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An Extra-Mathematical Program Explanation of Color Experience

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Abstract:
In the debate over whether mathematical facts, properties, or entities explain physical events (in what philosophers call "extra-mathematical" explanations), Aidan Lyon's (2012) affirmative answer stands out for its employment of the program explanation (PE) methodology of Frank Jackson and Philip Pettit (1990). Juha Saatsi (2012; 2016) objects, however, that Lyon's examples from the indispensabilist literature are (i) unsuitable for PE, (ii) nominalizable into non-mathematical terms, and (iii) mysterious about the explanatory relation alleged to obtain between the PE's mathematical explanantia and physical explananda. In this paper, I propose a counterexample to Saatsi's objections. My counterexample is Frank Jackson's (1998a) program explanation for color experience, which I argue needs recasting as an extra-mathematical PE due to its implicit reliance on reflectance, a property that suffers conceptual regress unless redefined with Fourier harmonics. *Pace* Saatsi, I argue that this recast example is an authoritative PE, non-nominalizable, and minimally esoteric. Important for the indispensability debate at large, moreover, is that my counterexample reifies Fourier harmonics without the Enhanced Indispensability Argument (an argument to which Lyon applies PE as a premise). Indispensabilists have long overlooked the conditionalization of a limited mathematical realism on property realism, and my counterexample to Saatsi exploits this conditionalization.

Keywords: extra-mathematical explanation; Enhanced Indispensability Argument; Program Explanation; mathematical nominalism; mathematical realism; color objectivism; reflectance physicalism; Fourier analysis

1. Introduction

In the debate over whether mathematical facts, properties, or entities[1] could explain physical phenomena, Aidan Lyon's (2012) affirmative answer stands out for its employment of the "program explanation" (PE) methodology of Frank Jackson and Philip Pettit (1990). Program explanation is a form of ontic (Saatsi 2016, 1046) or "constitutive explanation" (Pettit 1993, 69), whereby real properties in the world serve as explanantia for a physical event that is the explanandum.[2] Essential to PE theory, moreover, is that programming properties explain *without*

[1] I treat these three terms interchangeably, context permitting.

[2] Some may object that PE can be construed as an epistemic explanation that advances human understanding rather than picking out worldly ontology (thanks to XXXXX for this point), but to engage that controversy in this paper would be a digression.

causing the explanandum. This feature of PE undergirds some explanations from mental properties ("attitudinal contents") and sociological properties ("group-cohesion") (Jackson and Pettit 1990, 115),[3] and Lyon adds to these applications the primeness of integers 13 and 17 program-explaining the North American *Magicicada's* 13- or 17-year dormancy period.

The cicada explanandum-question, more precisely, is why cicada nymphs live in the ground for 13 or 17 years before emerging, mating, and dying, given ecological conditions that constrain dormancy periods to 12 to 18 years.[4] Lyon argues that avoiding periodic co-emergence with the life cycles of predators proves evolutionarily advantageous to the cicadas, and that prime periods of cicada emergence minimize such intersections. Hence Lyon sees an "extra-mathematical"[5] explanation to obtain between the primeness property of number theory and the actual dormancy periods of cicadas. Primeness "programs" the periods in a sense to be articulated in section 2, just as other mathematical properties program patterns and events in additional examples of mathematical indispensability, including Plateau's laws for soap films, the Bridges of Königsberg, the hexagonality of honeycomb, etc. (Lyon 2012).

More than merely explain events via mathematical properties, however, Lyon (2012) reifies explanatory mathematics, running his PE conclusion as a premise in the Enhanced Indispensability Argument (EIA):[6]

> [B1] We ought to be committed to the existence of all and only the entities that are explanatorily indispensable to our best scientific theories.
>
> [B2] Mathematical entities are explanatorily indispensable to our best scientific theories.

[3] More recently, Shea (2018, §8.3) endorses program explanations of human behavior.

[4] For biological and ecological details, see Lyon (2012, 561, 567, n. 10), Baker (2005, §2), or Berenstain (2017, §3.4).

[5] "Extra-mathematical explanation is the . . . mathematical explanation of physical facts" (Baker and Colyvan 2011, 326).

[6] The following text of B1 and B2 is Lyon's (2012, 572) paraphrase of Baker (2009), in my formatting.

The conclusion being that mathematical entities—the integers 13 and 17 in the cicada case—exist (Lyon 2012, 573). Lyon understands his substitution of PE for premise B2 in the EIA to advance the indispensability debate in favor of mathematical realism, since PE secures for mathematics an explanatory versus a merely descriptive role in our best scientific explanations (Lyon 2012, 572).

While I sympathize with Lyon's two-part argument (Part 1: extra-mathematical program explanation; Part 2: EIA), the first part faces powerful objections from Juha Saatsi (2012; 2016). In this paper, I argue that Saatsi's objections fail against an extra-mathematical program explanation that Lyon and other indispensabilists have overlooked: Frank Jackson's (1998a) program explanation for the human experience of seeing surface colors. In addition to answering Saatsi's "Part 1" criticisms, my analysis of Jackson's counterexample also appears timely for "Part 2" EIA concerns, since Saatsi has recently declared debate on the EIA (of which program explanation is a feasible premise) to have "reached a serious impasse" (Knowles and Saatsi 2019, §1). Circumventing this impasse, I argue that program explanation reifies some mathematical entities *without* the EIA, specifically the mathematical entities that I find indispensable to Jackson's PE for color experience; those entities are Fourier harmonics, infinite-duration monochromatic sinusoids. Fourier harmonics must be real, I will argue, if the dispositional property of "reflectance" central to Jackson's PE is to manifest in a conceptually coherent sense.[7]

Two conclusions follow, then, from my identification and defense of an extra-mathematical program explanation for color experience (hereafter EMPEC). The first is that

[7] I identified this conceptual coherence problem for reflectance, and its Fourier solution, in [SELF-CITATION REDACTED.]

Saatsi's criticisms of extra-mathematical PE are not universally applicable and decisive; EMPEC stands as a counterexample to them. The second conclusion is that some program explanations reify the mathematics indispensable to PE explanantia without the EIA, since the *property* realism inherent to program explanation entails a limited mathematical realism in those cases. To clarify, and to anticipate an objection to this second conclusion, it will always be (too) easy to accuse me of ascribing reflectance only *for* the purpose of generating a program explanation, and hence to deny that I reify mathematical entities without the EIA after all. Despite this perennially proximate objection, I will dig in my heels and insist that when I find real harmonics indispensable to reflectance ascription, the cash value of that indispensability stops at property ascription, and that this stoppage is my prerogative as a metaphysician to identify. Analogously, an action theorist might hold that a man whose normal job is to pump water into a house *is* just doing his job and is *not* murdering the house's occupants, even if he knows that the water supply has been poisoned (cf. Anscombe 1963, §§24-25). Reflectance happens to occupy the role of PE explanans, but that *role* is not *why* real harmonics are indispensable to reflectance ascription. Real harmonics are instead indispensable to reflectance ascription inside and outside of explanatory contexts, and this point is that which philosophers have overlooked in indispensability debates.

