ARE THERE ULTIMATELY
FOUNDED PROPOSITIONS? *

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ABSTRACT

Can we find propositions that cannot rationally be denied in any possible world without assuming the existence of that same proposition, and so involving ourselves in a contradiction? In other words, can we find transworld propositions needing no further foundation or justification? Basically, three differing positions can be imagined: firstly, a relativist position, according to which ultimately founded propositions are impossible; secondly, a meta-relativist position, according to which ultimately founded propositions are possible but unnecessary; and thirdly, an absolute position, according to which such propositions are necessary. In this short essay I show that under the premise of modal logic S5 with constant domain there are ultimately founded propositions and that their existence is even necessary, and I will give some reasons for the superiority of S5 over other logics.

Key words: logic S5, modal logic, ultimately founded propositions, relativism, absolute.

¿EXISTEN PROPOSICIONES CON FUNDAMENTO ÚLTIMO?

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RESUMEN

¿Podemos encontrar proposiciones que no se puedan negar racionalmente en ningún mundo posible sin asumir la existencia de esa misma proposición y, por ello, implicándonos en una contradicción? En otras palabras, ¿podemos encontar proposiciones transmundanas que no necesiten más fundamento o justificación? Básicamente, se pueden pensar tres posiciones: en primer lugar, una posición relativista que sostiene que las proposiciones con fundamento último son imposibles. En segundo lugar, una posición meta-relativista según la que, las proposiciones con fundamento último son posibles pero, innecesarias; y, en tercer lugar, una posición absoluta en la que este tipo de proposiciones son necesarias. En este breve ensayo, muestro que bajo la premisa de la lógica modal S5 con un dominio constante, existen proposiciones con fundamento último y que su existencia es, aún más, necesaria. Adicionalmente, ofreceré algunas razones a favor de la superioridad de la lógica S5 sobre otras lógicas.

Palabras clave: lógica S5, lógica modal, proposiciones de fundamentación última, relativismo, absoluto.

1. Introduction

THE PHILOSOPHICAL PROJECT OF ULTIMATE FOUNDATION aims at a proposition that cannot rationally be denied by one without presupposing the existence of that same proposition and, as a consequence, contradicting oneself. Therefore, this proposition needs no further foundation and justification[1]. This being said, three positions seem to be possible: first, a relativist position according to which it is impossible that there is such an ultimately founded proposition; second, a meta-relativist position according to which such a proposition is possible, but not necessary; and thirdly, an absolutist position according to which such a proposition is necessary.

Although the idea of ultimate foundation has always intrigued philosophers, in 20th century philosophy it has hardly been advocated in either the analytical or continental tradition[2]. But, recently, Vittorio Hösle (1990: 152-59) put forward a somewhat new argument for an ultimate foundation. Unfortunately, we have reason to believe that its second part is unsound (see Damschen, 2005), that at least in some cases it is formulated in a way much too vague to be convincing (see Grundmann, 1993: 330-37; Graeser, 1995), and Hösle does not specify what modal system he actually avails himself of, although his argument is based on modal concepts. However, I find his argument to be an important and fruitful step towards a clear concept of an ultimate foundation. Since his basic thought strikes me as attractive, I will suggest a new transcendental-modal argument for an ultimate foundation based on the modal system S5. In conclusion, I will discuss three necessary meta-conditions, which, if fulfilled, would allow for a successful continuation of the project of ultimate foundation, and I will also give some reasons for the superiority of S5 over other logics.

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[1] For a detailed account of various concepts and arguments for an ultimate foundation (sometimes also called 'ultimate justification') see e.g. Hösle, 1990; Grundmann, 1993 and Illies, 2003.

[2] An exception is Hilary Putnam's argument for an a priori truth (in Putnam, 1983: 98-114).

2. Definition and Criterion

TO BEGIN WITH, LET US DEFINE AN ULTIMATELY founded proposition[3] as a proposition that is characterised by the following three logical properties: it is (a) necessarily true; (b) necessarily synthetic; and as well (c) necessarily a priori. This definition can be motivated by the following criterion: a proposition is ultimately founded if it is not rationally possible to assert that (under certain not necessarily false conditions) it could be rightly denied (see Hösle, 1990: 153). Four notes on the definition and the criterion are called for:

First, an ultimately founded entity cannot have any contingent properties. It is not enough that it is contingently true, contingently synthetic, or contingently a priori, but it must also have these properties always and in every possible world W^*. Thus, it must have each of these three properties necessarily. Second, since necessarily true propositions are true in every possible world W^*, and the possibility of rationally denying a proposition p requires that a possible world W is conceivable in which not-p is true, only necessarily true propositions satisfy the criterion mentioned. Third, we are interested in meaningful propositions. Since analytical propositions are not meaningful (because of their analyticity), we have to look for non-analytical, that is synthetic, propositions. Finally, the property of being ultimately founded includes an epistemic factor: what is in no need of further justification can only be known in a non-empirical way. This kind of knowability is expressed by the term 'a priori'[4].

