The Kantian Framework of Complementarity[1][2]

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In the literature on the Copenhagen interpretation of quantum mechanics, not enough attention has been directed to the similarities between Bohr's views on quantum mechanics and Kant's theoretical philosophy. Too often, the connection is either ignored, downplayed, or denied outright. This has, as far as a proper understanding of Bohr's views is concerned, been detrimental, for it has contributed to the common misconception of Bohr as either a positivist or a pragmatist thinker.[3] In recent years, however, there has been a growing number of commentators attentive enough to note the important affinities in the views of these two thinkers (for instance, Honner 1982, MacKinnon 1982, Shimony 1983, Kaiser 1992, Chevalley 1994, Faye 2008). All of these commentators are, I believe, correct; however the picture they present to us of the connections between Bohr and Kant is one that is painted in broad strokes. It is open to the criticism that these affinities are merely superficial.

The contribution that I intend to make in this essay, therefore, is to

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[3]Baggot (2004), for example, does not mention Kant at all in relation to Bohr. Folse (1985), on the other hand, flatly denies any similarities whatsoever that are not merely superficial. Baggot and Folse both view Bohr as a pragmatist. For examples of positivist construals of Bohr, see: Popper (1982), Bunge (1955a,b).

provide a closer, structural, analysis of both Bohr's and Kant's views that makes these connections more explicit. In particular, I will demonstrate the similarities between Bohr's argument, on the one hand, that neither the wave nor the particle description of atomic phenomena pick out an object in the ordinary sense of the word, and Kant's requirement, on the other hand, that both 'mathematical' (having to do with magnitude) and 'dynamical' (having to do with an object's interaction with other objects) principles must be applicable to appearances in order for us to determine them as objects of experience. I will argue that Bohr's 'Complementarity interpretation' of quantum mechanics, which views atomic objects as idealizations, and which licenses the repeal of the principle of causality for the domain of atomic physics, is perfectly compatible with, and indeed follows naturally from a broadly Kantian epistemological framework.

There are exegetical difficulties with respect to both Bohr and Kant. Their writings are dense and are considered to be obscure by many. Interpreting Kant has become something of an industry in philosophy. As for Bohr, J.S. Bell writes of him: "While imagining that I understand the position of Einstein ... I have very little understanding of the position of his principal opponent, Bohr" (2004 [1981], p. 155). Abner Shimony writes: "I must confess that after 25 years of attentive—and even reverent—reading of Bohr, I have not found a consistent and comprehensive framework for the interpretation of quantum mechanics" (1985, p. 109). I do not pretend to have succeeded, where these and other eminent physicists and philosophers have failed, in resolving all of the problems that go along with giving a comprehensive and consistent interpretation of Bohr's philosophical position. Bohr is known to have thought highly of the Pragmatist philosophy of William James, and Bohr's philosophy represents, in all likelihood, a combination of Jamesian and Kantian strands (although even this is likely an oversimplification). In this essay it is the Kantian aspects of Bohr's views that I will focus on; I do not, however, believe this is the whole story.[4]

Understanding the Kantian aspects of Bohr's thought is important because, although Bohr's and Kant's philosophies do diverge ultimately, they nevertheless share (as I will argue) a common epistemological framework. Any interpretation of Bohr should, therefore, *start* with Kant. Further, comparing Kant and Bohr is also invaluable for our interpretation of Kant. By

[4]For more on Bohr and William James, see, e.g, Folse (1985, p. 49-51).

asking the question 'how can a Kantian make sense of quantum mechanics?', one gains valuable insight into the implications of the principles of quantum mechanics for Kantian philosophy—in particular, what the uncertainty relations, if accepted, entail for the applicability of Kant's principle of cause and effect.

The essay is structured as follows. In section 1, I give a detailed exegesis of the main negative theses of Complementarity, as stated by Bohr. I focus, primarily, on two works in this section: the Como paper (1928), and the response (1935) to the EPR paper. In the Como paper, we find Bohr's argument that the wave description of atomic phenomena must be denied physical reality. In the response to the EPR paper, we find a similar argument, this time against a particle description. The aim of section 2 is to show how Bohr's views fit comfortably into a broadly Kantian framework. Thus I show how the reasons Bohr gives for denying physical reality to the wave and particle descriptions would be reasons for Kant as well. Specifically, I show how Kant's requirement that mathematical and dynamical principles must be applicable to appearances rules out, as it does for Bohr, granting physical reality to either the wave or particle descriptions of elementary phenomena. I also show how Bohr's positive theses of Complementarity—his characterization of the positive use we can make of the concepts of causality and of position bears a resemblance to Kant's doctrine of the antinomies and to Kant's ideas on the regulative use of reason as he puts them forth in the *Critique of Judgement*. The third and final section of the paper is devoted to actual and potential objections to my interpretation of Bohr. In the process of responding to these objections, I will show how Kantian ideas influenced Bohr by way of his close friendship with the philosopher Harald Høffding.

1. Complementarity

Heisenberg wrote his uncertainty paper in 1927. Drawing on his famous γ-ray microscope thought experiment, and presupposing a particle interpretation of elementary objects, Heisenberg argued that it is in principle impossible to precisely determine both the position and momentum of an elementary particle at any one time. On the one hand, the inevitable disturbance (the Compton effect) of the electron by the high-frequency photon beam needed to determine its position precludes a precise determination of the particle's momentum. On the other hand, using a low-frequency beam to determine the particle's momentum results in an uncertainty regarding the particle's position. Heisenberg showed that as the uncertainty in the determination of one parameter approaches 0, the uncertainty in the determination of the other parameter approaches infinity. He also demonstrated an analogous uncertainty relationship between energy and time.