My roadmap for this paper is to review program explanation in section 2, and Jackson's PE in section 3. I then motivate the mathematization of Jackson's PE in section 4, arguing along the way that Jackson's reflectance realism reifies harmonics without the EIA, and I argue in section 5 that Saatsi's objections to extra-mathematical PEs fail against EMPEC. Section 6 concludes.

2. The Brass Tacks of Program Explanation

Helpful to understanding program explanation are Jackson and Pettit's (1990, 108) criteria for programming properties, which are properties that possess causal "relevance" but lack causal efficacy (I omit criteria 1 and 2 for brevity):

> 3. A property F is not causally efficacious in the production of an effect *e* if these three conditions are fulfilled together.
>   (i) there is a distinct property G such that F is efficacious in the production of *e* only if G is efficacious in its production;
>   (ii) the F-instance does not help to produce the G-instance in the sense in which the G-instance, if G is efficacious, helps to produce *e*; they are not sequential causal factors;
>   (iii) the F-instance does not combine with the G-instance, directly or via further effects, to help in the same sense to produce *e* (nor of course, vice versa): they are not coordinate causal factors.

An example of a programming property (F) is the "temperature" of boiling water; *ex hypothesi*, temperature does not cause the cracking of a sealed glass container of boiling water, the momentum (G) of such-and-such a water molecule does (110). Temperature neither 'combines' with nor 'precedes' molecular momentum to crack the glass, but temperature *programs* the cracking by making "probable" that some-molecule-or-other will have the momentum to crack the glass (114).[8] Hence an epistemic advantage accrues to the program-explanationist, Jackson and Pettit (1990, 117) claim, over the process-explanationist who studies (G) causes alone. The theorist who knows that the water was boiling possesses a better explanation, with stronger modal information about what *would have* happened had *this* molecule not attained high

[8] The probability-raising function of programming properties, which Jackson and Pettit (1990) sometimes describe with the verb "ensures" (114), has been a controversial tenet of program explanation. Thalos (1998, 286-289ff) criticizes the notion that a probability-raising factor in an explanation could be non-causal, and Macdonald and Macdonald (2007, §3) reach a similar conclusion. I review Saatsi's (2012; 2016) objections to the "ensuring" relation in section 5, but cannot otherwise assess the merits of PE as a kind of explanation in the scope of this paper.

momentum, than does the process-explanationist who studies only glass-bonds and molecular velocities.

In a similar vein, Lyon (2012) claims that the primeness (F) of integers 13 and 17 "ensures the instantiation of a causally efficacious property" (566) or sequence of properties (G), by which the cicadas consistently settle on a 13- or 17-year dormancy period. Primeness neither 'combines' causally with the cicadas' environmental stressors, nor does primeness causally 'precede' those stressors. Primeness rather ensures or makes probable the existence of a *minimum* period of lifecycle intersection between cicadas and their predators, given an environment and its resources. Had the ecological history of cicadas been different than it actually was, for example, their lifecycle periods would probably still be prime, or become prime (567-568).

An additional point, which will be important later, is that Jackson and Pettit (1990) allow program explanations whose programming properties *fail* to impart modally strong information about G-processes to nevertheless *be* program explanations (116).[9] Programming properties lack modally strong information when they fail to indicate what events would have transpired had the actual G-processes not taken place. An example is the "fragility" (F) of a vase whose microstructure cracks-like-so (G) upon impact. Fragility trivially programs the microstructure, because fragility ensures without precession or combination that the microstructure cracks-like-so (G), but knowledge of fragility (F) yields no *better* explanation of the cracking than does knowledge of (G) (116), because fragility does not make probable that something besides (G) would have happened in (G)'s place. Putting this observation more succinctly, Bliss and

[9] Jackson and Pettit (1990, 116) remark: "A program explanation of an event *may* provide information which the corresponding process explanation does not" (emphasis mine). I take this "may" qualifier to mean that program explanations do not necessarily provide the sought-after modally superior information.

Fernández (2010, §§5-6) recognize fragility (F) to be a higher-order property than cracking-like-so (G), where an object's higher-order property is the object's property-of-having a lower-order property. Generalizing on this insight, I call modally uninformative program explanations (whether they involve multi-ordered properties or not) "inert" or "impoverished,"[10] and I refer to them in my discussion of color experience PEs.

3. Programming Color Experience

3.1 Reducing color to a physical property

The event in the world that Jackson (1998a) seeks to explain is the human experience of seeing color, and the disposition that he finds to program this experience is the disposition of objects to look (e.g.) red to normal observers in standard conditions.[11] Important to immediately distinguish, then, is (a) the perceptual disposition to look red, from (b) another disposition utilized in the philosophy of color: the *reflectance* disposition to which some color objectivists ontologically *reduce* color (Byrne and Hilbert 2003; Hilbert 1987). While these two dispositions are distinct, exhaustively detailing why would take me too far afield[12] (but see the next paragraph). The point for my paper is that Jackson takes both dispositions to obtain in the world (Jackson 1998b, 87), but he only program-explains with the perceptual disposition as an

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[10] Bliss and Fernández (2010) give an accessible and convincing heuristic for distinguishing impoverished from non-impoverished PEs, in part by diagnosing multi-ordered programming chains, but because those authors' conclusions sometimes contradict Jackson and Pettit (1990, and elsewhere) about particular cases, I decline to elaborate Bliss and Fernández's account, to avoid internal debates about PE proper. That *some* PEs are impoverished, all parties agree, and that point is what I need for my argument.

[11] These conditions are spelled out by Hardin (1988: Chap. 2). One may note that by making objects the bearers of the disposition to look red, Jackson rules out various subjectivist ontologies of color, according to which color experience might be programmed or caused by observers, brains, or minds, etc.

[12] Helpful on this topic is Byrne (2001), responding to McGinn (1996).

explanans, whereas I argue that dispositional reflectance is also program-explanatory, in an extra-mathematical sense, in Jackson's PE for color experience.

Let me begin by distinguishing both dispositions slightly. While an apple may (according to arguments I will not provide) possess the disposition to look red to normal observers under normal conditions, and may even *saliently appear* to possess this disposition (Byrne 2001, 244), those two claims say virtually nothing about what the color red might *be* ontologically. An influential attempt to fill this lacuna, by identifying surface colors with sets of reflectances, is David R. Hilbert's (1987). He takes reflectance to be dispositional, and defines it as follows:

> There is a well-known dispositional property of objects . . . . This is the surface spectral reflectance of an object. . . . To measure the surface spectral reflectance . . . the ratio of the flux of incident light to the flux of reflected light is measured for each wavelength. Surface reflectances, thus conceived, are stable properties of objects. (Hilbert 1987, 1037-1041)[13]

In section 4, I make trouble for Hilbert's use of "flux" or optical average power in his definition of reflectance (hereafter SSR, for "surface spectral reflectance"), but for now the point to grasp is that dispositional SSR plausibly obtains on surfaces whether anyone is looking at them or not, and whether those surfaces are illuminated or not. This idea, in turn, supports the conceptual hypothesis that the surface color "red" *just is* a surface's SSR profile that obtains in the dark and under all illuminants, etc. From these assumptions, color objectivists then relate dispositional SSR *to* the perceptual disposition to look colored, by empirical correlation (Byrne 2001, 244). If objects with the disposition to look green all possess similar reflectance profiles (that differ

