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A Counterexample to Deflationary Nominalism

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1. Introduction: Azzouni and Colyvan on mathematical realism

In the debate between Platonists and nominalists about mathematical ontology, Jody Azzouni (2012b; 2010; 2009; 2004a; 2004b) defends a "deflationary nominalism"; deflationary in that mathematical sentences are true in a non-correspondence sense,[1] and nominalist because mathematical terms—appearing in sentences of scientific theory[2] or otherwise—refer to nothing at all. In this paper, I focus on Azzouni's positive account of what *should* be said to exist. The quaternary "sufficient condition" (Azzouni 2004b: 384) for posit[3] existence, Azzouni (2012b: 956) calls "*thick epistemic access*" (hereafter TEA), and in this paper I argue that TEA surreptitiously reifies some mathematical entities. The mathematical entity that I argue TEA reifies is the Fourier harmonic, an infinite-duration sinusoid applied throughout contemporary engineering and physics. The Fourier harmonic exists for the deflationary nominalist, I claim, because the harmonic plays what Azzouni calls an "epistemic role" (see section 2) in the commonplace observation of macroscopic entities, for example in viewing a vase with the human eye. Thus, I present

[1] More precisely, Azzouni's deflationism interprets truth as nothing above and beyond the "generalization" expressed by the Tarski biconditional (e.g.): "Snow is white" is true iff snow is white (Azzouni 2010: 19). Hence what redeems that biconditional, in Azzouni's account, is neither strictly correspondence, nor coherence, nor indispensability of the truth idiom to language. On the other hand, Azzouni rejects truth pluralism (see Azzouni 2010: §§4.7-4.8). The best articulation of Azzouni's deflationary account of truth in science, mathematics, and applied mathematics may be Azzouni (2009), but see also Azzouni (2010: Chap. 4). The details will not concern me in this paper.

[2] Azzouni (2014) understands scientific theories to be "linguistic entities" (2995) "written in natural languages supplemented with additional technical vocabulary" (2994).

[3] Posits are the alleged referents of singular terms.

a counterexample to deflationary nominalism, from assumptions that the deflationary nominalist holds or should accept. I support this counterexample by a positive argument, what I call a "second way" (the first way being Azzouni's) to ascribing a posit an epistemic role.

Mark Colyvan (2010) has already criticized TEA for admitting existent mathematical entities,[4] but he argues from an assumption that the deflationary nominalist denies on independent grounds (as Azzouni 2012b: 962-963 rightly objects). I avoid Colyvan's objectionable assumption, but before explaining how, it helps to understand a bit of Azzouni's terminology, which has developed somewhat over the years (cf. Azzouni 1994). The posits referenced by singular terms come in three varieties for the deflationary nominalist: "thick" (e.g., elephants or molecules that someone has detected with their senses or instruments), "thin" (e.g., elephants that presently exist according to scientific theory but have not been detected), and "ultrathin" or not existing in any sense, and whose terms are referentially empty (Azzouni 2004a: 128-129). Azzouni treats mathematical entities as ultrathin.

Colyvan (2010) argues straightforwardly against this taxonomy. Specifically, he appeals to Azzouni's (2004a: 138) criteria for thin-posithood. Those criteria were[5] that the posit exhibit the "Quinean virtues" of "simplicity, familiarity, scope, fecundity, and success under testing" (128) in scientific theory, and that there be a "defeasibility

[4] By an argument different from his influential contributions on explanatory indispensability (Colyvan 2001).

[5] Azzouni (2012b; 2004b) has developed his account of posit existence to focus on TEA conditions (see section 3 below), de-emphasizing the Quinean virtues mentioned in this sentence of the main text (and prevalent in Azzouni 2004a).

condition" or reason that the posit could not be thickly[6] detected. Colyvan (2010) argues that mathematical entities fulfill the Quinean virtues, and that their "'excuse clause'" (i.e. defeasibility condition) for eluding detection is their abstract nature (288).[7] In reply, Azzouni (2012b) rejects Colyvan's excuse clause as "philosophical" and not "scientific" (963), since Azzouni thinks that thin-posit discriminations should hail *from science* (962). He additionally cites an independent reason for doubting the existence of mathematical abstracta, an argument that he calls the "epistemic role puzzle" (hereafter ERP; p. 963, footnote omitted), which I discuss in section 2. Thus Azzouni (2012b) takes Colyvan's (2010) mathematical reification attempt to fail.

My argument for a limited mathematical realism differs from Colyvan's (2010), in that I attempt to meet the demands of the ERP as they have been codified into TEA criteria. Whereas Colyvan's "philosophical" excuse for thin-posithood appeals to the allegedly abstract nature of mathematical posits, I analyze the function of mathematical posits within TEA. I argue that some mathematical posits *qua mathematical*—*viz*., in a sense different from spatiotemporal abstractness[8]—prove indispensable to achieving TEA, or to forging[9] TEA to a "thick" posit like a vase by ordinary visual perception. More precisely, I argue that the infinite duration of the Fourier harmonic is a

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[6] The "thickness" of epistemic access tracks the "thickness" of the posit accessed, such that only "thin" access would be had to a thin posit (and no access to an ultrathin posit).

[7] Colyvan (2010) motivates this conclusion in a more nuanced and compelling way than I have summarized here, by appealing to "borderline" (290) cases of posit thinness that need not be elaborated for the present discussion.

[8] Mathematical characteristics differing from spatiotemporal abstractness include the "primeness" and "oddness" of 3. Azzouni (2009) agrees that "the symptoms of being mathematical" need not include "being 'outside of space and time' . . ." (165).

[9] Azzouni consistently uses this word to describe how an agent comes to stand in a "thick" epistemic relation to a posit (Azzouni 1997: 477, 480, 483; 2004a: 147, 150, 173; 2009: 149; Azzouni and Bueno 2016: 813). "Forge" lacks a technical definition, and just means "establish" or "achieve."

mathematical property (or dimension) indispensable to ascribing reflectance as a property of vases; reflectance being a property that Azzouni (2010: 30) implies to obtain on the surfaces of vases, and to facilitate the forging of TEA to them.[10] Thus, in this paper I commit the deflationary nominalist to the thin-posithood of the mathematical entity that is the Fourier harmonic, and I provide an excuse clause different from Colyvan's (2010) for why we cannot or do not thickly detect the Fourier harmonic in reflectance applications.

