_p$, then $\alpha > \beta, \alpha \triangleright \beta \in \mathsf{L}$; if $\alpha \in \mathsf{L}$, then $\neg\alpha \in \mathsf{L}$. The additional connectives $\wedge, \vee, \Diamond$ can be introduced in the usual way.

Note that, if the formulas of L_p are called *propositional*, the formation rules of L do not allow non-propositional formulas to occur in the scope of $\Box, >, \triangleright$, although such formulas can occur in the scope of $\neg$. This limitation is functional to the probabilistic semantics. Although adopting unrestricted formation rules would raise no technical problem in the modal semantics, here we aim at establishing a connection between the two versions of the evidential account, which is easier with a shared syntax.

37. Sextus Empiricus, *Outlines of Scepticism*, II, 111, in Sextus Empiricus 2005, p. 96.
38. Iacona 2023 develops precisely this idea under the label 'Stoic Thesis'.

In the probabilistic version of the evidential account, the acceptability of $p \triangleright q$ is defined in terms of a probabilistic measure of incompatibility expressed as follows for any probability distribution P:

DEFINITION 1 If $P(p \wedge \neg q) \leq P(p)P(\neg q)$ and $P(p)P(\neg q) \neq 0$, the incompatibility between p and $\neg q$ is

$$1 - \frac{P(p \wedge \neg q)}{P(p)P(\neg q)}$$

Otherwise, the incompatibility between p and $\neg q$ is 0.

This definition contemplates two cases. In the first case, p and $\neg q$ are not positively correlated, and the formula provided represents the *mutual relative reduction of credibility* according to P: it is equivalent to $(P(p) - P(p|\neg q))/P(p)$, the proportion of the initial probability of p that is cancelled out by the downward jump to $P(p|\neg q)$, which in turn is identical to $(P(\neg q) - P(\neg q|p))/P(\neg q)$, the proportion of the initial probability of $\neg q$ that is cancelled out by the downward jump to $P(\neg q|p)$. (The same quantity is also equivalent to 1 minus a popular probabilistic measure of the "coherence" between p and $\neg q$, originally introduced by Shogenji.[39]) In the second case, either p and $\neg q$ are positively correlated or $P(p)P(\neg q) = 0$. Note that, whenever p and $\neg q$ are incompatible to some degree greater than 0, the incompatibility between p an q is 0, for p and q are positively correlated.[40]

The degree of acceptability of $p \triangleright q$ can be equated with the degree of incompatibility between p and $\neg q$ as specified by definition 1. That is, $p \triangleright q$ intuitively holds when $p \triangleright q$ is highly acceptable, which means that the degree of incompatibility between p and $\neg q$ is high. Conversely, $p \triangleright q$ intuitively does not hold when the degree of incompatibility between p and $\neg q$ is low, or equivalently when p and $\neg q$ are only slightly incompatible, or not at all.

Since the formula in definition 1 yields a value greater than 0 only if $P(q|p) > P(q)$, we get that the degree of acceptability of $p \triangleright q$ is strictly positive only if p increases the probability of q. Unlike in Douven's theory, however, positive probabilistic relevance *per se* is not sufficient for $p \triangleright q$ to be highly acceptable, not even in case $P(q|p)$ itself is high. Just as it is possible for p and $\neg q$ to be fully independent, and thus fully compatible, while $P(q|p)$ is high, so that $p \triangleright q$ has zero acceptability, it is also possible that p and $\neg q$ are only very mildly

39. Shogenji 1999.
40. This measure of the incompatibility between p and $\neg q$ is identical to the measure of argument strength from p to q defined in Rips 2001 and to Bayesian confirmation as partial entailment in Crupi and Tentori 2013, Crupi and Tentori 2014.

incompatible, so that $p \rhd q$ is still not quite acceptable. The following is a plausible illustration, as concerns a fair coin that is tossed 20 times:

(6) If there is at least one head out of 20, then there are at least 8 heads out of 20.

In (6), the conditional probability of the consequent given the antecedent is demonstrably high, and slightly higher than its unconditional probability. Yet this does not seem to make (6) highly acceptable, for the inferential connection is arguably too weak. As long as 'reason' is understood as 'sufficient reason', there is a plausible sense in which the antecedent of (6) does not provide a reason for accepting its consequent.[41]

Now we will show how the probabilistic analysis of incompatibility just illustrated can be incorporated in a coherent probabilistic semantics for L. In line with a tradition initiated by Adams, we will define a valuation function for formulas based on the probability of their propositional constituents. The function V, which can be understood as a measure of acceptability, is defined as follows for any probability distribution P over L_p:

DEFINITION 2

1 For every $\alpha \in L_p$, $V_P(\alpha) = P(\alpha)$;

2 $V_P(\Box\alpha) = \begin{cases} 1 & if P(\alpha) = 1 \\ 0 & otherwise; \end{cases}$

3 $V_P(\alpha > \beta) = \begin{cases} P(\beta|\alpha) & if P(\alpha) > 0 \\ 1 & if P(\alpha) = 0; \end{cases}$

4 $V_P(\alpha \rhd \beta) = \begin{cases} 1 - \frac{P(p \wedge \neg q)}{P(p)P(\neg q)} & if P(p \wedge \neg q) \leq \ P(p)P(\neg q) \neq 0, \\ 0 & otherwise; \end{cases}$

5 $V_P(\neg\alpha) = 1 - V_P(\alpha)$.

Clause 1 says that the degree of acceptability of any propositional formula α relative to P amounts to the probability assigned to α by P. Clause 2 says that $\Box\alpha$ takes either 1, the maximal value, or 0, the minimal value, depending on whether or not $P(\alpha) = 1$. Clause 3 says that the value that V_P assigns to $\alpha > \beta$ is the conditional probability of β given α, with the proviso that $V_P(\alpha > \beta) = 1$ if $P(\alpha) = 0$. This is the suppositional conditional as defined by Adams.[42] Clause 4 is

41. This is not to deny that an intelligible notion of *in*sufficient reason can be defined, namely, a reason that positively contributes to credibility but is not quite enough for inference.

42. Adams 1968. About the stipulation that $V_P(\alpha > \beta) = 1$ if $P(\alpha) = 0$, see Adams 1998, p. 150.

the crucial one, as it specifies the value of $\alpha \triangleright \beta$ in accordance with definition 1. Finally, clause 5 defines negation in the classical way, as it entails that $V_P(\neg\alpha) = 1$ when $V_P(\alpha) = 0$, and that $V_P(\neg\alpha) = 0$ when $V_P(\alpha) = 1$.[43]

Once the function V is defined as above, one can apply the rest of Adams' formal machinery. In particular, one can stipulate that the uncertainty of a formula α relative to a probability distribution P — call it $U_P(\alpha)$ — is $1 - V_P(\alpha)$, and define logical consequence — indicated by the symbol $\models_p$ — as the relation that obtains when the sum of the uncertainties of the premises is higher than or equal to the uncertainty of the conclusion for any probability assignment.

DEFINITION 3

$\alpha_1, ...\alpha_n \models_p \beta$ iff $U_P(\alpha_1) + ... + U_P(\alpha_n) \geq U_P(\beta)$ for any P.