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[3] I use this term in the following way: Propositions and sentences are different. A proposition is what is expressed by a sentence; it is its meaning. In some passages I call a proposition a 'statement' if it can be true or false. If a statement or a proposition with a truth value is performed by an (truth-)asserting force, I call the proposition a 'propositional content' and the whole act a 'speech act of assertion' or simply an 'assertion'.

[4] This difference between necessity as an ontological concept and apriority as an epistemological concept was supported by Saul Kripke (1980: 34-39).

3. The Transcendental-Modal Argument

I<small>T IS THE AIM OF THE FOLLOWING</small> *transcendental-modal argument* to prove that it is necessary that there is at least one ultimately founded proposition[5]. The argument is formulated in the language of the modal predicate logic S5, and uses axioms and theorems of this modal system. Let the existential quantifier '∃x' range over the domain of all propositions α (that is, statements which have a definite truth-value). The argument has two parts. In the first part (steps 0 to 9) we examine whether it is possible that a subject of a speech act is asserting in a consistent way that it is impossible that there is at least one ultimately founded proposition (0). On the basis of a speech act analysis in step (1) and an analysis of presuppositions in steps (2) and (3), the propositional content of the assertion (0) proves to be a pragmatically inconsistent and, therefore, false proposition. Hence, proposition (9) —which is contradictory to the propositional content of (0)— is true. Because the first part of the argument is an analysis of presuppositions, it will be called a *transcendental argument*. The second part of the argument leads from proposition (9) to proposition (15) using theorems and rules of S5. Thus the second part of the argument is called a *modal proof* in S5[6].

[5] This argument follows in many steps an argument that was given by Vittorio Hösle in a non-formal way (Hösle, 1990: 152-59). Hösle's argument was reconstructed by Miriam Ossa (2007: 64-80), who proves that Hösle's argument is valid in S5 —a solution that was also given independently by me to Hösle (see the preface to the French edition in Hösle, 2004: 9) in private correspondence. However, my argument is in many important ways different from the good and helpful reconstruction done by Ossa: (1.) Hösle and Ossa did not adopt the theorem that I use in step 10, and they also did not adopt the proof from step 10 to 12 which replaces Hösle's argumentation that he uses in the second part of his argument (Ossa, 2007: 69-79). (2.) I showed elsewhere that the underlying idea of the second part of Hösle's argument is probably not sound (see Damschen, 2005). (3.) The most important difference lies in the interpretation of the starting point of the proof: Ossa interprets 'There is an ultimately founded entity' as 'There is something which is a sentence and *necessarily* true' (see Ossa 2007: 64), whereas I interpret it as 'There is a proposition which is *necessarily* [true (T) & synthetic (S) & a priori (A)]'. The difference is obvious. In Ossa's case to be a sentence could be contingent; in my case all of the properties are necessary. If one accepts that ultimately founded entities cannot have any contingent properties, Ossa's version does not seem appropriate to prove an ultimately founded entity. Moreover, being necessarily true alone (Ossa) does not rule out the possibility that the ultimately founded entity is analytic. (4.) There are, however, more differences: one of them is the problem of constant and varying domains, which occurs in predicate modal logic, another is my use of the assertion stroke which shows that the question of an ultimately founded proposition is inevitably connected to the question of the *ontological status of the one who* asserts this proposition.

[6] In some ways, my argument is closer to a former version of Hösle's argument (Hösle, 1987

3.1. The Transcendental Argument

LET US ALSO ASSUME THAT ANY POSSIBLE RELATIVIST thinks and claims the proposition 'It is impossible that there are ultimately founded propositions' to be true (let us take '⊢' as a symbol for claim in the logic of speech acts (in analogy to Frege's judgement stroke):

>   (0) ⊢ It is impossible that there are ultimately founded propositions.

According to the definition of 'ultimately founded', this means:

>   (0′) ⊢ It is impossible that there is at least one proposition that is
>   necessarily [true (T) & synthetic (S) & a priori (A)].

