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Is Fourier Analysis Conservative over Physical Theory?

By: Nicholas Danne, dann0027@umn.edu

Abstract:
Hartry Field argues that conservative rather than true mathematical sentences facilitate deductions in nominalist (i.e., abstracta-free) science without prejudging its empirical outcomes. In this paper, I identify one branch of mathematics as nonconservative, for its indispensable role in enabling nominalist language about a fundamental scientific property, in a fictional scientific community. The fundamental property is electromagnetic reflectance, and the mathematics is Fourier analysis, which renders reflectance ascribable, and nominalist reflectance claims utterable, by this community. Using a recent characterization of conservativeness by Kenneth Boyce, I argue that infinitudes can be rendered inherently mathematical and non-nominalizable in the fictional community, and that rendering infinitudes inherently mathematical for all real communities would yield a convincing counterexample to Fieldian conservativeness.

Main text:

I.

Hartry Field (1980, 13) famously argues that the mathematics suitable for application to science should be conservative, in the sense that conjoining[1] mathematics to nominalist science should yield no conclusions not derivable from nominalist premises alone. Field (2016) defines nominalism as the non-existence of "abstract entities," and he assumes that mathematical entities "would be abstract if they existed" (1). Hence, because he takes seriously[2] the Quinean admonition against "'doublethink'" (vii)—quantifying over the posits of fundamental physical theory while withholding belief in the posits—Field prioritizes the dispensability of mathematics to science as a defense of nominalism. Shoring up that defense is his conservativeness claim,

[1] Field (2016, xxii) speaks of set-theoretic "union" rather than conjunction (logical or otherwise), but my source below (Boyce 2020) uses "conjunction," so I follow suit; nothing about my argument hinges on this terminology.

[2] Or did in the Field (1980) formulation of his views. Field (2016, vii-viii) explains how his concern has shifted from refuting Platonism (realism) about mathematical existence to refuting Platonism about mathematical objectivity, e.g., the notion that "there's an objectively correct answer to the size of the continuum" even if we do not yet know what it is (viii). My arguments apply regardless of this shift.

i.e., that the dispensable mathematical language is not even true, but only conservative, and thus bound to have nothing inherently to do with "what the physical world is like" (xxii).[3]

In this paper, I argue that if one could show that the infinite is an inherently mathematical notion, then at least one branch of mathematics is nonconservative over physical theory—Fourier analysis, which incurs widespread application across science and engineering. The antecedent of my conditional thesis I will not even try to defend, because doing so would require a separate paper. The Fieldian, for example, expects a mathematical infinitude to be or involve abstract objects—including those that would exist despite the existence of any particular *physical* infinitude (cf. Field 1980, 95), whereas René Guénon (2001) denies that the infinite is mathematical *or* physical; and whereas Brouwer might relegate infinitudes to intuited sequences alone (Körner 2009, 146), Jody Azzouni (1994) affirms that infinitudes are mathematical because "not first-order definable" (3).[4] My alternative approach, which I offer strictly for the sake of argument, treats as mathematical what is excluded or missing from the nominalized scientific and commonsensical worldview N* of interest to the Fieldian (see below). Granted, this demarcation is imprecise. If God is excluded from N*, I need not call God mathematical. The point, rather, is that scholars have understood Field to argue "that the background mathematical theories which are *in fact* used in science are conservative" (Boyce 2020, 13), without always explicating what is *in fact* the content of nominalist background theory N*. My paper attempts to drive a wedge between these two facts of the matter: if infinitudes are not part of the N* that I select for a thought experiment in §II, then incidentally, as far as that nominalist view is concerned, Fourier analysis is nonconservative. Here someone might object that my

[3] Yablo (2005, §4) points out that Field leaves unexplained the applicability of conservative mathematics to science, and that if the nominalist could explain such applicability, then (in)dispensability would be a "red herring" in the defense of nominalism. This insight reemerges in footnote 30 below.

[4] See also Azzouni (2009, 146), on infinitesimals as mathematical.

setup is artificial and unfair, and that every nominalist claim ever tendered or that could be tendered should be included in N* when nominalizing a piece of mathematics. I grant the objection while maintaining that it misses the point: the point is not that the infinite has never appeared in any N* (on the contrary, it has), but rather that *if* an argument could be made that those appearances are disingenuous or plain faulty, then my application of Fourier analysis to science would provide a striking counterexample to conservativeness.[5]

Now for a word on the consequent of my conditional thesis. When I argue that "Fourier analysis" is nonconservative over physical theory, I focus on a precise operation within Fourier analysis known as the Fourier transform: the superposition of weighted and phase-shifted "harmonics"—infinite-duration monochromatic sinusoids—into finite-duration pulses[6] that satisfy Dirichlet conditions.[7] The Fourier transform perspicuously relates the frequency and time domains of analysis, and so researchers employ it to ascertain the frequency components of pulses, the bandwidths of filters, the states of physical systems, and more. Before outlining my use of the Fourier transform, however, I must at last formalize what I mean by "conservativeness," borrowing Kenneth Boyce's (2020) paraphrase of Field:

> **Conservation:** For any background mathematical theory, MT*, that is suitable for use in science, the conjunction of MT* with any body of non-mathematical statements, N*, has as logical consequences all and only the same non-mathematical statements as does N*. (Boyce 2020, 13, formatting mine)

[5] I pass over without exploiting Shapiro's (1983) insight that Fieldian nominalization accommodates only semantic rather than deductive conservativeness. I understand my Fourier example as more amenable to models than to derivations, and so I assume it to defeat semantic conservativeness, but I undertake no formal investigation of this distinction. Anticipating my work in this paper is Liston (1993, 444), who suspects that Fourier analysis is non-conservative over physical theory, but who works out no concrete example. Another criticism of conservativeness bereft of counterexamples is Melia (2006).

