On the Existence of Spacetime Structure†

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ABSTRACT

I examine the debate between substantivalists and relationalists about the ontological character of spacetime and conclude it is not well posed. I argue that the so-called Hole Argument does not bear on the debate, because it provides no clear criterion to distinguish the positions. I propose two such precise criteria and construct separate arguments based on each to yield contrary conclusions, one supportive of something like relationalism and the other of something like substantivalism. The lesson is that one must fix an investigative context in order to make such criteria precise, but different investigative contexts yield inconsistent results. I examine questions of existence about spacetime structures other than the spacetime manifold itself to argue that it is more fruitful to focus on pragmatic issues of physicality, a notion that lends itself to several different explications, all of philosophical interest, none privileged *a priori* over any of the others. I conclude by suggesting an extension of the lessons of my arguments to the broader debate between realists and instrumentalists.

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†I owe a great debt to Howard Stein's papers "Yes, but. . . : Some Skeptical Remarks on Realism and Anti-Realism" and "Some Reflections on the Structure of Our Knowledge in Physics", both of which inspired the paper's spirit. I am not sure whether Prof. Stein would endorse the paper's methods. I have hopes he would.

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[W]e must bear in mind that the scientific or science-producing value of the efforts made to answer these old standing questions is not to be measured by the prospect they afford us of ultimately obtaining a solution, but by their effect in stimulating men to a thorough investigation of nature. To propose a scientific question presupposes scientific knowledge, and the questions which exercise men's minds in the present state of science may very likely be such that a little more knowledge would shew us that no answer is possible. The scientific value of the question, How do bodies act on one another at a distance? is to be found in the stimulus it has given to investigations into the properties of the intervening medium.

James Clerk Maxwell
"Attraction", *Encyclopædia Brittanica* (9th ed.)

[B]etween a cogent and enlightened "realism" and a sophisticated "instrumentalism" there is no significant difference—no difference that *makes* a difference.

Howard Stein
"Yes, but...—Some Skeptical Remarks on Realism and Anti-Realism"

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1 Introduction

The revival of the debate in recent years in the broader community of philosophers over the ontic status of spacetime can trace its roots, in part, to its revival in the community of physicists. Belot

(1996) and Belot and Earman (2001), for instance, claim that philosophers ought to take the debate seriously because many physicists do. I do not think that fact suffices as a good reason for philosophers to take the debate as interesting, much less even well posed, and so enter into it. The active work of physicists on our best physical theories should provide the fodder for the work of the philosopher of physics most of the time. Sometimes, however, the physicists are confused or just mistaken, and it is then our job to try to help set matters straight. I believe that is the case here.[1]

Other philosophers in recent work have taken inspiration from the traditional debates themselves. Maudlin (1993), for instance, after a *prècis* of the debate in the 17th and 18th centuries and Kant's attempt to sidestep it, concludes, "[G]ranting that the world is *an sich* a spatiotemporal object, we must face a fundamental problem: Are space and time entities in their own right?" In this paper, I dispute that "must."

A virtue of Maudlin's approach, which his work shares with that of many other contemporary philosophers no matter their inspiration, is the foundation of his arguments on the structures of our best physical theories and the use of those structures to guide metaphysical argument. I think the method falls short, however, in so far as it treats those structures in abstraction from their uses in actual scientific enterprises, both theoretical and experimental. This lacuna leaves the debate merely formulaic, without real content, at the mercy of clever sophistications without basis in real, empirically grounded scientific knowledge in the fullest sense.

Stein (1994, p. 1) sums up the situation as I see it admirably. I quote him at length, as he says it better than I could:

> [L]et me . . . hazard a rough diagnosis of the reason why some things that are (in my view) true, important, and obvious tend to get lost sight of in our discussions. I think "lost sight of" is the right phrase: it is a matter of perspective, of directions of looking and lines of sight. As at an earlier time philosophy was affected by a disease of system-building—the *ésprit de système* against which a revulsion set in toward the end of the last century—so it has (I believe) in our own time been affected by an excess of what might be called the *ésprit de technique*. . . : a tendency both to concentrate on such matters of detail as allow of highly formal systematic treatment (which can lead to the neglect of important matters on which sensible even if vague things can be said), and (on the other hand), in treating matters of the latter sort, to subject them to quasi-technical elaboration beyond what, in the present state of knowledge, they can profitably bear. [W]hat I have described can be characterized rather precisely as a species of scholasticism. . . . In so far as the word "scholasticism," in its application to medieval thought, has a pejorative connotation, it refers to a tendency to develop sterile technicalities—characterized by ingenuity out of relation to fruitfulness; and to a tradition burdened by a large set of standard counterposed doctrines, with stores of arguments and counterarguments. In such a tradition, philosophical discussion becomes something like a series of games of chess, in which moves are largely drawn from a familiar repertoire, with occasional strokes of originality—whose effect is to increase

[1] See Curiel (2009) for extensive arguments to this effect on closely related matters, and for a defence of this claim as a fruitful philosophical attitude.

the repertoire of known plays.

In the spirit of Stein's diagnosis, rather than something formally sophisticated I'm going to propose something crude and simple: in order to try to avoid the sort of sterility that purely formal technical elaboration can lead to, we should look at the way that spacetime structures are used in practice to model real systems in order to try to make progress on issues closely related to those treated in the standard debate. For I do think that there are important, deep questions that we can make progress on in the vicinity of that debate, questions of the sort that Maxwell alludes to in the passage I quoted as one of this paper's epigraphs. As Maxwell intimates, however, in order for such questions to be investigated profitably, they must be such as to support and stimulate "the investigation of nature." And that, I submit, can be accomplished only when the questions bear on scientific knowledge in all its guises, as theoretical comprehension and understanding, as evidential warrant and interpretative tool in the attempt to assimilate novel experimental results, as technical and practical expertise in the design and performance of experiments, and as facility in the bringing together of theory and experiment in such a way that each may fruitfully inform the other.

To that end, in this paper I will argue that the way to find the philosophically and scientifically fruitful gold in the metaphysical dross is to formulate and address the questions in a way that explicitly makes contact with both the theoretical and the experimental aspects of our best current knowledge about the kinds of physical system at issue. One way of trying to do that is to pose and investigate the questions explicitly in the context of what I will call an investigative framework: roughly speaking, a set of more or less exactly articulated and fixed theoretical structures for the modeling of physical systems, along with a family of experimental practices and techniques suited to the investigation of the type of systems the theoretical tools appropriately model, in the way the theory actually models them. Different investigative frameworks, as I show by constructive example, provide different natural criteria with which to render determinate content to the question of the ontic status of spacetime, with none privileged *sub specie æternitatis* over any of the others. Those different criteria yield different answers to the question, suitably formulated in the given frameworks. This should not be surprising, I think. After all, different sorts of scientific investigations naturally assume and rely on different relations between individual spacetime points and metrical (and other forms of spatiotemporal) structure, and it is those relations that are supposed to serve as the criteria for existence of individual spacetime points; the mathematical formalism of the theory does not by itself fix a univocal relation with clear *physical* significance between points of the spacetime manifold and geometrical structures, both local and global ones, that live on the manifold. I therefore dispute not only the force of Maudlin's "must," but even more the cogency of the demand itself, baldly formulated.

I begin in §2 with an examination of a popular argument, the so-called Hole Argument, that seems to urge a form of relationalism. I do this for two reasons. First, because advertence to the argument has become something of a mannerism in the debate, it must be confronted; I conclude that it has no bearing one way or another on the issues the debate purports to address. Second, I discuss it because it yields a useful schema for the production of concrete criteria in the terms of which one can try to explicate the difference between substantivalists and relationalists, such as it is. I use that schema—whether the identification of spacetime points must depend on the prior stipulation of metrical structure—to frame the argument of the subsequent two sections of the paper. In each of those two sections I make the schematic criterion concrete in the context of a particular form of investigative framework so as to construct two arguments with contrary conclusions, one in support of something like relationalism and the other something like substantivalism to show that one can make the debate concrete in any of a number of precise, physically significant ways, none *a priori* privileged over the others, and that those ways will not in general agree in their consequences.[2]

The opposed arguments and contrary conclusions of §§3–4, in conjunction with the dismissal of the Hole Argument, do not decisively refute the claim that there is a single, canonical way to explicate the idea of a spacetime point and so to enter into debate over the existence of such a thing. As I urge in §5, they strongly suggest it is a question best settled in the context of a particular form of investigation. The investigation itself in tandem with pragmatic considerations and æsthetic predilections will guide the investigator in settling the form of the question and so the search for its answer. For a given spacetime theory, and even a given model within the theory, depending on one's purposes and the tools one allows oneself, either one can treat spacetime points as entities and individuate and identify them *a priori*, or one can in any of a number of ways construct spacetime points as factitious, convenient pseudo-entities, as it were. Nothing of intrinsic physical significance hangs on the choice, and so *a fortiori* science cannot guide us if we attempt to choose *sub specie æternitatis* between the alternatives—such a choice must become, if anything, an exercise in scholastic metaphysics only.

In §6, I extend the discussion to a host of other types of spacetime structure, such as Killing fields and topological invariants. The attempt to formulate criteria for the physicality of such other structures adds weight to the conclusion that such questions require concrete realization in the context of something akin to real science in order to acquire substantive content. I conclude in §7 with a brief attempt to show that the arguments of this paper ramify into the debate between realists and instrumentalists more generally, by dint, in part, of the picture of science the arguments implicitly rely on. The overarching lesson I draw is that metaphysical argumentation abstracted from the pragmatics of the scientific enterprise as we know it—science as an actually achieved state of knowledge and as an ongoing enterprise of inquiry—is vain. Very little of real substance can be

[2] I do not know of anyone in the literature who adopts exactly the schematic criterion I propose to found my two arguments. (Perhaps Hoefer 1996, 1998 comes the closest.) I use it because I think it captures the flavor of the criteria that are often stipulated when one or the other position is being argued for or against, *viz.*, schematically speaking, that the question of the existence of spacetime points boils down to the relation of those points to some fixed, underlying geometrical structure, such as the metric. (See, *e.g.*, Earman 1989, Maudlin (1990, 1993), Butterfield 1989, Rynasiewicz 1994, Belot 1999, Dorato 2000, Huggett 2006, Pooley 2006, Pooley 2013, Belot 2011.) This is all I require for the overall argument of the paper. I use this particular schema, moreover, as only one example of the sort of criterion one could with some justification rely on in this debate, not because I think it is canonical or privileged in some way, but because it is popular and has a lot to say for it *prima facie*. My hope is that showing how the debate breaks down when this particular criterion is used will, at the least, strongly suggest that it would similarly break down no matter what sort of purely formal criterion of that sort one used. DiSalle (1994), DiSalle (2006) is a notable example of a contemporary philosopher who takes an approach much more sympathetic to my own. (See Friedman 2007 for a thoughtful discussion of DiSalle's work.) Robert Geroch (in private conversation) is a notable example of a contemporary physicist who does so. Dorato (2006) is an interesting case of a philosopher who agrees with me that the contemporary debate is not well posed, but thinks there is a best answer to a proper reformulation of the debate. Rynasiewicz (1996) agrees with me that the contemporary debate is not well posed, but he uses arguments I would not completely endorse.

learned about the nature of the physical world by studying only theoretical structures in isolation from how they hook up to experimental knowledge in real scientific practice, as is the endemic practice in the current debate. In particular, tracking the alleged ontological commitments of a theory based on an analysis of its formal structure alone is not a viable approach to the issue, as we cannot know what structures the theory provides have real physical significance, and what sort of real physical significance they do have, unless we understand how the theory is successfully applied in practice.

The constructions I found the arguments on require the use of advanced mathematical machinery from the theory of general relativity. The format of the paper does not allow for an introduction to most of it. (For the interested reader, Wald 1984 or Malament 2012, for example, contains comprehensive coverage of all material required.) I have tried to segregate it as much as possible so that those who do not want to trudge through it will not have to while still following the general argument. For those who do want to skip most of the technical material, I recommend the following: in §2, ignore the sketch of the Hole Argument (the second and third paragraphs of the section), but read the rest; in §3, read the first two paragraphs and the last one; in §4, read the first two paragraphs (including definition 4.1), and the final two paragraphs. (The remainder of the paper should not pose strenuous technical difficulties.) This course will convey almost the entirety of my argument, bar supportive details the technical material purports to provide.