Bohr accepted the validity of the uncertainty relations, but disagreed with Heisenberg over their significance. For (the young) Heisenberg, the significance of the uncertainty relations is epistemic; they express a limitation on what we can know about elementary objects (which are presupposed to be particles). For Bohr, the uncertainty relations signify something deeper. They express the fact that the fundamental 'classical concepts' which both the particle and wave description of elementary objects presuppose (spatiotemporal concepts, on the one hand, and dynamical concepts, on the other) are inapplicable in the atomic domain, and that therefore a definition of the object in terms of these parameters is precluded. Let us work our way towards this conclusion. In his Como paper, Bohr writes:

> ... [quantum theory's] essence may be expressed in the so-called quantum postulate, which attributes to any atomic process an essential discontinuity, or rather individuality, completely foreign to the classical theories and symbolised by Planck's quantum of action (1928, p. 580).

Contrasted with the classical theories, here, is the irreducibly 'discrete' nature of atomic processes; the fact that, according to quantum theory, the observed state of an elementary object changes discontinuously with time. What this implies, Bohr goes on to say, is that in our observations of the results of experiments, the interaction with the 'agency of observation' (i.e., the experimental apparatus) is an ineliminable part of our description of phenomena (in his later writings, Bohr calls this an "essential wholeness." *Cf.* 1958b, p. 72).

That last step may seem like an inferential leap, but it is comprehensible in light of the common classical assumption that change is continuous. This is an assumption which we also find in Kant. For Kant, the presentations of time and space are continuous, infinitely divisible, quantities (CPR, B211).[5]

[5]References to the *Critique of Pure Reason* (CPR) are to the Guyer-Wood translation (1998 [1781]), while references to the *Critique of Judgement* (CJ) are to the Pluhar translation (1987 [1790]). Page numbers for Kant's works are as in the standard German

No matter how small, every ‘piece’ of space or time always presupposes a possible further intuition of space or time within its boundaries. Kant’s principle of cause and effect, now, tells us that every series of perceptions has some objective ordering according to which it progresses in time. But since time is infinitely divisible, so is the progression of perceptions (CPR, B255). All *change* associated with a possible experience is continuous, therefore, and this is something we must presuppose a priori, according to Kant. Bohr’s argument tacitly makes use of this assumption. Thus, on the classical conception of nature, change is continuous. Yet the state transitions of elementary objects are irreducibly discontinuous. It follows from this that, from a classical point of view, something is ‘missing’ from our description. What is ‘missing’, according to Bohr, is a clean distinction between the experimental apparatus and the object of our investigations; the ‘agency of observation’ is, in some sense, *a part* of what we observe.

Is it the case, then, that quantum mechanical descriptions of phenomena are not objective? No, quantum mechanical descriptions of phenomena, like classical descriptions, are objective. However what is different is that for the classical (but not for the quantum) case it is always possible to determinately describe (and correct for) the *interaction* between apparatus and object. There are always three components involved in my description of an object: first, there is the experimental apparatus which I use to investigate the object; second, there is the object of investigation; third, there is the interaction between the apparatus and the object. In my description of a classical object, I am able to describe the interaction between apparatus and object and I am able to compensate for this interaction in my description; I am able to distinguish the object ‘as it really is’ from the object ‘as it appears’. But this is *not* possible for atomic phenomena. Although we must make some ‘subject-object’ distinction—some ‘cut’ in what we observe—it is an *arbitrary* cut—one in which the interaction between apparatus and object cannot be disentangled from our description of the object.

One might object that there is some arbitrariness to the cut we make in the classical realm as well, for one will have distinct objects of inquiry depending on the context of the investigation.[6] But what is different is that in

edition. In the case of the first *Critique*, “A” denotes the first and “B” the second edition, as usual.

[6]For example, sometimes it may be useful to describe a billiard ball as a single ‘particle’ in order to describe its interactions with other billiard balls in the context of a game being

the classical case, as we correct for the interaction with the apparatus in our description of the object, we are constrained by the (according to Bohr) criteria for its independent reality: a precise location in space-time and a precise account of its interaction with other objects. In the quantum case, however, we cannot account for the interaction with the apparatus in a way that leaves the object with definite position/time and momentum/energy parameters. Our language nevertheless requires some distinction, so we arbitrarily impose one.

Bohr expresses all of the foregoing in the following concise paragraph:

> Now the quantum postulate implies that any observation of atomic phenomena will involve an interaction with the agency of observation not to be neglected. Accordingly, an independent reality in the ordinary physical sense can neither be ascribed to the phenomena nor to the agencies of observation. After all, the concept of observation is in so far arbitrary as it depends upon which objects are included in the system to be observed. Ultimately every observation can of course be reduced to our sense perceptions. The circumstance, however, that in interpreting observations use has always to be made of theoretical notions, entails that for every particular case it is a question of convenience at what point the concept of observation involving the quantum postulate with its inherent 'irrationality' is brought in. This situation has far-reaching consequences. On one hand, the definition of the state of a physical system, as ordinarily understood, claims the elimination of all external disturbances. But in that case, according to the quantum postulate, any observation will be impossible, and above all, the concepts of space and time lose their immediate sense. On the other hand, if in order to make observation possible we permit certain interactions with suitable agencies of measurement, not belonging to the system, an unambiguous definition of the state of the system is naturally no longer possible, ... (1928, p. 580).