Spelling out my argument requires some groundwork. Section 2 reviews the ERP and its function as a premise alongside TEA in Azzouni's argument for deflationary nominalism. Section 3 then outlines the conditions for TEA, and section 4 presents my argument[11] for the indispensability of the Fourier harmonic to reflectance ascription. In section 5, I answer the ERP with respect to the Fourier harmonic, proposing a "second way" to an epistemic role (Azzouni's first way appearing in section 3). I also respond to the "coding" objection of Azzouni and Bueno (2016), which despite my "second way," would nominalize the harmonic to ultrathin status. Section 6 concludes.

2. The Epistemic Role Puzzle and Thick Epistemic Access

Pivotal to Azzouni's deflationary nominalism about mathematical entities, and to its taxonomy of thick, thin, and ultrathin posits, is his "epistemic role puzzle" (ERP):[12] the observation that numbers play no "epistemic role" in mathematical practice, in

[10] In recent work, Azzouni (2017; 2012a) explicitly rejects property realism. My arguments of this paper remain relevant, however, because the recent Azzouni (2017: Chap. 8) endorses TEA without clearly extirpating property reference within TEA. As I explain in due course, removing property reference from TEA is no trivial matter.

[11] Elaborated in [SELF-CITATION REDACTED].

[12] Discussed in several works, including Azzouni (1994: I, §7; 2000; 2010: §1.3; 2015; 2016).

contradistinction to the entities of realist science that play an epistemic role in scientific practice. On what an epistemic role amounts to, Azzouni (2016) is the most explicit:

> the official concern of the puzzle is this: notice that our standard *epistemic practices* have certain accompaniments: methods of recognizing the *epistemic artifacts* that our means of access to the objects in question have *because of* those means of access. (Azzouni 2016: 12)

"Epistemic artifacts," in Azzouni's account, "are the ways that our means of access to objects distort our impressions of the properties of those objects" (5), for example the way that squinting one's eyes (Azzouni 2004b: 383) increases the optical resolution of an object's surface. Hence unlike Benacerraf's (1973) Dilemma against Platonism, which asks how we can possess mathematical knowledge despite the acausal nature of mathematical abstracta, the ERP asks why mathematics lacks "an ancillary science" that investigates mathematics' own epistemic artifacts (Azzouni 2016: 12), a question that pertains even if mathematical objects are *not* abstracta.[13]

Azzouni employs the ERP and TEA as premises for deflationary nominalism. The argument[14] (what I call the "Ultrathin Mathematics Argument") can be paraphrased as follows:

> **Criterion:** "anything that exists is mind- and language-independent"[15]
>
> **TEA:** "*we* recognize that an object is mind- and language-independent" when "it has an *epistemic role*"[16]
>
> **ERP:** mathematical entities lack an epistemic role

[13] McEvoy (2012) contends that the ERP reduces to Benacerraf's Dilemma unless the ERP is conjoined with premises that render the ERP redundant. Azzouni (2016) convincingly counterargues that McEvoy overstates his case. The debate does not affect my paper, which focuses on the workings and conditions of TEA rather than those of ERP, although my focus on TEA provides a "second way" of answering the ERP.

[14] Summarized in Azzouni (2016: 9-10).

[15] Text quoted from Azzouni (2016: 9).

[16] Text quoted from Azzouni (2016: 9-10).

Conclusion: mathematical entities do not exist

Azzouni (2016) acknowledges that the Ultrathin Mathematics Argument may appear to prove only that theorists lack "'*reason to believe*'" in mathematical entities (10), and not that they do not exist. He urges the stronger **Conclusion**, however, for the same reason that we do not say (without strain) that we lack reason to believe in "hobbits" or in "Santa Claus"; we instead "say" with aplomb that hobbits and Santa do not exist (10).[17]

This appeal to language use, and to what we *say*, to derive the defeasible[18] ontological **Conclusion** from the epistemic premise **TEA** follows, in my view, from Azzouni's "linguistic" arguments for mathematical nominalism more generally (Azzouni 2015: 1149). That is, while I must pass over them in this space, I accept for the sake of discussion Azzouni's extensive efforts to show that fictional characters like hobbits and Santa Claus exist in no sense at all,[19] and that "there is" in the vernacular fails to pick out hobbits or numbers in first-order regimented theories (Azzouni 2004a: Chapter 3). To reiterate my disclaimer from section 1, I accept Azzouni's independently argued accounts of deflationary truth and natural-language science that render the Ultrathin Mathematics Argument more cogent than I have outlined it. I argue instead that the constitutive principles of the **TEA** premise, which I list in the next section, falsify the **ERP** premise with respect to the Fourier harmonic, and thus falsify the **Conclusion** of the Ultrathin

[17] See Azzouni (2015) for arguments that we should not be "agnostic" about mathematical ontology, for a somewhat different reason.

[18] Azzouni (2016: 10): "surely the fact that I've no reason to believe in [hobbits] is compatible both with my being able to draw the conclusion: there are none of these things *and* I might be wrong about this."

[19] Azzouni (2010: Chapters 1 and 3; 2004a: Chapter 3).

Mathematics Argument with respect to the Fourier harmonic. The applicability of my argument to mathematical entities besides the Fourier harmonic is a topic for another occasion.