2 The Hole Argument

In recent times, several physicists and philosophers have construed Einstein's infamous Hole Argument so as to place it at the heart of questions about the ontic status of spacetime points. Its lesson, so claimed, is that one cannot identify spacetime points without reliance on metrical structure, that there is no "bare manifold of points", as it were, under the metric field,[3] though Einstein himself originally formulated the Hole Argument to highlight what he regarded as problems of indeterminism for any generally covariant theory.[4]

This, in brief, is the argument. Fix a spacetime $(\mathcal{M}, g_{ab})$. For ease of exposition, we stipulate that the spacetime be globally hyperbolic, and so possesses a global Cauchy surface, Σ. (We could do without this condition at the cost of unnecessary technical details.) Say that we know the metric tensor on Σ and on the entire region of spacetime to its causal past, $J^-[\Sigma]$. (Note that $J^-[\Sigma]$ contains Σ.) It is known that this forms a well set Cauchy problem, and so there is a solution to the Einstein field equation that uniquely extends g_{ab} on $J^-[\Sigma]$ to a metric tensor on all of $\mathcal{M}$, yielding the original spacetime we fixed.[5] In particular, the solution to the Cauchy problem fixes the metric on the region to the causal future of Σ, $J^+[\Sigma]$. Now, let ϕ be a diffeomorphism that is

[3]See, *e.g.*, Belot (1996) and Gaul and Rovelli (2000).

[4]See Einstein (1914) and Einstein and Grossmann (1914) for two versions of the original argument, Norton (1989, 1993) for historical and critical discussion, and Earman and Norton (1987) for the introduction of the argument to the contemporary philosophical debate.

[5]This is not, strictly speaking, accurate. If no restrictions are placed on the form of the metric, then in general the initial-value problem is not well set. Indeed, even a few known "physical" solutions to the Einstein field equation possess no well set initial-value formulation, for example those representing homogeneous dust and some types of perfect fluid. (See, *e.g.*, Geroch 1996.) We can ignore these technicalities for our purposes, though it may raise a serious problem for those who worry about indeterminism in the theory, one which, to the best of my knowledge, has not been addressed in the literature.

the identity on $J^-[\Sigma]$ and smoothly becomes non-trivial on $J^+[\Sigma] - \Sigma$. No matter what else one takes the significance of the diffeomorphism invariance of general relativity to be, at a minimum it must include the proposition that the application of a diffeomorphism to a solution of the Einstein field equation yields another, possibly distinct solution. Apply ϕ to g_{ab} (but not to M itself); this yields a seemingly different metric—a different "physical state of the gravitational field"—on $J^+[\Sigma] - \Sigma$, in the sense that the same points of $J^+[\Sigma] - \Sigma$ now carry (in general) a different value for the metric. This is the crux of the issue, that the diffeomorphism applied to the metric has yielded a different tensor field in the sense that the same points of the spacetime manifold now carry a different metric tensor than before.

We now face a dilemma, the argument continues (Earman and Norton 1987): we can either hold that the fixation of the metric on $J^-[\Sigma]$ does not determine the metric on $J^+[\Sigma] - \Sigma$, a radical form of seeming indeterminism, or else we can conclude that spacetime points in some sense have no identifiability or existence or what-have-you independent of the prior fixation of the metric tensor. The argument concludes that the denial of the independent existence of spacetime points is the lesser of the evils (or, depending on one's viewpoint, the greater of the goods).[6]

I want to make a crude and simple proposal, for it seems to me that the debate has lost sight of a crude and simple, and yet fundamentally important, fact: just because the mathematical apparatus of a theory appears to admit particular mathematical manipulations does not *eo ipso* mean that those manipulations admit of physically significant interpretation, much less that those apparently mathematical manipulations are even coherent in and of themselves.[7] One has the mathematical structure of the theory; one is not free to do whatever it is one wants with that formalism and then claim, with no foundation in practice, that what one has done has physical import.[8] Once one

[6] Though it does not seem to be recognized in the literature, there are two different versions of the argument used by different investigators. The one I rehearse here can be thought of, in a sense, as a generalization of the other. The more specialized form, which Einstein himself formulated and used, assumes that spacetime has a region of compact closure, the nominal hole, which is devoid of ponderable matter (*i.e.*, in which the stress-energy tensor vanishes) though it itself is surrounded by a region of non-trivial stress-energy; the diffeomorphism is then stipulated to vanish everywhere except in the hole, and the argument goes more or less as in the general case, with the emendation that now it is the distribution of ponderable matter that does not suffice to fix the physical state of the gravitational field. (Earman 1989, for example, uses the more general argument, whereas Stachel 1993 uses the more specialized form.) I think the specialized form of the argument introduces a dangerously misleading red herring, *viz.*, physical differences between regions of spacetime with non-vanishing stress-energy and those without. There seems to me no principled way within the context of the theory itself to distinguish between such regions in a way that bears on metaphysical or ontological issues. One of the regions, that with stress-energy, has non-trivial Ricci curvature; the other does not, though it may have non-trivial Weyl curvature. That difference by itself, the only one formulable strictly based on the theory, can tell us nothing in the abstract about the ontic status of the spacetime manifold. The introduction of the difference seems rather to bespeak an old prejudice that material sources should suffice to determine the physical state of associated fields, but this is not true even in classical Maxwell theory. Indeed, the issue seems much less of a problem in general relativity, for in the case of the Maxwell field we cannot determine a *physically* unique solution without imposing boundary conditions; otherwise, we are always free to add a field with vanishing divergence and curl to a solution to yield another that will have different physical effects on charged bodies. In general relativity, one does not need to do anything of the sort to determine a physically unique solution, so long as the initial data is well behaved in the first place. (See, *e.g.*, Wald 1984, ch. 10, pp. 243–268.)

[7] Weatherall (2014), whose conclusions I endorse, argues vigorously that the sort of manipulation employed in the standard form of the Hole Argument does not make even mathematical sense. For the sake of argument, however, I will assume here that it does. (If one likes, one can take that assumption as being in the service of a *reductio*.)

[8] Stachel (1993, p. 149) describes the attitude in the literature towards arbitrariness nicely:

> A current trend among some philosophers of science is toward what I will call "the fetishism of mathe-

has the mathematical formalism in hand, one must determine what one is allowed to do with it, "allowed" in the sense that what one does respects the way that the formalism actually represents physical systems. A simple example will help explain what I mean: adding 3-vectors representing spatial points in Newtonian mechanics. This shows the need for an investigative context for the fixing of what counts as admissible manipulations of the mathematical formalism, for as a physical operation adding spatial points makes no sense (there is no sense to be had from the idea of linearly superposing two different spatial points in Newtonian theory as a representation of a physical state of affairs), but for the purposes of computing factitious quantities such as the center of mass, it does make sense (though, again, not as an operation that has a physical correlate in the world).

General relativity, in its usual incarnation, is formulated with the use of differential manifolds with pseudo-Riemannian metrics. It does not *ipso facto* follow that every well formed mathematical operation one can perform on a manifold with such a metric has physical significance. It arguably makes mathematical sense to apply a diffeomorphism of the manifold to the metric only, and not to the underlying manifold at the same time. That fact by itself does not imbue the operation with physical significance. It is exactly considerations such as the Hole Argument highlights that show how diffeomorphisms ought to be applied to solutions of the Einstein field equation so as to have physical significance. When one applies a diffeomorphism, one must apply it to both the manifold and the metric. No other procedure has physical content.[9]

The Hole Argument is obviated by the fact that the application of ϕ to the manifold *cum* metric results only in a different presentation of the same intrinsic metrical structure. All observers, no matter which diffeomorphic presentation of the manifold *cum* metric they use in their respective models, will agree on what is of intrinsic physical significance in the possible interaction of physical systems. (Are those two bodies in physical contact? Is heat flowing from this one to that or vice-versa? Can a light-signal be sent from this to that? Is gravitational radiation present? And so on.) There is no logical or physical contradiction in taking different diffeomorphic presentations of the manifold *cum* metric each as the representation of the same physical structure. One must simply stipulate that, in the context of general relativity, the application of a diffeomorphism to the metric is a *physically* well defined procedure only when one also applies it to the (given presentation of the) manifold itself. The worry about determinism thus evaporates, doing away with the dilemma. How one then goes on to try to characterize the ontic nature of spacetime points, if that is the sort

matics." By this I mean the tendency to assume that all the mathematical elements introduced in the formalization of a physical theory must necessarily correspond to something meaningful in the physical theory and, even more, in the world that the physical theory purports to help us understand.

[9] If one adopts a certain definition of a differential manifold, *viz.*, that it is an equivalence class of "diffeomorphic presentations", then one will say that the proposed operation does not make even purely mathematical sense. ($\mathbb{S}^2$, for example, can presented as a certain submanifold of $\mathbb{R}^3$, or as a certain submanifold of a 17-dimensional hyperbolid, or simply as a manifold in its own right; $\mathbb{S}^2 \times \mathbb{R}^2$ can be presented, as here, as a direct product of manifolds, or as $\mathbb{R}^4$ with a line removed; and so on.) In this case, "pushing tensors around on the manifold by a diffeomorphism without also pushing the points around", as required by the Hole Argument, is not an unambiguous notion, for strictly speaking manifold points are defined only up to diffeomorphism in the first place. I do in fact accept that definition of a differential manifold, but I am trying to be as charitable as possible to the proponents of the debate and the arguments standardly deployed in its carrying out, so I am willing to grant for the sake of argument that the required manipulations make mathematical sense. In any event, it is not only philosophers who explicitly attempt to manipulate manifolds and objects in them, in the context of general relativity, in the way the Hole Argument requires; see, *e.g.*, Pons and Salisbury (2005) for physicists explicitly doing so.

of thing one is into, may be influenced by this restriction on the applicability of diffeomorphisms to solutions of the Einstein field equation, or it may not. The point of fundamental importance is that this restriction results from *both* pragmatic and semantic considerations about the way that one may employ the formal apparatus the theory provides so as to respect how solutions to the Einstein field equation represent physically possible spacetimes in practice—how it is that the formal structures of the theory acquire real physical meaning.

In sum, I do not see why the Hole Argument drives one to conclude that one should or should not attribute some form of existence to spacetime points independent of the metrical structure. There is no logical or physical contradiction, for example, in taking the image of a point under the action of ϕ to be "the same spacetime point" as its pre-image, as depicted in a different presentation of spacetime, irrespective of metrical structure. In this case, a spacetime point would be something like an equivalence class of ordinary mathematical points under the relation of being related by a diffeomorphism. An exact formulation that avoids having this idea collapse into triviality—given any finite number of points on a manifold, there is a diffeomorphism that maps those points onto any permutation of them, which seems to leave one with a single equivalence class containing all points—requires some refinement. One could do something like the following: a spacetime point is a physical entity that one can uniquely, or at least adequately and reliably, individuate and identify by what is of intrinsic physical significance at the physical event that occupies it, no matter the diffeomorphic presentation of the manifold of events; it is an entity, in other words, individuated and identified by the equivalence class of physical events under diffeomorphic presentation.[10] If one wants to respond that bare spacetime points *per se* even with what are tantamount to unique labels attached are dependent on physical phenomena under this definition and inobservable to boot, and so unnecessary in the formulation of physical theory, so as to conclude that they have no independent metaphysical existence of one sort or another, I would not necessarily disagree, but neither should I think that one requires the Hole Argument to make the point, for the game of the Hole Argument is that one cannot identify spacetime points in the absence of metrical structure. One need not invoke or rely on metrical structure to make the sort of identification I suggest, as I will show by construction in §4.

The basis for my rejection of the Hole Argument, that a proper understanding of diffeomorphism invariance and the way to properly implement it as a formal procedure vitiates it, rests on a deeper point. I think the most unproblematic and uncontroversial claim one can make about diffeomorphic freedom is that it embodies an irremediable mathematical arbitrariness in the apparatus provided by general relativity for the modeling of physical systems: the choice of the presentation of the spacetime manifold and metric one uses to model a physical system is fixed only up to diffeomorphism.[11] There are restrictions on how one can apply diffeomorphisms to solutions in

[10] Such a characterization would not necessarily rely on metrical structure at a point since, in general, one needs to fix the physical state on an open neighborhood of a point in order to fix the metric structure at that point by way of the Einstein field equation; one cannot solve the Einstein field equation "point by point", as it were. The easiest way to see this is to note the non-uniqueness of vacuum solutions. This is intimately bound up with the fact that the value of the stress-energy tensor at a point does not determine the value of the Weyl tensor (conformal structure) at that point.

[11] Einstein (1924) makes the point himself: "The fact that the general theory of relativity has no preferred spacetime coordinates which stand in a determinate relation to the metric is more a characteristic of the mathematical form of the theory than of its physical content."

practice in order for that application to be consistent with the physical content of the theory, and those restrictions may have philosophical significance, but they may not as well. By itself, that there is arbitrariness tells us nothing of interest about the theory.