In the following and to make things more clearly structured, for 'true (T) & synthetic (S) & a priori (A)' there shall simply be 'D' (definite):

>   (1) ⊢ (~ ◊ ∃x □Dx)

>   ⊢ It is impossible that there is at least one proposition that is
>   necessarily definite.

By replacing the modal operator of the possibility (~ ◊) by the one for the necessity (□ ~) we may proceed to:

>   (2) ⊢ (□ ~ ∃x □Dx)

>   ⊢ It is necessary that there is no proposition that is necessarily definite.

Already for S4 □ p ≡ □□ p is valid; thus the content of statement (2), i. e. □ ~ ∃x □Dx, is itself necessary:

>   (3) □ □ ~ ∃x □Dx

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and 1994) than to the revised three-part version from 1990, because the former version is a two-part argument with steps from impossibility to possibility and from there to necessity (see Hösle, 1987: 255-5; 1994: 286-7), and its transitions from step one to two and step two to three use two different concepts of contradictions similar to that of my argument (see Hösle, 1987: 253-5; 1994: 272-3).

>It is necessary that it is necessary that there is at no proposition that is necessarily definite.

If somebody accepts claim (2), this shows that he/she assumes that the propositional content of (2) is true. The statement of necessity which makes the propositional content of claim (2) is itself necessary, however, as shown by (3). Thus, the content of the claim in (2) may either be a necessarily *wrong* or a necessarily *true* statement.

If there is the assumption that the content of the claim in (2) is necessarily wrong, there follows (by employing the modal descent T "ab necesse ad esse valet consequentia") that there is possibly at least one ultimately founded proposition ($\square\sim\square\sim\exists x \ \square Dx \supset \lozenge \exists x \ \square Dx$). If the other, alternative assumption is true that the propositional content of the claim in (2) is necessarily true, it is true that there is at least one ultimately founded proposition ($\exists x \ \square Dx$). For, the content of (2) itself is a necessarily true synthetic proposition a priori: first, it is —as assumed— a necessarily true proposition. Second, the propositional content of (2) is not analytic, as the property D is not included in the concept of the proposition and the negation of the propositional content of (2) does not create a logical contradiction. On the assumption of the existential quantifier ranging over the domain of all possible propositions in conjunction with the domain being constant, it is impossible that in any possible world W there exists a proposition the concept of which includes the property D. Thus, it is necessary that the propositional content of (2) is necessarily non-analytic. Third, the propositional content of (2) can also necessarily only be known non-empirically, as it is a statement of impossibility. If we assume that 'non-analytic' and 'non-empirically knowable' together mean the same as 'synthetically a priori', under the second assumption the propositional content of (2) thus shows exactly the qualities as claimed by $\exists x \square Dx$. Also here, by employing the modal descent, there finally follows that there possibly exists at least one ultimately founded proposition ($\exists x \ \square Dx \supset \lozenge \exists x \ \square Dx$).

Thus, with the help of transcendental reflexion it becomes clear that for both of the two possible alternatives the claim of (2) presupposes that

there is possibly at least one ultimately founded proposition[7]. Thus, we may proceed to the claim that $◊ ∃x □Dx$ is the case:

$(4) ⊢ (◊ ∃x □Dx)$

⊢ It is possible that there is at least one proposition that is
        necessarily definite.

By replacing the modal operator of the possibility ($◊$) by that of the necessity ($\sim □ \sim$) there follows from (4):

$(5) ⊢ (\sim □ \sim ∃x □Dx)$

⊢ It is not necessary that there is no proposition that is
        necessarily definite.

At this point we can apply the law of introducing conjunctions from the logic of speech acts (Vanderveken 1991, 70):

$(6) ⊢ (A_p)$

$⊢ (B_p)$

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$⊢ (A_p \& B_p)$

Then, from (2), (5), and (6) there follows the following claim:

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[7] That is only one group of the presuppositions that one has to consider while trying to understand adequately and completely 'I am asserting that ($□ \sim ∃x □Dx$)'. Here, I am examining the status of the propositional content of the assertion (2). There are two other presuppositions of (2) I can only mention here; I will discuss their connection to the assertion-semantic presuppositions elsewhere. These two *performative* presuppositions lead us to two notorious transcendental arguments concerning the status of the *subject* of the assertive speech act and concerning the *assertive speech act* of the subject. The first type of argument is on the performative existence-presupposition of the subject of the speech act (*I* am asserting that p). The second type is on the practical rules that a successful assertion aiming at truth presupposes (I *am asserting* that p). Both types are discussed in an exemplary way by Christian Illies (2003); he tries to unite both types of transcendental argument. I think it is a promising challenge to combine his two arguments with the transcendental-modal argument of this paper because all three are based on the same structure, the assertion 'I am asserting that p'.