[13] As useful as this quote is, it contains infelicitous wording inconsistent with the rest of Hilbert (1987). Namely, Hilbert appears in this quote to place incident flux in the numerator of the reflectance ratio; but no one does that; reflectance is always the ratio of reflected-to-incident flux per wavelength.

appropriately from red, blue, yellow, and orange profiles, etc.), then color objectivism is *pro tanto* vindicated.[14]

The important point for this paper is that while Jackson accepts that SSR is a real disposition (Jackson 1998b, 87), he denies its role as the ontological reduction base for surface color, because colors are supposed to *cause* color experience (Jackson 1998a, 86),[15] and dispositions are non-causal. One reason that Jackson finds dispositions non-causal we saw in section 2: dispositions are non-causal higher-order properties (F) of entities that possess causal, categorical base properties (G);[16] he would find it "ontologically extravagant" to attribute causality to an impoverished higher-order property like fragility or SSR (Jackson 1996, 202).[17]

Secondly and more specifically, Jackson (1996) takes dispositions to be non-causal due to the mutability of physical laws across possible worlds. Some "poisonous" categorical microstructure might not kill humans in worlds with different "laws of nature," Jackson claims, but were poisonous*ness* a causal disposition, it would remain "intimately connected" to swallowing and human death "in every world" (203). Hence Jackson denies that "properties . . . have [their] causal powers essentially" (203), or independently of laws, but he interprets any dispositions lacking a categorical base *to* possess their causal powers essentially, with manifestations that transpire "uncaused" (204). Jackson (1998b) rejects the ontological

[14] Notoriously, reflectance profiles and perceptual dispositions anticorrelate in many cases, and color objectivists must handle this problem in nuanced ways outside my purposes to describe (Byrne and Hilbert 2003).

[15] See McFarland and Miller (1998; 2000) for objections to this assumption, which they claim goes undefended by Jackson.

[16] On the distinction between categorical and dispositional properties, see Schrenk (2017, Chap. 2).

[17] An anonymous reviewer asks why impoverished dispositions should be reified at all, and especially for the sake of ontic explanation. This question is a deep one about the fundamental motivations for program explanation, that I will be unable to answer authoritatively in this space. I defer to Bliss and Fernández's (2010, §1) brief history of PE as a defense of the autonomy of special sciences like sociology, psychology, and economics.

reduction of color to SSR, therefore, for not going far enough: color is (ontologically) the disjunctive, microstructural, categorical *base* of SSR rather than SSR *qua* disposition (110).[18]

3.2 Explaining color experience with SSR

Returning to the PE framework, Jackson (1998a) treats the perceptual disposition to look red (D_R) as a real property that programs the disjunctive microstructural base (G) that is redness, and this disjunctive base, *qua* disjunctive, programs a particular event of looking red in addition to causing it (87). The disjunctive base increases the probability that *some* such base causes *this* experience, but the disjunctive set furthermore straightforwardly causes the event, as the "depth" of a wound within some range causes death (87).[19]

Enter my insight about dispositional SSR. What Jackson (1998a) omits to mention is that the categorical base of the disposition to look red *could in principle be* the categorical base of dispositional SSR; for microstructure is microstructure, and it is an empirical question whether the correlations that Hilbert (1987) and other color objectivists tender between human color reports and SSR values further correlate to or circumscribe a nameable set of microstructures.[20] Hence just as Jackson and Pettit (1990, 115) allow programmatic chains through multiple non-causal properties to a terminating causal property (F1→F2→G), one may ask why the disposition to look red (D_R) could not program SSR, which would in turn program the disjunctive causal base (G) of both dispositions: (D_R→SSR→G).

[18] Hilbert (1987, 2089-2092) and Byrne and Hilbert (2003, 53; cf. 20, n. 25) deny on mostly empirical grounds that color reduces ontologically to microstructure; I return to this point, although it does not affect my argument and I need not take sides in the dispute.

[19] See McFarland and Miller (1998; 2000) and Wright (2003), for criticism.

[20] I remind the reader that I am not defending color objectivism or providing any reason to believe it. The view is controversial, as Gert (2017, Chap. 3) recently argues on scientific and philosophical grounds.

This expansion of Jackson's PE for color experience appears benign and minimally costly (save a minor loss of simplicity), since he already understands disposition D_R to be causally relevant but not causally efficacious, and because inserting another programming property (SSR) into the (D_R→G) chain leaves the programmatic status of D_R unchanged. The advantage of adding the SSR programming property, moreover, is that it enhances the *scientific credibility* of Jackson's PE for color experience. For while it is plausible that microstructure (G) somehow causes color experience, it is *im*plausible to suppose that (G) achieves this feat by some means besides reflecting light. (For example, it is implausible that G causes color experience by generating longitudinal sound waves!)[21] Jackson all but affirms the reality of the (D_R→SSR→G) chain, furthermore, when he concedes:

> We know that objects have dispositions to look one or another colour [D_R], *that they have dispositions to modify incident and transmitted light in ways that underlie their dispositions to look one or another colour* [SSR], [and] that they have physical properties [G] that are responsible for both these dispositions . . . . (Jackson 1998b, 87, emphasis and brackets mine)

While the context of this quote does not mention PE, all the elements of the (D_R→SSR→G) programming chain are present, and it is scientifically reasonable to employ that expanded chain.

For completeness and fairness to Jackson, however, it pays to examine why he withholds explicit endorsement of the (D_R→SSR→G) programming chain. The main reason, as I interpret him, is that he harbors dim prospects for Hilbert's *specific method* of correlating sets of SSR values to color experiences.[22] For Jackson (1998b) suspects that SSR properties and G

[21] And as XXXXXX points out, scientific investigation outside the human-visible spectrum will have to rely on electromagnetic reflectance of *some* sort; for example to detect the quantum-physical or chemical behavior of microstructure (G).

[22] Hilbert (1987, 1944 ff.) correlates color experiences to "triple[s] of integrated reflectances," whereby the SSR value corresponding to an objective color is the scaled sum of integrated reflectance values (measured from objects) at the three main wavelength bands of human retinal sensitivity. The classic resource on this procedure is Land (1977), who is intriguingly *not* a color objectivist.

properties will anticorrelate in principle (111). Whereas the SSR profiles of red objects, for example, should closely resemble the profiles of pink objects (neither of which should resemble the profiles of green or blue objects), "[t]here is no reason to think the physical property we are latching onto when some particular thing looks red is similar to that we are latching onto when some particular thing looks pink . . ." (111). Jackson anticipates that colors causing similar experiences (e.g., two slightly distinguishable hues of red) could lack "distal commonalities" in their causal bases to such an extent that "[r]edness" would no longer be "the property in common to red things," and color eliminativism or a perceptual error theory about color would result (112). Thus, as I understand Jackson, he would rather ignore SSR correlations and assume that color is a "not excessively disjunctive" set of (G) bases (108).