A comparison is edifying. Classical mechanics as embodied in either Lagrangian or Hamiltonian mechanics has a similar arbitrariness, slightly different in each formulation of the theory. In Lagrangian mechanics, one is free to choose the Lagrangian function itself on the tangent bundle of configuration space up to the addition of a scalar field derived from a closed 1-form on configuration space (or, in more traditional terms, up to the addition of a total time-derivative of a function of configuration coordinates) without changing the family of solutions the Lagrangian determines.[12] In Hamiltonian mechanics, one is free to choose any symplectomorphism between the space of states and the cotangent bundle of configuration space, *i.e.*, one may choose, up to symplectomorphism, any presentation of phase space (or, in more traditional terms, any complete set of canonical coordinates), without changing the family of solutions the Hamiltonian function determines.[13] One feels no lack of understanding of Lagrangian mechanics, no lacuna in its conceptual resources, merely because one is free to choose the form of the Lagrangian with wide latitude; just so, in Hamiltonian mechanics one is not driven to investigate the ontic status of points in phase space or of the physical quantities whose values one uses to label those points, which ones get nominated 'configuration' and which 'momentum', merely because one is free to choose whatever symplectomorphism one likes in its presentation. Consider the fact that one can run an argument analogous to the Hole Argument in the context of Hamiltonian mechanics, substituting "phase space" for "spacetime manifold", "symplectomorphism" for "diffeomorphism" and "symplectic structure" for "metric". Does that show anything of intrinsic physical significance? No serious person would argue so. And in this case, it would be manifestly absurd to "apply a symplectomorphism only to the symplectic structure and not the underlying manifold": in general the underlying manifold is a cotangent bundle and the symplectic structure is the canonical one on it; pushing the symplectic structure around on its own will yield a new symplectic structure that is *not* the canonical one, and so one manifestly unphysical for the purpose of formulating Hamilton's equation.

The choice of Lagrangian or the choice of symplectomorphism rests on nothing more than pragmatic considerations of the type adumbrated by Carnap (1956) in his discussion of the choice of a linguistic framework for the investigation of philosophical and physical problems.[14] One chooses on the basis of nothing more than what puts one at ease in any of a variety of ways, from pragmatic considerations such as what will be simple or useful for a particular investigation, to those based on historical custom and æsthetic predilection. It is clear that the existence of inevitable, more or less arbitrary, non-physical elements in the presentation of the models of a theory by itself does not require of one a decision on the ontic status of any entities putatively designated by the mathematical structures of either Lagrangian or Hamiltonian mechanics. More to the point, it is clear in these cases that the physical significance of the theory's models is not masked or polluted by the unavoidable arbitrariness in the details of their presentations.

[12] See, *e.g.*, Curiel (2012).
[13] *Op. cit.*
[14] This is not to say that I consider the choice of a Lagrangian or a symplectomorphic presentation of phase space to be the choice of a Carnapian linguistic framework, only that the sorts of considerations that go into each choice are similar.

In the same way, the diffeomorphic freedom in the presentation of relativistic spacetimes does not *ipso facto* require philosophical elucidation, in so far as it in no way prevents us from focusing on and investigating what is of true physical relevance in systems that general relativity models, what one may think of as the intrinsic physics of the systems, so long as one respects the pragmatic conditions for the application of diffeomorphisms to solutions. It is neither formal relations nor substantive entities that remain invariant when one applies a diffeomorphism to a relativistic spacetime; it is the family of physical facts the spacetime represents. (This line of thought already strongly suggests that the debate between substantivalists and relationalists is not well posed.) One may represent those facts in a language some of whose primitive terms designate "spacetime points" or not. Further, one may want to restrict the attribution of existence to what has intrinsic physical significance in the context of our best physical theories. Then again, one may not. It is irrelevant to our capacity to use them in profitable ways in science and, more important, to our comprehension of those facts and our understanding of the role they play in our broader attempts to comprehend the physical world.

In the end, however, the most serious problem I have with the Hole Argument, and all other arguments analogous to it, comes to this: nothing I can see militates in favor of taking the Hole Argument as bearing on the ontic status of spacetime points, *just because* the Hole argument by itself provides no independent, clear and precise criterion for what "existence independent of metrical structure" comes to. That idea has no substantive content on its own. In the next two sections, I will show this by exhibiting two plausible, precise criteria for what the idea may mean in the contexts of two different types of investigation, which in the event lead respectively to opposed conclusions.

3 Limits of Spacetimes

In this section, I propose an argument in favor of the view that one cannot identify spacetime points in the absence of metrical structure, and so, *a fortiori*, that one cannot attribute to the spacetime manifold any existence independent of that structure; the provision of a precise criterion for the existence of spacetime structure, grounded in both the structure and the application of physical theory, grounds the argument. In the event, two criteria natural to the investigative context will suggest themselves, a weaker one based on the idea of the identifiability of spacetime points and a stronger one based on their existence (in a precise sense).

To treat a spacetime as the limit, in some sense, of an ancestral family of continuously changing spacetimes is one of the ways of embodying in the framework of general relativity two of the most fundamental and indispensable tools in the physicist's workshop: the idealization of a system by means of the suppression of complexity, so as to render the system more tractable to investigation; and the enrichment of a system's representation in a theory by the addition (or reimposition) of complexity previously ignored (or ellided) in the model the theory provides for the system. As a general rule, the fewer degrees of freedom a system has, the easier it becomes to study. Schwarzschild spacetime (figure 3.1) is far easier to work with than Reissner-Nordström (figure 3.2) in large part because one ignores electric charge, and there is a natural sense in which one can think of Schwarzschild spacetime as the limit of Reissner-Nordström as the electric charge of the

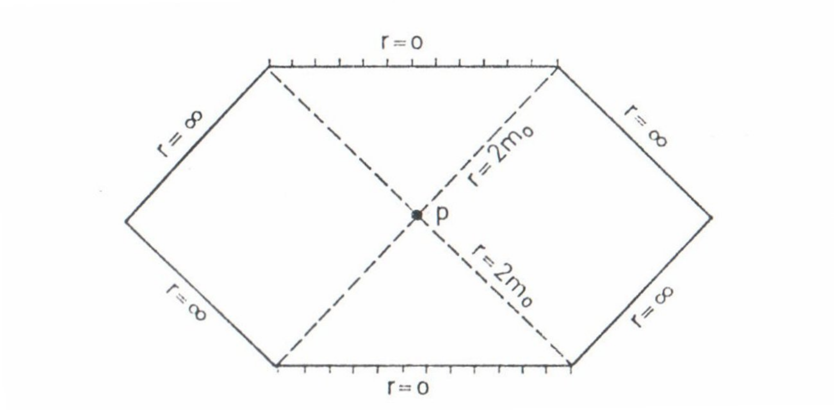

Figure 3.1: Carter-Penrose diagram of Schwarzschild spacetime. Each point in the diagram represents a 2-sphere in the spacetime manifold. (This diagram is taken from Geroch 1969.)

central black hole decreases in magnitude to zero.[15] Contrarily, as a general rule the more degrees of freedom one includes in a system's model, the more phenomena that the system manifests the model can represent, and with greater accuracy (or at least fineness of detail). A generic representation of such a limiting process can provide a schema of both of these theoretical tools respectively, depending on whether one enlarges or shrinks the number of degrees of freedom in the limiting process. As we will see, what in the idealized model one may reasonably identify and attribute existence to may depend in sensitive ways on the character of the more complex or simpler models one starts from and the nature of the limiting process itself. This fact drives the argument I propose. I will first discuss in some detail two examples of such a limiting process in order to motivate the two precise criteria I propose for the existence of spacetime points independent of metrical structure.

Before diving into the examples, however, I first characterize in the abstract the limiting process itself. I use the construction of Geroch (1969) (whose exposition I closely follow), which I only sketch, to capture it. (I simplify his construction in non-essential ways for our purposes, and gloss over unnecessary technicalities.) Consider a 1-parameter family of relativistic spacetimes, by which I mean a family $\{(\mathcal{M}_\lambda, g^{ab}(\lambda))\}_{\lambda \in (0,1]}$, where each $(\mathcal{M}_\lambda, g^{ab}(\lambda))$ is a relativistic spacetime with signature $(+, -, -, -)$ for $g^{ab}(\lambda)$. (It will be clear in a moment why I work with the contravariant form of the metric tensor.) In particular, I do not assume that $\mathcal{M}_\lambda$ is diffeomorphic to $\mathcal{M}_{\lambda'}$ for $\lambda \neq \lambda'$. The problem is to find a limit of this family, in some suitable sense, as $\lambda \to 0$. To solve the problem in full generality, we will use a geometrical construction, gluing the manifolds $\mathcal{M}_\lambda$ of the family together to form a 5-dimensional manifold $\mathfrak{M}$, so that each $\mathcal{M}_\lambda$ is itself a 4-dimensional submanifold of $\mathfrak{M}$ in such a way that the collection of all of them foliate $\mathfrak{M}$.[16] λ becomes a scalar

[15]Schwarzschild spacetime is the unique spherically symmetric vacuum solution to the Einstein field equation (other than Minkowski spacetime); it represents a spacetime that is empty except for an electrically neutral, spherically symmetric, static central body or black hole of a fixed mass. Reissner-Nordström is the generalization of Schwarzschild spacetime that allows the central structure to have an electric charge. See, *e.g.*, Hawking and Ellis (1973, ch.5, §5) for an exposition.

[16]In general what will result is not a foliation in the strict sense of differential topology, but is close enough to warrant using the term for simplicity of exposition.

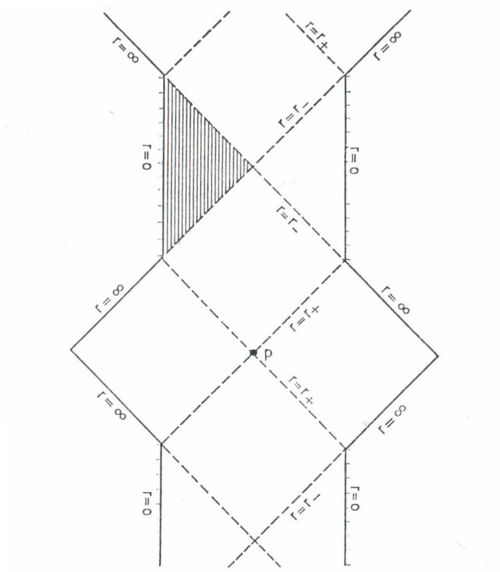

Figure 3.2: Carter-Penrose diagram of Reissner-Nordström spacetime. Each point in the diagram represents a 2-sphere in the spacetime manifold. (This diagram is taken from Geroch 1969.)

field on $\mathfrak{M}$, and the metrics $g^{ab}(\lambda)$ on each submanifold fit together to form a tensor field g^{AB} on $\mathfrak{M}$, of signature $(0, +, -, -, -)$. (I use majuscule indices for objects on $\mathfrak{M}$.) The gradient of λ on $\mathfrak{M}$ determines the singular part of g^{AB}: $g^{AN}\nabla_N\lambda = 0$. (This is why I work with the contravariant form of the metric; otherwise, we could not contravect its five-dimensional parent in any natural way with the gradient of λ.) Note that g^{AB} by itself already determines the submanifolds $\mathcal{M}_\lambda$ (*viz.*, as the surfaces defined by $g^{AN}\nabla_N\lambda = 0$), and that it does so in a way that does not fix any identification of points among them. In other words, the structure I posit does not allow one to say that a point in $\mathcal{M}_\lambda$ is "the same point in spacetime" as a point in a different $\mathcal{M}_{\lambda'}$ (as I shall discuss at some length below).

To define a limit of the family now reduces to the problem of the attachment of a suitable boundary to $\mathfrak{M}$ "at $\lambda = 0$". A *limiting envelopment* for $\mathfrak{M}$, then, is an ordered quadruplet $(\hat{\mathfrak{M}}, \hat{g}^{AB}, \hat{\lambda}, \Psi)$, where $\hat{\mathfrak{M}}$ is a 5-dimensional manifold with paracompact, Hausdorff, connected and non-trivial boundary $\partial\hat{\mathfrak{M}}$, $\hat{g}^{AB}$ a tensor field on $\hat{\mathfrak{M}}$, $\hat{\lambda}$ a scalar field on $\hat{\mathfrak{M}}$ taking values in $[0, 1]$, and Ψ a diffeomorphism of $\mathfrak{M}$ to the interior of $\hat{\mathfrak{M}}$, all such that

1. Ψ takes g^{AB} to $\hat{g}^{AB}$ (*i.e.*, Ψ is an isometry) and takes λ to $\hat{\lambda}$
2. $\partial\hat{\mathfrak{M}}$ is the region defined by $\hat{\lambda} = 0$
3. $\hat{g}^{AB}$ has signature $(0, +, -, -, -)$ on $\partial\hat{\mathfrak{M}}$

This makes precise the sense in which $\hat{\mathfrak{M}}$ represents $\mathfrak{M}$ with a boundary attached in such a way that the metric on the boundary ($\hat{g}^{AB}$ restricted to $\partial\hat{\mathfrak{M}}$) can be naturally identified as a limit of the metrics on the $\mathcal{M}_\lambda$ (g^{AB} on $\mathfrak{M}$). I call $\{(\mathcal{M}_\lambda, g^{ab}(\lambda))\}_{\lambda\in(0,1]}$ an *ancestral family* of the spacetime represented by $\partial\hat{\mathfrak{M}}$, and I call $\partial\hat{\mathfrak{M}}$ the *limit space* of the family with respect to the given envelopment. In general, a given spacetime will have many ancestral families, and an ancestral family will have many different limit spaces. For the sake of convenience I will often not distinguish between $\mathfrak{M}$ and the interior of $\hat{\mathfrak{M}}$. (Although it is tempting also to abbreviate '$\partial\hat{\mathfrak{M}}$' by '$\mathcal{M}_0$', I will not do so, because part of the point of the construction is that different spacetimes can have the same ancestral family.)