3. Thick Epistemic Access and Ordinary Visual Perception

In explaining the epistemic role of posits generally construed, Azzouni contrasts two examples in a passage worth quoting at length:

> Should *S* see an urn, and think, "that's an urn," crucial to his thought being about *that urn* are (nonconceptualized and nonrepresentational) facts about perception that are (at least partly) involved in the relationship between *S* and the urn. One therefore cannot simply replace the urn with a vase in a thought experiment (corresponding to the referential-order thought experiment above about 1, 2, 3 . . . and 1, 2, 3 . . .), and have everything go swimmingly. The relationship between *S* and that urn is based partly on the perceptual interactions between *S* and that urn. It's those perceptual interactions that indicate (in part) "the epistemic role" of the urn itself [. . .]. For when we engage in a detailed study of the perceptual abilities of *S*, what emerges is a description of—to put it roughly—the sorts of things *S* is capable of distinguishing by perception (and why). At this point, the actual (and perhaps dispositional) properties possessed by the *urn* become relevant [. . .]. (Azzouni 2010: 30)

The number puzzle (1, 2, 3 . . . and 1, 2, 3 . . .) referenced in this passage illustrates the ERP. If the alleged referents of numerical terms were "swapped" clandestinely, then mathematical practice would allegedly proceed unabated, in a way that the study of urns could not proceed if they were swapped with vases. (I have already directed readers to auxiliary debate about the ERP in footnote 13.) I focus instead on Azzouni's passing but implicatory remark about property ascription, namely that some properties of the vase[20] facilitate perception of it, and thus partially constitute—as I will explain in this section—

[20] Azzouni (2010: 30) discusses both urns and vases, but I focus on vases for their familiar role in philosophical discussions of another "dispositional" property: fragility (Schrenk 2017: §3.1).

the TEA forged between the human perceiver and the vase. I will eventually argue that the Fourier harmonic plays an epistemic role *via* the ascription of dispositional reflectance[21] to the vase, a property that renders the vase perceptible.

Reflectance is a good candidate property responsible for the vase's perceptibility,[22] for two reasons. The first is Azzouni's (2005) implicit concession that reflectance could play a role in color perception (101-102, 105), although he doubts that color *reduces* to reflectance as a natural kind (105). The second reason follows from the first, namely the established philosophical pedigree of reflectance in perceptual theory (Byrne and Hilbert 2003; Jackson 1998; Hilbert 1987), despite ongoing controversies about whether color reduces ontologically to reflectance.[23] Thus to be clear, my thesis has nothing to do with whether human-visible colors plausibly reduce to sets of reflectances.[24] I claim only that reflectance is ostensibly a surface property to which radar systems respond, and a property that conditions much of the ambient light striking the human retina; thus reflectance likely occupies a role within TEA, whether TEA be forged by the human visual system, or by a radar system, etc.

Before listing the conditions of TEA, to show how reflectance fits among them, it pays to recall the deflationary nominalist's **Criterion** for posit existence (section 2). While TEA is a sufficient condition for posit existence (Azzouni 2004b: 384), **Criterion** is a necessary condition: for a posit to exist, it must be "mind- and language-

[21] I say "dispositional" because Azzouni (2010: 30) does. My argument applies equally to categorical renderings of reflectance, like Frank Jackson's (1998: Chap. 4; 1996). On the difference between dispositional and categorical properties, see Schrenk (2017: Chapter 2).

[22] And not exclusively so; a perceptible vase must also possess a "shape," "mass," or "surface," perhaps.

[23] A recent critic of this reduction is Gert (2017: Chapters 1 and 3).

[24] This reduction is defended by Byrne and Hilbert (2003), and Hilbert (1987).

independent" (Azzouni 2016: 9). A clear antonym of mind- and language-independence, in Azzouni's (2012b) account, is the quality of being *stipulated* (955).[25] Thus any posit's properties that facilitate TEA should be non-stipulated to obtain at or on the posit, and the empirically defeasible[26] necessary conditions of TEA, which I sometimes call "ingredients," go some way toward precluding such stipulation:[27]

1) **Robustness:** Properties or entities observed can diverge from what or how a theory predicts them to be, or from what observers "believe about what they'll observe." Alternatively: "what instruments detect greatly outstrips what theories predict" instruments to detect (383).

2) **Refinement:** "[T]heory-free" means exist for "adjusting and refining observations" (383), or for "adjusting and refining instruments and what they reveal . . ." (384). Such theory-free methods just are those pre-scientific methods by which we discern various regularities in the world, such as by "squint[ing]" our eyes (383).

3) **Monitoring:** "What's observed can be monitored . . . over time . . ." (383).

4) **Grounding:** "Certain properties of the object observed can be used to explain why, and in what respects, observed things can be observed" (383). That is, we can "study . . . how the instrumental access to items reveals properties of what's being studied" (384, footnote removed).

One may notice that the term "properties" appears in the first and fourth TEA ingredients. Posits are **robust** if they or their properties exhibit characteristics that surprise theorists, or if such surprises are possible in principle. Properties also **ground** the existence of a

[25] For example, Azzouni (2004a: 56-57) holds that the properties of Mickey Mouse are stipulated, not discovered. Hence the fictional character Mickey exists "in *no sense at all*" (57).

[26] Azzouni (2012b: 956-957): "It's an *empirical claim* that the only way we have to discover anything about ontologically-independent objects involves epistemic processes that must include appropriate sensory or instrumental interaction either with those objects, with objects they have affected, or with other suitably theoretically-related objects."

[27] I paraphrase the following list from Azzouni (2004b: 383-384), quoting where appropriate. The boldface titles of the four conditions I take from Azzouni (2004a: 129).

real posit, by providing a reflexive mechanism by which an observer can discern that properties of the posit render the posit observable.

Colyvan explains **Grounding** in two helpful passages. In the first, he says, "I can tell that a jet is moving across the sky by observing its vapour trail and seeing that the leading edge of the trail is advancing across the sky" (Colyvan 2005: 221). The non-stipulated reflexivity here is between the object that produces vapor trails (perhaps for theoretical reasons believed on independent grounds), and the dynamicity of the vapor trail that indicates objectual movement. Another example is Colyvan (2010: 288): "we can identify the heart in a chest x-ray because its relative density means that it appears as a region of greater x-ray absorption and this, in turn, enables us to determine other properties of the heart, such as its size." Here the reflexivity is between the heart's density and the kind of instrumental procedure (x-rays) that distinguishes the heart from other, non-heart objects.

My argument to bestow upon Fourier harmonics an epistemic role by ascribing reflectance to a vase, an ascription that reifies Fourier harmonics as thin posits, exploits both the **Robustness** and **Grounding** ingredients of TEA. Before laying out that argument, however, I will first argue that the deflationary nominalist must apply the Fourier harmonic in order to ascribe reflectance to surfaces at all.