This definition implies that, in a valid argument, the acceptability of the conclusion is guaranteed to be high enough as long as the premises are themselves highly acceptable.[44]

The semantics just outlined is essentially the probabilistic semantics originally formulated by Crupi and Iacona except for one feature. In their original formulation, Crupi and Iacona stipulated that, for the limiting cases in which $P(q) = 1$ or $P(p) = 0$, and thus $P(p)P(\neg q) = 0$, incompatibility is maximal (i.e., 1), for then $P(p \wedge \neg q) = 0$. As a result, $p \triangleright q$ turns out to follow from $\Box q$, as well as from $\neg\Diamond p$.[45] In other words, conditionals with impossible antecedents or necessary consequents are treated as vacuously acceptable, that is, as cases of absolute incompatibility understood as $P(p \wedge \neg q) = 0$. This treatment is in line with an established tradition, which includes Adams, Stalnaker, and Lewis, and relies on the assumption that the impossibility of the conjunction of the antecedent and the negation of the consequent is sufficient for the truth of a conditional.

However, as (4) shows, some cases in which $p \wedge \neg q$ is impossible are potentially contentious in an inferentialist perspective, for they are cases in which there is no clear intuition to the effect that p provides a reason for q. More generally, if an inferentialist theory of conditionals validates *Necessary Consequent*, the principle according to which $\Box q$ entails $p \triangleright q$, the theory thereby implies that p — just like anything else — is a reason for q merely in virtue of the necessity of q. Similarly, if an inferentialist theory of conditionals validates *Impossible Antecedent*, the principle according to which $\neg\Diamond p$ entails $p \triangleright q$, the theory thereby

43. Note that, when $V_P(\neg\Box\neg\alpha) = 1$, we get that $V_P(\Box\neg\alpha) = 0$, which means that $\neg\alpha$ is not necessary, hence that α is possible. This shows that $\Diamond\alpha$ can be defined in the usual way as $\neg\Box\neg\alpha$.

44. Adams 1966.

45. See Crupi and Iacona 2022b and Crupi and Iacona, 2021.

implies that p is a reason for q — or for anything else — merely in virtue of its impossibility. For example, the conditional obtained by contraposing (4) would be an instance of the latter principle.

In other words, Necessary Consequent and Impossible Antecedent imply that p and $\neg q$ can be absolutely incompatible merely in virtue of some property — impossibility or necessity — that belongs to one of them independently of the other. This goes against a thought that may naturally be associated with the Chrysippus Test, namely, that the incompatibility between p and $\neg q$ is *relational*: what is wrong with the combination of p and $\neg q$ must somehow depend on p and $\neg q$ taken together, that is, it must not arise from p or $\neg q$ taken separately.[46] For example, in (1) the antecedent and the negation of the consequent form an impossible combination, while there is nothing wrong with each of them taken separately. So their incompatibility is relational in a sense that we do not find in (4).

In order to preserve relationality in this sense, absolute incompatibility should *not* be equated with the condition that $P(p \wedge \neg q) = 0$. The class of cases in which the former holds — due to the combination of p and $\neg q$ — should be a proper subset of the class of cases in which the latter holds. In the probabilistic semantics outlined above, this is obtained by not stipulating that the incompatibility between p and $\neg q$ is 1 when $P(p) = 0$ or $P(q) = 1$. If the incompatibility between p and $\neg q$ is assumed to be 0 in those cases, as in definition 1, we get that only some of the cases in which $P(p \wedge \neg q) = 0$ are cases of absolute incompatibility. More precisely, p and $\neg q$ are absolutely incompatible when $P(p \wedge \neg q) = 0$ but $P(p)P(\neg q) \neq 0$, as in (1), so the value of the formula in definition 1 is 1.

The rationale for this choice is somehow opposite to the rationale that one would employ for Crupi and Iacona's original definition. Instead of assuming that anything is a reason for a necessary truth, it is assumed that *nothing* is a reason for a necessary truth, given that there is no interesting sense in which something can support a necessary truth. Similarly, instead of assuming that an impossible truth is a reason for anything, it is assumed that it is a reason for *nothing*, for again there is no interesting sense in which an impossible truth can support something.[47]

8 EVIDENTIAL ACCOUNT: MODAL VERSION

In the modal version of the evidential account, the truth of $p \triangleright q$ is defined in terms of a disjunctive condition that makes explicit the dis-

46. This is essentially the point made in Nelson 1930, p. 443.
47. As explained in Lenzen 2023, the claim that nothing can follow from an impossible truth can be ascribed to the followers of Robert of Melun (ca 1100-1167).

tinction between absolute incompatibility and relative incompatibility. The definition goes as follows:

DEFINITION 4 p and $\neg q$ are incompatible iff either there are no worlds in which p is true and q and false, and

(a) p is true in some world;

(b) q is false in some world;

or there are worlds in which p is true and q is false and

(c) p and q have the same value in some of the closest worlds;

(d) in the closest worlds in which p is true, q is also true;

(e) in the closest worlds in which $\neg q$ is true, $\neg p$ is also true.

The first disjunct defines absolute incompatibility as the impossibility that p and $\neg q$ are jointly true, provided that such impossibility does not depend on p being impossible or q being necessary. Here (a) and (b) rule out cases of vacuous truth, thus warranting that the incompatibility between p and $\neg q$ essentially depends on the relation between p and $\neg q$. This is in line with the amendment explained in the previous section in connection with the probabilistic semantics.[48]

The second disjunct defines relative incompatibility in terms of three conditions. (c) requires that p and q have the same value — hence p and $\neg q$ have different values — in some of the closest worlds. One way to make sense of this condition is the following: if p and $\neg q$ are relatively incompatible, meaning that their combination is a remote possibility, p and q must be relatively compatible, meaning that their combination is a near possibility, so it is reasonable to rule out that p and q have different values in all the closest worlds. (d) expresses the Ramsey Test, and implies that $\neg q$ is false in the closest worlds in which p is true. Note that, given (d), the only interesting case ruled out by (c) is that in which p is false and q is true in the closest worlds. (e) expresses what Crupi and Iacona call the *Reverse Ramsey Test*, and implies that p is false in the closest worlds in which $\neg q$ is true. To say that (c)-(e) are jointly satisfied is to say that the combination of p and $\neg q$ is a remote possibility.

Definition 4 thus says that p and $\neg q$ are incompatible just in case either they are absolutely incompatible or they are relatively incompatible. When the first disjunct holds, we say that p is a conclusive reason for q. When the second disjunct holds, we say that p is a

48. Priest 1999, p. 145, considers an account of conditionals that equates their truth conditions with the first disjunct. Lenzen 2022 ascribes such account to Chrysippus. The same account is discussed in Gherardi and Orlandelli 2021 and advocated in Raidl and Gomes 2023.

defeasible reason for q. Note that the first disjunct can be satisfied even if (c) is not fulfilled, which means that absolute incompatibility does not entail relative incompatibility. Therefore, as far as definition 4 is concerned, being a conclusive reason for q does not entail being a defeasible reason for q.