Before giving an example of the construction and putting it to work, I discuss one of its features, that it parametrizes not only the metrics but also the spacetime manifolds themselves. Geroch (1969, p. 181) himself states in illuminating terms the reason behind this.

> It might be asked at this point why we do not simply take the $g^{ab}(\lambda)$ as a 1-parameter family of metrics on a given fixed manifold $\mathcal{M}$. Such a formulation would certainly simplify the problem: it amounts to a specification of when two points $p_\lambda \in \mathcal{M}_{\lambda'}$ and $p_{\lambda'} \in \mathcal{M}_\lambda$ $(\lambda \neq \lambda')$ are to be considered as representing "the same point" of $\mathcal{M}$. It is not appropriate to provide this additional information, for it always involves singling out a particular limit, while we are interested in the general problem of finding all limits and studying their properties.

To make the force of these remarks clear, consider the attempt to take the limit of Schwarzschild spacetime as the central mass goes to 0. In Schwarzschild coordinates, using the parameter $\lambda \equiv M^{-1/3}$ (the inverse-third root of the Schwarzschild mass), the metric takes the form

$$\left(1 - \frac{2}{\lambda^3 r}\right) dt^2 - \left(1 - \frac{2}{\lambda^3 r}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\phi^2) \tag{3.1}$$

This clearly has no well defined limit as $\lambda \to 0$. Now, apply the coordinate transformation

$$\tilde{r} \equiv \lambda r, \quad \tilde{t} \equiv \lambda^{-1} t, \quad \tilde{\rho} \equiv \lambda^{-1}\theta$$

In these coordinates, the metric takes the form

$$\left(\lambda^2 - \frac{2}{\tilde{r}}\right) d\tilde{t}^2 - \left(\lambda^2 - \frac{2}{\tilde{r}}\right)^{-1} d\tilde{r}^2 - \tilde{r}^2(d\tilde{\rho}^2 + \lambda^{-2}\sin^2(\lambda\tilde{\rho})\, d\phi^2)$$

The limit $\lambda \to 0$ exists and yields

$$-\frac{2}{\tilde{r}} d\tilde{t}^2 + \frac{\tilde{r}}{2} d\tilde{r}^2 - \tilde{r}^2(d\tilde{\rho}^2 + \tilde{\rho}^2\, d\phi^2)$$

a flat solution discovered by Kasner (1921). If instead of that coordinate transformation we apply the following to the original Schwarzschild form (3.1),

$$x \equiv r + \lambda^{-4}, \quad \rho \equiv \lambda^{-4}\theta$$

then the resulting form also has a well defined limit, which is the Minkowski metric. The two limiting processes yield different spacetimes because it happens behind the scenes that "the same points of the underlying manifold get pushed around relative to each other in different ways". Because the coordinate relations of initially nearby points differ in different coordinate systems, those differences get magnified in the limit, so that their final metrical relations differ. Thus, the limits in the different coordinates yield different metrics.

In the language I introduced above, we should say that the difference between the two limits consists in the different identifications each makes among the points of different $\mathcal{M}_\lambda$. That is why it is inappropriate to work with a fixed manifold from the start. To do so determines a unique limit, but we want to allow ourselves different ways to take the limit, so that our ideal scientist can ignore different facets of the complex system under study, and so produce different idealized models of it.[17] For example, she may want to take the limit of Reissner-Nordström spacetime as the mass goes to zero while leaving the electric charge fixed rather than taking the limit as the electric charge vanishes, or she may want to take the limit in a way that does not respect the spherical symmetry of the initial system in order, *e.g.*, to study small perturbations of the original system.

To characterize the metrical structure of the limit space using structure of members of the ancestral family, I introduce one more construction. An orthonormal tetrad $\xi(\lambda)$ at a point $p_\lambda \in \mathcal{M}_\lambda$ is a collection of 4 tangent vectors at the point mutually orthogonal with respect to $g_{ab}(\lambda)$. Let γ be a smooth curve on $\mathfrak{M}$ nowhere tangent to any $\mathcal{M}_\lambda$ such that it intersects each exactly once. γ then is composed of a set of points $p_\lambda \in \mathcal{M}_\lambda$, one for each λ. A *family of frames* along γ is a family of orthonormal tetrads, one at each point of the curve such that each vector in the tetrad is tangent to its associated $\mathcal{M}_\lambda$, whose members vary smoothly along it. In general, a family of frames will have no well defined limit in $\hat{\mathfrak{M}}$ as $\lambda \to 0$, *i.e.*, there will be no tetrad $\xi(0)$ at a point of $\partial\hat{\mathfrak{M}}$ that the family $\xi(\lambda)$ converges to; in this case, I say the family is *degenerate*. It is always possible, however, given a tetrad $\xi(0)$ at a point on the boundary to find some family of frames that does converge to it.

Now, fix $\xi(0)$ at $p_0 \in \partial\hat{\mathfrak{M}}$ and a family of frames $\xi(\lambda)$ that converges to it. We can represent the metric tensor $g_{ab}(\lambda)$ in a normal neighborhood of p_λ in $\mathcal{M}_\lambda$ using the normal coordinate system that $\xi(\lambda)$ defines in the neighborhood. The components of the metric with respect to these coordinates converge as $\lambda \to 0$, and the limiting numbers are just the components of $g_{ab}(0)$ at p_0 with respect to the normal coordinates that $\xi(0)$ defines. In this way, we can characterize all structure on the limit space based on the behavior of the corresponding structures along the family of frames in the ancestral family.

We are finally in a position to use this machinery to costruct concrete examples. Consider a family $\{(\mathcal{M}_\lambda, g^{ab}(\lambda))\}$ of Reissner-Nordström spacetimes each element of the family having the same fixed value M for its mass and all parametrized by their respective electric charges λ, which converge smoothly to 0.[18] Construct their envelopment. One can now impose a natural collection of families of frames on the family, with the limit space being Schwarzschild spacetime.[19] Now, comparison of figures 3.1 and 3.2 suggests that something drastic happens in the limit. All the points in the throat of the Reissner-Nordström spacetimes (the shaded region in the diagram) seem to get swallowed by the central singularity in Schwarzschild spacetime—in some way or other, they vanish. Using our machinery we can make precise the question of their behavior in the limit $\lambda \to 0$

[17]Of course, sometimes is is appropriate for the scientist to take the limit of a family of metrics on a fixed background manifold. An excellent example is in the statement and proof of the geodesic theorem of Ehlers and Geroch (2004). In fact, they give an illuminating discussion of this very issue on p. 233.

[18]I ignore the fact that electric charge is a discrete quantity in the real world, an appropriate idealization in this context.

[19]The frames are natural in the sense that they conform to and respect the spherical and the timelike symmetries in all the spacetimes. One could use this fact to explicate the claim that Schwarzschild spacetime is the canonical limit of Reissner-Nordström spacetime, in the sense that it is what one expects on physical grounds, whatever exactly that may come to, in the limit of vanishing charge while leaving all else about the spacetime fixed.

in the envelopment.

Consider the points in the shaded region in figure 3.2, between the lines $r = 0$ and $r = r^-$. (r is the radial coordinate in a system that respects the spacetime's spherical symmetry; the coordinate values r^- and r^+ define boundaries of physical significance in the spacetime, which in large part serve to characterize the central region of the spacetime as a black hole.) Fix a natural family of frames along a curve in $\mathfrak{M}$ composed of points q_λ each of which lies in the shaded region in its respective spacetime. It is straightforward to verify that the family of frames along the curve does not have a well defined limit: roughly speaking, the curve runs into the Schwarzschild singularity at $r = 0$. In this sense, no point in Reissner-Nordström spacetime to the future of the horizon $r = r^-$ has a corresponding point in the limit space. To sum up: one begins with a family of Reissner-Nordström spacetimes continuously parametrized by electric charge, which converges to 0, and constructs the envelopment of the family; one constructs the limit space by a choice of families of frames; the collection of families of frames enforces an identification of points among different members of the family of spacetimes, including a division of those points that have a limit from those that do not; and that identification, in turn, dictates the identification of spacetime points in the limit space (which points in the ancestral family lie within the Schwarzschild radius, *e.g.*, and which do not). Thus one can identify points within the limit Schwarzschild spacetime, one's idealized model, only by reference to the metrical structure of members of the ancestral family; one can, moreover, identify points in the limit space with points in the more complex, initial models one is idealizing only by reference to the metrical structure of the members of the ancestral family as well. It is only by the latter identification, however, that one can construe the limit space as an idealized model of one's initial models, for the whole point is to simplify the reckoning of the physical behavior of systems in particular spatiotemporal regions of one's initial models, and most of all at individual spacetime points of one's initial models.

One can, moreover, use different families of natural frames to construct Schwarzschild spacetime from the same ancestral family, with the result that in each case the same point of Schwarzschild spacetime is identified with a different family of points in the ancestral family. More generally, different families of frames will yield limit spaces different from Schwarzschild spacetime, with no canonical way to identify a point in one limit space (one idealized model the theoretician constructs) with one in another. In other words, the identification of points in the limit space depends sensitively on the way the limit is taken, *i.e.*, on the way the model is constructed. In consequence, in so far as one conceives of Schwarzschild spacetime as an idealized model of a richer, more complete representation, one can identify points in it only by reference to the metrical structure of one of its ancestral families, and one can do that in a variety of ways.

Now, say one wants to treat slightly aspherical, almost Schwarzschildian spacetimes as a complexification of Minkowski spacetime, in order to study how asphericities affect metrical behavior.[20] Because the limit spacetime will be almost Schwarzschildian, its appropriate manifold is still $\mathbb{R}^2 \times \mathbb{S}^2$, the natural topology of Schwarzschild spacetime. In this case, in one intuitive sense points will "appear", because the topology of Minkowski spacetime is $\mathbb{R}^4$, so in some sense one

[20]One ought not confuse the idea of complexification I employ here—the making of a model more complex by the introduction of new representational structure—with the idea bandied about in other contexts in mathematical physics often also called 'complexification', in which one takes a mathematical structure based on the real numbers and extends it to one based on the complex numbers.

must "compactify two topological dimensions" to derive a Schwarzschildian spacetime as a more complex limit. There are many ways to effect such a compactification; all the simplest, such as Alexandrov compactification, work by the addition of an extra point or set of points to the topological manifold to represent, intuitively speaking, the bringing in of points at infinity to a manageable distance from everything else.[21] The difficulty of these issues, however, is underscored by the fact that one can also think of this as a case in which points rather *disappear*: $\mathbb{R}^2 \times \mathbb{S}^2$, after all, is homeomorphic to $\mathbb{R}^4$ with a line removed! Thus one could use an ancestral family every member of which is $\mathbb{R}^4$ but that has as limit space the manifold of Schwarzschild spacetime presented as the manifold $\mathbb{R}^4$ with a line removed.[22]

In this example, we will consider the attempt to introduce a central, slightly aspherical body by physical construction in a Minkowskian laboratory, as an experimentalist might do it. For the sake of concreteness, let us say that our experimentalist will, in his representation of the experiment, use an Alexandrov compactification of $\mathbb{R}^4$ to yield $\mathbb{R}^2 \times \mathbb{S}^2$ as the presentation of the manifold of the limit space. The physical construction will proceed in infinitesimal stages, with a tiny portion of matter introduced at each step distributed in a slightly aspherical way (keeping, in an intuitive sense, the aspherical shape of the body the same), and an allowance of a finite time to allow the ambient metrical structure to settle down to an almost Schwarzschildian character before the next step is initiated, until the central body's mass reaches the desired amount. (Intuitively, the finite time period allows the perturbations introduced by the movement of the matter in and its distribution around the central body to radiate off to infinity.) One can represent this process with a limiting ancestral family of Geroch's type in a more or less obvious way, starting with Minkowski spacetime, *viz.*, the empty, flat laboratory, and each member of the ancestral family representing the laboratory at a particular stage of the construction, when a bit more matter has been introduced and the perturbations have settled down.