4. The Mathematics of Ascribing Dispositional Reflectance

4.1 A metaphysical problem with ascribing reflectance

Harmonic realism follows, in deflationary nominalism, from a particular metaphysical problem[28] with ascribing reflectance as a real property of surfaces. The problem is that the most philosophically accessible definition of reflectance (Hilbert 1987; Byrne and Hilbert 2003) happens to be an operational definition:

> There is a well-known dispositional property of objects . . . . This is the surface spectral reflectance [SSR] of an object. . . . To measure the surface spectral reflectance . . . the ratio of the flux of incident light to the flux of reflected light is measured for each wavelength. Surface reflectances, thus conceived, are stable properties of objects. (Hilbert 1987: 1037-1041)[29]

This definition might not appear operational at first glance (quite the opposite, considering its language about "stable properties of objects"), but in this section I shall argue that Hilbert's definition (hereafter "pulse-SSR" or "Hilbertian SSR") functions only as an operational definition of reflectance, and that the attempt to ascribe Hilbertian SSR as a property *of vases* fails. Only a reflectance defined in terms of Fourier harmonics (which Hilbert's definition lacks) can be ascribed to vases and other surfaces.

Here's why. Firstly, "flux" in Hilbert's definition means average power in watts (Hilbert 1987: 1033-1042; cf. Germer et al. 2014), but only in the colloquial sense that "average power" could *be* nonzero for finite-duration pulses of light. Signal theory, which omits the colloquialism employed by Hilbert and some spectrophotometrists,[30]

[28] Detailed in [SELF-CITATION REDACTED].

[29] Hilbert describes the reflectance ratio somewhat infelicitously in this passage, suggesting that incident flux comprises the numerator of the ratio; that insinuation is wrong, and inconsistent with the rest of Hilbert (1987); the SSR ratio is reflected/incident flux.

[30] To be clear: calculating the "average power" of finite-duration signals is a very common and useful practice in science and engineering, but a woefully bad practice for metaphysicians to adopt, as I argue in this section.

permits only *infinite-duration* signals to possess nonzero average power, since the energy of the signal for which average power is computed is itself an integration over infinite time (Haykin and Van Veen 1999: 20-21).[31] By this crucial difference, signal theory accounts for a classical (non-quantum) behavior of light (Hirlimann 2005: 31) that undermines the attempt to ascribe pulse-SSR to surfaces. That behavior of light I call "harmonic dispersion," the inverse relationship of a pulse's bandwidth to its duration, or the ubiquitous, empirical, and well-documented tendency of monochromatic light to become heterochromatic as its pulse duration decreases (Stingl et al. 1995; Deng et al. 2005).

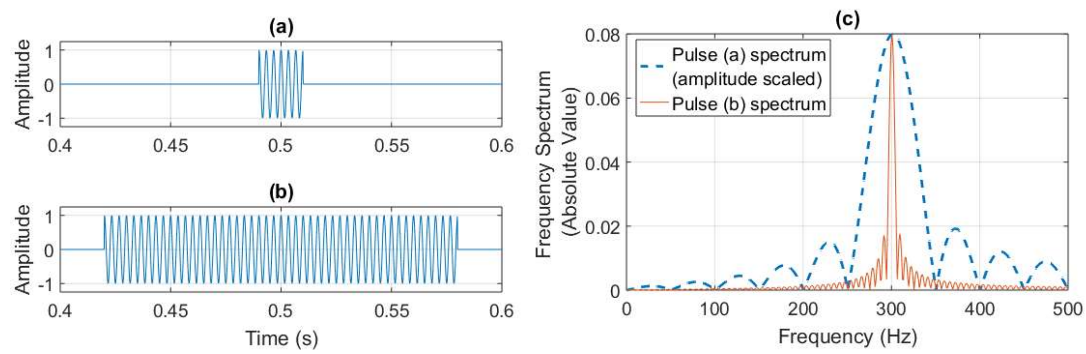

Figure 1: Harmonic Dispersion

Figure 1 illustrates harmonic dispersion at a carrier frequency (300 Hz) far below that of light (for ease of modeling), nevertheless indicating the dramatic dispersion (c) that occurs at all carrier frequencies. The point to notice is that pulses (a) and (b) differ only in their durations, but exhibit a considerable difference in their bandwidths (c).

[31] That is, signal theorists classify any signal possessing an "average power" to also have infinite energy (Haykin and Van Veen 1999: 21).

Hilbert's definition rightly allows a surface's reflectance profile[32] to possesses different values at different wavelengths, but only pulse (b) propagates at anything close to one wavelength (300 Hz);[33] pulse (a) propagates as a wide "envelope" of wavelengths, per the dotted line in (c).

Thus, harmonic dispersion generates what I call the "Vicious Reflectance Regress" (VRR) against Hilbertian SSR. This regress refers to the precipitous collapse of the pulse-SSR value alleged to obtain at a surface, for a given wavelength. Assume, for example, an optical 5 W pulse centered[34] at 800 nm. Empirical data confirms that when the duration of such a pulse falls below 1 picosecond (ps; 10^{-12} seconds), the pulse's bandwidth grows as wide as 100 nm (Deng et al. 2005; Stingl et al. 1995). Thus, if this pulse propagates into a perfectly reflecting mirror (SSR = 1 at all wavelengths), what is the "average power" of light expected to reflect at 800 nm? The pulse-SSR theorist ostensibly needs to answer: 5 W. But I argue that the answer cannot be 5 W, since the original 5 W pulse is spectrally redistributed by harmonic dispersion (Figure 1).

Assume very roughly, then, that only 80% of the 5 W pulse actually reflects at 800 nm (the remaining 1 W dispersing to neighboring frequencies). Can we say that the average power of this reflected, 800 nm *component* of the original pulse is 4 W (80% of 5 W)? Again, I say no, because that 4 W "component" is itself a finite-duration pulse (what else could it be?), and *ex hypothesi*, pulses disperse their frequency content when they are short-duration. Thus the 4 W pulse really only propagates with 3.2 W (80% of 4

[32] A "reflectance profile" is the set of (or a plot of the set of) a surface's reflectance values (between 0 and 1) across the human-visible range of wavelengths; for an example, see Byrne and Hilbert (2003: 9, Fig. 1).