Now we will set out a preferential semantics for L that incorporates the modal analysis of incompatibility just illustrated. Let us start with the definition of model:

DEFINITION 5 A *m-model* for L is a quadruple $\langle W, F, \prec, v \rangle$, where

- W is a non-empty set
- F assigns to each $x \in W$ a subset W_x of W
- $\prec$ assigns to each $x \in W$ an irreflexive and transitive relation $\prec_x$ on W_x
- v assigns to each $x \in W$ and $\alpha \in L_{pa}$ one element of $\{0, 1\}$

W is a set of worlds. F is a function that determines a sphere of accessibility W_x for each $x \in W$. $\prec$ is a function that assigns to each $x \in W$ an order of preference. We interpret $y \prec_x z$ as saying that y is preferred to z relative to x, or equivalently that y is strictly closer than z relative to x. Given this order of preference one can define, for any $A \subseteq W$, a set $Min_x(A)$ of *x-minimal* worlds as follows:

DEFINITION 6 $Min_x(A)$ is the set of all $y \in A \cap W_x$ such that there is no $z \in A \cap W_x$ such that $z \prec_x y$.

For the sake of simplicity, it is useful to write $Min_x(\alpha)$ for $Min_x(A)$ when A is $||\alpha||$, the set of worlds in which α is true. When A is W_x itself, one can simply write Min_x.[49]

Although definitions 5 and 6 apply to a wide variety of structures, m-models can be constrained in different ways. In particular, the following conditions deserve attention:

(Uni) $W_x = W$.

(LA) If $||\alpha|| \cap W_x \neq \emptyset$, then $Min_x(\alpha) \neq \emptyset$.

(SC) $Min_x = \{x\}$.

(Uni) is *Universality*: every world is accessible from any world. As long as this condition holds, one can simply write $\langle W, \prec, v \rangle$ instead of $\langle W, F, \prec, v \rangle$, given that F is inert. (LA) is the *Limit Assumption*, which ensures that we always reach x-minimality for every α, ruling out

49. Here we follow Giordano et al. 2009, where a preferential semantics is defined for a wide class of conditional logics.

infinitely descending chains. (SC) is *Strong Centering*: it requires that any world other than x is strictly further away from x than x itself. In what follows, for dialectical purposes, we restrict consideration to m-models that satisfy these three conditions, although weaker constraints would be equally compatible with the definitions provided below. For example (SC) could be dropped or replaced by the weaker requirement that $x \in Min_x$.

The truth of a formula in a world x in a m-model is defined as follows:

DEFINITION 7

1 $[\alpha]_x = 1$ iff $v(x, \alpha) = 1$, for $\alpha \in L_{pa}$;

2 $[\neg\alpha]_x = 1$ iff $[\alpha]_x = 0$;

3 $[\alpha \supset \beta]_x = 1$ iff $[\alpha]_x = 0$ or $[\beta]_x = 1$;

4 $[\Box\alpha]_x = 1$ iff $[\alpha]_y = 1$ for all $y \in W_x$.

5 $[\alpha > \beta]_x = 1$ iff for every $y \in Min_x(\alpha)$, $[\beta]_y = 1$;

6 $[\alpha \triangleright \beta]_x = 1$ iff either there is no $y \in W_x$ such that $[\alpha]_y = 1$ and $[\beta]_y = 0$, and

(a) $[\alpha]_y = 1$ for some $y \in W_x$;

(b) $[\beta]_y = 0$ for some $y \in W_x$;

or there is $y \in W_x$ such that $[\alpha]_y = 1$ and $[\beta]_y = 0$, and

(c) some $z \in Min_x$ is such that $[\alpha]_z = [\beta]_z$;

(d) for every $z \in Min_x(\alpha)$, $[\beta]_z = 1$;

(e) for every $z \in Min_x(\neg\beta)$, $[\neg\alpha]_z = 1$.

Clauses 1-5 are standard. In particular, clause 5 defines the suppositional conditional as understood by Stalnaker and Lewis. Clause 6 is the crucial one, as it specifies the truth conditions of the evidential conditional in accordance with definition 4.

Logical consequence, indicated by the symbol $\models_m$, is defined in the usual way as preservation of truth in every world in every model:

DEFINITION 8 $\Gamma \models_m \alpha$ iff there is no m-model and x such that $[\beta]_x = 1$ for every $\beta \in \Gamma$ and $[\alpha]_x = 0$.

The semantics just outlined differs in two crucial respects from the modal semantics originally formulated by Crupi and Iacona. One is that clause 6 of definition 7 includes (a) and (b) in the first disjunct, as explained above, so it does not equate absolute incompatibility with the impossibility that the antecedent is true and the consequent is false. The other is that clause 6 of definition 7 includes (c) in the second disjunct, while the earlier modal definition only contains two conditions equivalent to (d) and (e). This second difference, which specifically concerns relative incompatibility, is motivated by the need to overcome a problem that affects the earlier modal definition.

If one examines the original formulation of the two versions of the evidential account, one will find a disturbing asymmetry between them. Although this asymmetry may easily remain undetected, as it does not affect the convergence of the two versions on several important principles of conditional logic, it becomes evident when one considers the suppositional conditionals $p > q$ and $\neg q > \neg p$ in relation to $p \triangleright q$. According to the earlier probabilistic definition, $p \triangleright q$ entails each one of $p > q$ and $\neg q > \neg p$, but it is not itself a logical consequence of those conditionals taken together. If $p \triangleright q$ is highly acceptable, then a high degree of mutual relative reduction of credibility between p and $\neg q$ also entails that both $P(\neg q|p)$ and $P(p|\neg q)$ are low, so that both $P(q|p)$ and $P(\neg p|\neg q)$ are high, making each of $p > q$ and $\neg q > \neg p$ highly acceptable. On the other hand, one can have $P(q|p)$ and $P(\neg p|\neg q)$ very high and still no degree whatsoever of mutual relative reduction of credibility between p and $\neg q$, simply because p and $\neg q$ are probabilistically independent. A conditional such as (3) is a perfect illustration of this scenario. The situation is different with the earlier modal definition. In fact, it is a straightforward consequence of that definition that $p \triangleright q$ *does* follow from $p > q$ and $\neg q > \neg p$.

The source of the problem is, we submit, that the earlier modal definition is too weak in its characterization of relative incompatibility. While still crucially compatible with the possible joint truth of p and $\neg q$, the modal criterion of relative incompatibility has to be strictly stronger than the joint truth of $p > q$ and $\neg q > \neg p$. More precisely, it must *not* obtain merely because the joint *falsity* of p and $\neg q$ is distinctively likely, as in the class of examples illustrated by (3). If the definition is amended by adding (c), as we suggest, one can prevent the incompatibility condition from obtaining when (d) and (e) are satisfied only because p is very unlikely and q is very likely for independent reasons, as in the case of (3).[50] Conversely, it is instructive to figure out what, if any, would have been the probabilistic counterpart of the earlier modal definition. As it turns out, if one takes the acceptability of $\alpha \triangleright \beta$ to be $P(\beta|\alpha) + P(\neg\alpha|\neg\beta) - 1$ (truncated at 0 for negative values), then one reproduces the main relevant properties of that

50. The additional condition (c) can still be phrased in terms of suppositional conditionals: at least one of $\neg p > q$ and $q > \neg p$ must be false. In this sense, the way in which the revised semantics rectifies the old one implies a distinct anti-irrelevance clause, namely that p makes a difference for q or $\neg q$ makes a difference for $\neg p$ in Rott's terms.

definition in a probabilistic setting, including its key shortcoming: $\alpha \triangleright \beta$ now becomes a logical consequence of the pair of suppositional conditionals $\alpha > \beta$ and $\neg\beta > \neg\alpha$ and, relatedly, (3) receives a very high degree of acceptability despite being a missing-link conditional.