Bohr must still explain exactly why the classical concepts are not appli-

played. If one wanted to undertake a chemical analysis of the ball, on the other hand, it would be more useful to describe it as a collection of molecules.

cable to elementary objects. He writes:

> The fundamental contrast between the quantum of action and the classical concepts is immediately apparent from the simple formulae which form the common foundation of the theory of light quanta and of the wave theory of material particles. If Planck's constant be denoted by h, as is well known,
>
> $$E\tau = I\lambda = h, \quad . \quad . \quad . \quad (1)$$
>
> where E and I are energy and momentum respectively, τ and λ the corresponding period of vibration and wave-length. In these formulae the two notions of light and also of matter enter in sharp contrast. While energy and momentum are associated with the concept of particles, and hence may be characterised according to the classical point of view by definite space-time co-ordinates, the period of vibration and wave-length refer to a plane harmonic wave train of unlimited extent in space and time (1928, p. 581).

In other words, in each case (i.e., for light and matter), Planck's constant relates two incompatible quantities. In the first relation, E is associated with the concept of a particle given with definite spatiotemporal coordinates, while τ is associated with a wave-train "of unlimited extent", not conceptualizable with respect to definite space-time coordinates. The case is the same for I and λ. Bohr's point is that it does not make sense to picture an object to ourselves that is, as the above relations express, *both* given at some definite spatiotemporal location *and* of unlimited extent in space and time. Nevertheless, physical theory does provide us with the resources we need to get around this difficulty, whether we assume a wave or a particle description of the object. The problem, as we shall see, is that neither description is determinate.

For the case of the wave description, we can do this by means of the superposition principle. Bohr writes:

> Only with the aid of the superposition principle does it become possible to obtain a connexion with the ordinary mode of description. Indeed, a limitation of the extent of the wave-fields in space and time can always be regarded as resulting from the interference of a group of elementary harmonic waves (1928, p. 581).

What Bohr is describing here is essentially Schrödinger's picture of the wave packet (i.e., a wave, manifesting particle-like properties) moving through space and time. However, although the superposition principle enables us to construct a description of an object in this way, it necessarily involves an element of indeterminacy with regard to that object.

> Rigorously speaking, a limited wave-field can only be obtained by the superposition of a manifold of elementary waves corresponding to all the values of ν and σ_x, σ_y, σ_z. But the order of magnitude of the mean difference between these values for two elementary waves in the group is given in the most favourable case by the condition
>
> $$\Delta t \Delta \nu = \Delta x \Delta \sigma_x = \Delta y \Delta \sigma_y = \Delta z \Delta \sigma_z = 1 \qquad [1a]$$
>
> where $\Delta t, \Delta x, \Delta y, \Delta z$ denote the extension of the wave-field in time and in the direction of space corresponding to the co-ordinate axes (Bohr, 1928, p. 581).

Here, ν refers to the frequency, and $\sigma_x, \sigma_y, \sigma_z$ refer to the wavenumbers for the elementary waves in the directions of the coordinate axes. All else equal, the broader the spread of wavenumbers/frequency in the wave group, the more determinate the spatiotemporal extent of the resultant packet, and vice versa. Now, according to the de Broglie relations, $E = \hbar\nu$, $I = \hbar\sigma$, where $\hbar = h/2\pi$ is the reduced Planck's constant. If we multiply equation (1a) by $\hbar$, this gives us Heisenberg's uncertainty relations:

$$\Delta t \Delta E = \Delta x \Delta I_x = \Delta y \Delta I_y = \Delta z \Delta I_z = \hbar \qquad (2)$$

which give the upper bound on the accuracy of momentum/position determinations with respect to the wave-field.

Thus, as the wave-field associated with the object gets smaller —as we 'zoom in', so to speak, on its position and time coordinates—the possibility of precisely defining the energy and momentum associated with the object decreases in proportion. And the opposite is also true: in order to determine the object's momentum (or energy), we require a larger wave-field—we need to 'zoom out'—but this foregoes a precise determination of the object's position. 'Zooming in' and 'zooming out', however, are associated with different experimental arrangements. For the case of the γ-ray microscope, they are associated with the finite size of the microscope's aperture; the uncertainty in the position and momentum of the electron arises, not because of the interaction between two determinate entities (a photon and an electron), but rather because certain experimental arrangements, well-suited for precisely determining momentum, preclude *the definition* of the object in terms of continuously changing spatiotemporal coordinates, and vice versa.

> Indeed, a discontinuous change of energy and momentum during observation could not prevent us from ascribing accurate values to the space-time co-ordinates, as well as to the momentum-energy components before and after the process. The reciprocal uncertainty which always affects the values of these quantities is, as will be clear from the preceding analysis, essentially an outcome of the limited accuracy with which changes in energy and momentum can be defined, when the wave-fields used for the determination of the space-time co-ordinates of the particle are sufficiently small (Bohr, 1928, p. 583).

Thus no one experimental setup allows for an *exact* definition of the object in terms of both quantities. One experiment can, at most, give us a picture of "unsharply defined individuals within finite space-time regions" (Bohr, 1928, p. 582).

Now let us consider a particle description of the elementary object. To simplify somewhat the example Bohr presents in his response to the EPR paper (1935),[7] consider two variations of a one-slit experiment, where a photon is directed at a small opening in a thin diaphragm, on the other side of which is a photographic plate. In one version of the experiment, the diaphragm is not rigidly attached to the experimental apparatus. When we direct the photon at the diaphragm, it exchanges momentum with the apparatus as it passes through the slit, which we measure by the amount of recoil we observe in the diaphragm. However the recoil of the diaphragm makes it impossible to precisely determine the position of the slit, and hence of the particle, at the moment of impact. "... we lose, on account of the uncontrollable displacement of the diaphragm during each collision process

[7] In the Como paper, Bohr simply tells us that similar remarks apply to particles, but does not elaborate in great detail (1928, p. 582).

with the test bodies, the knowledge of its position when the particle passed through the slit" (Bohr, 1935, p. 698).