In response, I am not sure that we should demarcate usefully heterogeneous disjunctions of causes by *absconding* from measurement technologies like the spectrophotometry that measures SSR. As I claimed two paragraphs ago, microstructures must be reflective in *some* sense, if we are to learn very much about them (even non-visually). The SSR property in the (D_R→SSR→G) programming chain fills that generic "reflectance" role, without committing the program explanationist to Hilbert's specific correlative method. Due to the ubiquity of reflectance throughout science and common sense, the best-credentialed version of Jackson's PE for color experience will follow the template of (D_R→SSR→G). Hence, I slightly expand Jackson's PE from, "the disposition to look red ensures that some-G-disjunct-or-other causes this red experience" (the D_R→G chain), to "the disposition to look red ensures that some-G-disjunct-or-other *qua reflective* causes this red experience" (the D_R→SSR→G chain). My emended PE does not render dispositional SSR causal, but only renders the PE appropriately specific about the G-process.

Taking stock, I have argued that (D_R→SSR→G) is an entirely reasonable and scientifically motivated version of Jackson's PE for color experience. As I argue in [SELF-CITATION REDACTED], however, there is a conceptual regress about SSR (equally applicable to its base G) that renders it unfit for ascribing as a property of anything. Unfit, because self-contradictory: the received definition of SSR turns out to mean something analogous to "non-cubical cube," and I deny that such a conceptually incoherent property could cause or explain color experience. The swiftest way to block this regress is to mathematically redefine SSR, which I do in the next section; this redefinition, in turn, renders Jackson's PE for color experience extra-mathematical, transforming the (D_R→SSR→G) chain, with some modifications, into EMPEC (see section 1).[23]

4. Mathematizing and Ascribing Reflectance

4.1 The Vicious Reflectance Regress

As I argue in [SELF-CITATION REDACTED], the main problem with ascribing SSR is that Hilbert's (1987) influential definition of it (given in section 3.1 above) turns out to be an operational definition that launches a regress of the reflectance value at any given wavelength. Specifically, the regress is in the ratio of reflected to incident flux, both flux quantities asymptotically plunging to zero and sending the SSR value in question to 0/0, an undefined and meaningless quantity unsuitable for ascribing as a *real* PE explanans. Granted, Hilbert's

[23] An anonymous reviewer wonders if the program explanation for color experience is simply backward, since the "more fundamental" categorical base (quoting my reviewer) seems to explain why a disposition obtains or manifests. Key to answering this objection, I think, is to point out that the *singular* event explained, such as the single momentous water molecule, or the single experience of seeing red, does *not* by itself reveal the state of a liquid as boiling, or the property of an object to look red in normal conditions to normal observers, respectively; an inference from multiple events may be needed to ascribe programming properties, or to draw modal inferences about what *would* have happened due to possible alternative events, but I do not know how Jackson and Pettit would answer the objection.

definition of SSR does not *appear* operationalist, since he treats the flux ratio as the value of a disposition that he calls a "stable propert[y] of objects" (1037-1041). But I will argue that Hilbert's definition of SSR in terms of finite-duration measured fluxes is not stable, but regressive. Stable SSR (I shall argue) is most readily defined in terms of infinite-duration (*viz*. mathematical) electromagnetic propagations.

Here's why. Hilbert follows spectrophotometric convention in defining "flux" as the average power in watts of a finite-duration signal (Hilbert 1987, 1033-1042; cf. Germer et al. 2014), but ascribing "average power" to a finite-duration signal is itself an operational colloquialism unsuitable for the metaphysics of property ascription. Yes, "average power" is profitably computed for finite-duration signals throughout science and engineering, but by an analogous turn of phrase, checking "what the thermometer says" saves lives in hospitals, governs power plant output, and feeds millions in cafeterias and restaurants. "What the thermometer says" is an operational definition of temperature, a more physically reduced definition being the mean kinetic energy (MKE) of substrate molecules. Analogously, I argue in this section that Hilbert's finite-duration pulse-SSR[24] is operational, and the metaphysician wishing to reduce color to a physical disposition needs some other kind of reflectance.

Interesting to note in this vein (and crucial to my argument), is that orthodox signal theorists reserve the ascription of average power to signals of *infinite energy*, which usually take the form of periodic signals of *infinite duration* (Haykin and Van Veen 1999, 20-21).[25] Why

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[24] By "pulse," I mean a finite-duration sinusoid like Figure 1 (a) below, although my arguments apply to other-shaped pulses used in laboratories, such as Gaussian and hyperbolic-secant.

[25] Formally, $E = \int_{-\infty}^{\infty} x^2(t)\, dt$ for a continuous-time signal $x(t)$ with total energy E (Haykin and Van Veen 1999, 20). The signals susceptible to average power ascription possess "infinite energy," therefore, because they tend to be periodic (Haykin and Van Veen 1999, 21), which means defined "for all *t*" (Haykin and Van Veen 1999, 18), which in signal theory means for all of positive and negative temporal infinity; and integrating a squared periodic signal forever, by the equation just cited, yields infinite energy.

(historically, strategically) signal theorists reserve average power ascriptions to infinite-energy signals is a question for further research, but for the present I can attest that by this discipline signal theorists at least *avoid* the per-wavelength regress that I generate for finite-duration "flux" in this section. The regress manifests in a straightforward thought experiment incorporating one final insight about light propagation that Hilbert (1987) overlooks. That insight I call "harmonic dispersion," the widely-documented empirical law of nature whereby electromagnetic pulse duration relates inversely to pulse bandwidth.[26] (Said another way, harmonic dispersion is what prevents the periods of finite-duration signals from being well-defined, per the discussion of footnote 25.)

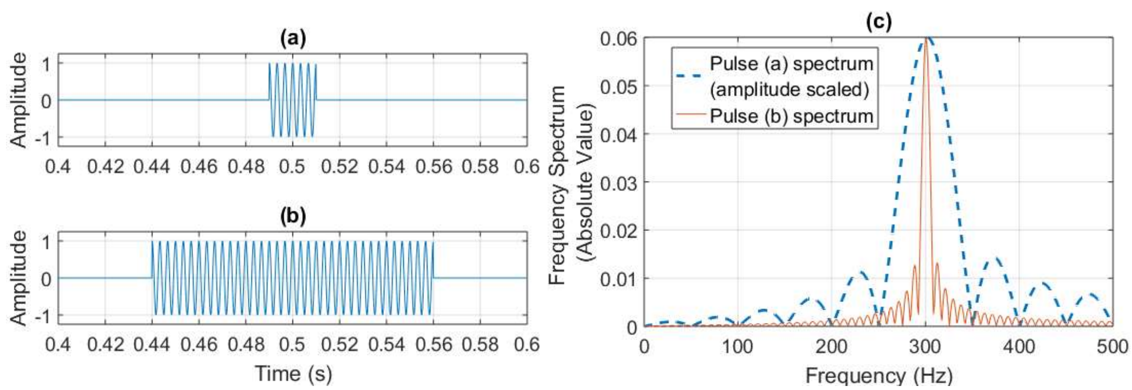

Figure 1: Harmonic Dispersion

An anonymous reviewer helpfully asks, "Isn't average power just the ratio of total power and total duration, and do finite signals not possess both quantities?" I answer that the "total power" of a signal is only well-defined *per wavelength* if its period is well-defined in the orthodox sense, which means defined for all of positive and negative temporal infinity. Otherwise, the average power of a continuous-time signal is $P_{avg} = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x^2(t)\, dt$ (Haykin and Van Veen 1999, 20), the period T again being well-defined only throughout infinite time. Because finite-duration signals do *not* possess well-defined periods T, their period limit is taken to infinity by the equation just cited, and their average power is zero. Hence technologists who ascribe "average power" to finite-duration signals are *pretending* that the period T of those signals is well-defined, when it is not.