[6] By "pulses," I mean the waveforms of Figure 2 (a) and (b), below. Laboratories generate and measure pulses with more complex profiles than the simple sinusoids of Figure 2, but this difference is irrelevant, as I later argue.

[7] Dirichlet conditions fall outside the scope of this paper (see Haykin and Van Veen 1999, 172).

Here we see that conservativeness is a *logical* property, although what is meant by logical, as in *a priori* or necessary, is not entirely clear.[8] As I alluded one paragraph ago, furthermore, Boyce suggests that the content of N* includes "any" non-mathematical statement; presumably "any" that could possibly be conceived.[9] Thus, to conclude that Fourier analysis is nonconservative, I will argue that adding Fourier-mathematical sentences[10] to nominalist sentences N* about physical reality entails nominalist sentences about physical reality not derivable from N* alone. Particularly, I argue that nominalists studying light propagation require Fourier mathematics to assert *nominalist* claims about surface reflectance, one of the most fundamental physical properties in science. Such nominalist claims include, "This surface is reflective," and, "Surface A is more reflective than surface B"; for brevity, I call these claims Reflectance Claims.

II.

To show the nonsensicality of Reflectance Claims in nominalized[11] scientific theory, I introduce a thought-experimental world named Shineland, whose inhabitants speak a mathematics-free English, and who have developed physical theory N*.[12] Shinelanders are simple folk, in that they occupy themselves mostly with studying the average power of light that

[8] Even Field's (1980) chapter 9, "Logic and Ontology," does not elaborate the nature of logic.

[9] And by whom? Kitcher's (1988) Ideal Theorician?

[10] I treat sentences, assertions, equations, expressions, etc., as the same thing, context permitting.

[11] That is, *mathematically* nominalized, as I assume throughout. I allow realism or nominalism about properties and laws.

[12] An anonymous reviewer questions the need to posit an imaginary world besides the actual world of Earth. My main reason for doing so is to avoid the dismissal that Field (1985, 242) gives to a number of unspecified counterexamples to his conservativeness claim, on grounds that the background scientific theories in those counterexamples are not mathematically nominalist. Field finds some remarkably straightforward theorems of mereology to refer to sets (242, 250), for example, and so I strive to construct as mathematics-free a world as possible, for the sake of argument.

impinges on surfaces, and the average power of light that propagates away from those surfaces after they are impinged. For the chief industry of Shineland is reflective signage manufacturing, and the ratio of average powers just described is a popular definition of electromagnetic reflectance.[13] In mathematized English, the average power of a finite-duration pulse is:

$$P_{avg} = \frac{1}{T}\int_{-T/2}^{T/2} x^2(t)\,dt \qquad [1]^{14}$$

where T is the fundamental period of the propagating signal, x is the amplitude of that signal, and t is time. The units of average power are watts, but Shinelanders do not refer to numerically quantified watts (e.g., a "5W" pulse). Shinelanders instead nominalize pulse power values into wattages that are **between** others, and **congruent** with still others, in the Fieldian (1980) sense.[15] Indeed, Shinelanders nominalize the entire average power integral (Equation 1), replacing any reference to divisions by "2" and powers of "2" within it, as well as any mathematical characterizations of the signal, such as $x(t) = \cos(\omega t)$,[16] the details of which I must bypass in this paper.[17]

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[13] Popular in Earthling spectrophotometry and philosophy of perception, the latter influence due to Hilbert (1987, Ch. 3) and Byrne and Hilbert (2003; 2004; 2007). Philosophically, Byrne and Hilbert treat reflectance as the ontological reduction base for surface color, but I remain neutral about such controversies in this paper.

[14] Equation 1 is from Haykin and Van Veen (1999, 21).

[15] Very roughly, Field's (1980) nominalism expunges natural numbers from science by pointing out that one need not say that a pot of boiling water is "100 °C," if one can instead specify its temperature as lying **between** one object's temperature and another object's temperature, or as possessing a temperature difference with another object that is **congruent** with the difference in temperature between two other objects. Substituting these "objects" with spacetime points, as Field does, generalizes scientific laws.

[16] ω represents angular frequency, which is 2π times the center or carrier frequency (in Hertz) of a simple (co)sinusoid.

[17] On the nominalization of integrals, which Field (1980) did not attempt, see Field (2016, §0.7).