Now, consider at the beginning of the process a small patch of space in the laboratory not too far from the position where the central body will be constructed. We want to try to track, as it were, the spacetime points in that patch during the enlargement of the central body because we plan to investigate, say, how the metrical structure in regions at that spatiotemporal remove from a central aspherical body differ from each other for different masses of the central body. (Because the Einstein field equation is nonlinear, and there is no exact symmetry, one cannot just assume that slightly aspherical spacetimes will scale in any straightforward way with increases in the central mass.) There are several ways one might go about trying to track the region as the construction progresses. One obvious, simple way is by the triangulation of distances from some "fixed" markers in the laboratory. Because the metrical structure within the lab is constantly changing, however, and doing so in very complex ways during the periods when new matter is being introduced and distributed, and the concomitant metrical perturbations are radiating away, there is no canonical way of implementing the triangulation procedures; in fact, the different ways of doing so are exactly captured by the different families of frames one can fix to identify points among the members of the ancestral family of spacetimes (which in this case, recall, now respectively represent the

[21] See, *e.g.*, Kelley (1955) for an account of methods of compactification, including the Alexandrov type.

[22] This is a concrete instance where thinking of two different diffeomorphic presentations of the same manifold—in this case, $\mathbb{R}^2 \times \mathbb{S}^2$ and $\mathbb{R}^4$ with a line removed—as different manifolds leads to obvious difficulties, if not downright confusions.

spacetime region enclosed by the laboratory at different stages of the construction of the central body). According to some of the concrete implementations of the triangulation procedure, *i.e.*, according to different families of frames one uses to identify points among the several members of the ancestral family, the patch one tries to track will end up inside the central body; according to other procedures, it will end up outside the central body. In consequence, what one means by "the set of spacetime points composing a small region at a fixed spatiotemporal position relative to the central body" will depend sensitively on how one fixes and tracks relative spatiotemporal positions, which is to say, depends sensitively on one's knowledge of the spacetime's metrical structure.[23]

We are finally in a position to offer a precise criterion for "existence of spacetime points independent of metrical structure" natural to the investigative contexts we have considered. There are in fact two natural criteria that suggest themselves, one weaker than the other. The first, suggested by the example of complexification and stated somewhat loosely, is

Definition 3.1 *Points in a spacetime manifold* have existence independent of metrical structure *if there is a canonical method to identify spacetime points during gradual modifications to the local spacetime structure.*

My discussion of the example of complexification shows that, in this context and using this criterion, spacetime points do not have existence independent of metrical structure.

Now, based on the discussion of simplification, I propose a second criterion, stronger than the first and formulated more precisely and rigorously. Fix an envelopment of a limiting family with a definite limit space. I say that a point in $\mathcal{M}_1$ with an associated degenerate family of frames *vanishes* (or that the point itself is a *vanishing point*) with respect to the given family of frames. I say that a point in $\partial\hat{\mathfrak{M}}$ *appears* if there is no family of frames that converges to it.

Definition 3.2 *Points in a spacetime manifold* have existence independent of metrical structure *if no specification of a family of frames in any ancestral family of the spacetime has vanishing or appearing points.*

I do not demand that one be able to identify in a preferred way a spacetime point in the limit with any point of any member of one of its ancestral families, much less for all its ancestral families; this allows us to hold on to diffeomorphic freedom in the presentation of the limit space. I do not even demand that the criterion hold for every possible spacetime model—perhaps in some spacetimes it makes sense to attribute existence to spacetime points independent of metrical structure, whereas in others (say, completely homogeneous spacetimes) it does not. I demand only that, for a given spacetime, one not be able to make points in any of its ancestral families vanish and not be able to make points in it, as the limit space, appear. This attempts to capture the idea that, when

[23] One might object that, in this example, the experimentalist is really trying to track "the same points through space over time", not "the same spatiotemporal points in different spacetimes". In fact, though, since the goal of the investigation is to determine how global metrical structure in slightly aspherical spacetimes differ for different values of the central mass, it is natural for the experimentalist to consider each static phase of the laboratory—the period after the last bit of mass has been added and the perturbations have settled down, but before the next bit of mass is added—as a separate spacetime in its own right, for the purposes of comparison. An appropriate analogue is the so-called "physical process" version of the First Law of black-hole mechanics Wald and Gao 2001; Wald 1994, where one must identify two separate spacetimes (in the sense of two different solutions to the Einstein field equation) that differ in that one conceives of the one as the result of a dynamical evolution of the other, even though there is no concrete representation of that evolution as occurring in a single spacetime.

we construct a spacetime model and treat it as an idealized representation of a more complex system—as it always is—then we can reliably identify spacetime points in our model with points in the more complex system, albeit up to diffeomorphic presentation. If we cannot do this irrespective of the more complex model we start from, then we cannot without arbitrariness and artifice regard results of an investigation in the context of the idealized model as relevant to the physics of the more complex system, for we will be unable to identify the regions in the more complex system that the results of the idealizing investigation pertain to. The example of Schwarzschild spacetime as a limit of a family of Reissner-Nordström spacetimes clearly does not satisfy the criterion, for there are points that vanish in the limiting procedure (*e.g.*, those in the shaded region of figure 3.2). One may suspect that the existence of singular structure in the two spacetimes fouls things up. The following result, however, establishes in a strong sense that no spacetime satisfies the criterion, *i.e.*, that its failure is universal and depends on no special properties of any spacetime model.

Every spacetime has at least one ancestral family, the trivial one consisting of the continuous sequence of itself, so to speak. Construct an envelopment $\mathfrak{M}$ for it, with it itself as the limit space, and apply a slight twist, so to speak, to every metric in every model in the family so as to render each model non-isometric to any other, *i.e.*, so as to render the family non-trivial. (One can make this idea precise in any of a number of simple ways.) On a curve in $\mathfrak{M}$, fix a family of frames that has a well defined limit on $\partial\hat{\mathfrak{M}}$. Now, define a family of Lorentz transformations along that curve, one transformation at each point, such that the family varies smoothly along the curve, and such that when one applies each transformation to the tetrad at its point, the result is a family of frames that has no well defined limit. (One can always do this; for example, the Lorentz transformations can cause the tetrads to oscillate wildly as $\lambda \to 0$.) The points of the ancestral family along that curve have no corresponding point in the limit space defined by the resulting family of frames. This proves

Proposition 3.3 *Every spacetime has a non-trivial ancestral family with vanishing points. Every non-trivial ancestral family has a limit space with respect to which some of its points vanish.*

In consequence, in every relativistic spacetime we treat as an idealized model in the context of this sort of scientific investigation, we can attribute existence to individual spacetime points (or not), only by reference to the metrical structure of the ancestral family we use to construct the model, and the limiting process we choose for the construction.

An obvious objection to the relevance of these arguments to the ontic status of spacetime points is that I deal here only with idealizations and approximations, not with "a real model of real spacetime". But we never work with anything that is not an idealization—it's idealizations all the way down, young man, as part of the human condition. If you can't show me how to argue for the existence of spacetime points independently of metrical structure using our best scientific theories *as they are actually used in successful practice*, then you are not relying on real science to ground your arguments. You are paying only lip-service to the idea that science should ground these sorts of metaphysical issues.

4 Pointless Constructions

The argument of §3 yields a conclusion that holds only in a limited sphere, *viz.*, those investigations based on the idealization of models of spacetime by means of limits. One may wonder whether it could be parlayed into a more general argument. I do not think so. Indeed, I think there is *no* sound argument to the effect that no matter the context of the investigation one can identify spacetime points or attribute existence to them only by reference to prior metrical structure. Sometimes, in some contexts, one can attribute existence to them and identify them without any such reference. To show this, I will present an argument that all the structure accruing to a spacetime, considered simply as a differential manifold that represents the collection of all possible (or, depending on one's modal predilections, actual) physical events, can be given definition with clear physical content in the absence of metrical structure. The argument takes the form of the construction of the point-manifold of a spacetime, its topology, its differential structure and all tensor bundles over it from a collection of primitive objects that, when the construction is complete, acquires a natural interpretation as a family of covering charts from the manifold's atlas, along with the families of bounded, continuous scalar fields on the domain of each chart. That idea yields the following precise criterion the argument will rely on.

Definition 4.1 *Points in a spacetime manifold* have existence independent of metrical structure *if the manifold can be constructed from a family of scalar fields, the values of which can be empirically determined without knowledge of metrical structure.*

The basic idea of the construction is simple. I posit a class of sets of rational numbers to represent the possible values of physical fields, with a bit of additional structure in the form of primitive relations among them just strong enough to ground the definition of a derived relation whose natural interpretation is "lives at the same point of spacetime as". A point of spacetime, then, consists of an equivalence class of the derived relation. The derived relation, moreover, provides just enough rope to allow for the definition of a topology and a differential structure on the family of all equivalence classes, and from this the definition of all tensor bundles over the resultant manifold, completing the construction. The posited primitive and derived relations have a straightforward physical interpretation, as the designators of instances of a schematic representation of a fundamental type of procedure the experimental physicist performs on physical fields when he attempts to ascertain relations of physical proximity and superposition among their observed values. An important example of such an experimental procedure is his use of the observed values of physical quantities associated with experimental apparatus to determine the values of quantities associated with other systems, those he investigates by use of the apparatus. This interpretation of the relations motivates the claim that the constructed structure suffices, for our purposes, as a representation of spacetime in the context of a particular type of experimental investigation as modeled by mathematical physics, and is not (only) an abstract mathematical toy.

I begin the construction by laying down some definitions. Let $\mathbb{Q}$ be the set of rational numbers. A *simple pointless field* ϕ (or just *simple field*) is a disjoint union $\biguplus_{p \in \mathbb{Q}^4} f_p$, indexed by the set $\mathbb{Q}^4$, such that

1. every $f_p \in \mathbb{Q}$

2. there is an $f_p \in \phi$ for every $p \in \mathbb{Q}^4$
3. there are two strictly positive numbers B_l and B_u such that $B_\mathrm{l} < |f_p| < B_\mathrm{u}$ for all $p \in \mathbb{Q}^4$
4. the function $\bar{\phi} : \mathbb{Q}^4 \to \mathbb{Q}$ defined by $\bar{\phi}(p) = f_p$ is continuous in the natural topologies on those spaces, except perhaps across a finite number of compact three-dimensional boundaries in $\mathbb{Q}^4$

Our eventual interpretation of such a thing as a candidate result for an experimentalist's determination of the values for a physical field motivates the set of conditions. That we index ϕ over $\mathbb{Q}^4$ means that we assume from the start that the experimentalist by the use of actual measurements and observations alone can impose on spacetime at most the structure of a countable lattice indexed by quadruplets of rational numbers (and even this only in a highly idealized sense); in other words, the spatiotemporal precision of measurements is limited. Condition 1 says that all measurements have only a finite precision in the determination of the field's value. Condition 2 says that the field the experimentalist measures has a definite value at every point of spacetime. Condition 3 says that there is an upper and a lower limit to the magnitude of values the experimentalist can attribute to the field using the proposed experimental apparatus and technique; for instance, any device for the measurement of the energy of a system has only a finite precision, and thus can attribute only absolute values greater than a certain magnitude, and the device will be unable to cope with energies above a given magnitude. Condition 4 tries to capture the ideas that (local) experiments involve only a finite number of bounded physical systems (apparatuses and objects of study), and that classical physical systems bear physical quantities the magnitudes of which vary continously (if not more smoothly), except perhaps across the boundaries of the systems.

Fix a family Φ of simple pointless fields. The *link at* p, λ_p, is a set containing exactly one element from each simple field in Φ such that all the elements are indexed by p, the same quadruplet of rational numbers. One link, for example, consists of the set of all values in the fields in Φ indexed by $(3/17, 2, -3001\frac{90}{91}, 2)$. A *linked family of simple pointless fields* $\mathfrak{F}$ is an ordered pair (Φ, Λ) where Φ is a countable collection of simple fields, and Λ is the family of links on Φ, a *linkage*, complete in the sense that it contains exactly one link for each $p \in \mathbb{Q}^4$. The idea is that the values of the simple fields in the same link all live "at the same point of spacetime", namely that designated by p. One can think of the linkage as a coordinate system on an underlying, abstract point set.

We are almost ready to define the point-structure of the spacetime manifold. We require only two more constructions, which I give in an abbreviated fashion so as to convey the main points without getting bogged down in unnecessary technical detail. Let $\mathfrak{F} = (\Phi, \Lambda)$ be a linked family containing all simple fields; we call it a *simple fundamental family*. Let $\hat{\mathfrak{F}} = (\hat{\Phi}, \hat{\Lambda})$ be another. We want a way to relate the linkages of the two, so as to be able to represent the relation between the coordinate systems of two different charts on the same neighborhood of the spacetime manifold, or on the intersection of two neighborhoods. A *cross-linkage* on a simple fundamental family is an ordered triplet $(O, \hat{O}, \chi)$ where $O \subseteq \mathbb{Q}^4$ and $\hat{O} \subseteq \mathbb{Q}^4$ are open sets, such that either both are the null set or else both are homeomorphic to $\mathbb{Q}^4$, and χ is a homeomorphism of O to $\hat{O}$. The link $\lambda_p \in \Lambda$ for $p \in O$, then, will designate the same point in the underlying manifold as $\hat{\lambda}_{\chi(p)} \in \hat{\Lambda}$ for $\chi(p) \in \hat{O}$; in this case, we say the links *touch*. If O and $\hat{O}$ are the null set, then the represented neighborhoods do not intersect. (We do not require that the values of the scalar fields in the two different simple fundamental families be numerically equal at any given point, as the two scalar fields may represent different physical quantities, *e.g.*, a component of the fluid velocity and a component of the shear-stress tensor of a viscous fluid.) One can extend the idea of a cross-linkage to an arbitrary number of simple fundamental families in the obvious way. (To make the idea precise we would need to index the collection of families, and so on, but I think it is clear enough without going through the bother.) We would then identify a point in an underlying abstract point-set as an equivalence class of links under the equivalence relation "touches".