On the other hand, if the diaphragm is rigidly fixed to the rest of the apparatus, then as the photon passes through the slit, whatever momentum it exchanges with the diaphragm is completely absorbed by the apparatus (Bohr, 1935, p. 697). Like the case for the wave picture, then, we have on our hands two experimental arrangements, one of which is compatible with a precise position determination; the other compatible with a precise momentum determination; however each of these *excludes* a determination of (and hence a definition of the object in terms of) the other parameter. "Indeed we have in each experimental arrangement suited for the study of proper quantum phenomena not merely to do with an ignorance of the value of certain physical quantities, but with the impossibility of defining these quantities in an unambiguous way" (Bohr, 1935, p. 699).

2. A Kantian View of Complementarity

Let us stop and reflect; consider the result of some experiment, say the mark on a photographic plate. The mark itself is a classical object. It has definite spatiotemporal coordinates, and it causally interacts in a definite way with its surroundings. However, *this* description of the phenomenon—of the mark *as a mark* on a photographic plate and nothing more—includes the photographic plate. To go further and describe the mark as a mark that has been left by some *independently existing object* that has interacted with the plate is what we desire to do, for this allows us to unify the marks resulting from different experiments as being different manifestations of the same independently existing object. Our goal is to 'get at' reality—the thing behind the phenomena—as it exists independently of the conditions of our experiments. We do this by eliminating the interaction between apparatus and object from our description of the latter.

From a Kantian perspective, in order to describe the object behind the phenomena as some independently existing[8] object that we can possibly experience we must determine it according to both *mathematical* and *dynamical*

[8]That is, in the sense of its being the same object in different experimental contexts. We can never abstract, on Kant's view, *completely* from the subjective conditions of observation (space and time), of course.

principles (CPR, B198-B294).[9] The former say that in order for anything to appear to us, it must be apprehended as having, determinately, both an extensive (length, breadth, etc.) and an intensive magnitude (i.e., a degree). But that something appears to us is, by itself, not enough to determine this something as a physically real object. To do this, we must apply the dynamical principles. The dynamical principles are regulative for appearances,[10] i.e., they are principles, not for the apprehension, but for the connection of appearances in time; they presuppose that an appearance has already been apprehended in accordance with the mathematical principles. These dynamical principles state, first, that all change presupposes something permanent; second, that all change must occur according to the law of cause and effect; third, that all substances that are perceived as simultaneous are in mutual interaction.[11]

To determine an appearance as an object of a possible experience, therefore, we require that at a determinate instant in time, it has a determinate extent (constrained by the mathematical principles) and hence a determinate position in space, and that there is a law (subject to the dynamical principles) by which it dynamically interacts with its surroundings in and through time. This means that any description of elementary objects that purports to pick out an object of possible experience must ascribe to the object *both* a determinate position *and* a determinate momentum parameter.

Now to visualise the object, we make use, say, of the superposition principle. But by this means it is impossible to obtain *both* an exact position and an exact momentum determination (likewise for energy and time). It is possible to obtain an exact position determination, but in that case we completely forego a determination of the object's momentum, and vice versa. We can, however, get something like a 'complete' object (i.e., one in which both causal and spatiotemporal parameters are present in some sense) by making

[9]The mathematical principles are the *Axioms of Intuition* and *Anticipations of Perception*; the dynamical principles are the *Analogies of experience* and the *Postulates of empirical thought as such*.

[10]This sense of regulative should not be confused with the sense that Kant uses with respect to the 'ideas of reason'. There the distinction is between that which is constitutive or regulative with respect to experience as a whole. Here, he uses regulative not in the context of experience in general, but in the context of particular objects of experience.

[11]Here, I only consider the *Analogies*, as the *Postulates* are not directly relevant for our discussion.

our position and momentum determinations *inexact*—"unsharply defined". But in that case, although our description is objective, it is no longer the description of an object of possible experience (i.e., something physically real that we could possibly experience), for Kant—for in order for it to be physically real, we must assign determinate values to both parameters. Instead, the object is what Kant calls a noumenon, or abstract object.

To clarify: according to Kant, a concept of the understanding must be understood both in terms of its form and in terms of the content to which it can be applied. We can think of the form of a concept as analogous to a mathematical function, e.g., $f(x) = 2x + 4$. Now a determinate result can be obtained for this function only if something is filled in for x. By itself, the function only represents a form for the determination of a variable. Likewise for a concept: without *determinate* content, a concept gives us no *determinate* cognition. "Without [an object] it has no sense, and is entirely empty of content, even though it may still contain the logical function for making a concept out of whatever sort of *data* there are." (CPR, B298).

The concept of a noumenon is the concept of something *indeterminate*—analogous to x in the mathematical equation. The function above *cannot* be applied to x itself, but only to a value that has been filled in for x. Similarly for concepts: cognition of an object of possible experience requires that a concept be applied to a determinate, not indeterminate, intuition. A concept of some causal mechanism corresponds to a rule for the progression of perceptions *in time*, and the concepts of the understanding, in general, correspond to rules that must be applied to *our* sensible forms of intuition, space and time, which are always given determinately.