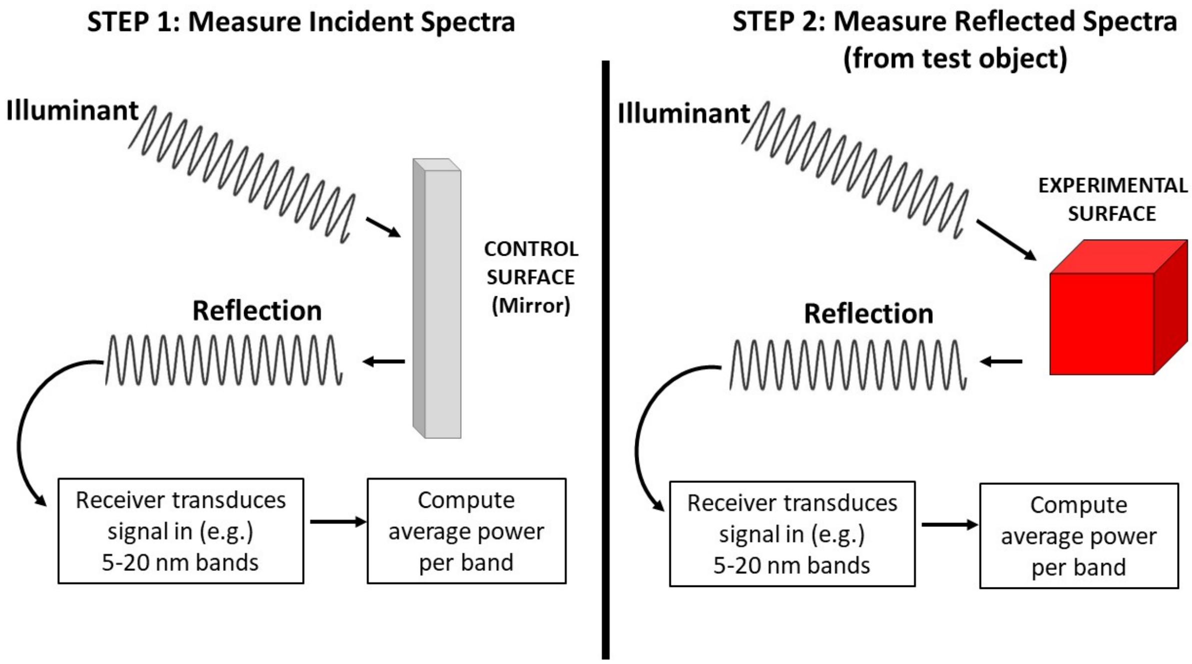

Figure 1: Pulse Average Power Measurements

Figure 1 depicts (in mathematized English) how Shinelanders (and Earthlings) measure pulse average power, *en route* to measuring reflectance. The reflectance of the shaded cube in Figure 1, for example, is the ratio of the average power measured in Step 2 to the average power measured in Step 1. Scientists compute this unitless ratio "per wavelength" across the human-visible band (400-800 nm), and the reflectance profile for the shaded cube (which is red) likely peaks around 650 nm. Thus, given Equation 1 and the Shinelanders' nominalization techniques, Reflectance Claims about the cube and the mirror in Figure 1 appear unproblematic. With pulse average power measurements in hand, the reflectance ratios of objects are easily compared, and so a given cube may be called "more reflective" than another, or indeed "reflective" at all. Why, then, do Shinelanders dismiss Reflectance Claims as nonsensical, and refuse to utter them?

The answer is that when you study the "average powers" of finite-duration pulses as a matter of habit, you begin to realize that there is no sound way to define "reflectance" in terms of them. Through their constant scrutinization of electromagnetic pluses, the Shinelanders invariably notice what I call "harmonic dispersion," the empirically confirmed[18] inverse relation of each pulse's duration to its bandwidth. It is the reality of harmonic dispersion, I will argue in §III, that renders Reflectance Claims nonsensical, when one defines reflectance in terms of pulse "average powers."

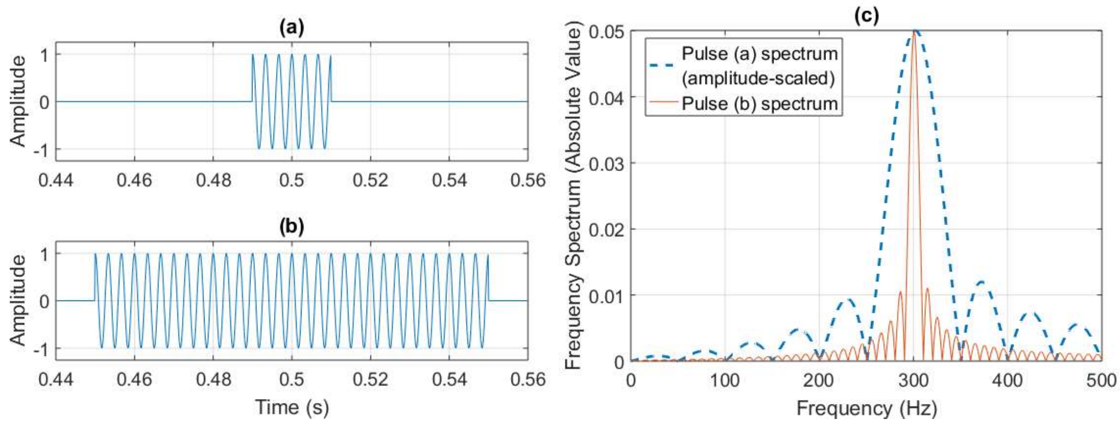

Figure 2: Harmonic Dispersion

Figure 2 depicts harmonic dispersion in mathematical terms, for pulses (a) and (b) with arbitrarily low carrier frequency of 300 Hz (for ease of modeling). The point to notice is that although input signals (a) and (b) enter the analysis as monochromatic, differing only in their durations, the spectral content of the two signals differs considerably (graph c).[19] The longer the duration of a monochromatic pulse, the narrower its bandwidth (as the solid-trace profile is narrower than the dotted profile, in graph c), and so the ability of a finite-duration pulse to propagate 'per wavelength' is compromised as soon as it is properly conceptualized. I say more

[18] Across many scientific domains on Earth. See Stingl et al. (1995) for a pronounced example.