[26] Hirliman (2005, 31) explicitly denies that this inverse relationship reduces to a duality of quantum physics, and emphasizes instead that harmonic dispersion obtains squarely within classical electromagnetics. Hence my applying harmonic dispersion against Hilbert's (1987) macroscopic account of reflectance appears fair, since I am not foisting on his macroscopic account a remote quantum problem.

Figure 1 depicts harmonic dispersion for pulses of sub-optical carrier or center frequency (for ease of modeling).[27] Pulse (b), whose duration is six times that of pulse (a), exhibits the greater monochromaticity of the two in graph (c), which depicts the "per wavelength"[28] power of the longer pulse (solid line) as more tightly concentrated about center frequency 300 Hz than is the power of the shorter pulse (dotted line). The upshot of Figure 1, I will argue, is that Hilbertian per-wavelength pulse-SSR is a contradiction in terms like "non-cubical cube," because pulses do not propagate "per wavelength"; a pulse instead propagates as an envelope of wavelengths (each trace of Figure 1 (c) is an "envelope"), and any per-wavelength components of such envelopes are themselves finite-duration, and so likewise propagate as envelopes, *ad infinitum*. Thus, *contra* Hilbert (1987), there is no *pulse* flux that dissipates at (say) 650 nm on the visible spectrum; something besides a "pulse" dissipates at 650 nm (see remainder of this section, and section 4.3), just as something besides "what the thermometer says" is what temperature *is*.

I launch the pulse-SSR regress (the "Vicious Reflectance Regress," or VRR) by assuming that an optical pulse centered at 650 nm,[29] with average power 5 Watts (W) and sub-picosecond[30] duration, propagates toward a perfect mirror with SSR = 1 across the human-visible band (400-800 nm). Laser researchers report such pulses to exhibit harmonic dispersion as wide as 100 nm (Stingl et al. 1995; Deng et al. 2005), so we can assume that our 650 nm pulse dissipates appreciable power up to 700 nm, and down to 600 nm, analogous to the 225 Hz and 375 Hz sideband dissipations of the dotted-line spectrum in Figure 1 (c). For calculational convenience,

[27] Visible light propagates in the hundred-terahertz (THz; 10^{12} Hz) range, but harmonic dispersion occurs at all usable frequencies and pulse durations, including those of radio and radar.
[28] "Frequency" and "wavelength" are interchangeable in physical optics. Frequency is speed of light divided by wavelength.
[29] As the pulses of Figure 1 are centered at 300 Hz.
[30] 1 ps = 10^{-12} seconds.

I assume that 20% of the 650 nm pulse dissipates at non-650 nm wavelengths (the accuracy of this estimate will matter little, because a regress is a regress).

The crucial, and apparently easy question at this point is: "How many watts will reflect from the perfect mirror at 650 nm?" According to the parameters I have laid down, the answer appears to be 80% of 5 W = 4 W. But the answer cannot be 4 W, I contend, because that 4 W, 650 nm component of the original pulse *itself* possesses a sub-picosecond duration (what other duration could it have?). Hence by the empirical law of harmonic dispersion, that 650 nm, 4 W "component" must itself disperse power to its sideband wavelengths and become only 3.2 W at 650 nm, a pulse of short duration which must re-disperse *its* power to yield only 2.56 W at 650 nm, which must re-disperse *its* power to yield only 2.05 W at 650 nm, *ad infinitum*.

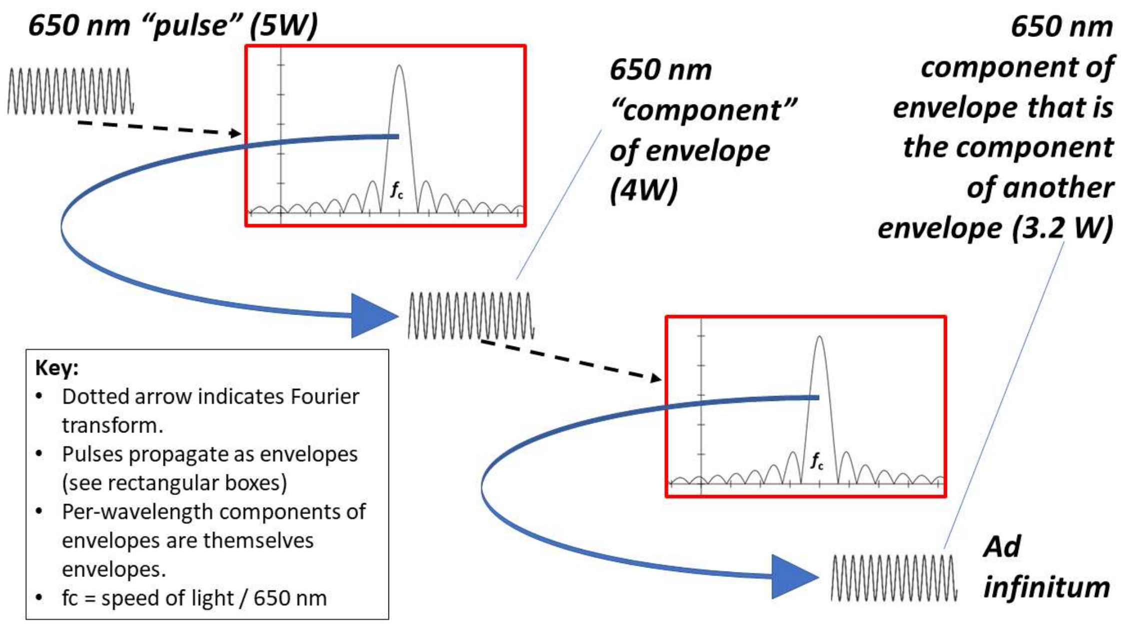

Figure 2: The Vicious Reflectance Regress.

Figure 2 depicts the VRR for our assumed 650 nm pulse. Contra Hilbert and our assumption of the last two paragraphs, the pulse-SSR value of the perfect mirror at 650 nm is not 1, but is rather asymptotically approaching the meaningless, undefined, and non-ascribable value of 0/0, since the incident and reflected "fluxes" both asymptotically regress to 0 Watts.[31]

Important for philosophical consideration is that I have not shown the VRR to be a *physical* regress (since we see ourselves in mirrors somehow), but I do not need to. The scientist likely measures 4 W at 650 nm, and thinks nothing further of it, but whatever the scientist measures is not a *pulse*, by the argument just given.[32] The pulse-SSR value at 650 nm asymptotically approaches 0/0, and I do not find a property with that value to be a plausible explanans in Jackson's program explanation for color experience (and recall that I argued in section 3 that reflectance or SSR *is* a plausible explanans of that PE).