But now consider an elementary particle. According to the uncertainty relations, it is impossible *in principle* to describe the particle's momentum with any degree of precision without a corresponding loss of precision with regards to its spatiotemporal coordinates. It follows that in order to describe it using *both* position/time (spatiotemporal) and momentum/energy (dynamical) parameters, the *spatiotemporal* parameters associated with it must be made *indeterminate*. In fact, both the spatiotemporal and dynamical parameters must be made indeterminate, but it is the fact that the spatiotemporal parameters must be made indeterminate that is the key, for now, on a Kantian picture, the dynamical principles (whether or not we ascribe determinate dynamical parameters) are strictly speaking no longer applicable, for the dynamical principles always presuppose a *determinate* appearance in space and time apprehended in accordance with the mathematical principles.

The upshot of all of this is that since there is no determinate spatiotemporal magnitude to apply the dynamical principles to, we cannot complete our description of the object according to the Kantian criteria for objects of possible experience. Therefore the 'object' corresponding to our description, on Kant's view, is not *physically* real.

Bohr reaches the same conclusion regarding the physical reality of our descriptions of elementary objects:

> ... a sentence like "we cannot know both the momentum and the position of an atomic object" raises at once questions as to the physical reality of two such attributes of the object, which can be answered only by referring to the conditions for the unambiguous use of space-time concepts, on the one hand, and dynamical conservation laws, on the other hand. (1949, p. 211).

The issue is not the existence of atomic objects as such (it is undeniable that something gives rise to the phenomena we observe), but whether our fundamental spatiotemporal and dynamical concepts are literally applicable to them. Evidently, according to both Bohr and Kant, they are not. And yet these 'ordinary' concepts, for Bohr, are also *necessary* concepts. The experimental apparatus (a voltmeter, say) is always a piece of classical equipment which communicates classical information about what we assume to be (using classical criteria) an independently existing object. The concept of observation itself, therefore, presupposes the classical concepts.

> Here, it must above all be recognized that, however far quantum effects transcend the scope of classical physical analysis, the account of the experimental arrangement and the record of the observations must always be expressed in common language supplemented with the terminology of classical physics. (Bohr, 1948, p. 313).

> The main point here is the distinction between the *objects* under investigation and the *measuring instruments* which serve to define, in classical terms, the conditions under which the phenomena appear. (Bohr, 1949, pp. 221-222).

We require the classical concepts, not only to observe, but also to communicate experimental results:

> ... the requirement of communicability of the circumstances and results of experiments implies that we can speak of well defined experiences only within the framework of ordinary concepts (Bohr, 1937, p. 293).

The situation seems hopeless. We require the classical criteria in order to observe a physical object and to communicate the experience; yet, the classical criteria cannot fulfil their intended function in the atomic domain, for they mutually exclude each other. Ironically, it is the uncertainty relations that save us. They guarantee that we *can* nevertheless achieve a unified description by 'patching together' the mutually exclusive dynamical and spatiotemporal descriptions of the object under different experimental conditions. "The apparently incompatible sorts of information about the behaviour of the object under examination which we get by different experimental arrangements can clearly not be brought into connection with each other in the usual way, but may, as equally essential for an exhaustive account of all experience, be regarded as "complementary" to each other" (Bohr, 1937, p. 291). The uncertainty relations guarantee that a causal description can never contradict a spatiotemporal description—that the two can be used in a complementary way—for any experiment intended to *determinately* establish the object's spatiotemporal coordinates *can tell us nothing* about its dynamical parameters, and vice versa.

> the proper rôle of the indeterminacy relations consists in assuring quantitatively the logical compatibility of apparently contradictory laws which appear when we use two different experimental arrangements, of which only one permits an unambiguous use of the concept of position, while only the other permits the application of the concept of momentum ... (Bohr, 1937, p. 293).

We are not licensed, however, to take the next step and ascribe physical reality to this 'patched together' object of our descriptions, for the object is not real but abstract, and its classical attributes are idealizations.

> From the above considerations it should be clear that the whole situation in atomic physics deprives of all meaning such inherent attributes as the idealizations of classical physics would ascribe to the object (Bohr, 1937, p. 293).

It is not too difficult to make sense of this from a Kantian point of view. Again, the concept of a noumenon is the key—this time in its positive signification. When we speak of noumena in the negative sense, as we did above, we say that they are *not* objects of sensible intuition, and hence not physically real in the sense that a noumenon cannot be an object of a possible experience for us. Even in this merely negative signification, however, they still have a use; and it is this negative signification of noumena which Kant emphasises in the *Critique of Pure Reason*.

> I call a concept problematic that contains no contradictions but that is also, as a boundary for given concepts, connected with other cognitions, the objective reality of which can in no way be cognized. The concept of a **noumenon,** i.e., of a thing that is not to be thought of as an object of the senses but rather as a thing in itself (solely through a pure understanding), is not at all contradictory; for one cannot assert of sensibility that it is the only possible kind of intuition. Further, this concept is necessary in order not to extend sensible intuition to things in themselves, and thus to limit the objective validity of sensible cognition ... (CPR, B310-311).