[19] The "spectral content" in graph (c) is the Fourier transform of finite-duration pulses (a) and (b): $G(f) = \frac{AT}{2}\{\text{sinc}[T(f - f_c)] + \text{sinc}[T(f + f_c)]\}$, for pulses of amplitude A, period T, and center or carrier frequency f_c (Haykin 1989, 37. *Ibid.*, 18, defines $\text{sinc}(x) = \sin(\pi x)/\pi x$). For clarity, I omit from Figure 2 the negative-frequency harmonics generated by "sinc[$T(f+f_c)$]," and the per-wavelength phase plots that partially define the harmonic distribution.

about proper conceptualization in the next section, but for now it is important to reject any mischaracterizations of the inverse duration-bandwidth tradeoff as an anomaly like "measurement noise," an artifact of mathematical representation, or a quantum-physical duality or uncertainty misapplied to the macroscopic domain. Harmonic dispersion is instead a mainstay of classical (Maxwellian) physics (Hirlimann 2005, 31), thoroughly confirmed in the laboratory (Stingl et al. 1995; Deng et al. 2005). Hence, I treat harmonic dispersion as an empirical datum in Shineland.

III.

The simplest way to see the conceptual problem with the received view of reflectance, which I hereafter call "pulse-reflectance," is to begin computing it within some context.[20]

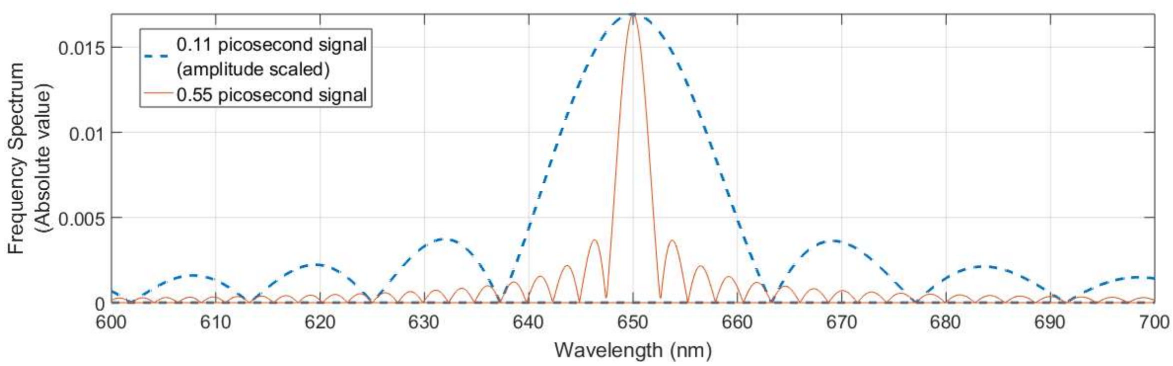

Figure 3: Sub-picosecond Harmonic Dispersion

Figure 3, for example, represents realistic spectra for short pulses in the visible light range. It does not matter that these pulses may be too short for humans/Shinelanders to perceive with the unaided eye, since it is scientific property ascription and not phenomenology which is at issue. The key point, again, is to notice the large peaks at 630 nm and 670 nm in the spectrum of the

[20] Danne (2020) investigates this problem with more tutorial and detail than I can recreate here.

short pulse (dotted trace), which are mostly negligible in the spectrum of the longer pulse (solid trace). Assuming for the sake of argument a perfect mirror with reflectance value 1 at all wavelengths, and assuming that the 11 picosecond (ps; 10^{-12} seconds) pulse of Figure 3 propagates toward the mirror with an average power of 5W, how much power can be expected to reflect at 650 nm? A glance at Figure 3 suggests that the answer is not 5W. A working estimate is that 80% or 4W reflects at or near 650 nm, while 20% or 1W dissipates at wavelengths tens of nanometers away from 650 nm.

Thus, the conceptual difficulty of pulse-reflectance is twofold: firstly, the 650 nm pulse enters the analysis as a monochromatic finite-duration simple sinusoid (as in Figure 2a), not as a wave packet.[21] So in response to the question, "How much of a 5W, 650 nm pulse would reflect from a perfect mirror at 650 nm?",[22] the answer in our working example is paradoxically not 5W, but 4W, despite the mirror being perfect. One might rejoin that the initial assumptions for the computation are wrong, deny that simple sinusoids exist in nature, and insist that only wave packets propagate in nature; but this correction only ushers in the second difficulty. For wave packets, too, undergo harmonic dispersion (Stingl et al. 1995), which as in the case of simple sinusoids, is less the longer the wave packet extends in space and time (cf. Figures 2 and 3). Hence an informed response to the question, "How much of a 5W, 650 nm pulse would reflect from a perfect mirror at 650 nm?", is, "It depends on pulse duration," and *that* answer has nothing apparently to do with the definition of reflectance laid down in §II above. In §II, the average powers of pulses were assumed sufficient to define reflectance as a static or time-invariant property of surfaces. Pulse durations were supposed to 'come out in the wash' as

[21] A wave packet is a heterochromatic propagation of finite duration, such as a Gaussian pulse.