9 TEST CASES AND PROBABILISTIC RELEVANCE

This section shows how the evidential account, in the two versions outlined in sections 7 and 8, meets Challenges 1-3 in a fully satisfactory way, thus overcoming the problems raised in connection with the other three accounts discussed.

Let us start with Challenge 1. The probabilistic version of the evidential account provides correct predictions about (1)-(4). Definition 1, given plausible background assumptions, implies that (1) and (2) are highly acceptable while (3) and (4) are unacceptable. More precisely, (1) has acceptability 1 because the probability of the conjunction of its antecedent and its negated consequent is 0; (2) is highly acceptable due to a strong mutual reduction of credibility between its antecedent and the negation of its consequent; (3) has acceptability 0 because the conditional probability of its consequent given its antecedent equals the unconditional probability of its consequent; (4) has acceptability 0 because the probability of its consequent is 1. Similar results are obtained with the modal version of the evidential account. Definition 4, given some plausible background assumptions, implies that (1) and (2) are true while (3) and (4) are false. More precisely, (1) is true in virtue of the first disjunct; (2) is true in virtue of the second disjunct, for (c)-(e) are jointly satisfied; (3) is false because the first disjunct does not hold and (c) is also not satisfied; (4) is false because its consequent is necessarily true.

As explained in sections 7 and 8, these predictions about (1)-(4) crucially differ from those implied by Crupi and Iacona's original formulation of the evidential account, for the earlier probabilistic definition makes (4) maximally acceptable, and the earlier modal definition makes (3) and (4) true.

Now consider Challenge 2. The probabilistic version meets this challenge for the following reason: for a conditional to have a certain positive degree of acceptability, the probability of the consequent given the antecedent must be higher than it would be otherwise. In order to show that the modal version meets this challenge as well, we will show that if $\alpha \triangleright \beta$ is true in a world in a given m-model, then α is indeed positively relevant for β in a suitably defined probabilistic counterpart of that model. Let us start with the following definitions:

DEFINITION 9 A *basic probability assignment* B on a countable set of worlds W is a assignment to every $x \in W$ of a value greater than 0 and such that $\sum_{x \in W} B(x) = 1$.

DEFINITION 10 A *p-model* for L is a triple $\langle W, B, v \rangle$, where

- W is a non-empty countable set
- B is a basic probability assignment on W
- v assigns to each $x \in W$ and $\alpha \in L_{pa}$ one element of $\{0, 1\}$

Note that a p-model $K = \langle W, B, v \rangle$ induces a full probability distribution P_K on L_p (the propositional part of L) where $P_K(\alpha) = \sum_{x \in W, V(\alpha, x) = 1} B(x)$ and V extends v to any propositional formula α following standard principles, e.g., $V(\beta \wedge \gamma, x) = 1$ iff $V(\beta, x) = V(\gamma, x) = 1$. Conditional probabilities are as usual: $P_K(\alpha|\beta) = P_K(\alpha \wedge \beta)/P_K(\beta)$ for any pair of propositional formulas α and β, provided that $P_K(\beta) > 0$. Conversely, given a full probability distribution P_K on L_p, one can identify a corresponding p-model as follows: let W include all maximal sets of literals for distinct atomic formulas in L (a literal is an atomic formula or the negation thereof), then posit $B(\alpha) = P_K(\alpha)$ for each $\alpha \in L_{pa}$, and $v(\alpha, x) = 1$ if and only if $\alpha \in x$.

Now we want to establish a formal connection between m-models and p-models by relying on two stipulations. First, to preserve the analogy with m-models, we will treat P_K as a property that belongs to formulas relative to worlds: trivially, for every formula α and every $x \in W$, the value of $P_K(\alpha)$ in x is nothing but $P_K(\alpha)$. Second, we will focus on a subclass of p-models which match the preference relation that characterizes m-models according to an order-of-magnitude rule, namely, such that $x \prec y$ if and only if $P(x) \geq rP(y)$ for some integer r. Let us start by defining a distinctive kind of basic probability assignments, which we call *finite order of magnitude* assignments, or simply *FOM* assignments:

DEFINITION 11 B is a FOM probability assignment on a countable set W iff B is a basic probability assignment on W and there is a relation $\preceq$ on W and two natural numbers $n > 0$ and $r > 1$ such that:

(i) $A_1, \ldots, A_n \subseteq W$ and $x \sim y$ iff $x, y \in A_i$ for some $1 \leq i \leq n$;

(ii) $x \sim y$ iff $B(x) = B(y)$;

(iii) if $x \prec y$ and no z is such that $x \prec z \prec y$, $B(x) = r \sum_{w \sim y} B(w)$.

As (i) implies, the expression $x \sim y$ is intended to mean that x and y are of the *same level*. That is, $\sim$ partitions W into a finite number of equivalence classes. (ii) equates sameness of level with sameness of probability. (iii) defines the relation between the probability of x and the probability of its immediate successors in terms of r, the order of magnitude of B. We say that x *immediately precedes* y when $x \prec y$ and there is no z such that $x \prec z \prec y$.

The following example may help to illustrate definition 11. Let $W = \{x, y, z, w, k\}$. Let B be a basic probability assignment on W such that $B(x) = 36/50$, $B(y) = B(z) = 6/50$, $B(w) = B(k) = 1/50$. This is a FOM probability assignment on W: (i) and (ii) are satisfied because we have three subsets of W, namely, $\{x\}, \{y, z\}, \{w, k\}$, such that $y \sim z$ and $w \sim k$; (iii) is satisfied for $r = 3$. So we get that $x \prec y \sim z \prec w \sim k$.

One important structural feature of FOM probability assignments is that any world x turns out to be at least $r - 1$ times more probable than all worlds that are strictly less probable than x (equivalently, ranked as strictly further away by $\preceq$) taken together.

FACT 1 Let B be a FOM probability assignment. Let $\preceq$ be a relation on W corresponding to B. Let r be the order of magnitude of B. Then for any $x \in W$, $B(x) > (r-1)\sum_{x \prec y} B(y)$.

Proof. Let $x \in W$. First consider the case in which there is no y such that x immediately precedes y. In this case we get the result desired because $\sum_{x \prec y} B(y) = 0$, hence $(r-1)\sum_{x \prec y} B(y) = 0$, whereas $B(x) > 0$. Now consider the case in which x is exactly one level below, that is, some y is such that x immediately precedes y and no z is such that $y \prec z$. Here we have that $B(x) = r\sum_{y \sim w} B(w) = r\sum_{x \prec w} B(w) > (r-1)\sum_{x \prec w} B(w)$. Next, suppose there are y and z such that x immediately precedes y and y immediately precedes z, but no k is such that $z \prec k$. In this case, $B(x) = r\sum_{y \sim w} B(w) = (r-1)\sum_{y \sim w} B(w) + \sum_{y \sim w} B(w)$. Moreover, we have that $\sum_{y \sim w} B(w) \geq B(y)$, whereas $B(y) > (r-1)\sum_{y \prec w} B(w)$, as proven in the previous case. As a consequence, $B(x) > (r-1)\sum_{y \sim w} B(w) + (r-1)\sum_{y \prec w} B(w) = (r-1)\sum_{x \prec w} B(w)$. The same kind of reasoning can be reiterated until a final level is reached. □

The notion of a FOM assignment can be used to define a special kind of p-models:

DEFINITION 12 A *FOM p-model* is a p-model $\langle W, B, v \rangle$ such that B is a FOM assignment.