Kant's negative sense of noumena is well-known. Less well-known outside the circle of Kant scholarship is that noumena also have a positive signification, as objects of *nonsensible* intuition (CPR, B308-309); i.e., as objects of an intuition which is intellectual. In this sense, we call them ideas or 'concepts of reason'. Kant writes: "Concepts of reason serve for **comprehension**, just as concepts of the understanding serve for **understanding** (of perceptions)" (CPR, A311/B367). Concepts of the understanding correspond to rules for synthesizing the manifold of intuition. Thus 'chessboard', for example, corresponds to a rule according to which this particular bit of white, that particular bit of black, etc. can be associated in one representation. Concepts of reason, on the other hand, do not correspond to rules for associating sensible intuitions; rather, they correspond to rules for associating *concepts of the understanding* (CPR, B377). They are regulative concepts in the sense that we use them to *connect* the understanding's concepts (which in our case would be the various descriptions of phenomena, i.e., the 'marks', observed in the context of individual experiments) together in a coherent way in the context of our overall experience. Kant's main discussion of the regulative use of the ideas of reason is not in the *Critique of Pure Reason* but in the *Critique of Judgement*.

> All other pure concepts the critique relegates to the ideas, which are transcendent for our theoretical cognitive power, though that certainly does not make them useless or dispensable, since they serve as regulative principles: they serve, in part, to restrain the understanding's arrogant claims, namely, that (since it can state a priori the conditions for the possibility of all things it can cognize) it has thereby circumscribed the area within which all things in general are possible; in part, they serve to guide the understanding, in its contemplation of nature, by a principle of completeness—though the understanding cannot attain this completeness—and so further the final aim of all cognition. (CJ, 5:167-168).

Thus, these regulative ideas guide our research. They prompt us to search for relations between our concepts—species-genus relations, relations of part to whole, etc.—and they (indirectly) serve to expand the realm of our cognition. At certain times these regulative ideals may conflict, as in the case of the conflict between the mechanistic and teleological conceptions of nature. The key to resolving these conflicts lies in the fact that even in their positive signification, regulative ideas do not yield cognition. All cognition must refer to the conditions for a possible experience. Ideas of reason are

> referred to a concept, according to an objective principle, but these ideas still can never yield cognition of the object ... Rational ideas are *transcendent* concepts; they differ from concepts of the understanding, which are called *immanent* because they can always be supplied with an experience that adequately corresponds to them. (CJ, 5:342).

And since we cannot cognize anything by them, then so long as these ideas are used merely as methodological principles they are compatible with each other.

Now the classical spatiotemporal and dynamical concepts, when they transcend possible experience, become ideas—they become the classical dynamical and spatiotemporal idealizations at the heart of the mechanistic conception of nature (*Cf.* CJ, §§69-78).[12,13] In the realm of atomic physics, however, these dynamical and spatiotemporal idealizations are incompatible in the sense that we cannot use them to describe a classical object. The uncertainty relations tell us that a precise determination of one type of parameter entirely excludes any determination whatsoever of the other type; therefore, they cannot be used to determine an object of possible experience, which requires a determination of both. But precisely because they say nothing about the objects of possible experience in this sense, they are compatible with the objects of possible experience—the results of our experiments—just so long as we understand that when we use these ideas in our description of nature it is only a manner of speaking; we may only speak 'as if' these ideas apply to our observations.

> We must be clear that, when it comes to atoms, language can be used only as poetry. The poet, too, is not nearly so concerned with describing facts as with creating images and establishing mental connections (Heisenberg, 1971, p. 41, recollecting a conversation with Bohr from years before).

Those familiar with Kant should immediately recognize the strategy being employed here. When confronted with other areas (biology and ethics, for instance) of human inquiry where the mechanistic conception (on his view) is either inadequate or inappropriate, Kant appeals to his doctrine of the antinomies to show that competing conceptions (e.g., freedom and determinism, mechanism and teleology) are merely ideas, and that they are compatible with each other if treated as such (*Cf.* CPR, B566-567, B586, CJ, §§69-78).

[12]Among Kant scholars, there is debate over whether Kant's discussion of the principle of causality is consistent between the first and third critiques, for in the first Critique the principle of causality is considered as being constitutive for any possible experience, while in the third it is considered as regulative for experience as a whole. On one side, there are commentators, such as John Zammito (1992, pp. 222-224), who believe that the status of the principle of causality actually changes between the first and third critiques. On the other, there are commentators, such as Henry Allison (1991), who view the change to be merely with regard to a point of view; i.e., causality is considered in the 'narrow' sense in the first critique (in which it is constitutive), and in the 'wide' sense (as having universal applicability) in the third (in which it is determined to be merely regulative).

[13]See, also, Pringe (2009) who argues that these are intended, by Bohr, to be 'symbolic analogies' with classical concepts.

3. Concerns and Objections

One might object that a Kantian should not feel herself committed to anything like Complementarity, for one may opt to view the uncertainty relations as an expression of the temporary state of our ignorance with regard to elementary particles, and not as a final word. This is correct. A Kantian need not follow Bohr. However, if, as a Kantian, one does accept the uncertainty relations, then something like Complementarity must be the result—this is what it was my intention to show in this paper. Indeed, as I have shown, it is because one starts from within the Kantian framework that the motivation for Complementarity arises. It is unclear what Kant himself would have thought, but the following discussion of the mechanistic versus the teleological conceptions of nature may give us a clue.

> ... I *ought* always to *reflect* on these events and forms *in terms of the principle* of the mere mechanism of nature, and hence ought to investigate this principle as far as I can, because unless we presuppose it in our investigation [of nature] we can have no cognition of nature at all in the proper sense of the term. But none of this goes against the second maxim—that on certain occasions, in dealing with certain natural forms (and, on their prompting, even with all of nature), we should probe these and reflect on them in terms of a principle that differs entirely from an explanation in terms of the mechanism of nature ... (CJ, 5:387-388).