[22] One need not be troubled by the bad scientific grammar of this question. Electromagnetic pulses do not reflect from surfaces like tennis balls bounce from them; pulses are absorbed by the reflective surface and the reflective surface emits a new pulse in turn. My shorthand here does not affect my argument.

theorists performed pulse integrals and constructed unitless reflectance ratios. But from the example worked in this section, such washouts clearly do not happen, inferably for pulses of *any* finite duration, and the Shinelanders recognize as much. They consider Reflectance Claims bosh,[23] because finite-duration *pulses* neither propagate nor reflect 'per wavelength'. Something besides finite-duration electromagnetic pulses reflect that way, if anything does.

IV.

A good candidate for the "something" that propagates per-wavelength is the mathematical entity that I have been depicting throughout this paper—the Fourier harmonic introduced in §I. Indeed, every reference to the 650 nm, 630 nm, or 670 nm component of a signal's spectrum in Figure 3 is reference to a Fourier harmonic of that wavelength with amplitude depicted on the vertical axis. For pulses and wave packets are not themselves 'composed'[24] of finite-duration pulses of various frequencies, but rather of *harmonics* weighted and phase-shifted such that their sum yields the pulse/wave packet in question. Harmonics superbly achieve 'per wavelength' propagation because they are immune to harmonic dispersion—their infinite duration guarantees their constant bandwidth (cf. Haykin and Van Veen 1999, 236-237).

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[23] Then how do they succeed in reflective signage manufacturing? By circumlocution. Glass A performs better in highway signs than Glass B, not because A is "more reflective," but because drivers see A better than B at night. And why do drivers see A better than B? Here the Shinelanders reference average power ratios computed for wide frequency bands under low-dispersion conditions, but a real reflectance property so defined they refuse to acknowledge, because of the two conceptual incongruities mentioned in the present section of the main text.

[24] I use scare-quotes to isolate *physical* wave packets from the mereological structure that Field finds mathematical in footnote 12 above. Presumably, one could hold that light propagates harmonically, but that mathematized mereologies are not true of the physical world, including Shineland. Thus, the nominalist science of Shineland may require a disclaimer that wave packets are nothing over and above the harmonics that superimpose to form them. This disclaimer instrumentalizes wave packets for the sake of rendering reflectance coherent and utterable, a tradeoff that some physicists of Earth or Shineland may find problematic. I further discuss instrumentalist tradeoffs about reflectance in the main text below.

Thus, there appears a way to restore Reflectance Claims to nominalist Shinelander scientific language. Reflectance must be redefined from pulse-reflectance (§II) to harmonic-reflectance. Harmonic-reflectance remains the per-wavelength unitless ratio of average powers, but *only the average powers of* infinite-*duration signals*, or harmonics. This redefinition eliminates the two conceptual problems encountered with pulse-reflectance in §III, because (1) a 5W, 650 nm "pulse" is now immediately understood as a wave packet of harmonics, whose 4W component at 650 nm reflects 4W from the perfect mirror as expected, and (2) because a 4W harmonic will always reflect 4W from a perfect mirror, no matter how spatiotemporally long or short the pulse/wave packet into which the harmonic superimposes. Assuming harmonic-reflectance, perfect mirrors are perfect again, and Reflectance Claim language flourishes in Shineland, about many imperfect reflectors.

Signal theorists might recognize the foregoing as a very long way of saying that the received definition of reflectance in philosophy is a rather poor one for property realist metaphysics, despite the utility of pulse-reflectance in applied science. In that vein, something should be said about the reticence of Shinelanders to utter Reflectance Claims when their science utilizes pulse-reflectance, versus their openness to Reflectance Claims in a science employing harmonic-reflectance. For some might object that pulse-reflectance has *always* been an overtly instrumentalized scientific property, and that Shinelander puritanism about coherent concepts, speech, and reference (a kind of anti-instrumentalism) should not be permitted to beg questions about which mathematical sentences are conservative.

To put first things first, I agree with my imagined objector that the instrumentalized status of pulse-reflectance appears decisive. The reason why is that "average power" has always strictly, truly, or most properly applied only to infinite-duration signals. Equation 1 is shorthand for the more generalized

$$P_{avg} = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\, dt \qquad [2]^{25}$$

and within orthodox signal theory, period *T* is defined "for all *t*" (Haykin and Van Veen 1999, 18), or across infinite time. Thus, theorists who use Equation 1 to compute the average power of a finite-duration signal are only *pretending* that *T* is well-defined, when it is not. On the contrary, Equation 2 reveals that finite-duration signals (possessing finite energy) possess zero average power (Haykin and Van Veen 1999, 20-21),[26] and so the pulse-reflectance that traffics in finite-duration signals is instrumentalist.