(7) ⊢ (□ ~ ∃x □Dx & ~ □ ~ ∃x □Dx)

⊢ It is necessary that there is no proposition that is necessarily
        definite and it is not necessary that there is no proposition
        that is necessarily definite.

By (7) it is shown that the propositional content of claim (1) is wrong,
as the propositional content of claim (2), which is an implication of (1),
and its presupposition, the propositional content of claim (5), are in open
contradiction to each other. Thus, the proposition that is contradicting
proposition (1) is true:

(8) ~ ~ ◊ ∃x □Dx

It is not impossible that there is at least one proposition that is
        necessarily definite.

This again is equivalent to the proposition

(9) ◊ ∃x □Dx

It is possible that there is at least one proposition that is necessarily
        definite.

3.2. The Modal Proof

BY PROPOSITION (9), OUR DEMONSTRATION HAS shown the possible existence of
ultimately founded propositions. Now, how could we show that ultimately
founded propositions do really and even necessarily exist?

   Already for S4 with constant domain the following interesting theorem
can be proven (see Fitting & Mendelsohn 1998: 137):

(10) ∃x □Φx ⊃ □∃x □Φx

If there is at least one x that is necessarily Φ, then it is necessary
        that there is at least one x that is necessarily Φ.

If (10) is a theorem of the modal predicate calculus, we may apply the rule DR3 (see Hughes & Cresswell 1996: 35):

(11) $\vdash_{S5} \alpha \supset \beta \Rightarrow \vdash_{S5} \lozenge \alpha \supset \lozenge \beta$ [rule DR3]

If 'If α, then β' is a theorem in modal logic S5, then also 'If it is possible that α, then it is possible that β' is a theorem in S5.

Then there follows from (10) and (11):

(12) $\lozenge \exists x \, \Box \Phi x \supset \lozenge \Box \exists x \, \Box \Phi x$ [10, rule DR3]

If it is possible that there is at least one x that is necessarily Φ, then it is possible that is is necessary that there is at least one x that is necessarily Φ.

If we insert property D for property Φ and if we apply a modus ponens to (9) and (12), we get:

(13) $\lozenge \Box \exists x \, \Box Dx$ [9, 12, Φ/D, MP]

It is possible that is is necessary that there is at least one proposition that is necessarily definite.

Now for S5 the theorem S5(1), which is crucial for our argument, is valid:

(14) $\lozenge \Box p \supset \Box p$ [theorem S5(1)]

It states that a proposition is necessary if it is possibly necessary. By way of modus ponens we get from (13) and (14):

(15) $\Box \, \exists x \, \Box Dx$ [13, 14, MP, p/$\exists x \, \Box Dx$]

It is necessary that there is at least one proposition that is necessarily definite.

In this way, we have achieved the goal of our argument[8]. On the basis of the modal system S5 and a constant domain, there is, thus, not only at least one proposition that is necessarily true and synthetically a priori, but it is also necessary that this is the case. Thus, it was shown at the same time that there is a methodically controllable way to show that ultimately founded propositions do necessarily exist. The ontological commitments we require to show the necessity of ultimately founded propositions are, on the one hand, the possibilist interpretation of the existential quantifier or the assumption of a constant domain of the possible worlds *W**; and, on the other hand, the modal logic S5.

4. Metaconditions and the Logic of the Argument

EVEN IF THE TRANSCENDENTAL ARGUMENT HERE PRESENTED is convincing and the modal proof of this paper is sound in S5, we have to consider further problems, because ultimately founded entities must not depend on contingent conditions. There are, however, at least three *meta-conditions* of the transcendental-modal argument itself which seem to be contingent at first glance:

(i) First, using S5 for the interpretation of the modal operators seems to be a mere contingent condition. For this reason a supporter of an ultimate foundation has to show that S5 is precisely that modal system that is necessarily preferable to all other systems of modal logic in matters of ultimate foundation[9]. We need a *criterion* for this (see Plantinga, 1974). This criterion cannot be located outside of logic because every reason for a preference of S5 to all other logical systems has itself a logical structure; after all, to give a reason is itself a logical relation, namely 'x is a reason for y'. Then the criterion can only be *within* logic. So we need to search for that logic within the boundaries of which an ultimate foundation is necessary. The only possibility is that S5 is founded and reconstructed by *itself*. Hösle (1990: 164) points out that such a self-foundation and self-reconstruction of a logical system is necessarily circular in a certain way. This circularity must not be confused with a petitio principii because if it were a petitio

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[8] From (15), sentence (16) $\exists x \,\Box Dx$ follows by employing the modal descent and sentence (17) $\Box \,\Box \,\exists x \,\Box Dx$ by employing $\Box \, p \equiv \Box\Box \, p$.