Although both the mechanistic and the teleological conceptions are thought of as 'complementary' ideas which guide our investigation of nature, priority is clearly given, nevertheless, to the mechanistic conception. The use of the teleological conception is reserved only for 'certain occasions' in which the mechanistic conception is either inapplicable (perhaps only temporarily) or inappropriate. It is likely that Kant would have been more conservative than Bohr, i.e., that he would not have accepted the uncertainty relations as final. In that case, one way to interpret Bohr's Complementarity doctrine is as an attempted refutation of what he took to be Kantian philosophy, with its overemphasis on the mechanistic conception of nature. Indeed, this is one way to reconcile Bohr's oft-cited criticisms of 'a priorism' (*Cf.* Folse 1985, pp. 217-221) with his insistence on the bedrock-like status of the classical concepts.

Potentially problematic for my reading, however, are statements like the following: "... no experience is definable without a logical frame and ... any apparent disharmony can be removed only by an appropriate widening of the conceptual framework" (Bohr, 1958b, p. 82), which lead Kaiser to write, of Bohr's view:

> ... there is also a very un-Kantian sentiment expressed in the end of Bohr's quotation: our formal frame might need to be *altered.* ... Kant viewed this formal frame, which includes the forms of intuition and the categories, as *a priori* and unalterable. Bohr followed a two-faculty format but he rejected *a priorism.* (Kaiser, 1992, pp. 222-223)

Yet Kaiser's interpretation is misleading, at best, for it seems to conflate Bohr's view with (the later) Heisenberg's. Heisenberg maintained that the gradual evolution of scientific concepts (or even the human species) would allow us to transcend our limitation to the classical concepts (*Cf.* Heisenberg 1959, p. 83, Heisenberg 1971, p. 124). This was not Bohr's view: "... it would be a misconception to believe that the difficulties of the atomic theory may be evaded by eventually replacing the concepts of classical physics by new conceptual forms" (1934, p. 16). And again:

> We must, in fact, realise that the unambiguous interpretation of any measurement must be essentially framed in terms of the classical physical theories, and we may say that in this sense the language of Newton and Maxwell will remain the language of physicists for all time (1931, p. 692).

What Bohr means by 'widening', then, is not a fundamental alteration of our basic conceptual framework, but an imaginative use of our framework's own resources in order to extend its reach. "Indeed, the development of atomic physics has taught us how, *without leaving common language*, it is possible to create a framework sufficiently wide for an exhaustive description of new experience" (1958a, p. 88, emphasis mine).

Specifically related to Kant, Folse objects that Kant was a 'subjectivist' philosopher, while Bohr's intention was to provide an objective description of experience. Folse writes:

> These facts have given rise to the view held by some of the most perceptive of Bohr's interpreters that his position contains Kantian elements supporting a subjectivistic reading of complementarity. Since Bohr specifically stated complementarity provides an objective description of experience, it would seem that virtually any such reading would be contrary to his intent ... (Folse, 1985, p. 217).

But this misinterprets Kant. Kant's theoretical philosophy revolves around the question of how to give an objective description of experience; thus in the second analogy, for instance, he takes great pains to show how our application of the principle of cause and effect to appearances must be such as to distinguish the *objective* succession of appearances from the *subjective* one (CPR, B232-257). If not subscribing to naïve realism amounts to being a subjectivist then Kant is guilty on all counts; however I do not think this is the type of subjectivism that Folse is referring to, for Bohr would be guilty of this charge as well. For Kant, possible experience is constrained by the forms of our intuition, space and time, and by the concepts by which we are able to combine these intuitions into one representation of an object. But this is no different from Bohr's insistence that we require classical concepts for the unambiguous description of experience.

Bohr was known to have admired the work of the American pragmatist William James, and this has been taken by Folse (1985, pp. 49-51, 217-221) to tell against a Kantian influence on Bohr, for James was sharply critical of Kant. As Kaiser points out, however, James' criticisms of Kant are all directed at Kant's a priorism and not at the other aspects of his philosophy. This is perfectly compatible with a picture of Bohr as accepting certain aspects of Kant's philosophy while rejecting others. It is certainly not without precedent for one philosopher to be influenced by two rivals: Kant himself was strongly influenced by both Newton and Leibniz; their rivalry did not stop him from incorporating aspects of both of their views into his own.

Indeed, many philosophers have borrowed from Kant without making themselves into carbon copies. The Neo-Kantian philosopher, Ernst Cassirer, for instance, rejects the a priori status of Kant's classical concepts (1956 [1936], pp. 194-195) while still maintaining a broadly Kantian epistemology; the intuitionist mathematician L.E.J. Brouwer was strongly influenced by Kant—Brouwer, like Kant, founds arithmetic on the pure intuition of time—yet Brouwer rejects the pure intuition of space in light of the development of non-Euclidean geometry. Both Reichenbach and Carnap began their careers as Neo-Kantians before turning towards logical empiricism in light of the developments in geometry and logic (Friedman, 2000; Glymour and Eberhardt, 2008). Frege, throughout his career, though critical of Kant's views on arithmetic, nevertheless believed Kant to be correct for the case of geometry, even in the wake of the modern developments.[14] After mercilessly skewering most of his own contemporaries and predecessors, Frege writes, of Kant: "I have no wish to incur the reproach of picking petty quarrels with a genius to whom we must all look up with grateful awe; I feel bound, therefore, to call attention also to the extent of my agreement with him, which far exceeds any disagreement" (Frege, 1980, §89). Brouwer's arch-rival, Hilbert, was also influenced by Kant. Hilbert, in the epigraph to his *Foundations of Geometry*, quotes Kant: "All human knowledge begins with intuitions, thence passes to concepts and ends with ideas" (Hilbert, 1902). All of these thinkers incorporated parts of Kantian philosophy into their own. Bohr was a contemporary of all of these men; further, he had access to Kantian ideas through his lifelong friend and mentor, the philosopher Harald Høffding, who himself was substantively influenced by Kant. Consider Høffding's analysis of Kantian philosophy, in light of our discussion of Complementarity:

> Experience not only implies that we conceive something in space and time, but likewise that we are able to combine what is given in space and time in a definite way, i.e. as indicated in the concepts of magnitude and causality. This is the only means of distinguishing between experience and mere representation or imagination. All extensive and intensive changes must proceed continuously, i.e. through every possible degree of extension and intensity, otherwise we could never be certain of having any real experience. Gaps and breaks must be impossible (non datur hiatus non datur saltus). The origin of each particular phenomenon moreover must be conditioned by certain other phenomena, ... Wherever there appear to be gaps in the series of perceptions we assume that further investigation will discover the intervening members. This demonstration of the *validity of the categories* of

[14] *Cf.* Merrick (2006) for more on the relation between Kant and Frege.

> magnitude and causality likewise involves a limitation: The validity of the categories can only be affirmed within the range of possible experience; they cannot be applied to things which from their very nature cannot become objects of experience (Høffding, 1922, 147-148).

Høffding, who was almost exclusively responsible for Bohr's philosophical education, was a close friend and colleague of Niels Bohr's father, Christian Bohr. The two of them, along with their colleagues Christian Christiansen and Vilhelm Thomsen, would often meet at each others' houses after sessions of the Royal Danish Academy of Sciences and Letters to discuss the scientific and philosophical issues that were brought up at those meetings. The gatherings continued in the Bohr home from the time Niels was 8 or 9 years old up until his father's death in 1911 (Faye, 1991, pp. 13-14), and Niels was likely given the opportunity to listen to their discussions. While in university, Bohr took the required introductory-level philosophy course with Høffding. Shortly after, Niels, his brother Harald, and a small group of other students from Høffding's classes began to meet to discuss philosophical issues. The "Ekliptica group," as it was called, attended Høffding's more advanced seminars and public lectures on philosophy and met regularly from around 1905 until at least 1909. Niels and Harald were both active participants at these meetings (Faye, 1991, p. 20). Høffding remained a close friend of Niels, and the two of them regularly corresponded and discussed philosophical questions up until Høffding's death in 1931 (Faye, 1991, ch. 3).

Høffding's own views were, as Faye relates, "... somewhere between Kant's notion of *a priori* categories and the theories of knowledge characteristic of pragmatism as they were developed by his contemporaries Charles S. Peirce and William James as well as Ernst Mach and James Clerk Maxwell." (1991, p. 12). Høffding held, like Kant, that causality is the criterion for reality. Høffding distinguished, however, between the concept and the principle of causality (Faye, 1991, pp. 81-82) (for Kant, these correspond to the constitutive principle of causality of the first critique, and the regulative principle of causality, or 'idea of mechanism', of the third). According to Høffding, cognition is only possible if we subsume particulars under the *concept* of causality, and thus bring them them into a stable continuous connection with the rest of our cognition (Faye, 1991, pp. 82-83). But since, considered as a *principle*, one cannot demonstrate the universal applicability of causality, it follows that not everything in our experience can be turned into cognition. There always remains an 'irrational' element, therefore, which we cannot comprehend into the totality of our experience (Faye, 1991, p. 85). As we have seen, these views are echoed by Bohr and find their source in Kant's epistemological framework.

A last objection that I will address, before concluding, is with regards to the common misconception of Bohr as a positivist. This conception of Bohr has been popularised by, among others, Karl Popper and Mario Bunge. I will not spend much time answering it here. In addition to directing the interested reader to Don Howard's illuminating article (2004) on the subject, I will simply point out that this is a view that Bohr (as recollected by Heisenberg) explicitly denied: "Positivist insistence on conceptual clarity is, of course, something I fully endorse, but their prohibition of any discussion of the wider issues, simply because we lack clear-cut enough concepts in this realm, does not seem very useful to me—this same ban would prevent our understanding of quantum theory" (Heisenberg, 1971, p. 208, recollecting an old conversation with Bohr).

One may, of course, ignore Bohr here and presume to understand him better than he understood himself. If one were to make such a claim, it would not be objectionable as such; however, given the current, and widely acknowledged, dearth of understanding with respect to Bohr's views on quantum mechanics, such a presumption should be regarded as highly dubious.

In this paper I have highlighted the parallels between Bohr's doctrine of Complementarity and Kant's theoretical philosophy. We have seen how Bohr's principle of complementarity and Kant's theoretical philosophy are common in their approach: that both approaches are centred around what each thinker took to be the limits of objective experience. We have seen how, in order to transcend these limits, Bohr appealed to what a Kantian would call noumena in the positive sense, or ideas of reason. We have seen how a Kantian (who does not deny the validity of the uncertainty relations), starting from the principles of Kantian philosophy, would be led to many of the same conclusions as Bohr. Finally, we have seen how the objections to the link between the two thinkers rest on either a misinterpretation of Kant, or on a misrepresentation of Bohr, or both.

Complementarity is the natural outcome of a broadly Kantian epistemological framework and a Kantian approach to natural science, conjoined with Heisenberg's uncertainty relations. There is a very strong similarity in spirit, if not in technical detail, between Bohr's and Kant's approaches to natural science, and I hope to have inspired the conviction that the further examination of these similarities (and differences) will lead us to a better understanding of both of these men.

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