To finish the preparatory work, we must move from rationals to reals. Fix a simple, fundamental family $\mathfrak{F}$. First, we attribute to $\mathfrak{F}$ the algebraic structure of a module over $\mathbb{Q}$. For example, the sum of two simple pointless fields ϕ and ψ in Φ is a simple pointless field ξ such that $x_p \equiv f_p + g_p$ is the value in ξ labeled by the index p, where $f_p \in \phi$ and $g_p \in \psi$. ξ is clearly itself a simple pointless field with a natural embedding in the linkage on $\mathfrak{F}$, and so belongs to Φ. Now, roughly speaking, we take a double Cauchy-like completion of Φ over both the points $p \in \mathbb{Q}^4$ and the values $f_{\hat{p}} \in \mathbb{Q}$, yielding the family $\bar{\Phi}$ of all disjoint unions of real numbers continuously indexed by quadruplets of real numbers.[24] This procedure makes sense, because every continuous real scalar field on $\mathbb{R}^4$ is, again roughly speaking, the limit of some sequence of bounded, continuous rational fields defined on $\mathbb{Q}^4$. We thus obtain what is in effect the family $\bar{\Phi}$ of all continuous real scalar fields on $\mathbb{R}^4$, though I refer to them as *pointless fields*, in so far as, at this point, they are still only indexed disjoint unions. The limiting procedure, moreover, induces on $\bar{\Phi}$ the structure of a module over $\mathbb{R}$ from that on Φ. Finally, in the obvious way, we take the completion, as it were, of *Lambda* using the same limiting procedure to obtain a linkage $\bar{\Lambda}$ on $\bar{\Phi}$. I call $\bar{\mathfrak{F}} = (\bar{\Phi}, \bar{\Lambda})$ a *fundamental family*. A cross-linkage on a pair of fundamental families is the same as for simple fundamental families, except only that one uses homeomorphisms on subsets of $\mathbb{R}$ rather than $\mathbb{Q}$. If we have two simple fundamental families with a cross-linkage on them and take limits to yield two fundamental families, then the nature of the limiting process guarantees a unique cross-linkage on the two fundamental families consistent with the original.

We can at last construct a real topological manifold from a collection of simple fundamental families. The basic idea is that a fundamental family represents the family of continuous real functions on the interior of a bounded, normal neighborhood of what will be the spacetime manifold. Because a spacetime manifold must be paracompact (otherwise it could not bear a Lorentz metric), there is always a countable collection of such bounded, normal neighborhoods that cover it. This suggests

Definition 4.2 *A* pointless topological manifold *is an ordered pair* $(\{\mathfrak{F}_i\}_{i\in\mathbb{N}}, \chi)$ *consisting of a countable set of simple fundamental families and a cross-linkage on them.*

To justify the definition, I sketch the construction of the full point-manifold and its topology. First, we take the joint limit of all simple fundamental families to yield a countable collection of fundamental families with the induced cross-linkage. A point in the manifold, then, is an equivalence class of links, at most one link from each family, under the equivalence relation "touches". The

[24] In order to get the completion we require, standard Cauchy convergence does not in fact suffice. We must rather use a more general method, such as Moore-Smith convergence based on topological nets. The technical details are not important. See, *e.g.*, Kelley (1955, ch. 2) for details.

set of links associated with one of the families, then, becomes a representation, with respect to the equivalence relation, of the interior of a compact, normal neighborhood in the manifold, and the fields in that family represent the collection of continous real functions on that neighborhood. The cross-linkage defines the intersections among all these neighborhoods, yielding the entire point-set of the manifold. By assumption, the collection of all such neighborhoods forms a sub-basis for the topology of the manifold, and so, by constructing the unique topological basis from the given sub-basis, the point-set becomes a true topological manifold. It is straightforward to verify, for example, that a real scalar field on the constructed manifold is continuous if and only if its restriction to any of the basic neighborhoods defines a field in the fundamental family associated with that neighborhood.

Now, to complete the construction, we can define the manifold's differential structure can in a straightforward way using similar techniques. First, demarcate the family of smooth scalar fields as a sub-set of the continuous ones, which one can do in any of a number straightforward ways with clear physical content based on the idea of directional derivatives. The family of all smooth scalar fields on a topological manifold, however, fixes its differential structure (Chevalley 1947). The directional derivatives themselves suffice for the definition of the tangent bundle over the manifold, and from that one obtains all tensor bundles.

After so much abstruse and, worse, tedious technical material, we can now judge whether the construction supports the argument I want to found on it. The use of $\mathbb{Q}^4$ to index a simple pointless field represents the fact that all points in a laboratory have been uniquely labeled by 4 rational numbers, say, by the use of rulers and stop-watches. Such an operation neither measures nor relies on knowledge of metrical structure, for it yields in effect only a chart on that spacetime region. (No assumption need be made about the "metrical goodness" of the rulers and clocks.) Neither does any other operation used in the construction pertain to metrical structure. One determines the values of the simple fields, for example, by use of physical observations, which do not themselves necessarily depend on knowledge of the ambient metrical structure. To illustrate the idea, consider the use of a gravity gradiometer to measure the components of the Riemann tensor in a region of spacetime, which exemplifies many of the ideas in the construction. The gradiometer is essentially a sophisticated torsion balance for measuring the quadrupole moments (and higher) of an acceleration field.[25] Its fixed center and the ends of its two rotatable axes continuously occupy at any given moment 5 proximate points, the attribution to which of values for linear and angular acceleration yields direct measures of the components of the Riemann tensor in a normal frame adapted to the position and motion of the instrument. One then identifies the spacetime points the parts of the instrument respectively occupy, and by extension those in the normal frame adapted to it, by the values of the components of the Riemann tensor and their derivatives in that frame, by the values of its scalar invariants, and so on.[26] One does not have to postulate a prior metric structure in order to perform the measurements and label the points, nor need one have already determined the metrical structure by experiment. Indeed, in the performance of the gradiometer measurements one determines much of spacetime's metrical structure. Because the facts of intrinsic physical significance that the values of the fields and the

[25]See, *e.g.*, Misner, Thorne, and Wheeler 1973, §16.5, pp. 401–402, for a description of the device and its use.

[26]See, for example, Bergmann and Komar (1960), Bergmann and Komar (1962) for a concrete, albeit purely formal, example of a procedure for implementing this idea.

relations among them embody (is this body in contact with another? does heat flow from that body to this or vice-versa?), moreover, remain invariant under the action of a diffeomorphism it follows that the equivalence classes we used to construct points does so as well. Thus, we can fix all the manifold structure, including metrical, only up to diffeomorphism, as we expect. This shows that the construction delivers everything we need and nothing more.

There is an obvious response to the argument based on this construction. One may object that, so far from the argument's having shown thatthe construction pushes us to attribute independent existence to spacetime points, it rather suggests that points are defined only by reference to prior physical systems, and hence exist in only a Pickwickian sense, dependent on the identifiability of those physical systems. This objection can be answered by, as it were, throwing away the ladder. Once one has the identification of spacetime points with equivalence classes of values of scalar fields, one can as easily say that the points are the objects with primitive ontological significance, and the physical systems are defined by the values of fields at those points, those values being attributes of their associated points only *per accidens*.[27] I do not pretend to endorse such a move, but I do not have to. My constructive argument is *ad hominem*.

5 The Debate between Substantivalists and Relationalists

I do not consider the idea of pointless manifolds deep or of great interest in its own right.[28] There are, I am sure, many other constructions in the same spirit. If one were so inclined, I suppose one could try to take something like it to give a precise way for a relationalist to characterize the spacetime manifold.[29] I am not so inclined, because I do not think the contemporary debate between the relationalist and the substantivalist has been well posed, and I am inclined to think it never will be in any interesting sense. That is what I take to be the force of the opposed constructions of §3 and §4, taken in tandem. They show that "dependence on prior metrical structure" is formal, *i.e.*, without substantive content until given explication in the framework of an investigative enterprise, even if that framework be given only in schematic form. Once one grants this, however, the game is up. Different investigative frameworks can and do yield natural criteria that lead to contrary conclusions.[30]

An amusing and poignant feature of the constructions shows this clearly: each yields a conclusion contrary to what the traditional debates would have led one to have expected based on the tools and techniques it employs. In the second, one uses independent values of physical quantities (a stock in trade of the relationalist) in order to identify and attribute existence to spacetime points

[27] Stachel (1993) provides an elegant tool for describing the result of such a construction as I propose and in particular this rebuttal to the proposed objection (though I should say his work is not related to a project such as this). In his terms, I have sketched the construction of an *individuating field* independent of the stipulation of metrical structure, *viz.*, a field or system of fields on spacetime that suffices for the identification of individual spacetime points.

[28] There are a few questions of potential interest that accrue to it. Is it possible to determine the topology of a non-compact manifold by the postulation of a finite number of simple fields? If so, does the minimum number depend on a topological invariant? Is it in any case greater than the number of fields we currently believe to have physical import?

[29] See Butterfield (1984) for a survey of some ways one might attempt such a project.

[30] This line of argument bears fruitful comparison to the ideas of Ruetsche (2011), though it was developed independently of her work.

without a prior assumption of metric structure; and in the first, one uses structures in mathematical physics that seem to presuppose the independent identifiability of spacetime points (a stock in trade of the substantivalist) in order to argue that in fact they are not identifiable without a prior postulation of metric structure. One may think that these features of the arguments make them, in the end, self-defeating, but I do not think that is so. In the first, one operates under the implicit assumption that the more complex models one idealizes are themselves only idealizations of yet more complex models. In the second, one implicitly assumes that, say, the gradiometer is small enough and the temporal interval of the measurement itself short enough to justify the use of the Minkowski metric in making the initial attributions of the magnitudes of spatiotemporal intervals in the experiment; one then uses this to bootstrap one's way to a more accurate representation of the metrical structure of spacetime, which is what is done in practice. I think that this facet of the arguments, perhaps more than anything else, illustrates the vanity of the traditional debate: one can use the characteristic resources and moves of each side to construct arguments contrary to it, once one takes the trouble to make the question precise.

Most damning in my eyes, the constructions show the futility of the debate, for they make explicit how very little one gains in comprehension or understanding by having taken the considerable trouble to have made the questions precise. Indeed, one may feel with justice that nothing has been gained, but rather something has been lost in a pettifoggery of irrelevant technical detail.[31]

Although I conclude the traditional debate is without real content, I think there is a related, interesting question one *can* give clear sense to: what in one's investigative framework is naturally taken to, or must one take to, have intrinsic physical significance? Even putting aside existence and ontology as emotive distractions, however, I do not think one can give even this question substantive sense in the abstract: the question is a formal template that one must give substance to by fixing the significance of its terms in presumably different ways in different particular contexts.

Consider one way to rephrase the question that may seem on its face to give it concrete content in abstraction from any schematic framework: what propositions would all observers agree on? One cannot answer this question in the abstract, or even give it definite sense, because one has not yet fixed the way that one will schematically represent the observer (or experimental apparatus) and the process of observation. In order to do so, one must settle many questions of a more concrete nature. Will one use the same theory to model the observation as one uses to model the system? Will one take the observer to be a test system, in the sense that the values of its associated physical quantities do not contribute to the initial-value formulation of the equations of motion of one's theoretical or experimental models? And so on. Until one settles such issues, one cannot even say with precision what any single observer can or will observe, much more what all will agree on. In this sense, even claims such as "in general relativity, only what is invariant under diffeomorphisms has intrinsic physical significance" have only schematic content. One must give definite substance to the "what" in "what is invariant"—substance that involves the forms of the physical systems at issue and the methods available for their probing and representation—before one can make the claim play any definite role in our attempts to comprehend the world. I take this to be the lesson of Stein (1977), *viz.*, that the way to proceed in these matters is the one Newton and Riemann relied on: we must infer what we can about the spatiotemporal structure of the world from the

[31] Jeremy Butterfield in particular has vigorously tried to convince me that I dismiss too readily the possible philosophical value of the technical constructions and arguments of §§3 and 4. I would like to think he is right.

roles it plays in characterizing physical interactions; and on this basis, neither substantivalism nor relationalism can claim any great victory.[32]

In the end, why should we ever have expected there to have been a single, canonical way to explicate the physical significance of the idea of a spacetime point, on the basis of which we might then attempt to determine whether such a thing exists or not in some lofty or mundane sense? What, after all, is lost to our comprehension of the physical world without such a unique, canonical explication? We purport after all, in these debates, to attempt to better comprehend the *physical* world. Hadn't we better ensure, then, that the terms of our arguments have the capacity to come in some important way into contact with the physical world by way of experiment and theory? Once we take that demand seriously, we find an orgiastic crowd of possible candidates to serve as concrete realizations of the question, some of which will be fruitful in some kinds of enterprises, others in others, and, most likely, several in none at all. Indeed, I am far from convinced that the question of the existence of spacetime points has ever itself been well posed. What possible difference could an answer to it make one way or another to the proper comprehension of the performance of an experiment or the proper construction of a model of a physical system in the context of general relativity?