[33] "Wavelength" and "frequency" are interchangeable terms in applied optics. Frequency is speed of light divided by wavelength.

[34] As the plots of Figure 1 (c) are "centered" at 300 Hz.

W) at 800 nm, *ad infinitum*. By inspection, this regress obtains no matter what dispersion percentage is originally picked,[35] and this regress is vicious, destroying the pulse-SSR property by rendering it conceptually incoherent, and by asymptotically driving any given pulse's per-wavelength "average power" to zero.

The VRR renders pulse-SSR conceptually incoherent because there is no principled stopping place within the infinite iterations (5 W, 4 W, 3.2 W, etc.) to construct the SSR ratio (of reflected and incident average powers—see Hilbert's definition opening this section). Nor does stopping the regresses for the numerator and denominator of the SSR ratio at the *same* iteration solve anything, since one can still ask why the arbitrary stopping point was picked, and why every given pulse's per-wavelength average power plunges to zero. Nor can one appeal to the "average power" that was "actually measured" in a laboratory to stop the regress; that proposal begs the question against the VRR, about which "pulse" in the regressive iteration was measured. No one knows. If the measurement device reads 4 W at 800 nm, one can justifiably *ask* why the value is not 3.2 W, and conversely; whatever is being measured by the device is not a *pulse*, by the argument just given. "Per-wavelength pulse-reflectance" is a contradiction in terms, like "non-cubical cube."

Philosophically, then, I conclude that pulse-SSR cannot be a property of vases, because ascribing to vases a property that is viciously regressive and conceptually incoherent is disingenuous at best, and vacuous at worst. Pulse-SSR can only be an operational property, a set of instructions about how to measure the colloquial "average

[35] This point matters, because all finite-duration pulses are dispersive, even the hyper-picosecond pulses. Their harmonic dispersion might not be as radical as Figure 1 (c), but will still obtain, and so perhaps a 99.999% regress ensues, which is nevertheless just as vicious as the 80% regress discussed in this section.

power" ratios of long-duration, (relatively) non-dispersive reflecting signals; pulse-SSR is not sufficiently well-defined to be a property of the vase.[36,37]

4.2 A solution to the Vicious Reflectance Regress

Blocking the VRR is where the Fourier harmonic comes into play. Fourier harmonics just are the infinite-duration signals that signal theorists use to compute average power, and which I have already represented in Figure 1 (c). Every point in either trace of Figure 1 (c) represents a harmonic possessing that trace point's plotted amplitude and frequency. All of the harmonics represented in a Figure 1 (c) trace superimpose without remainder, moreover, into that trace's corresponding pulse (a) or (b).[38]

The philosophical point to grasp is that due to their infinite duration, harmonics never disperse their frequency content; they are *immune* to harmonic dispersion, and they possess unity bandwidth, by definition. Thus, reflectance redefined as the per-wavelength efficiency of a surface to reflect harmonics is conceptually coherent and ascribable, since it never suffers the Vicious Reflectance Regress. No matter the duration of the "pulse" propagating into a mirror, the per-wavelength reflective efficiency of that mirror remains a stable, constant, and well-defined property if that property is harmonic-

[36] Nor is pulse-SSR sufficiently well-defined to be a property of the vase-and-impinging-light, an "extrinsic disposition" (Hoffmann-Kolss 2010; McKitrick 2003). The VRR militates against any definition of reflectance employing per-wavelength "pulses."

[37] See [SELF-CITATION REDACTED] for additional objections, with replies. Note that the wave-particle duality of light does not affect my argument, because photonic emission and absorption also occur "per wavelength" in finite time, launching the VRR.

[38] The Fourier composition includes harmonics of negative frequencies, and requires a wavelength-dependent phase shift, neither of which are shown; this detail does not affect my argument.

SSR. When the laboratory measures 4 W at 800 nm, it measures the average power of a finite-duration portion of a *harmonic* that is not cancelled-out in superposition by other harmonics (Figure 1). Hence the regressive chase to measure the 800 nm power of a "pulse" never begins. While mathematical representations besides Fourier analysis can model electromagnetic pulses (e.g., wavelets, Bessel functions), the philosopher ascribing "per-wavelength" reflectance to surfaces as a real property appears to have a particular need (due to the VRR) of signal components that are "per wavelength" in the most literal sense. The harmonic, a monochrome, is that very signal.[39]

In the next section, I argue that the Fourier harmonic possesses an "epistemic role" in the deflationary nominalist sense; not for the harmonic's utility in predicting or representing harmonic dispersion (as in Figure 1), but for its indispensability to ascribing reflectance to vases, and so to forging TEA to vases. Nominalize the harmonic by "approximating" it away from its infinite duration, my argument goes, and the Vicious Reflectance Regress ensues, destroying reflectance ascription to the vase, and disrupting the TEA alleged to obtain between the observer and the vase.

5. My Philosophical Excuse for Mathematical Realism

5.1 A second way to acquire an epistemic role

I am now in the position to argue that the Fourier harmonic possesses an "epistemic role" in the deflationary nominalist sense (section 2), because the harmonic proves indispensable to rendering dispositional reflectance ascribable to vases. That is, the harmonic proves indispensable for ascribing the reflectance property (section 4) that

[39] Many if not all wavelet bases, on the contrary, are heterochromatic, and will introduce dispersion effects (Deng et al. 2005; Mallat 1999: 546-547).

partially constitutes TEA to vases. Granted, a harmonic does not need to *exist* as the form of propagating light for harmonic-SSR to be ascribed; the harmonic needs to exist if harmonic-SSR ever incurs stimulation or manifests.[40] But some deflationary nominalists suggest that vases do manifest reflectance (see Azzouni 2010: 30), and so the epistemic role—if any—of the harmonic should be scrutinized.

In effect, I am introducing a second way that a posit (the harmonic) could acquire an epistemic role. The first way was explained in section 2: a posit possesses an epistemic role if it produces "epistemic artifacts," if we can *tailor* our observational interaction with the posit. This tailoring receives explicit codification as the **Refinement** ingredient of TEA, but it seems fair to say that epistemic artifactuality includes the reflexivity, temporality, and surprise of the other TEA ingredients (section 3). Hence the traditional way to argue that the Fourier harmonic possesses an epistemic role is to identify its epistemic artifacts, and I will venture the traditional way in section 5.2; here I propose that a given *posit* can possess an epistemic role for an observer's knowledge of *another posit*.