4.2 Two Intermediate Objections

Before proposing my mathematical redefinition of SSR that blocks the VRR, two objections merit dispatching.[33] The first is that I too severely demand completeness in the definition of a scientific property (reflectance); the second is that the VRR has nothing to do with *color* objectivism, or the ontological reduction that interests Jackson (1998a), since I run the VRR at picosecond durations in which humans cannot distinguish colors (Scase and Foster 1988). By the first objection, one could claim that other real properties, like the MKE definition

[31] One may object that an asymptotic approach to zero never reaches zero, and because numerator and denominator regress at identical iterative rates (an arbitrary assumption), the pulse-SSR ratio retains its value of 1 like we assumed. The problem with this response is that a 5 W pulse incident to the mirror still reflects with ≈ 0 W at 650 nm, or never really propagated with anything close to 5 W in the first place; both absurd conclusions.

[32] An anonymous reviewer suspects that the harmonic dispersion "becomes smaller and smaller" through its regressive iterations, allowing the 80% bulk of the first iteration to reflect as expected (4 watts), but this construal of the VRR is mistaken. What is getting smaller and smaller is the power component of the central wavelength of interest (650 nm), and not just the sidebands, nor just the 'noise' or 'error' of the signal.

[33] See additional objections to the VRR, with replies, in [SELF-CITATION REDACTED].

of temperature, is scale-bounded in its definition just as SSR appears to be (since the harmonic dispersion percentage for long-duration signals is nowhere near as high as the 20% assumed in section 4.1). In this vein, Peter Smith (1998) argues that "*there is no fact of the matter*" (39) about the value of a substrate's MKE temperature beyond a few decimal places (40-41), since molecular velocity through the vanishing sphere used to estimate molecular kinetic energy becomes undefined. I reply that the MKE problem is one of chasing asymptotic *accuracy*, unlike the VRR which destroys the reflectance value on both sides of the decimal place by sending the reflectance value to 0/0. Whereas for Smith the *error* becomes smaller and smaller to the point of incoherence (see my footnote 32), the VRR ruins the pulse-SSR concept by obliterating its value irrespectively of error margins.

Secondly, and for the same reason, it does no good to object that humans fail to discriminate colors in the sub-picosecond range, because the object of my investigation is the metaphysics of program explanation rather than perceptual theory. I have argued that pulse-SSR is too regressive and incoherent to ascribe as a *real* program explanans, and Jackson (1998a) does not treat any property in the color experience PE as operational; he posits real dispositions and bases responsive to natural laws in the actual world. Harmonic dispersion is undeniably one of those laws, and so the program explanationist needs a better-defined reflectance property than pulse-SSR. I propose such a redefinition next.

4.3 The indispensability of Fourier harmonics to reflectance ascription, and their reality

The most straightforward way to block the VRR is to heed the orthodox signal-theoretic meaning of "average power." In this vein, I redefine reflectance from the per-wavelength disposition to reflect "pulses" (section 4.1), to the per-wavelength disposition to reflect the

Fourier harmonics that superimpose without remainder into pulses; for due to their infinite duration, harmonics never disperse, and so never launch the VRR. I have already illustrated harmonic superposition, moreover, in Figure 1. Every plotted point in Figure 1 (c) indicates the amplitude of an infinite-duration harmonic oscillating at the single frequency listed on the horizontal axis. Superimposing all such infinite-duration harmonics yields the finite-duration pulses shown in Figure 1 (a) and (b) by the Fourier transform.[34]

Thus, when the laser scientist measures a "4 W" reflection at 650 nm, in my account (see section 4.1), she is measuring some portion of an infinite-duration optical *harmonic* at 650 nm, a portion not cancelled-out by its neighboring harmonics.[35] No matter the duration of the incident "pulse," the reflective efficiency at 650 nm is "stable" and unchanging, because only a constituent harmonic of a pulse, and not the pulse simply construed, is reflecting at 650 nm; the VRR is vanquished.

The appropriate program explanation for color experience, therefore, becomes ($D_R \rightarrow SSR_H \rightarrow G$), where SSR_H is the disposition of a surface to reflect optical harmonics instead of pulses. This programming chain is EMPEC, and interestingly, it reifies Fourier harmonics *without* the EIA, since SSR_H—if it is to be stimulated and manifest at all—must do so via *real* infinite-duration waves (harmonics), on pain of launching the VRR (sections 4.1-4.2).[36] This EIA-free mathematical realism contingent on property realism, however, is only the secondary

[34] The Fourier transform also requires a per-wavelength phase shift and negative-frequency harmonics, which I omit for clarity.

[35] Just as the portion of the pulse between 0.49 and 0.51 seconds in Figure 1 (a) is not cancelled-out by the dotted-trace harmonics in Figure 1 (c), but the regions of Figure 1 (a) outside 0.49 and 0.51 seconds *are* cancelled out by that superposition.

[36] I am admittedly being vague about what kind of mathematical realism follows from the reality of harmonic-SSR (SSR_H). Are the harmonics real as Aristotelian immanent universals, transcendent Platonic universals, or as something else? I suspect that I need not answer that question in this paper (since Lyon 2012 does not in his paper), but I would allow that the Fourier harmonics are being *instantiated* as Berenstain (2017, §5.3) explains that term.

thesis of my paper. My primary thesis is that EMPEC overcomes Saatsi's objections to extra-mathematical program explanation (cf. section 1), an argument to which I turn next.[37]

Before so turning, however, I must answer the objections of two anonymous referees, who take issue with my claim of reifying Fourier harmonics "without the EIA." One referee suspects that I am (a) "making an obvious mistake of reifying a mathematical description," while another referee constructs the dilemma that (b1) one can "resist being a realist . . . and make do with the explanatory usefulness of the proposed models [of harmonic-SSR]," or (b2) I am only a "realist about the [mathematical] explanantia . . . because of the explanatory role they play in the particular scientific context . . . ." I preempted objection (b2) in section 1, when I analogized my harmonic-reifying inference from reflectance ascription, to the *intention* to murder in a voluntary act known to be deadly. I might fire a gun in a dark alley, for example, only to stop an attack; my plans do not require that my attacker dies, but only that he stops; so if he dies, some would grant that I did not intend to kill. Similarly, I need not *intend* to explain color experience (or anything else!) when I coherently ascribe real SSR and its concomitant harmonics; the intention to explain incidentally follows in this paper about Jackson's work, but anyone could study

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[37] A number of objections to EMPEC must be bypassed in the interest of space. Some might wonder, for example, how the stimulus and manifestation of harmonic-SSR could really be infinitely long. I defer to Morganti (2013, 179) and Barrow (1998, Chap. 6) on the possibility of an infinitely-sized universe, through which infinite-duration light would have room to propagate. On whether a disposition with eternal manifestation would still be a "disposition," see Chakravartty (2013, 45) for the preliminaries of a positive answer.

Others might press me to explain why harmonic-SSR is not an unreal idealization of reflectance. Pincock (2014), for example, assumes an infinitely-deep ocean to calculate the phase speed of water waves, but he retracts that idealization at the end of his analysis as not ontologically referring; why cannot I do the same with the harmonics of harmonic-SSR? My simple answer is that Pincock faces no *conceptual regress* by withdrawing the infinitudes he posits, whereas I clearly do, in the form of the VRR.