On the other hand, and in defense of the Shinelanders, the supposition that electromagnetic reflectance is an instrumentalist property in Shinelander or Earthling science is a weighty and non-trivial one. A strong case can surely be made that a property as fundamental as reflectance, having survived the Fresnel-Maxwell paradigm shift, appearing throughout science at every scale between the atomic and the galactic, and underpinning the development of most any sundry technology, should not be considered a convenient fiction or non-truth-apt heuristic.[27] Fortunately, however, I do not need to make the case against instrumentalizing reflectance, because Field (2016) implicitly makes it for me. While he grants that "some features of . . . fundamental theories" could turn out to be instrumentalizations, he also advocates "minimiz[ing]

[25] Haykin and Van Veen (1999, 20).

[26] To clarify: if $x^2(t)$ in Equation 2 possesses infinite energy, or extends infinitely in both directions of time, then the value of the integral does not attenuate as *T* goes to infinity in the limit. Such signals of infinite energy are the only signals with nonzero average power, in orthodox signal theory.

[27] The *Bureau International des Poids et Mesures* appears to share this intuition, when they characterize the SI or International System of Units as a "description of nature" (BIPM, n.d.)—hardly the vocabulary of a studied instrumentalism. Reflectance is a close derivative of the SI candela, the intensity of light being the square of the amplitude of the signal whose average power is computed for reflectance.

the extent to which we rely on instrumental devices" (xlii, editing mine). Instrumentalizing as ubiquitous a property as reflectance without argument is not a minimization. Thus, *with a view to minimizing instrumentalism in fundamental scientific theory*, I argue in the next section that Fourier analysis, which enables Shinelanders to utter Reflectance Claims in good conscience, is nonconservative—for them and possibly everyone.

V.

Fourier analysis is nonconservative over physical theory, I now argue, because conjoining Fourier analysis with the nominalist sentences N* of Shineland (about the average powers of pulses) yields nominalist Reflectance Claims N^0 and N^1 not derivable from N*: "This surface is reflective" (N^0), and "Surface A is more reflective than surface B" (N^1). I have made N^0 and N^1 plausible for Shinelanders by redefining pulse-reflectance as harmonic-reflectance, thereby eliminating the two conceptual incongruities that rendered N^0 and N^1 meaningless in §III. Here, however, one might simply redouble the Fieldian (1980) objection that mathematics can be replaced by **betweenness** and **congruence** relations between spacetime points (see footnote 15 above), and that the Fourier transform likewise reduces to such relations. Does this reduction work? Michael Liston (1993) expects not, calling "likely . . . insurmountable" the difficulty of expressing the global relationships between harmonic amplitudes *via* Fieldian predicates like **betweenness** and **congruence** (444).[28] But even if Liston is wrong, and a Fieldian nominalization of the Fourier transform can be had, I argue in this section that the point

[28] An objection to both Liston and me at this point is that mathematical formalisms besides Fourier analysis represent light propagation, and might better avail themselves to nominalization. Of course, the onus is on the objector to make this suggestion plausible, and I do not find it initially plausible. Wavelets, for example, represent signals more compactly than Fourier harmonics do, but wavelet bases are heterochromatic; thus, employing them to describe 'per-wavelength' propagations will resurrect the conceptual incongruities of §III and frustrate the Shinelanders just as pulse-reflectance does.

is moot, and that Fourier analysis is still nonconservative *for the Shinelanders*, who in their calculation of pulse average powers had no reason to consider or believe in the infinitely-durative propagations that render Reflectance Claims coherent (and who for the sake of argument will perpetually struggle to express such ideas in any but mathematical terms). I challenge, in other words, the Fieldian assumption that infinitudes are always nominalizable into non-mathematical relations.

For maximal clarity, let us resume the **Conservation** terminology of §I, letting MT* represent the Fourier transform as a purely mathematical statement, and N* the sentences of the Shinelanders' nominalist science. A nominalist like Boyce (2020) might argue that Fourier analysis does *not* violate **Conservation**, because nominalist claim N^0 ("This surface is reflective") follows not from MT* & N*, but from M & MT* & N*, where M is a "mixed mathematical statement[]" (14) entailed by the conjunction of MT* and some non-mathematical statement(s) not already in N*.[29] Perhaps mixed-mathematical statement M is, "Light propagates isomorphically to transverse sinusoids," and from that mixed-mathematical assumption (along with MT* & N*), N^0 and N^1 follow, preserving the conservativeness of MT*. On this view, Fourier transform MT* remains the metaphorical ladder that we employ for deduction and then 'kick away' from our ontology, just as Field (1980) predicts.

In reply, I deny that the statement blocking the incongruities of §III about reflectance is mixed-mathematical (M). The statement performing incongruity-blocking work is MT*, the Fourier-transform insight that "per wavelength" optical properties, to be instantiable (in the opinion of Shinelanders), *must* be stimulated by and manifestations of infinite-duration harmonic propagations, on pain of conceptual incoherence. Despite our ability to 'kick away' the number

[29] Thanks to Kenneth Boyce for a similar suggestion (email correspondence, October 2020).