Consider, for example, Colyvan's jet-vapor example of **Grounding** (section 3). One **grounding** property in the jet example is the vapor trail, since its spatial elongation is our defeasible indicator that a metal airplane exists as a thick posit. That the vapor trail exists can be inferred by its own epistemic artifacts: it looks a little different if we squint

[40] I hereafter imply rather than repeat this important point. Any reference that I make to an "ascribed" property is always a locution for "an ascribed property whose stimulus or manifestation (in a dispositionalist or non-dispositionalist sense) has occurred, is occurring, or is expected to occur." For my purposes, the stimulus of reflectance is light impinging on a surface, and the manifestation is light propagating away from the surface. Hilbert (1987) says relatively little about the stimulation and manifestation of SSR; cf. Boghossian and Velleman (1989), Jackson (1996), Byrne (2001), and Pasnau (2009).

or hold up binoculars. But what property **grounds** the thick posit vapor trail? The answer is plausibly reflectance! By understanding reflectance, we can understand why different observational methods confirm the presence of water vapor, despite its possibly different appearances under those methods. Granted, Azzouni's (2010: 30) vase anecdote is a rare example of dispositional property ascription within the many of his references that I cite in this paper, and so I shall avoid suggesting that TEA can be forged to self-standing *properties* like reflectance, or that "properties" can be thick or thin posits.[41] That move would be metaphysical overkill for the deflationary nominalist. Instead, I point out that the jet plane kicks off epistemic artifacts *because* it generates vapor trails; a posit may generate epistemic artifacts through its **grounding** property. Hence my question is: does not the property that *bestows* an epistemic role on a posit (as the vapor trail bestows that role on the jet plane) *thereby* possess an epistemic role of its own? How can one bestow what one lacks? My "second way" of ascribing an epistemic role to a posit, then, is by showing that posit to be the **ground** of TEA to another posit. The Fourier harmonic **grounds** the vase that we see, because the harmonic's infinite duration—a *mathematical* property in Azzouni's account[42]—(a) "explain[s] why, and in what respects" the vase "can be observed" (Azzouni 2004b: 383), as well as (b) how "instrumental access" to the vase "reveals" its properties (Azzouni 2004b: 384).

Clauses (a) and (b) in the previous sentence re-quote the **Grounding** ingredient of TEA (section 3). The Fourier harmonic satisfies (a) because it explains why the vase can be observed: because harmonic-SSR is what makes *the vase* reflective (section 4). The

[41] Especially considering Azzouni's (2017) antirealism about properties (despite his retention of unchanged TEA criteria); see section 5.4 of this paper for discussion.

[42] Azzouni (1994) calls the "'infinite'" a "mathematical notion[] . . . [that is] not first-order definable . . ." (3).

Fourier harmonic satisfies (b) because it explains how human vision or radar systems "reveal" the vase to be reflective: vision and radar work, ostensibly, because the vase possesses harmonic-SSR. Thus, the Fourier harmonic possesses a **grounding** role, and so an epistemic role in perceiving vases. Such is my "second way" to ascribing an epistemic role.

Granted, the Fourier harmonic remains a "thin" posit because I did not claim to forge TEA to *it*, and so I need an excuse clause (section 1) regarding its undetectability. Unlike Colyvan's (2010: 288) appeal to spatiotemporal abstractness, I submit that the Fourier harmonic goes undetected because it propagates in zero-sum superposition outside the duration of the "pulses" that we take ourselves to manipulate (see Figure 1). An elaboration of the same insight is that even if we filter one frequency from a pulse with high precision, we finite beings cannot have "thick" access to its infinite duration *qua* infinite. Yes, Azzouni prefers that thin-posit discriminations hail from science (section 1), but I reply that according to my "second way" argument, Azzouni needs real harmonics to make viable the very TEA process that practicing scientists use to perform such discriminations.

5.2 Back to the first way: TEA ingredients for the Fourier harmonic

Before considering objections to the "second way" to an epistemic role, could the "first way" of ascertaining the epistemic artifacts of a Fourier harmonic succeed? Such an approach would involve verifying that all four TEA ingredients obtain between the observer and the Fourier harmonic itself. While I find it difficult to imagine what the epistemic artifacts of an infinitely-durative monochrome would be, one might suppose that an initial answer emerges from Azzouni's prior commitment to the reality of a behavior that the harmonic exhibits: superposition.

Specifically, Azzouni (2004a) endorses "[r]ecent experiments apparently illustrating thick epistemic access to superpositions of a particle . . ." (225, n. 3). The experiment referenced is that published by C. Monroe, D. M. Meekhof, B. E. King, and D. J. Wineland (1996),[43] which reports the manipulation and superposition of the quantum states of a Beryllium ion. I hypothesize that if these individual *quantum states* can be construed as (at least) thin posits, then because Fourier harmonics likewise superimpose into finite-duration pulses (thick posits referenced throughout science), then one may analogously claim TEA to *Fourier* superposition, and (at least) thin access to Fourier harmonics. One flaw with this analogy, however, is that the quantum states in superposition are values of position and "angular momentum" (Monroe et al. 1996: 1132), the latter of which can be detected *independently of superposition experiments* (Halliday et al. 1997: 1030-1031),[44] and even "thick[ly]" in the deflationary nominalist sense (Bueno and French 2018: 176). Indeed, it could be said that angular momentum **grounds** the quantum superposition, as the Fourier harmonic **grounds** the optical superposition, but a harmonic in its infinite duration is not observed by itself. Thus, a side-by-side comparison of the TEA ingredients for quantum and optical superposition would remain fraught with disanalogy.

[43] Thanks to XXXXXX for this information.

[44] I refer to the "Einstein – de Haas Experiment," in which a macroscopic iron cylinder rotates inside a current-carrying solenoid.