In other words, a FOM p-model is a p-model characterized by the properties of FOM assignments. In particular, we have what follows:

FACT 2 Let $\langle W, B, v \rangle$ be a FOM p-model. Let $\preceq$ be a relation on W corresponding to B. Let r be the order of magnitude of B. Then for any $x \in W$, $B(x) > (r-1)\sum_{x \prec y} B(y)$.

Proof. Directly from definition 12 and fact 1. □

Now we can define a correspondence relation between FOM p-models and m-models. More precisely, the relation holds between the two kinds of models relative to worlds, or equivalently, it holds between model-world pairs:

DEFINITION 13 Let $K = \langle W, B, v \rangle$ be a FOM p-model. Let $M = \langle W, \prec, v \rangle$ be a m-model. For any $x \in W$, we say that $\langle K, x \rangle$ *matches* $\langle M, x \rangle$ iff for any $y, z \in W$, $y \prec_x z$ iff $B(y) > B(z)$.

Given this definition, we can prove the following fact:

FACT 3 Let $M = \langle W, \prec, v \rangle$ be a m-model. Let $K = \langle W, B, w \rangle$ be a FOM p-model. Let $\langle K, x \rangle$ match $\langle M, x \rangle$. If $[\alpha \triangleright \beta]_x = 1$, then $P_K(\beta|\alpha) > P_K(\beta)$ in x.

Proof. Assume that $[\alpha \triangleright \beta]_x = 1$, and consider two cases.

Case 1. The first disjunct of clause 6 of definition 7 holds. In this case, there is no $y \in W$ such that $[\alpha]_y = 1$ and $[\beta]_y = 0$, and $[\alpha]_w = 1$ and $[\beta]_z = 0$ for some $w, z \in W$. As a consequence, by fact 2, $P_K(\alpha) > 0$, $P_K(\beta|\alpha) = 1$, and $P_K(\beta) < 1$ in x. So $P_K(\beta|\alpha) > P_K(\beta)$ in x.

Case 2 The second disjunct of clause 6 of definition 7 obtains. In this case, $[\alpha]_y = 1$ and $[\beta]_y = 0$ for some $y \in W$. Given (SC), $[\alpha]_x = [\beta]_x = 1$ or $[\alpha]_x = [\beta]_x = 0$, for the possibility that $[\alpha]_x \neq [\beta]_x$ is ruled out by (c). Consider the first disjunct (the second is similar). By fact 2, we then have that $P_K(\alpha \wedge \beta) > P_K(\neg\alpha \wedge \beta)$ in x. Moreover, by (e), any $z \in Min_x(\neg\beta)$ is such that $[\neg\alpha]_z = 1$. Given that $[\neg\beta]_w = 1$ for some $w \in W$, such z exists in W and $z \prec_x y$ for any y such that $[\alpha \wedge \neg\beta]_y = 1$. As a consequence, again by fact 2, $P_K(\neg\alpha \wedge \neg\beta) > P_K(\alpha \wedge \neg\beta)$ in x. So we have that $P_K(\alpha \wedge \beta)P_K(\neg\alpha \wedge \neg\beta) > P_K(\neg\alpha \wedge \beta)P_K(\alpha \wedge \neg\beta)$ in x, which implies that $P_K(\beta|\alpha) > P_K(\beta)$ in x by the following reasoning:

$$P_K(\alpha \wedge \beta)P_K(\neg\alpha \wedge \neg\beta) > P_K(\neg\alpha \wedge \beta)P_K(\alpha \wedge \neg\beta)$$

$$\frac{P_K(\alpha \wedge \beta)}{P_K(\alpha \wedge \neg\beta)} > \frac{P_K(\neg\alpha \wedge \beta)}{P_K(\neg\alpha \wedge \neg\beta)}$$

$$\frac{P_K(\beta|\alpha)P_K(\alpha)}{P_K(\neg\beta|\alpha)P_K(\alpha)} > \frac{P_K(\beta|\neg\alpha)P_K(\neg\alpha)}{P_K(\neg\beta|\neg\alpha)P_K(\neg\alpha)}$$

$$\frac{P_K(\beta|\alpha)}{P_K(\neg\beta|\alpha)} > \frac{P_K(\beta|\neg\alpha)}{P_K(\neg\beta|\neg\alpha)}$$

$$\frac{P_K(\beta|\alpha)}{1 - P_K(\beta|\alpha)} > \frac{P_K(\beta|\neg\alpha)}{1 - P_K(\beta|\neg\alpha)}$$

$$P_K(\beta|\alpha) - P_K(\beta|\alpha)P_K(\beta|\neg\alpha) > P_K(\beta|\neg\alpha) - P_K(\beta|\alpha)P_K(\beta|\neg\alpha)$$

$$P_K(\beta|\alpha) > P_K(\beta|\neg\alpha)$$

$$P_K(\beta|\alpha) > P_K(\beta)$$

□

Fact 3 shows how the modal version meets Challenge 2. Indeed, a formal connection between the truth of $\alpha \triangleright \beta$ in the modal semantics and the probabilistic relevance of the antecedent α for the consequent β can be spelled out as follows: the conditional probability of β given α is stricly higher than the probability of β in each p-model-world pair which matches a m-model-world pair making $\alpha \triangleright \beta$ true. So there is a clear sense in which the truth of $\alpha \triangleright \beta$ implies that α provides evidential support to β in a suitable probabilistic framework.

Our revised formulation of the Chrysippus Test rectifies a crucial mismatch between the modal and the probabilistic versions of the evidential account as found in Crupi and Iacona's original work. Of course, the modal and the probabilistic frameworks are technically and philosophically heterogeneous, so it would be unwise to expect a perfect alignment between them. But their convergence, in our view, supports the robustness of the core idea of incompatibility between antecedent and negated consequent, which can survive across other significant issues of legitimate theoretical disagreement.

10 LOGICAL PROFILE

The evidential account has no special trouble with Challenge 3, for the distinctive set of logical properties it delivers is particularly appealing for inferentialism. This is already clear from the earlier formulation of the account, so most of the relevant facts can be taken for granted. Here we will focus on the key logical principles mentioned above in connection with Challenge 3. As has been shown by Crupi and Iacona on the basis of the earlier probabilistic definition, Monotonicity, Right Weakening, and Conjunctive Sufficiency fail, while AND and OR hold, which is exactly as it should be. Their proofs of these facts do not essentially depend on the previous stipulation concerning vacuous cases, and can easily be adapted to definition 2.[51] The same results are obtained with the modal semantics outlined in section 8, as the following proofs show.