Lastly, to those who object that I fixate on a Fourier reduction base when there are other mathematical methods for representing propagating light, I reply that the onus is on the objector to show which mathematics predicts harmonic dispersion but avoids the VRR; wavelet analysis, for example, fails to block the VRR, since wavelet bases are manifestly heterochromatic with spectra resembling the "envelopes" of Figure 2. On the nominalizability of Fourier harmonics, see section 5.

reflectance ascription for its own sake and come to my harmonic conclusion. As contentious as the debate on action intentionality is,[38] I suspect that (b2) and my denial of it can each be reasserted until they cease to be interesting; hence I call my denial of (b2) a secondary point of this paper.

Objections (a) and (b1), on the other hand, come to the same claim—that I reify without warrant that which is only conceptually useful—and the supporting arguments for this kind of criticism are vast and nuanced. I reply that the context of program explanation blunts these criticisms—but again, not because I am operating in a context of scientific *explanation*, but because PE conveniently traffics in *property realism*. As I attempt to ascribe the real property "reflectance" for its own sake, one can accuse me of (a) reifying a mathematical description; but so much the worse for property realism! Without mathematics, one lacks coherent property-realist claims about reflectance (I have argued).

Thus, I find objection (a) to land against property realism before it lands against my alleged (mis)use of a representation of reflectance, because unlike other scientific representations depicting a world that we can "go out and touch," reflectance is supposed to be the property by which we visually interact with anything; we cannot "go out and look" *at reflectance* after tinkering with its representation. Reflectance is that whereby we look at anything, so defining this pre-experiential property with mathematics seems unobjectionable, and the real complaint behind (a) is that this pre-experiential property is then *reified* (along with its definitional mathematics) by the *property realist*. I remind the reader that Jackson and Pettit (1990) are the property realists; I am doing the pre-experiential mathematical work behind the scenes to make a particular program explanans coherent and non-regressive. Hence, I similarly dispel objection

[38] Tollefsen (2013) summarizes one such pernicious saga.

(b1), because Jackson and Pettit (1990) traffic in real properties rather than "models"; if Jackson and Pettit *should* traffic in models, then objection (b1) befalls those authors before it befalls me.

5. Can Mathematical Properties Program Physical Events?

With EMPEC now articulated, motivated, and defended on a first-pass level, I can argue that EMPEC meets Saatsi's (2016, §5; 2012) objections to Lyon's (2012) approach to extra-mathematical program explanation (section 1). Those objections are threefold: (i) that Lyon's examples like cicadas and Bridges of Königsberg do not fit the PE template, since their explananda patterns are not "events" (Saatsi 2012, 580); (ii) that Lyon's explanantia are nominalizable into non-mathematical terms (Saatsi 2012, 581-582); and (iii) that the explanatory relation of "making probable" or "ensuring" within PE, between mathematical entities and the physical world, is mysterious (cf. section 2). The argument from authority blocks the first of these objections (i) against EMPEC, since Jackson (1998a) is a co-progenitor of PE, and thus the author of a genuine PE for color experience whose explananda are genuine events.

EMPEC also resists objection (ii), I claim, but in a way that most parties to the indispensability debate have overlooked. Firstly, I *agree* with Saatsi that many if not all of Lyon's (2012) examples are nominalizable. With respect to the extra-mathematical program explanation of cicada life cycles, for example, Saatsi denies that primeness explains the 17-year period, since a "fact about time" could instead explain the period: the fact that "[f]or periods in the range 14 to 18 years the intersection minimizing period is 17," conjoined with other premises, entails that the cicadas emerge every 17 years (Saatsi 2016, 1050; cf. 2011, 149). The fact that time divides into units, in other words, and that concatenations of these units can be represented by (e.g.) sticks of various lengths, and that chains of length-17 sticks extend the longest before equaling the lengths of chains composed of length-14 or length-18 sticks, etc., reveals the evolutionary advantage of a 17-year life cycle without appealing to mathematics (cf. Saatsi 2011, 150 ff.).

When it comes to nominalizing the Fourier harmonics indispensable to reflectance ascription, however, the situation changes. Michael Liston (1993, §2) has argued persuasively for the practical impossibility of nominalizing the Fourier transform into "betweenness" and "congruence" relations like Hartry Field's (1980), but my resistance to Saatsi's nominalizing objection (ii) is different, and by now familiar. I argue that EMPEC withstands objection (ii) because EMPEC withstands objection (i). Nominalize the infinite duration of the harmonic into an unreality, or approximate that infinitude into a finitude, and reflectance ceases to be a real property (section 4.1), which undermines the *property realism* of PE as much as it undermines the viability of EMPEC as extra-mathematical (section 4.3). That is, by insisting that the harmonic can be nominalized, the Saatsian working within the PE framework incurs the awkward position of denying vastly more existents than she likely set out to deny. In the cicada case, the Saatsian denies a working PE because the property "primeness" is not a real existent, and the consequences for the world end there. In the EMPEC case, denying real harmonics amounts to denying the reality of reflectance—an astounding setback for all of science. Of course, the Saatsian might for independent reasons reject dispositional realism and prefer an epistemic account of (say) explanatory models for light propagation; but that avenue of argumentation only changes the subject, and renders Saatsi's (2012; 2016) previous criticisms of extra-mathematical program explanation somewhat superfluous, if the *property realism* of PE was the problem with those explanations all along.

To avoid the very strong consequence of telling Jacksonians and the rest of the world that reflectance is not a real property, perhaps one could mathematically nominalize reflectance into the disposition to reflect waves whose peak amplitudes are mutually "congruent" forever and ever in time (*viz*. infinitely), dispensing with the harmonic proper. But in reply, I can side with Azzouni (1994, 3) in understanding the infinite, under any guise, to be something quintessentially mathematical. This position renders even the "forever-congruent-amplitudes" nominalization of EMPEC extra-mathematical, pending an independent argument for the application of non-mathematical infinitudes to science and metaphysics; and advancing such an argument will not be trivial. To summarize, then, EMPEC withstands Saatsi's nominalization objection (ii) because either (a) the nominalization turns out to be alarmingly strong against realism about a basic scientific property (see previous paragraph), or (b) a compelling argument for non-mathematical infinitudes remains to be broached.