"100 (°C)" when discussing boiling water, we cannot 'kick away' the infinite duration of reflecting harmonics without losing reflectance itself (§III).[30] Here the Fieldian might counter-object that one need not 'kick away' the infinite when they dispense with the harmonic: the Fourier-analytical laws of harmonic dispersion and pulse propagation can (presumably but not yet actually) be expressed as ontologically dispensable relations between a posited infinity of spacetime points. As Field (2016) remarks, "surely it is consistent to maintain that there are infinitely many grains of sand but no numbers of functions or sets" (95, footnote omitted). He similarly accepts nominalist assertions about the **continuity** of a scalar quantity (Ch. 8, §A), and defines a nominalist "Inf" predicate for infinite quantities (101-102). His nominalization of Newtonian physics depends on such a posited infinity of spacetime points (Ch. 4, §I). Thus, perhaps the Shinelanders can nominalize infinite-duration harmonics by calling their consecutive periods mutually **congruent** "forever and ever" across time and space, or by way of some other nominalist predicate about infinity.

I reply with the major and minor points of my thesis. The minor point is that *ex hypothesi*, the Shinelanders have no concept of infinitude and lack the ability to develop that concept, as implausible as this cognitive or developmental condition might seem.[31] Hence

[30] In this vein, the Boycean argument threatens to prove too much. The M therein proposed ("Light propagates isomorphically to transverse sinusoids") appears to be what Field (1980) calls a "'bridge law[]'" (9), a sentence "that involve[s] the mathematical vocabulary and the physical vocabulary together" (10), and which renders a given portion of MT* *applied* rather than pure. Hence the Boycean must be careful not to allow bridge laws to trivially protect every sentence of MT* from claims of non-conservativeness, on pain of begging the question about the conservativeness of applied mathematics (cf. Melia 2006, §1b). If application guarantees **Conservation**, then dispensability is indeed a red herring for nominalism (Yablo 2005, §4).

[31] Indeed, nearly every clarificatory detail about Shineland renders it less and less plausible a world. Nurida Boddenberg has suggested (at a reading group hosted by Michael Stoeltzner at the University of South Carolina) that because Shinelanders recognize the reflectance *ratio* to possess any value between 0 and 1, they recognize the infinite before I (as their Oracle) introduce harmonics. Thus, I must assume that Shinelanders are peculiarly dense in this regard, and that they fail to recognize the rational continuum between 0 and 1, and the infinitesimal nature of *dt* in Equation 1. Shinelanders in fact nominalize the infinitesimal *dt* in Equation 1, but that fact does not entail that Shinelanders can nominalize the harmonic.

within the Shinelander scenario, talking about "forever and ever" just is mathematical language about the infinite. Fourier analysis is nonconservative for them, because (i) they lack infinity-concepts, and (ii) they prefer anti-instrumentalism about reflectance, a serious position even on Earth (cf. Hartenstein and Hubert 2021). Hence in proportion to the possibility of a place like Shineland existing, or in proportion to the possibility that Earthling concepts might have topped-out or met their limit in Shinelander concepts (which they ostensibly did not), the motivation emerges to recharacterize conservativeness as an empirically contingent, psychological, or epistemic property, rather than a logical property.

The more relevant and major point of my thesis is simply this: *Observe the dialectical progress hereto made*. If posited spatiotemporal infinitudes without abstracta (the Fieldian account of infinity) turns out in the actual world not to be the most defensible characterization of the infinitudes in mathematized science, and if the infinitudes of mathematized science could be convincingly rendered inherently mathematical (as abstracta or not),[32] then Fourier analysis appears nonconservative in the actual world, where theorists hold non-instrumentalized fundamental science in high regard. As I already disclaimed (§I), I can make no progress on such a recharacterization here, but I open the question to others. There remains dialectical value

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An anonymous reviewer similarly questions whether Shinelanders would know what a "frequency" is, since according to orthodox signal theory, "1 Hz is defined as once per second . . . . 'every second', i.e., forever." This point underscores how naively Shinelanders approach instrumentalized pulse-reflectance physics, but a counterpoint is that other Earthling theorists approach the instrumentalization just as naively, to no obvious practical detriment, since harmonics are almost never broached in discussions on color ontology, philosophy of perception, aesthetics, scientific realism, and the like. (Important exceptions in the philosophy of science include Wilson (2018, 233, 238), McGivern (2008, 68, n. 19), Liston (1994), and Sheldon (1985)). And of course, in Shineland there *is* a practical detriment: the loss of Reflectance Claim language.

[32] I do not think that abandoning abstracta in one's mathematics, to secure an inherently mathematical notion of infinity, begs the question against Field, because conservativeness proper is about non-truth before it is about non-abstracta; but of course, I would need to argue this point, were I to adopt it.

in the potential trap that I have identified for **Conservation**, based as it is on the universal ascription of a credentialled scientific property.

VI.

Field posits infinitudes as non-mathematical realities, to expunge many other, mathematical, entities from science. Presumably, recharacterizing the infinite as inherently mathematical would frustrate Field's dispensability program on its own terms. In this paper, I argued that an inherently mathematical notion of infinity would also frustrate Fieldian conservativeness, due to the indispensability of infinite-duration Fourier harmonics to securing nominalist sentences about non-instrumentalized electromagnetic reflectance. Are the prospects for an inherently mathematical (non-nominalizable) infinity any better than those for inherently mathematical natural numbers or trigonometric functions? The modest spate of positions mentioned in the second paragraph of §I suggests yes, but my strengthening of this claim must await another occasion.