5.3 A Colyvanian approach, and the Coding Role

As I leave aside the orthodox or "first way" defense of the Fourier harmonic's epistemic role, I also decline the Colyvanian (2010) approach of claiming that the Fourier harmonic fulfills the Quinean virtues (which it does[45]), but that the excuse for not detecting harmonics in the raw is that they exist only as superimposed within the finite-duration pulses that we manipulate.[46] That proposal, despite appealing to Quinean virtues that Azzouni has abandoned as criteria for thin-posithood,[47] remains vulnerable to the "coding" objection of Azzouni and Bueno (2016),[48] an objection that equally threatens the (identical) excuse clause of my "second way" argument in section 5.1. The point of the coding objection is that indispensably mathematical sentences can be used "assertorically"—or in a way that commits the user to their truth[49]—without the mathematical terms referring, and that sentences of scientific theory can be asserted with the understanding that their "mathematical [ultrathin] posits are proxying for something empirical that we can't otherwise describe" (Azzouni 2004a: 173).

An example of a sentence with coding terms is, "The average star has 2.4 planets" (Azzouni 2009: 157). The sentence can be used assertorically, despite "average stars" and rational numbers not existing, Azzouni (2009: §5) argues, because of what he calls a

[45] Michael Liston (2004; 1993) argues cogently for what amounts to a defense of the Quinean virtues of Fourier analysis.

[46] Note that this excuse clause is identical to that of the "second way" argument in section 5.1.

[47] For reasons outside the scope of this paper; see Azzouni (2012b).

[48] Discussed also by Azzouni (2009; 2004a: Chapters 8 and 9).

[49] The "assertoric use" of sentences (which I sometimes call "assertion") is a tenet of Azzouni's deflationary account of truth. Azzouni (2009) sees assertoric use as a sort of converse of our linguistic practice with the Tarski biconditional. As a sentence like "'Snow is white' is true" can be shorn of its truth idiom and replaced with "Snow is white" (Azzouni 2004a: 16), so Azzouni (2009) calls it an "empirical fact" that when we "*assertorically use*" (141) some sentence W, we incur logical commitment to the sentence "'W' is true." Azzouni's point is that assertoric use transpires in scientific contexts and other deductions and descriptions, but not in a stage play, a quotation of another's words, or in various other contexts (Azzouni 2009: §2).

"proxy norm" obtaining among scientific interlocutors. That is, because no one person can know all of science, the scientist uses "public" (152) sentences *D* assertorically for deduction and representation, with the implicit understanding that some of those sentences proxy for sentences *D** that other specialists could use assertorically (154). The point is not that *D** sentences could in principle always replace *D* proxies, but that *D** sentences enable one to draw appropriate implications from proxy sentences, including ontological implications (153-155).

As additional examples of proxy terms, Azzouni mentions "infinitesimals" and "the Dirac delta function" as "loosely-employed concepts" in science (154), concepts among which a deflationary nominalist might include the Fourier harmonic.[50] As I find the infinite-duration harmonic indispensable to reflectance ascription (section 4), moreover, so Azzouni and Bueno (2016) claim that scientifically recognized properties of "metal deform[ation]" depend indispensably upon "continua structural postulations" about real materials, a structure nevertheless "recognized [by scientists and some philosophers] . . . to be unreal" (794). Thus, against the supposition that Fourier harmonics exist hidden in superposition, the deflationary nominalist might call Fourier analysis a proxy language for whatever the electromagnetic field—or other presently obscure entity or process—is doing.[51] This objection appears to undermine the excuse clause both for the "Quinean virtue" argument of this section, and for my "second way" argument of section 5.1. If the harmonics indispensable to the stimulus and manifestation

[50] Although not shown in Figure 1, the frequency-domain representation of a single harmonic is a delta function with finite amplitude and unity bandwidth.

[51] This objection has a mechanical analogue: the "'third harmonic'" of a string does not vibrate, Liston (1993) clarifies, *the string* vibrates (451).

of SSR are just coding for something we cannot currently describe, then the harmonics are not thin posits, they are ultrathin.

5.4 Response to the coding objection

The coding objection to the thin-posithood of the Fourier harmonics that are indispensable to reflectance ascription appears sound and compelling,[52] because while it is one thing to claim that harmonics **ground** other posits and so possess an epistemic role (section 5.1), it is quite another to walk into a laboratory and assert that mirror A is "more reflective" than mirror B only if real mathematical entities (which possess infinite duration, by the way) are propagating through the room. I deny, however, that one can fairly, *universally* apply the coding objection (*viz*. the proxy norm) when identifying the very properties or entities by which **Grounding** obtains.

My denial hinges on an understanding of what kind of relation **Grounding** is supposed to be. At the end of the day, **Grounding** is "the [set of] detail-oriented scientific explanations (of how *this* specific property of *that* enables us to track it because of certain causal interventions we're consequently capable of) . . ." (Azzouni 2004a: 134). There is a lot to unpack in the previous sentence, and I cannot elaborate all of it in the remaining space of this paper, but the overriding point is that **Grounding** is an explanation, and I have not seen Azzouni endorse a specific account of explanation that either supports or undermines the notion that mathematical entities explain physical

[52] I answer later in this section whether the coding objection succeeds against the Colyvanian "Quinean virtue" argument of section 5.3, which, recall, does not involve reflectance ascription.

phenomena such as observation, the data accumulated through TEA, or the obtaining of TEA conditions like **grounding**.[53]

Indeed, Azzouni (1998: 12) simply punts on the question: "explanation operates at the sentential level, and is indifferent to how we tease out the ontological commitments of the sentences which provide the explanations we take seriously." Hence in declining to give an account of extra-mathematical explanation proper,[54] but freely allowing the proxy norm to quash the reification of mathematical posits indispensable to the science of continuum-bent metals (section 5.3), Azzouni appears to *assume* that mathematical posits are never going to incur an epistemic role, and so never play more than a representative or descriptive role in scientific explanations (the set of which includes **Grounding**). He says as much when he remarks in passing that mathematical entities do not explain physical phenomena, because mathematical entities do not exist (Azzouni 2012: 964). But when the *obtaining* of a **Grounding** explanation depends on a mathematical property like infinitude (as when Fourier harmonics **ground** human-visual TEA to vases by making reflectance ascribable), Azzouni's unargued dismissal of mathematics as non-explanatory begs the question, by deflationary nominalism's own lights.