Lastly, EMPEC withstands Saatsi's objection (iii), that the PE relation of "making probable" between mathematics in the explanans, and physical events in the explanandum, is mysterious and scarcely intelligible. If one dips a hollow wire cube into soap solution and removes it, soap film will coalesce within the cube in a repeatable, particular shape; Lyon (2012) program-explains this shape by a mathematical fact (Plateu's laws) about the film geometry that minimizes film surface area and hence the potential energy between soap molecules (563-564). Saatsi objects:

> But, of course, no real soap film actually instantiates the *exact* properties investigated by a mathematical theory of minimal surfaces (e.g. geometric measure theory): what we have are idealized mathematical models of real soap films that ignore forces other than those that keep the film together. More importantly, an explanatorily relevant mathematical property can always be instantiated (in this loose sense) in a physical system without being 'realized' in the causally efficacious properties related to Plateau's laws. (For instance, we might craft a model of soap film from some wire and a sheet of copper, also

> instantiating the minimal surface area for a given geometric configuration.) Thus, such a mathematical property does not *ensure* the instantiation of the relevant causally efficacious properties that ground a lower-level explanation of the shape . . . . (Saatsi 2012, 583)

If I understand Saatsi correctly, Plateau's laws do not "ensure" that *soap* does anything, because Plateau's laws can be just as well approximated in copper sheet and wire; thus, there is no "making probable" relation between Plateau's laws and soap. Similarly, Saatsi questions how "primeness" makes probable that cicadas will conduct their lives in a certain way, and how graph theory problematizes the pedestrian's attempt to cross of all of Königsberg's bridges in a continuous sequence without backtracking or repeating any crossing.

I reply that EMPEC de-mystifies the "making probable" relation between its explanantia mathematics and its explananda events by legitimately shifting the question from "What can mathematics do to the world?", to "What can mathematics do to our ascriptions of *real* properties?"—the latter question being more applicable than the former, and to that extent less objectionable. Indeed, I think that the latter question prompts skeptics of mathematical ontology to consider their own underdiscussed commitments to property ontology, and the closer-than-comfortable relationship that sometimes obtains between mathematics and property ascription (a relation different from that obtaining between mathematics and *phenomena*, the relation dominating indispensability debates). While I am not claiming that indispensably mathematical properties like reflectance are common, I think that indispensably mathematical property definitions are *extremely* common, and that we simply do not reify those properties. Consider, for example, the Pythagorean Theorem. Designing bridges with 3-4-5 triangle sections is likely crucial to the safety of urban commuters, but when pressed on whether the 3-4-5 triangle is *exactly* instantiated, most will admit that the steel sections are actually mashed or warped slightly into a 4-or-more-sided structure that distributes loads as a 3-4-5 triangle *would* distribute them, to high approximation.

One should not suppose, however, that I have forfeited the argument by weakening the "making probable" relation, for I do not think that I have weakened it at all. The Fourier harmonic program-explains color experience, because the harmonic *makes ascribable* the property (SSR_H) that renders plausible the causal efficacy of (G) (sections 3-4). If there are no harmonics, then there is no reflectance, and without reflectance, the hypothesis that (G) causes color experience becomes highly improbable. Hence, I think that making-probable sometimes amounts to making-ascribable, and one might ask what are the necessary and sufficient conditions for a property's ascribability. While I cannot make much headway on this interesting and deeply fundamental question in this space, I can at least remark that all parties to the indispensability and program explanation debates have hitherto *respected in practice* the conceptual coherence about property ascription that I value. Solids are cubical or non-cubical in these debates; explanatory solids are not the "non-cubical cubes" to which I analogized pulse-SSR in section 4.1. Granted, Dan Marshall (2021) examines views that might exclude my necessary condition of conceptual coherence for ascribability, since they hold that "non-cubical cubes," even if not instantiable, possess that property intrinsically; hence adherents of these views might call the property of being a non-cubical cube ascribed or ascribable. I reply that if the ascription of a property as fundamental, simple, and commonplace as reflectance is to be analogized to something as strange as a non-cubical cube, then property realism remains no *less* surprisingly implicatory than I have described it. In the meantime, section 4 reveals how important I find mathematics for regress-blocking in property ascription.[39]

[39] An anonymous reviewer recommends that I add the references Morganti (2015; 2014), Bohn (2009), and Schaffer (2003) to my footnote 37 discussion on the plausibility of an infinitely spatial universe, but I

One consequence of my thesis—that mathematics makes physical events probable by making program-explanantia ascribable—is that it sets the bar high for legitimate *extra-mathematical* program explanation; for in addition to passing Saatsi's criteria of (i) PE structure and (ii) non-nominalizability, the program-explainer must ascribe the mathematical explanans as actually and perfectly instantiated (for the PE to be extra-mathematical), and I think that such ascriptions are rare. Jackson and Pettit (1990, 110), for example, argue that the "squareness" of a peg with side-length equal to the diameter of a hole program-explains why the peg will never pass through the hole but will instead always collide with its circumference (and why this rule generalizes to all pegs and holes so related). Assuming that "squareness" is mathematical and non-nominalizable, it is arguably not perfectly instantiated, and so the PE is not extra-mathematical, despite appearances. Why or how approximately square pegs program collisions with holes of approximately correct diameters remains a curiosity that some may find unsatisfying, since geometrical facts are supposed to explain such predictable interactions of bodies whose dimensions approximate geometrical relations.

Nevertheless, EMPEC answers Saatsi's third objection (iii) by showing that the "making probable" relation is not always what's mysterious; *property realism* is mysterious and vastly more implicatory than indispensabilists and their critics have acknowledged (cf. section 4.3). While I have not identified a univocal meaning for the "making probable" relation, I do not see the need to do so. I have argued instead that making-probable can be as straightforward as making-ascribable (which is at least as straightforward as avoiding self-contradictory property concepts like "non-cubical cube"), and that actually ascribing real properties is one kind of

think that these authors even better present reasons not to 'fear' the VRR, and to accept it as ontologically fundamental without dire need of 'solution', especially not solution by mathematical realism. Developing this tolerant and even warm response to the VRR would of course require a separate paper, but the thesis is intriguing.

metaphysical heavy-lifting that entails a limited mathematical realism—whether ascribers realize it or not.

6. Conclusion

In this essay, I proposed a counterexample to Saatsi's (2012; 2016) tripartite objection to extra-mathematical program explanation (PE). My counterexample is Frank Jackson's (1998a) PE for color experience, which I argue is extra-mathematical for implicitly incorporating the reflectance property that suffers conceptual regress unless redefined with Fourier harmonics. I named this counterexample EMPEC, and I argued that it meets Saatsi's objections because (i) EMPEC's explananda are legitimate events, (ii) the Fourier harmonics indispensable to its explanantia are non-nominalizable (because of the regress mentioned in the previous sentence), and (iii) EMPEC renders the "making probable" relation of extra-mathematical program explanation no stranger than the conceptual and ontological steps needed to render some properties ascribable. One of these steps, I assumed throughout, is conceptual coherence, which I equate to non-self-contradictoriness about a property's definitional concept (an offending example being "non-cubical cube"). If a mathematical entity or property renders a program explanans conceptually coherent and ascribable, then that mathematical entity *thereby* renders some physical event(s) probable. This "ensuring" or "making probable" relation by a mathematical entity might turn out to be unintuitive or modally opaque: I see a red tulip because harmonics propagate at one frequency for eternity; but impoverished PEs are PEs after all (section 2).

A corollary of this result is that program explanation facilitates mathematical realism without the Enhanced Indispensability Argument, since ascribing properties is a self-intelligible act that can transpire absent any explanatory agenda. While the focus of indispensability discussions has shifted from confirmational holism and posit quantification to scientific explanation and scientific explainers (Marcus 2015, Chap. 7), I think that this step has in some cases been a step too far from metaphysics, since it leads interlocutors to ignore the ontological import of the *property* realism that many theorists appear to espouse in their unguarded moments.

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