FACT 4 $\alpha \triangleright \gamma \not\models_m (\alpha \wedge \beta) \triangleright \gamma$ (*Monotonicity* $\times$)

Proof. Let $W = \{a, b, c\}$, $a \prec_a b$, $b \prec_a c$, and

$[\alpha]_a = 1, [\beta]_a = 0, [\gamma]_a = 1$

$[\alpha]_b = 0, [\beta]_b = 0, [\gamma]_b = 0$

$[\alpha]_c = 1, [\beta]_c = 1, [\gamma]_c = 0$

Then $[\alpha \triangleright \gamma]_a = 1$ because there is $y \in W_x$ such that $[\alpha]_y = 1$ and $[\gamma]_y = 0$, namely c, and (c)-(e) are satisfied. Note that $Min_x = Min_x(\alpha) = \{a\}$

51. Crupi and Iacona 2022b, see Appendix.

and $Min_x(\neg\gamma) = \{b\}$. Instead, $[(\alpha \wedge \beta) \rhd \gamma]_a = 0$. In this case (c) is not satisfied because $Min_x = \{a\}$ and $[\alpha \wedge \beta]_a \neq [\gamma]_a$, and (d) is not satisfied because $Min_x(\alpha \wedge \beta) = \{c\}$ and $[\gamma]_c = 0$. □

FACT 5 Not: $\alpha \rhd \beta \models_m \alpha \rhd \gamma$ whenever $\beta \models \gamma$ (*Right Weakening* $\times$)

Proof. Let $W = \{a, b, c\}$, $a \prec_a b$, $b \prec_a c$, and

$[\alpha]_a = 1, [\beta]_a = 1, [\gamma]_a = 1$

$[\alpha]_b = 0, [\beta]_b = 1, [\gamma]_b = 0$

$[\alpha]_c = 1, [\beta]_c = 0, [\gamma]_c = 1$

First, note that $[\alpha \rhd (\beta \wedge \gamma)]_a = 1$. The reason is that $[\alpha]_c = 1$, $[\beta \wedge \gamma]_c = 0$, and (c)-(e) hold. (c) holds because $[\alpha]_a = [\beta \wedge \gamma]_a = 1$. (d) holds because $Min_a(\alpha) = \{a\}$ and $[\beta \wedge \gamma]_a = 1$. (e) holds because $Min_a(\neg(\beta \wedge \gamma)) = \{b\}$ and $[\neg\alpha]_b = 1$. Second, note that $[\alpha \rhd \beta]_a = 0$ in spite of the fact that $\beta \wedge \gamma \models \beta$. Since $[\alpha]_c = 1$ and $[\beta]_c = 0$, (c)-(e) must hold. However, (e) does not hold, because $Min_a(\neg\beta) = \{c\}$ and $[\neg\alpha]_c = 0$. □

FACT 6 $\alpha \wedge \beta \not\models_m \alpha \rhd \beta$ (*Conjunctive Sufficiency* $\times$)

Proof. Let $W = \{a, b\}$, $a \prec_a b$, and

$[\alpha]_a = 1, [\beta]_a = 1$

$[\alpha]_b = 1, [\beta]_b = 0$

In this case $[\alpha \wedge \beta]_a = 1$. However, $[\alpha \rhd \beta]_a = 0$ because $[\alpha]_b = 1$, $[\beta]_b = 0$, but (e) does not hold. □

FACT 7 $\alpha \rhd \beta, \alpha \rhd \gamma \models_m \alpha \rhd (\beta \wedge \gamma)$ (*AND* ✓)

Proof. Assume that $[\alpha \rhd \beta]_x = [\alpha \rhd \gamma]_x = 1$. Four cases are to be considered.
Case 1: the first disjunct of clause 6 of definition 7 holds both for $\alpha \rhd \beta$ and for $\alpha \rhd \gamma$. In this case there is no $y \in W_x$ such that $[\alpha]_y = 1$ and $[\beta]_y = 0$, there is no $y \in W_x$ such that $[\alpha]_y = 1$ and $[\gamma]_y = 0$, there is $y \in W_z$ such that $[\alpha]_y = 1$, there is $y \in W_x$ such that $[\beta]_y = 0$, and there is $y \in W_x$ such that $[\gamma]_y = 0$. It follows that there is no $y \in W_x$ such that $[\alpha]_y = 1$ and $[\beta \wedge \gamma]_y = 0$, there is $y \in W_x$ such that $[\alpha]_y = 1$, and there is $y \in W_x$ such that $[\beta \wedge \gamma]_y = 0$. Therefore, $[\alpha \rhd (\beta \wedge \gamma)]_x = 1$.
Case 2: the first disjunct of clause 6 of definition 7 holds only for $\alpha \rhd \beta$. In this case there is no $y \in W_x$ such that $[\alpha]_y = 1$ and $[\beta]_y = 0$, there is $y \in W_x$ such that $[\alpha]_y = 1$, there is $y \in W_x$ such that $[\beta]_y = 0$, and (c)-(e) hold for $\alpha \rhd \gamma$. From (c) we get that, for some $z \in Min_x$, either $[\alpha]_z = [\gamma]_z = 1$ or $[\alpha]_z = [\gamma]_z = 0$. If $[\alpha]_z = [\gamma]_z = 1$, then $[\alpha]_z = [\beta \wedge \gamma]_z = 1$, given that there is no $y \in W_x$ such that $[\alpha]_y = 1$

and $[\beta]_y = 0$. If $[\alpha]_z = [\gamma]_z = 0$, then $[\alpha]_z = [\beta \wedge \gamma]_z = 0$. From (d) we get that, for every $z \in Min_x(\alpha)$, $[\gamma]_z = 1$. This yields that, for every $z \in Min_x(\alpha)$, $[\beta \wedge \gamma]_z = 1$, given that there is no $y \in W_x$ such that $[\alpha]_y = 1$ and $[\beta]_y = 0$. Moreover, from (e) we get that, for every $w \in Min_x(\neg\gamma)$, $[\neg\alpha]_w = 1$, which entails that, for every $w \in Min_x(\neg(\beta \wedge \gamma))$, $[\neg\alpha]_w = 1$, given that there is no $w \in Min_x$ such that $[\beta]_w = 0$ and $[\alpha]_w = 1$. Therefore, $[\alpha \triangleright (\beta \wedge \gamma)]_x = 1$.
Case 3: the first disjunct of clause 6 of definition 7 holds only for $\alpha \triangleright \gamma$. This case is analogous to case 2.
Case 4: the first disjunct of clause 6 of definition 7 holds for neither of the two formulas. In this case (c)-(e) hold both for $\alpha \triangleright \beta$ and for $\alpha \triangleright \gamma$. From (c) we get that there is $z \in Min_x$ such that $[\alpha]_z = [\beta]_z$ and there is $w \in Min_x$ such that $[\alpha]_w = [\gamma]_w$. If $[\alpha]_z = [\beta]_z = 0$ or $[\alpha]_w = [\gamma]_w = 0$, then $[\alpha]_z = [\beta \wedge \gamma]_z = 0$ or $[\alpha]_w = [\beta \wedge \gamma]_w = 0$. If $[\alpha]_z = [\beta]_z = 1$ and $[\alpha]_w = [\gamma]_w = 1$, then $[\alpha]_z = [\beta \wedge \gamma]_z = 1$ and $[\alpha]_w = [\beta \wedge \gamma]_w = 1$ because (d) requires that $[\gamma]_z = 1$ and $[\beta]_w = 1$. Moreover, from (d) we get that every $z \in Min_x(\alpha)$ is such that $[\beta]_z = 1$ and $[\gamma]_z = 1$, so that $[\beta \wedge \gamma]_z = 1$. Finally, (e) yields that for every $z \in Min_x(\neg(\beta \wedge \gamma))$, $[\neg\alpha]_z = 1$. Therefore, $[\alpha \triangleright (\beta \wedge \gamma)]_x = 1$. □