References:

Azzouni, Jody. 1994. *Metaphysical Myths, Mathematical Practice: The Ontology and Epistemology of the Exact Sciences*. New York: Cambridge University Press.

---. 2009. "Evading Truth Commitments: the problem reanalyzed." *Logique & Analyse* 206: 139-176.

Boyce, Kenneth. 2020. "Mathematical application and the no-confirmation thesis." *Analysis* 80 (1): 11-20. doi:10.1093/analys/anz021

Bureau International des Poids et Mesures (BIPM). n. d. "Resolution 1 of the 26th CGPM (2018)." Accessed September 13, 2022. https://www.bipm.org/en/committees/cg/cgpm/26-2018/resolution-1

Byrne, Alex and David R. Hilbert. 2003. "Color Realism and Color Science." *Behavioral and Brain Sciences* 26: 3-64.

---. 2004. "Hardin, Tye, and Color Physicalism." *The Journal of Philosophy* 101 (1): 37-43. http://www.jstor.org/stable/3655778

---. 2007. "Truest Blue." *Analysis* 67 (1): 87-92.

Danne, Nicholas. 2020. "How to Make Reflectance a Surface Property." *Studies in History and Philosophy of Modern Physics* 70: 19-27. https://doi.org/10.1016/j.shpsb.2020.01.002

Deng, Yuqiang, Zubin Wu, Lu Chai, Ching-yue Wang, Keisaku Yamane, Ryoji Morita, Mikio Yamashita, and Zhigang Zhang. 2005. "Wavelet-transform analysis of spectral shearing interferometry for phase reconstruction of femtosecond optical pulses." *Optics Express* 13: 2120-2126, https://doi.org/10.1364/OPEX.13.002120

Field, Hartry. 1980. *Science Without Numbers: A Defense of Nominalism*. Princeton, NJ: Princeton University Press.

---. 1985. "On Conservativeness and Incompleteness." *The Journal of Philosophy* 82 (5): 239-260. Stable URL: http://www.jstor.com/stable/2026490

---. 2016. *Science Without Numbers: A Defense of Nominalism*. 2nd Edition. New York: Oxford University Press. Kindle Edition.

Guénon, René. 2001. *The Metaphysical Principles of the Infinitesimal Calculus*. Trans. Michael Allen and Henry D. Fohr. Hillsdale, NY: Sophia Perennis.

Hartenstein, Vera and Mario Hubert. 2021. "When Fields Are Not Degrees of Freedom." *The British Journal for the Philosophy of Science* 72 (1): 245-275.

Haykin, Simon. 1989. *An Introduction to Analog and Digital Communications*. New York: John Wiley & Sons, Inc.

Haykin, Simon and Barry VanVeen. 1999. *Signals and Systems*. New York: John Wiley & Sons, Inc.

Hilbert, David R. 1987. *Color and Color Perception: A Study in Anthropocentric Realism*. Center for the Study of Language and Information, Stanford University. Kindle Edition.

Hirlimann, C. 2005. "Pulsed Optics." In *Femtosecond Laser Pulses: Principles and Experiments*, 2nd edition, edited by Claude Rulliere, 25-56. Berlin: Springer-Verlag.

Kitcher, Philip. 1988. "Mathematical Naturalism." In *Essays on the History and Philosophy of Modern Mathematics*, *Minnesota Studies in the Philosophy of Science*, edited by W. Aspray and P. Kitcher, 11: 293–328.

Körner, Stephan. 2009. *The Philosophy of Mathematics: An Introductory Essay*. Mineola, NY: Dover Publications.

Liston, Michael. 1993. "Taking Mathematical Fictions Seriously." *Synthese* 95 (3): 433-458. http://www.jstor.org/stable/20117789

---. 1994. "How Abstract Objects Strike Us." *Dialectica* 48 (1): 3-27, https://doi.org/10.1111/j.1746-8361.1994.tb00102.x

McGivern, Patrick. 2008. "Reductive Levels and Multi-Scale Structure." *Synthese* 165: 53-75, https://doi.org/10.1007/s11229-007-9232-3

Melia, Joseph. 2006. "The Conservativeness of Mathematics." *Analysis* 66 (3): 202-208.

Shapiro, Stewart. 1983. "Conservativeness and Incompleteness." *The Journal of Philosophy* 80 (9): 521-531.

Sheldon, Neil A. 1985. "One Wave or Three? A Problem for Realism." *British Journal for the Philosophy of Science* 36 (4): 431-436. https://doi.org/10.1093/bjps/36.4.431

Stingl, A., M. Lenzner, Ch. Spielmann, F. Krausz, and R. Szipocs. 1995. "Sub-10-fs mirror-dispersion-controlled Ti:sapphire laser." *Optics Letters* 20 (6): 602-604.

Wilson, Mark. 2018. *Physics Avoidance: Essays in Conceptual Strategy*. New York: Oxford University Press.

Yablo, Stephen. 2005. "The Myth of the Seven." In *Fictionalism in Metaphysics*, edited by Mark Eli Kalderon, 88-115. New York: Oxford University Press.