I think there is a better question at hand: what mathematical structures "best" represent our experience of spatiotemporal localization? Again, this question cannot be answered in the abstract, for it depends sensitively on the answers to other, more or less independent and yet inextricable questions, such as: what mathematical structures best represent our experience of other features of spatiotemporal phenomena, such as the lack of absolute simultaneity, the orientability of space, *etc.*? And also questions such as: what structures for representation of various kinds of derivatives do we need to formulate equations of motion? And what structures for representation of Maxwell fields? And so on. One has to attempt to address these questions in a dialectical fashion, answering part of one here, seeing what adjustments that requires in other parts of the manifold of possible structures, so to speak, and so on. The answer to one of these questions in one context may be individual points of a spacetime manifold, to another question in another context it may be area and volume operators as in loop quantum gravity, and so on. It is to the investigation of such questions that I now turn.

6 An Embarassment of Spacetime Structures

The arguments of this paper extend themselves naturally beyond the realm of the debate over the existence of spacetime points, and do so in a way that sheds further light on the futility of that debate. There are many different senses one can give to the question whether some putative entity or structure of any type has real physical significance in the context of general relativity,

[32] DiSalle (1994, p. 274) trenchantly makes a very closely related point, one, indeed, that in large part may be viewed as foundational for my analysis:

> Since the work of Riemann and Helmholtz, however (not to mention Einstein), it should be clear that our claims about 'objective' spatiotemporal relations always involve assumptions about the physical processes we use for measurement and stipulations about how those processes are to indicate aspects of geometry.

each more or less natural in different contexts. For lack of a better term, I shall say that an entity (which, as we shall see, can encompass several different types of thing), purportedly represented by a theoretical structure, has *physicality* if one has a reason to take that structure seriously in a physical sense, *viz.*, if one can show that it plays an ineliminable role in the way that theory makes contact with experiment. Of course, as I stressed in §2, such an abstract, purely formal schema as "plays an ineliminable role in the way that theory makes contact with experiment" has no real content until one explicates it in the context of a more or less well delineated investigative framework. It is the examples that give the idea life.

6.1 Manifest Physicality

A Maxwell field, represented by the Faraday tensor F_{ab}, is manifestly physical. One important sense in which this is true turns on the fact that it contributes to the stress-energy tensor on the righthand side of the Einstein field equation. The Maxwell field itself possesses stress-energy, and in general relativity nothing is physical if not that.

Consider now a Killing field on spacetime, a vector field ξ^a that satisfies Killing's equation

$$\nabla_{(a}\xi_{b)} = 0 \tag{6.1}$$

and so generates an isometry, in the sense that $\pounds_\xi g_{ab} = 0$. In this guise, it seems not to possess the characteristics of a physical field, in so far as it enters the equations of motion of no manifestly physical system, such as a Maxwell field. In other words, it does not couple with phenomena we consider physical, does not contribute to the stress-energy tensor. Now, define the 2-index covariant tensor $P_{ab} \equiv \nabla_a\xi_b$. Equation (6.1) implies that it is anti-symmetric. Let us say that it happens as well to have vanishing divergence and curl, $\nabla_n P^{na} = 0$ and $\nabla_{[a}P_{bc]} = 0$, and so satisfies the source-free Maxwell equations. Is it *eo ipso* a true Maxwell field, and so physical? Not necessarily. There are always an innumerable number of 2-forms on a spacetime that satisfy the source-free Maxwell equations. At most, one of them represents a physical Maxwell field. If, however, it just so happened that P_{ab} were to represent the physical Maxwell field on spacetime—one known as a Papapetrou field in this case—the fact that one natural way to represent the field happened to generate an isometry would appear to be an accident, in the sense that no property of the field accruing to it by dint of its physicality, which is to say, by dint of its satisfaction of the Maxwell equations and concomitant coupling with other manifestly physical phenomena (such as spacetime curvature, by way of the Einstein field equation), depends on the satisfaction of equation (6.1) by ξ^a (except in the trivial sense that satisfaction of equation (6.1) is necessary for ξ^a to be a 4-vector potential for a Maxwell field). Still, ξ^a is a naturally distinguished geometrical structure in the physical description of spacetime, forms a part of the description of spacetime independent of the particulars of the physical constitution of any observed phenomena, in particular in so far as it places non-trivial contraints on a manifestly physical structure, the spacetime metric. In this sense, different from that pertaining to the Maxwell field, ξ^a is physical, for the Maxwell field, by contrast, is not naturally distinguished in this sense, but rather depends in an essential way on the peculiar physical constitution of a particular family of phenomena.

In what sense, though, is the metric manifestly physical? The metric does not itself contribute to the stress-energy content of spacetime, for one cannot attribute a localized gravitational stress-energy to it.[33] That is not to say that the metric does not appear in the stress-energy tensor of a given spacetime, for it is almost always required for the construction of the stress-energy tensor.[34] The stress-energy tensor of a Maxwell field, for example, is $F_{an}F^{n}{}_{b} + \frac{1}{4}g_{ab}F_{rs}F^{rs}$. (The metric appears not only explicitly in the second term, but also implicitly in both terms, raising the contracted indices.) The metric, however, is necessary both for posing the initial-value formulation of every possible kind of field that may appear in a relativistic spacetime, in particular all of those (such as the Maxwell field) that we regard as manifestly physical, and for formulating the equations of motion of the fields. In particular, the metric dynamically couples with other physical systems, *i.e.*, enters into interaction with them in the strong sense that there always exist terms in the equations of motion for any given field in which the metric appears as one factor and the tensor representation of the field as another. For the Maxwell field, the metric appears contravected with the Faraday tensor in the equation of motion representing the fact that its covariant divergence equals the charge-current density of matter.[35]

The metric, of course, can play other roles as well, just as a Killing field. A vacuum spacetime with non-zero cosmological constant has a stress-energy tensor equal to the metric times a constant. In this case, one plausible way of reading the Einstein field equation is to have the metric play simultaneously two distinct roles, one as the necessary ground of all spatiotemporal structure (embodied in the Einstein tensor) and the other as a component in the tensor representing the stress-energy content of spacetime, depending on contingent features of the ambient matter field, in this case, whatever field gives rise to the cosmological constant. Again, in the former sense, as ground of spatiotemporal structure, the metric is a naturally distinguished structure in any physical description of spacetime; in the latter sense, it rather depends on the peculiar, contingent physical constitution of a particular family of phenomena.

Consider the Riemann tensor. Again, it manifests physicality in several different ways, in different contexts. Perhaps the most important is in the equation of geodesic deviation, where it directly measures the rate at which infinitesimally neighboring geodesics tend to converge towards or diverge away from each other. In this case, the Riemann tensor's physicality consists in the fact that it encodes all information needed to model manifestly observable phenomena, *viz.*, the relative acceleration of nearby freely falling particles and the tidal force exerted between different parts of a freely falling extended body. Another important role it plays in general relativity is as the measure of the failure of the ambient covariant derivative operator associated with the spacetime metric to commute with itself when acting on vectors or tensors. Here, the physical interpretation is not clear, but one way of trying to explicate it is by considering the way that a tangent vector changes when parallel-propagated around an "infinitesimally small" loop.[36] The infinitesimal change in the vector when it returns to the initial point is directly proportional to the Riemann tensor. Still, it is difficult to say that this has real *physical* significance, in so far as one could implement such a

[33] See, *e.g.*, Curiel (2014a).

[34] Indeed, the only example I know of a stress-energy tensor for which the metric is not needed for its definition is the case of a null gas, for which only the conformal structure of spacetime is required.

[35] That the other defining equation for a Maxwell field, representing the fact that the Faraday tensor is curl-free, does not require the metric at all for its formulation—the exterior derivative is determined by the differential structure of the underlying manifold, and does not require any other structure at all for its definition—may push one to say that it is not a dynamical equation of motion, but rather a kinematical constraint.

[36] See Wald (1984, ch. 2, §3) for a thorough exposition.

mechanism and measure the result only in a spacetime with closed causal curves. And yet so much of the mathematical apparatus of general relativity depends on the fact that the ambient derivative operator is, in general, not flat (*i.e.*, fails to commute with itself), that it would be absurd to say that the Riemann tensor is not playing a physical role here. What exactly that role is, however, is not easy to pin down.

The Einstein tensor itself presents an interesting case. It has no straightforward geometrical interpretation.[37] It seems, moreover, to have no straightforward physical interpretation either—it enters into the equations of motion of no known fields; it measures no quantitative feature of any known physical phenomena; it does not represent a field possessing stress-energy; it constrains the behavior of no other manifestly physical structure; and so on. And yet it is the structure that matter fields couple to (via the Einstein field equation) in their role as source for spatiotemporal curvature. In this role, it dynamically couples with no individual matter fields, but rather only to the aggregate physical quantity "stress-energy" they all possess, and which, according to the fundamental principle of the fungibility of all forms of energy,[38] in no way differs qualitatively among all known fields. Again, then, it seems manifestly physical in some sense, but it is difficult to put one's finger clearly on that sense.

Global structures of various sorts (causal, topological, projective, conformal, affine, *et al.*) present interesting cases as well.[39] Consider the conformal structure of a spacetime. It governs and is embodied in the relative behavior of the null cones across all spacetime points. One natural interpretation of the null cones is as determining a finite, unachievable upper-limit for the velocities of material systems.[40] The fact that the null cones determine a topological boundary for the chronological future and past of every spacetime point also has a natural interpretation in the same vein: if the chronological future or past were topologically closed, then there would be a limiting upper velocity for massive bodies that would be *actually achievable* by a massive body using only a finite amount of energy. If one accepts these interpretative glosses, then the conformal structure has physicality in so far as it constrains the behavior of manifestly physical systems.

So, to sum up the notions of physicality mooted here are:

- contributes to T_{ab} (*e.g.*, Maxwell field)
- required for initial-value formulation of manifestly physical fields (*e.g.*, Maxwell field, g_{ab})
- dynamically couples to manifestly physical entities (*e.g.*, Maxwell field, g_{ab})
- dynamically couples to manifestly physical quantities that more than one type of physical system can bear (*e.g.*, Einstein tensor)

[37] See Curiel (2014b) for a discussion.

[38] See Maxwell (1877, ch. v, §97) and Maxwell (1888, chs. i, iii, iv, viii, xii) for illuminating discussion of this principle.

[39] I take a structure to be global if it is not local in the sense explicated by Manchak (2009, p. 55):

> [A] condition C on a spacetime is *local* if, given any two locally isometric spacetimes (M, g_{ab}) and (M', g'_{ab}), (M, g_{ab}) satisfies C if and only if (M', g'_{ab}) satisfies C.

I think Manchak's definition of "local" is superior, as judged by its physical significance in the context of general relativity, to the one I proposed in Curiel (1999, §5), though the latter may still be of interest in purely mathematical contexts, or in contexts of physical investigation that transcend the scope of a single theory.

[40] See, however, Geroch (2010) and Earman (2013) for dissenting arguments.

- acts as a measure of an observable aspect of manifestly physical entities (*e.g.*, Riemann tensor)
- enters the field equation of a manifestly physical structure (*e.g.*, Einstein tensor)
- constrains the behavior of a manifestly physical entity (*e.g.*, Killing field, conformal structure)
- plays an ineliminable, though physically obscure, role in the mathematical structure required to formulate the theory (*e.g.*, Riemann tensor, Einstein tensor)

I am confident there are yet more senses of physicality I have not touched upon.

6.2 Observability

One does not have to be an instrumentalist or an empiricist to accept that the possible observability of physical phenomena is one of the most fundamental reasons we have to think such things are physical in the first place. The question of the observability of various kinds of global structure in general relativity, therefore, poses particularly interesting problems for arguments about physicality. Manchak (2009), Manchak (2011) shows that, in a precise sense, local observations can never suffice to determine the complete global structure of spacetime in general, and in particular cannot determine whether a spacetime is inextendible or stably causal (Manchak 2011, p. 418, proposition 3). Nonetheless, there remain several things to say and ask about the matter of physicality here.