FACT 8 $\alpha \triangleright \gamma, \beta \triangleright \gamma \models_m (\alpha \vee \beta) \triangleright \gamma$ (*OR* ✓)

Proof. Assume that $[\alpha \triangleright \gamma]_x = [\beta \triangleright \gamma]_x = 1$. Four cases are to be considered.
Case 1: the first disjunct of clause 6 of definition 4 holds both for $\alpha \triangleright \gamma$ and for $\beta \triangleright \gamma$. In this case there is no $y \in W_x$ such that $[\alpha]_y = 1$ and $[\gamma]_y = 0$, there is no $y \in W_x$ such that $[\beta]_y = 1$ and $[\gamma]_y = 0$, there is $y \in W_x$ such that $[\alpha]_y = 1$, there is $y \in W_x$ such that $[\beta]_y = 1$, and there is $y \in W_x$ such that $[\gamma]_y = 0$. It follows that there is no $y \in W_x$ such that $[\alpha \vee \beta]_y = 1$ and $[\gamma]_y = 0$, there is $y \in W_x$ such that $[\alpha \vee \beta]_y = 1$, and there is $y \in W_x$ such that $[\gamma]_y = 0$. Therefore, $[(\alpha \vee \beta) \triangleright \gamma]_x = 1$.
Case 2: the first disjunct of clause 6 of definition 7 holds only for $\alpha \triangleright \gamma$. In this case there is no $y \in W_x$ such that $[\alpha]_y = 1$ and $[\gamma]_y = 0$, there is $y \in W_x$ such that $[\alpha]_y = 1$, there is $y \in W_x$ such that $[\gamma]_y = 0$, and (c)-(e) hold for $\beta \triangleright \gamma$. Note that, since there is $z \in W_x$ such that $[\beta]_z = 1$ and $[\gamma]_z = 0$, there is $z \in W_x$ such that $[\alpha \vee \beta]_z = 1$ and $[\gamma]_z = 0$. To see that (c) holds for $(\alpha \vee \beta) \triangleright \gamma$, given that it holds for $\beta \triangleright \gamma$, consider $z \in Min_x$ such that $[\beta]_z = [\gamma]_z$. If $[\beta]_z = [\gamma]_z = 1$, then also $[\alpha \vee \beta]_z = [\gamma]_z = 1$. If instead $[\beta]_z = [\gamma]_z = 0$, then $[\alpha]_z = 0$ by the initial assumption about $\alpha \triangleright \gamma$. So, $[\alpha \vee \beta]_z = [\gamma]_z = 0$. To see that (d) holds for $(\alpha \vee \beta) \triangleright \gamma$, given that it holds for $\beta \triangleright \gamma$, it suffices to think that $Min_x(\alpha \vee \beta) \subseteq Min_x(\alpha) \cup Min_x(\beta)$ and by the initial assumption about $\alpha \triangleright \gamma$ we have that $[\gamma]_z = 1$ for every $z \in Min_x(\alpha)$. Finally, (e)

holds for $(\alpha \vee \beta) \triangleright \gamma$, again because it holds for $\beta \triangleright \gamma$ plus the initial assumption about $\alpha \triangleright \gamma$. Therefore, $[(\alpha \vee \beta) \triangleright \gamma]_x = 1$.
Case 3: the first disjunct of clause 6 of definition 4 holds only for $\beta \triangleright \gamma$. This case is analogous to case 2.
Case 4: the first disjunct of clause 6 of definition 7 holds for neither of the two formulas. In this case (c)-(e) hold both for $\alpha \triangleright \gamma$ and for $\beta \triangleright \gamma$. To see that (c) holds for $(\alpha \vee \beta) \triangleright \gamma$, suppose first that for some $z \in Min_x$, either $[\alpha]_z = 1$ or $[\beta]_z = 1$. Then $[\gamma]_z = 1$, given that (d) holds for $\alpha \triangleright \gamma$ and $\beta \triangleright \gamma$. So, $[\alpha \vee \beta]_z = [\gamma]_z = 1$. Now suppose that for all $z \in Min_x$, $[\alpha]_z = 0$ and $[\beta]_z = 0$. Then, since (c) holds for $\alpha \triangleright \gamma$ and $\beta \triangleright \gamma$, we get that some z is such that $[\alpha \vee \beta]_z = [\gamma]_z = 0$. To see that (d) holds for $(\alpha \vee \beta) \triangleright \gamma$, given that it holds for $\alpha \triangleright \gamma$ and $\beta \triangleright \gamma$, it suffices to think that $Min_x(\alpha \vee \beta) \subseteq Min_x(\alpha) \cup Min_x(\beta)$. Finally, also (e) holds for $(\alpha \vee \beta) \triangleright \gamma$, since it holds for $\alpha \triangleright \gamma$ and $\beta \triangleright \gamma$. Therefore, $[(\alpha \vee \beta) \triangleright \gamma]_x = 1$. □

In addition to the facts just considered, both versions of the evidential account validate some distinctive principles established by Crupi and Iacona in the original framework, such as *Contraposition*: $\alpha \triangleright \beta$ entails $\neg\beta \triangleright \neg\alpha$. So the new definitions substantially converge with the earlier definitions. The key difference is that the new definitions have non-classical implications. As we have seen, they do not validate Necessary Consequent and Impossible Antecedent. Another example is *Supraclassicality*, the principle according to which $\alpha \triangleright \beta$ holds whenever $\alpha \models \beta$. While the earlier definitions validate this principle, the same does not hold for the new definitions. What the latter validate is a weaker principle that may be called *Restricted Classicality*: if $\alpha \models \beta$, $\Diamond\alpha$, and $\Diamond\neg\beta$, then $\alpha \triangleright \beta$. What holds for Supraclassicality also holds for closely related results, such as *Identity*, the principle according to which $\alpha \triangleright \alpha$. While the earlier definitions validate Identity, the new definitions validate a weaker principle, that is, $\Diamond\alpha, \Diamond\neg\alpha \models \alpha \triangleright \alpha$, as only the latter follows from Restricted Classicality.

11 A RECENT CRITIQUE

In a recent article by Rott, the evidential account of conditionals has been the target of thorough scrutiny. Rott considers simplified models in which four worlds instantiate the four possible combinations of truth values for the antecedent and consequent of a conditional, and uses these models to formulate two crucial objections, which we will call the *sufficiency objection* and the *necessity objection*. Both objections can be effectively illustrated by reference to medical scenarios.