[9] See e.g. the critical comments by Divers and Melia, 2002.

principii then there would always be a possible alternative. S5, however, the logic of my transcendental-modal argument, has no alternative in matters of ultimate foundation. Therefore, this type of circularity seems to be neither vicious nor avoidable. I call the problem underlying these questions the *problem of a criterion for, a self-justification of and a self-reconstruction of modal predicate logic S5*.

(ii) Second, there is the important problem of which role exactly the epistemic subjects play within the interpretation of modal-logical systems —the epistemic subjects who think of possible worlds. If these epistemic subjects themselves are only contingent then the interpretation of S5 and, thus, the transcendental-modal argument possibly depends on contingent conditions. To be more precise, there are two questions: is conceivability a necessary or only a sufficient condition for metaphysical possibility and necessity (see Yablo, 1993; Chalmers, 2002)? And if it is a necessary condition: is it necessary that there are epistemic subjects who conceive possible worlds? Can we think that there is no one who conceives possible worlds? I call this problem the *problem of the epistemic impregnation of conceivability*.

(iii) Thirdly and in addition to the points just made, it is a particularly good question whether the supposition that there is at least one possible epistemic subject asserting the propositional content of (1) is contingent. Is it necessary that there are entities that by virtual asserting the propositional content of (1) let the transcendental argument begin? In addition, what is our own role if we think about these entities and the possibility of asserting the propositional content of (1)? Consider the following: if something is necessary it is true in all possible worlds. Of course, you cannot doubt in a rational way your own existence and the existence of an ultimately founded entity except by presupposing the existence of exactly one epistemic subject who doubts something, namely at least the existence of yourself. Moreover, you cannot even think about a possible subject who doubts the existence of an ultimately founded entity without presupposing your own existence as the thinker of it. Whenever you doubt something or think of one who could do that, you must at the same time presuppose your own existence as a doubting being or as a being thinking this thought. But it seems that this does not entail that there is someone in *all* possible worlds who performs these acts of doubt or thinking. I call this last problem the *problem of beginning*.

So, if it is necessary that a proposition x exists such that x is necessarily true, synthetic and a priori, then the logically necessary conditions (i), (ii) and (iii) are also metaphysically necessary and (iv) the transcendental-modal argument in S5, as presented in this paper, is sound. To avoid a misunderstanding, with respect to the use of the expression 'necessary', let me stress that the conditions (i), (ii) and (iii) are formally necessary conditions, but that their content is, in each case, also a statement of metaphysical or even modal necessity ('it is necessary that p'). In this paper, I have only tried to show that the fourth condition is fulfilled. But, I think that it is possible with supporting arguments to show that conditions (i) to (iii) are also fulfilled. So, the transcendental-modal argument functions as the inner core of a stronger argument in favour of an ultimate foundation that has solved (i) the problem of a criterion for, a self-justification of and a self-reconstruction of modal predicate logic S5, (ii) the problem of the epistemic impregnation of conceivability, and finally (iii) the problem of beginning.

Disclosing the ontological commitments that are necessary for the first and the second step of the argument precisely determines the desideratum for the last step: we must find an argument to support our preference of modal logic S5 to other logics. Just a few hints how we could proceed here[10]. The question whether there are ultimately founded propositions presupposes that an acceptable answer shall be valid not only for a possible world *W* but for every possible world *W**, since the idea of an ultimately founded proposition that is only relatively founded is obviously contradictory. A proposition is ultimately founded only if in every possible world *W** it is a necessarily true, synthetic proposition a priori. Then, however, it is true that that what is possible does not change from world to world. "If a state of affairs *S* is possible, then it is necessarily possible; that is, possible with respect to every possible world" (Plantinga, 1974: 54). The same is analogously true for that what is necessary: if a state of affairs *S* in respect of at least one possible world *W* is necessary, it is necessary in every possible world *W**, i. e. necessarily necessary. Only a logic whose accessibility relation is reflexive, transitive and symmetrical preserves the meaning of the modal operators in all possible worlds. The only logic which meets these three demands is the modal logic S5.

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[10] I propose a complete metaphysical proof for this in the second part of my "Logik der Letztbegründung und Letztbegründung der Logik" (forthcoming).

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