Take, for example, the Euler number of the spacetime manifold, a global topological structure.[41] It is a topological invariant that, in part, constrains the possible existence of everywhere non-zero vector fields on a manifold. That an even-dimensional sphere, for example, possesses no everywhere non-zero vector field (and indeed no Lorentzian metric) follows directly from the computation of its Euler number. If we were to live in a world whose underlying manifold possessed a non-trivial Euler number, and so could support no physical process that would manifest itself as an everywhere non-zero vector field, this would constitute a physical fact about the world in an indubitable sense. It is not clear to me, however, whether in some precise sense the Euler number of the spacetime manifold could ever be determined by direct observation.

The orientability of spacetime is an example of a global topological structure that seems to be strictly inobservable in an intuitive sense. This follows from the fact that one can construct an orientable manifold from any non-orientable one by lifting the structures on it to a suitable covering space, which is automatically orientable. The lift of the spacetime metric to a covering manifold, however, would yield a representation of exactly the same physical spacetime as the original: every physical phenomena in the one has an isometric analogue, as it were, in the other, and vice-versa. Whether or not a spacetime manifold is simply connected, moreover, seems to be in the same boat, for the universal covering manifold of a manifold is guaranteed to be simply connected.[42]

[41] See, *e.g.*, Alexandrov (1957, ch. VIII).

[42] In order for a manifold to possess a universal covering manifold, it must be semi-locally simply connected. Intuitively, this means that it cannot contain "arbitrarily small holes". More precisely, it means that every point in the space has a neighborhood such that every loop in the neighborhood can be continuously contracted to a point. (The contraction need not occur entirely with the given neighborhood.) The so-called Hawaiian Ear-Ring is an example of a topological space that is not semi-locally simply connected (Biss 2000). Whether or not a spacetime

Nonetheless, I think those answers about the possible observability of a manifold's orientability and simple connectedness may be too pat. If one were to observe that any member of a certain family of closed, physically distinguished spatiotemporal loops could not be continuously deformed into any member of another family of closed, physically distinguished spatiotemporal loops, one would have shown that the spacetime manifold is not simply connected.[43] Similarly, if one could show that to parallely propagate a fixed orthonormal tetrad around a given closed spatiotemporal loop would result in its inversion, one would have demonstrated that spacetime is not orientable. I personally have no idea what sorts of experiment could show either of those things. The history of physics, however, if it shows us nothing else, does show us never to underestimate the ingenuity of experimentalists, no matter what the theoretician may tell them is impossible to observe or measure.

The first Betti number of the spacetime manifold offers another interesting example of this sort. The first Betti number of a topological space is the number of distinct connected components it has; any manifold with a first Betti number greater than one is *ipso facto* not connected. Say that we posited a non-connected spacetime manifold. According to the principles of general relativity, any phenomena in one component would be strictly inobservable in any other.[44] By this criterion, it makes no sense to attribute physicality to regions of spacetime disconnected from our own.

So, are these possibly inobservable global structures physical? Well, it seems to me that in one sense they are, and in others they are not. The only lesson I want to draw here is that questions of this sort require in-depth investigation sensitive both to the technical details of the mathematics and to the physical details of how such structures may and may not bear on other phenomena we think of as manifestly physical, even if they turn out to be indubitably inobservable.[45]

6.3 Physicality and Existence

What I have discussed so far in this section, I submit, are philophically rich, scientifically significant questions and arguments, of the sort Maxwell mentions in the epigraph to this paper. Insight into and progress on any of the questions would constitute real progress in our attempts to understand the world in a scientific sense. The sterility of the current debate between substantivalists and relationalists is shown in the fact that no questions it addresses has scientific value in the sense of Maxwell—it has spurred no work with direct scientific, as opposed to purely metaphysical, import.

manifold is semi-locally simply connected presents us with yet another type of question related to physicality: strictly speaking, there is no physical need for a manifold to possess a universal cover, and it is difficult, to say the least, to see what other physical relevance being semi-locally simply connected could have; and yet the construction of the universal cover is such an extraordinarily useful theoretical device that one wants to demand that a candidate spacetime manifold be semi-locally simply connected. What status does such a demand have? A purely pragmatic one?

[43] Thus giving the lie to the old chestnut that one cannot prove a negative existential statement.

[44] Perhaps one could posit some form of quantum entanglement among phenomena in the different components. The ramblings of many theorists of quantum gravity notwithstanding, such a possibility lies so far beyond the ambit of current well entrenched experimental technique and well founded theoretical knowledge as to render it incomprehensible as physics. By the nature of the case, for instance, we could perform no direct experiments on the putatively entangled phenomena in the postulated other component to verify the entanglement beyond a shadow of a doubt.

[45] The family of phenomena in relativistic spacetimes grouped under the rubric "singular stucture" (or "singularities") provides on its own a rich and diverse selection of examples, which I do not have room even to sketch here. See Curiel (1999) for an extended discussion.

Still, No matter how convincing or interesting or philosophically rich these examples and arguments may be, one might still want to respond that they show nothing about the possible *existence* of spatiotemporal entities, and so in the end they do not bear on the debate between substantivalism and relationalism. I do not think that is the correct lesson to leave with, though. I take physicality to be a necessary condition for the attribution of existence to a theoretical entity. If there are many possible ways an entity can manifest physicality, therefore, and one can show that different entities manifest some but not others of them, then it follows that it is meaningless to attribute existence *simpliciter* to such theoretical entities. If there are two entities each manifesting a different type of physicality, then, in so far as each is a necessary condition for existence, if one attributes existence to those entities, it must be of a different sort for each. Thus, in so far as one wants to make sense of the idea of "existence" in the context of physical entities purportedly represented by theoretical structures (if that is the sort of thing one likes to do), it cannot be univocal. To paraphrase Aristotle, existence is said, if at all in physics, in many ways.

What light, if any, does all this shed on the cogency of the traditional debate about the ontic status of spacetime? I think quite a bit. A spacetime point is not physical in any of the ways I have explicated: there is no such thing as an initial-value problem for them; there is no equation of motion for them; no property of theirs dynamically couples to any physical field; and so on. How, then, is one supposed to try to answer the question of whether or not they exist in any way that purports to be grounded in physics?

7 Valedictory Remarks on Realism and Instrumentalism

I think my conclusions about the vanity of metaphysical argumentation abstracted from the pragmatics of the scientific enterprise carry over into the general debate over realism and instrumentalism. Indeed, I consider the argument about relationalism and substantivalism to be an instance of the more general form of argument one can give for existence claims about entities and structures in science. I will consider two examples to make the point, the first somewhat sophisticated, the second quite simple.

Consider, first, the Unruh effect.[46] The effect, roughly speaking, is as follows. (We discuss it only in the context of a special case, but this does not affect the point.) Consider two observers in Minkowski spacetime pervaded by a scalar quantum field in its vacuum state. Each observer carries a simple particle-detector coupled to the field, with two states: an excited state ("particle observed"), and a ground state ("particle not observed"). Both detectors are initially in the ground state. The first observer follows a geodesic, and so does not accelerate; in this case, quantum field theory predicts that the particle detector will remain in the ground state, *i.e.*, the probability that he will detect any particles is zero, as one would expect on physical grounds, since the background field is in the vacuum state. The second observer, however, begins to accelerate. Now, there is a high probability that her detector *will* change from the ground to the excited state; she will "see particles". That is the Unruh effect. Even though the two observers disagree on whether there are particles or not, they both agree that the state of the second particle detector changes, so there is a physical fact of the matter in that sense.[47] Now, the bit of most interest to us is that the

[46] See Wald (1994) for a rigorous exposition of the phenomenon.

[47] Roughly speaking, the resolution of the paradox turns on the fact that an accelerating system in Minkowski

fluctuations in the field that determine the change in the state of the detector do not contribute to the definition of the stress-energy tensor. All observers, both inertial and accelerating, will still conclude that the ambient stress-energy tensor is that of the vacuum state. Is Unruh radiance, then, physical or not? Is it "real" radiation? Well, in the sense that it is a phenomenon that all observers will agree on, one that manifests itself in directly observable effects, yes; in the sense that it does not contribute to the stress-energy of spacetime, no.

Now, consider the question "do electrons exist?" On its face, it seems immune to the sorts of problems I raise about the ontic status of spatiotemporal structure. Surely one can attribute canonical significance to the question "do electrons exist?" independent of investigative framework? In fact, one cannot. Think of the different contexts in which the concept of an electron may come into play, and the natural ways in those contexts one may want to attribute physicality (or not) to electrons. A small sample:

- as a component in a quantum, non-relativistic model of the Hydrogen atom
- as an element in the relativistic computation of the Lamb shift
- as a possible "constituent" of Hawking radiation in an analysis of its spectrum
- as a measuring device in the observation of parton structure from deep inelastic scattering of electrons off protons, as modeled by the Standard Model

In the first case, one may want to attribute physicality to the electron in so far as its associated quantities enter into the initial-value formulation of the system's equations of motion; in the second, one may base the attribution on the fact that one identifies the electron as the bearer of definite values for the kinematic Casimir invariants of spin and mass; there is no good definition in general of an electron in the third, because there is no unambiguous, physically significant definition of "particle" in quantum field theory on a curved spacetime, and so *a fortiori* no way to attribute physicality to such a thing;[48] in the fourth and final case, one can attribute physicality to the electron because one can associate localized charge, spin and lepton number with the mass-energy resonance that represents it. Now, one cannot even formulate in a rigorous, precise way (or, indeed, often not even in a loose and frowzy way) the criterion for physicality in any of these frameworks in the terms of at least some of the others.

It follows that even in this case any formulation of the question in abstract terms, such as "what all observers agree on" or "what has manifestly observable effects" or "what couples with other systems we already think of as physical" or "what is essential to the formulation of the theory", remains empty until one renders content to it by the fixation of a framework, even if only schematic. To be clear, I do not claim that one must always make the investigative framework of

spacetime occupies a negative energy-state: the accelerating detector, in dropping to an energy level beneath that of the ambient vacuum, registers the vacuum as having positive energy, which the accelerating observer interprets as its having "detected a particle"; the inertial observer, however, accounts for the drop in the accelerating detector's energy by concluding that it *emitted* a particle, and so changed its state. If one likes, one may take this as one way to make precise the idea that "particle" is not a natural notion in quantum field theory, and is indeed at times not only not useful but downright obfuscatory.

[48] In essence, this is because one has no privileged group of timelike symmetries in a generic spacetime, as one has in Minkowski spacetime, on which to ground the notion of a particle. See Wald (1994) for a detailed explanation.

one's work explicit, only that one ought to recognize it must be there in the background, specifiable when push comes to shove, as it will from time to time.

In the picture I have implicitly relied on in the construction of my arguments, the structure of physics may be thought of as something like a differential manifold itself, with different techniques and concepts that find appropriate application in different sorts of investigation, and even in similar sorts of investigation of different subject matters, all covering their own idiosyncratic patches of the global manifold, consonant with each other when they overlap but with none necessarily able to cover the entirety of the space. In that vein, I am confident there are many other interesting senses one can render to the idea of the physicality of putative entities and structures represented by our best physical theories, variously useful or at least illuminating in investigations of different sorts. In some of those senses, one will rightly, or at least usefully or suggestively, say those things are physical. In others, one will not. The words we use to further all the sorts of scientific and philosophical investigations we pursue do not matter, only the concepts behind the words, some of which find natural application in some investigations and some of which do not.

This is not instrumentalism. Among other things, I neither make nor rely on any claim about how one ought to understand the structures of our best theories as formal systems, the terms and relations with which we formulate them, and their broader or deeper relation to the world itself, only about how we ought not understand them. The greatest physicists have always, it seems to me, had the capacity to to think in both realist and instrumentalist ways about both the best contemporary theories and the most promising lines of theoretical attack as they were being developed. Often, they held both sorts of views in their minds at the same time, keeping many avenues open, sometimes moving forward along one, sometimes switching to another, sometimes straddling the line, as best befit the demands of the investigation, with a concomitant gain in richness of conception and depth of thought.[49] In some contexts and for some purposes it is most useful to conceive, think and speak in realist terms, and in others to do so in instrumentalist terms. They are both good in their place, and neither is correct *sub specie æternitatis.*

I am not against asking questions that, in traditional terms, seem to bear on issues of realism and instrumentalism. I am against the focus on the questions as meaningful and valuable in themselves, without regard to the roles they may or may not play in the ongoing enterprise of our scientific attempts to comprehend the physical world. That focus, it seems to me, leads only to a sterile form of ideological back-and-forth that has all but crowded out of this realm the possibility of formulating and addressing questions of real scientific and philosophical clarity and value. I take that to be the thrust of the epigraph from Maxwell at the head of this paper.

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[49] Stein (1977, unpub.) forcefully argues this line of thought.

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