The first scenario involves a medication and recovery from a disease. The four possibilities can be denoted as rm, $r\overline{m}$, $\overline{r}m$, $\overline{rm}$, where

r indicates that recovery occurs and m indicates that the medication is administered. Thus, rm means recovery upon treatment, $r\overline{m}$ means spontaneous recovery, $\overline{r}m$ means failed recovery upon treatment, and $\overline{rm}$ means failed spontaneous recovery. In general, the ranking of such possibilities may vary in many ways. Following Rott, however, we will restrict consideration to a case in which $rm \prec r\overline{m} \sim \overline{r}m \prec \overline{rm}$. Here, the most plausible combination is recovery upon treatment and the least plausible combination is failed spontaneous recovery, with the other two combinations in between. A sensible FOM probability assignment matching this ranking is defined by $P(rm) = 98/113$, $P(\overline{r}m) = 7/113$, $P(r\overline{m}) = 7/113$, and $P(\overline{rm}) = 1/113$, which is a good approximation of figures employed by Rott himself.[52] This arrangement reflects a situation of this kind: the disease is benign (as, say, bronchitis), because recovery is generally probable (about 93% of the cases); the medication (say, antibiotics) is reasonably effective, because recovery conditional on treatment is more frequent than it is without treatment (93.3% vs. 87.5%); and the medication is also widely administered to ill patients (about 93% of the cases).

Rott's sufficiency objection arises in this scenario considering the following sentence:

(7) If Ann does not recover from bronchitis, she has taken antibiotics.

Since the disease is benign and the treatment is widespread, the negated antecedent and the consequent of (7) are both very likely. As a consequence, (7) is true according to the earlier modal definition. This is a serious problem for that definition, for the antecedent of (7) does not support its consequent in any intuitive sense. Rott's conclusion is that Crupi and Iacona's characterization of the evidential conditional is not sufficient to capture support.

Rott's sufficiency objection is compelling and can only be effectively overcome, we suggest, through the amendment explained in section 8. Note that, intuitively, there is no incompatibility between the antecedent and negated consequent of (7): failed recovery is in agreement, if anything, with lack of treatment. Definition 4 accords with this intuition: (7) turns out false because (c) is violated. The only reason why the combination $\overline{rm}$ is implausible is that rm happens to be most plausible, not because the antecedent and negated consequent of (7) really are at odds with each other. In fact, (7) is essentially a slightly more extreme variant of (3), where once again the new modal semantics rectifies a mismatch between the old modal semantics and the probabilistic analysis. For completeness, note that in the target models the corresponding suppositional conditional and its contraposed are both true (because (d) and (e) in definition 4 are satisfied)

52. See example 6 in Rott 2023b.

and both highly acceptable (because the conditional probability of the consequent given the antecedent is $7/8$ in both cases). This notwithstanding, (7) does not pass the revised Chrysippus Test, much as it has zero acceptability in the probabilistic version of the evidential account, just because the probability of the consequent is slightly *de*creased by the antecedent. So the sufficiency objection is met.

Let us now turn to Rott's necessity objection. The target scenario involves a clinical condition and a diagnostic test. The relevant possibilities can be denoted as cp, $c\overline{p}$, $\overline{c}p$, $\overline{cp}$, where c indicates that the condition is present and p indicates that the test result is positive. In the standard medical terminology, cp amounts to a true positive test outcome, $c\overline{p}$ to a false negative, $\overline{c}p$ to a false positive, and $\overline{cp}$ to a true negative. Here again, in general, the ranking of such possibilities may vary in many ways. Following Rott, however, we will restrict consideration to a typical 'base-rate neglect' arrangement. A paradigm case of this kind arises when the condition of interest (say a SARS-CoV-2 infection) is rare and the related diagnostic test (say the nasal swab) is useful but fallible, thus yielding the following ranking: $\overline{cp} \prec \overline{c}p \prec cp \prec c\overline{p}$. A sensible FOM probability assignment matching this ranking is defined by $P(\overline{cp}) = 512/585$, $P(\overline{c}p) = 64/585$, $P(cp) = 8/585$, and $P(c\overline{p}) = 1/585$, which is a good approximation of figures employed by Rott himself.[53] This neatly reflects the pattern we want to discuss: the condition of interest is quite rare, because $P(c) = 0{,}2\%$, and the diagnostic test scores well on both sensitivity and specificity, because $P(p|c) = P(\overline{p}|\overline{c}) \approx 89\%$.

As a preliminary remark about this kind of scenarios, consider the following sentence:

(8) If Ann has a positive swab test, she is infected by SARS-CoV-2.

Most people in base-rate neglect scenarios tend to think (mistakenly) that in (8) the probability of the consequent given the antecedent, $P(c|p)$, is high, whereas in our case it is no more than $1/9 \approx 11\%$. For similar reasons, there may be a misguided first impression that (8) is acceptable. Interestingly, (8) turns out to be rejected in all accounts of conditionals considered so far, including purely suppositional views. In fact, modal theories require that $cp \prec \overline{c}p$, which contradicts the model (a true positive is in fact less likely than a false positive, due to the rarity of the condition). Similarly, probabilistic theories classify (8) as not acceptable either in qualitative terms (as in Douven's account, because $P(c|p) < 1/2$) or by the assignment of a very low degree of acceptability (about $0{,}1$ in the probabilistic version of the evidential account).

53. See example 2 in Rott 2023b.

Rott's necessity objection arises in this scenario considering the contraposed of (8), namely:

(9) If Ann is not infected, she will have a negative swab test.

The point of Rott's second line of criticism is that a sentence like (9) does not hold in the target model according to the evidential account. However, he contends, the antecedent does support its consequent here, showing that Crupi and Iacona's characterization of the evidential conditional is not necessary to capture support.

In this case we diverge from Rott's conclusions, though, for we are not willing to grant that (9) should hold. Surely (9), unlike (8), does hold in the suppositional interpretation, but this is no reason to conclude that it should hold on an inferentialist reading, for otherwise (3) should hold as well. (9), unlike (3), involves support as an increased probability of the consequent, but this is also not enough to conclude that (9) must hold, for otherwise (8) should hold as well. As pointed out in section 7, suppositional acceptability and probability increase together may still not suffice for a conditional to hold in the evidential sense because the support relation is too weak. Much like (6) above, (9) illustrates such implication, for the probability of the consequent increases from about 88% to about 89%, so that the corresponding graded incompatibility (the degree to which the antecedent actually contributes in ruling out the falsity of the consequent) is unimpressive. Note also that the key point why (9) fails in the evidential account, namely the ranking $\bar{c}p \prec cp$, is the same that is plausibly overlooked by someone who mistakenly regards (8) as compelling, thereby falling to the base-rate fallacy. So there might be a common root for the impression that (8) is sound and the intuitive appeal that some inferentialists may perceive in (9).[54]

All in all, we believe that there is a plausible sense of 'reason' — the sense articulated by the evidential account — in which the antecedent of (9) is *not* a reason for its consequent in the situation described. Of course, this is not to deny that there may be some other plausible sense of 'reason' in which the antecedent of (9) is in fact a reason for its consequent. After all, the pretheoretical notion of reason is vague, so it can be made precise in more than one way. But at least it is an open question how (9) is to be treated in an inferentialist theory of

54. One way to insist that (9) should be regarded as highly acceptable is to notice that the difference between the conditional probability of the consequent given the antecedent and the conditional probability of the consequent given the negated antecedent is large (89% vs. 11%, respectively). One reason why we consider this point inconclusive arises from the similarity between (9) and (6) above: in (6) the difference at issue is even larger (almost 92%) and yet it is apparent, we submit, that the inferential connection between antecedent and consequent is in a sense very weak. We thank an anonymous reviewer for raising this issue.

conditionals, a question that we do not regard as settled simply by an appeal to intuitions. Perhaps the interest of examples such as (9) and (6) is precisely that they highlight possibly diverging options within an inferentialist framework.

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