[53] For an introduction to the ongoing controversy over whether mathematics can explain physical phenomena, see Marcus (2015: Chap. 7).

[54] "Extra-mathematical explanation is the . . . mathematical explanation of physical facts" (Baker and Colyvan 2011: 326).

Can a mathematical posit be thin?	→	Consult the TEA conditions, to see if the posit possesses an epistemic role.	→	Fourier harmonics indispensable to the obtaining of the TEA condition of **Grounding** possess an epistemic role (section 5.1).	→	Then render those mathematical posits ultrathin by the proxy norm (ignoring TEA conditions). Why? Because mathematical posits do not exist (*petitio principii*).

Table 1: A question-begging application of the proxy norm

Table 1, albeit free of "explanation" language (which I provide below), outlines how I think that appeals to the proxy norm can beg the question about the thin-posithood of a mathematical entity. Simply put, consulting and ignoring TEA conditions in the same argument, on the same question, is invalid. An alternative rendering of the same point is that categorically denying that mathematical entities could explain physical phenomena (Azzouni 2012: 964), but then appealing to the *proxy norm* to preserve that denial when mathematics proves indispensable to the ascription of properties within a **grounding** explanation of physical phenomena, is to deny without argument that mathematical entities explain physical phenomena. The proxy norm, in other words, is not an argument that mathematical posits do not exist, but an implicit assumption that the TEA conditions successfully preclude the existence of mathematical posits. My reflectance counterexample challenges that assumption, and doubling-down on an assumption (the proxy norm) against a counterexample is begging the question.

The upshot is that the proxy norm has its place. The proxy norm is non-question-begging when the mathematized properties that it mathematically nominalizes are "coherent in themselves,"[55] or do not *cease to be properties* when their definitional

[55] I quote this useful expression from an anonymous reviewer.

mathematics are expunged, but only cease to be well-described or tractable; examples include the complex properties of continuum-bent metal (as far as I understand them—see section 5.3),[56] and the cumbersomely-described property by which every 10 stars tend to possess a combined total of 24 orbiting planets (section 5.3). Reflectance is different. It ceases to be a property at all (it becomes like a non-cubical cube—see section 4.1) when its harmonics are nominalized away, and while this result might incline one to reflectance antirealism (so long, dispositional pulse-SSR), that inclination can additionally topple **Grounding** explanations for TEA, a step too far for the deflationary nominalist, or at least for the proxy norm (see Table 1).[57]

To tie up one loose end, then, I find the proxy norm *rightly* applied against the free-standing Fourier harmonics alleged to exist in undetectable zero-sum superposition in the Colyvanian "Quinean virtue" argument of section 5.3. That example does not obviously involve **grounding**, nor does the superposition in question clearly implicate other TEA conditions to mark out the harmonic's epistemic role (section 5.2). Mathematized physical properties that play **grounding** roles *and* suffer conceptual

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[56] I do not mean to minimize the possibly physical and explanatory significance of the mathematized properties analyzed by Azzouni and Bueno (2016), which may or may not play **grounding** roles, or suffer conceptual regress when mathematically nominalized, like reflectance does. Ascertaining such a **grounding** role and/or threat of regress is simply outside the scope of this paper. Additional mathematized properties discussed by Azzouni include continuum-divisible space (Azzouni 2004a: Chapters 8 and 9), and the continuum-defined "background geometry" of string theory (Azzouni 2009: 161).

[57] Hence it bears mentioning that Azzouni's (2017; 2012a) antirealism about properties does not affect any of the foregoing problems or arguments. The word "properties" still features prominently in the TEA conditions, and Azzouni (2017: Chap. 8) endorses TEA without listing revised or property-free TEA conditions. Indeed, Azzouni (2017: Chap. 6) embraces an antirealism about physical *surfaces*, giving reflectance no place to inhere, but so long as the "per wavelength" dimension of light propagation remains scientifically important, nominalism about Fourier harmonics is bound to land optical theorists in conceptual regress (section 4). For the record, I suspect that optical properties besides reflectance suffer a conceptual regress analogous to the VRR; they include the dispositional refractivity implicitly endorsed by Chakravartty (2007: Chapters 2-3), and the surface plasmon resonance ascribed by Bursten (2018).

regress when mathematically nominalized are exempt from the proxy norm, as are any purely mathematical properties that **ground** in some case but resist mathematical nominalization, as infinitude does (see footnote 42), and possibly others do.[58]

6. Conclusion

My goal in this paper has been to identify an epistemic role for some mathematical entities, not by satisfying all four TEA conditions with respect to those entities, but by showing them indispensable to the ascription of properties that facilitate TEA to "thick" posits, and by arguing that the mathematical entities perform a **grounding** role in such TEA (section 5.1). I identified this **grounding** and thus epistemic role for the Fourier harmonic by arguing that its infinite duration is indispensable to blocking a conceptual regress of the reflectance property that Azzouni (2010: 30) implicitly ascribes to vases. I then argued that appealing to the deflationary nominalist "coding" objection of Azzouni and Bueno (2016), to discount the harmonic term as a non-referring proxy for some to-be-had non-mathematical theory about propagating light, amounts to a *petitio principii* within the **Grounding** context (section 5.4). One target for further research is a scientifically-informed account of what it means for properties to be sufficiently "ascribed" to perform their **grounding** role(s) in

[58] Suitable for footnote-length mention here is how unhelpful it would be to call Fourier harmonics an "idealization" in the deflationary nominalist sense. Azzouni (2005) refers to idealizations as "falsifications" (34), and as a process of "systematically excluding phenomena (such as friction) from explicit consideration to make derivations tractable" (30, n. 16). As I have argued in section 5.4, I do not have a tractability problem with reflectance, but a conceptual coherence problem. Thus, idealizing harmonics would not rescue reflectance ascription, and the **Grounding** condition between the viewer and the vase would still be obliterated, an unwanted result for the deflationary nominalist.

deflationary nominalism. Such an account could alter my present conclusion, which is that the Fourier harmonic exists as a thin posit.

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