JUSTIFICATION OF RULES IN QUANTIFICATION LOGIC

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Inspite of the fact that all logical systems are governed by rules, the rules by themselves do not, or even perhaps cannot, decide *effectively* the validity or truth of all logical systems. Beside the rules some logical systems require human skill and ingenuity for deciding their validity. Natural deduction (quantification logic) is one of the most powerful methods of logical demonstration in which human skill and ingenuity are required. It is true that unlike other logical systems, qualification logic is primarily concerned with a special set of procedures, namely, the procedures of 'Universal Instantiation'(UI); 'Universal Generalisation'(UG); Existential Instantiation '(EI)'; and 'Existential Generalisation' (EG) and the rules governing them. There procedures are thought to be a distinctive feature of quantification logic since on the basis of these procedures other logical systems can be marked off. It is true that although a good many logicians have put forward explanations of the procedures mentioned above and the rules governing them, but hardly any one of them has provided an adequate account of the quantification procedures. Some logicians, namely, G. Gentzen, S. Jaskowski and W. V. O. Quine have devised these procedures on their own rationale. Yet some problems about the import of the procedures as well as their justification in respect of the rules governing them are still lurking within the system. J. L. Mackie finds a lot of discrepancies in the justification of these procedures. However, I would not enter into the debated issue in the present paper, but I do hope to take it up in a future paper. In the present paper, I would try to explain, as clearly as I can, the justification of rules in quantification logic with special reference to Copi's *Symbolic Logic*.

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Before I enter into an elaborate justification of the procdures of EI, EG, UI, and UG, I shall try to explain the symbolic device of the rules in the loigc of quantification. The simplified forms of UI, EI, UG and EG are given below :

(i) Universal Instantiation : (UI) : $\dfrac{(\mu)\ (\phi\mu)}{\therefore \phi\nu}$

(ii) Existential Instantiation : (EI) : $\dfrac{(\exists\mu)\ (\phi\mu)}{\therefore \phi\nu}$

(iii) Universal Generalisation (UG) : $\dfrac{\phi\nu}{\therefore (\mu)(\phi\mu)}$

(iv) Existential Generalisation (EG) : $\dfrac{\phi\nu}{\therefore (\exists\mu)(\phi\mu)}$

Let us first clarify the symbolic notations - what do they mean or what do they stand for ? Here Copi uses the Greek letter phi (ϕ) along with either the Greek letter mu (μ) or Greek letter nu (ν). The letter 'ϕ' stands for attributes or predicates; the letter 'μ' stands for individual variables exclusively and the letter 'ν' stands for either individual variables or individual constants. Copi, however, presupposes two basic conventions regarding 'μ' and 'ν'. They are : [2] (i) "mu ('μ') denotes individual variables exclusively whereas nu ('ν') can denote either an individual variable or an individual constant" (ii) "The expression ϕμ denotes any proposition or propositional function. The expression ϕν denotes the result of replacing every free occurrence of μ in ϕμ by ν, provided that if ν is a variable, it must occur free in ϕν at all places that μ occurs free in ϕμ (If ϕμ contains no free occurence of μ, then ϕν and ϕμ are identical. The variables ν and μ may, of course, be the same. If they are, in this case too, ϕν and ϕμ are identical."

On the basis of these two conventions proposed by Copi regarding ν and μ, let us explain in greater detail the procedures of UI, EI, UG and EG in turn.

UNIVERSAL INSTANTIATION : (UI) : $\dfrac{(\mu)\ (\phi\mu)}{\therefore \phi\nu}$

In the above symbolic schema of UI '(μ)' stands for universal quantifier, such as '(x)'. In 'ϕμ' 'μ'denotes individual variables governed by ('μ') and in 'ϕν' 'ν' denotes either individual variable/s or individual constant/s derived from '(μ)(ϕμ)' by applying UI. '(μ)(ϕμ)' is a symbolic schema for universal proposition having no free occurence of 'μ' in '(μ)(ϕμ)'. Now, we can deduce 'ϕν' from '(μ)(ϕμ)' by instanticating all 'μ' variables in '(μ)(ϕμ)' either in terms of individual variables or in terms of individual constant. Let us suppose we have the universal proposition '(x)(Mx ⊃Wx)' ['(μ)(ϕμ)']. We can get 'Ma ⊃ Wa'[ϕν] from this proposition by instantiating all occurrences of the individual variable 'x' [μ in ϕμ] in terms of individual constant 'a'[ν in ϕν]. Similarly, we can deduce the propositional function 'My ⊃ Wy' [ϕν] from (x) (Mx ⊃ Wy) ['(μ)(ϕμ)'] by instantiating all occurrences of the variable 'x' [μ in ϕμ] in terms of the variable 'y' [ν in ϕν]. When we get 'Ma ⊃Wa', from '(x) (Mx ⊃ Wx)', we get a proposition (closed sentence in Quinean sense) from another proposition. In such a case both '(μ)(ϕμ)' and 'ϕν'stand for proposition. But they differ in the sense that 'μ' in 'ϕμ' stands exclusively for individual variables and 'ν' in 'ϕν'stands for individual constants. But when we have 'My ⊃ Wy' from '(x)(Mx ⊃ Wx)' we have a propositional function from a proposition. Here although '(μ)(ϕμ)' and 'ϕν' are different but 'μ'and 'ν'are identical in the sense that both of them stand for individual variables.

It may be noted that there can be no free occurence of μ in '(μ)(ϕμ)'but it could be the case that we find there in any number of free occurrence of 'μ' in 'ϕμ'. For example, in (x)(Mx ⊃ Wx) ['(μ)(ϕμ)'] every occurrence of 'x' (μ) is bound, but in 'Mx ⊃ Wx' (ϕμ) no occurrence of 'x' ('μ') obtains as bound. It may also be the case that there we find a mixture of free as well as bound occurrences of μ in ϕμ. For example, in 'Fx ⊃ (∃*x*)*(Gx V Hy)*' ['ϕμ'] the first occurrence of 'X'('μ') is free, but the second and third occurrences of 'μ' in 'ϕμ' are bound. The second occurrence is bound as a part of the existential quantifier '(∃*x*)' and the third occurrence is also bound as it lies within the scope of the quantifier '(∃*x*)'.

But the procedure of UI can be applied legitimately subject to the following stipulations:

(i) UI must be applied uniformly

In quantification logic we find two poles, one of which is called Instantiation and the other Generalisation. We get either one of them in terms of other. The application of UI removes a quantifier and the application of UG brings the quantifier back. By principle of unifrom insantitation we mean that when we apply the rule of UI on a proposition we instantiate all occurrences of the variable governed by the quantifier in terms either individual variable (may be the same variable governed by the quantifier or different variable) or individual constant. Let us consider the following proposition :

$$(x)[(Mx \,.\, Px) \supset (Mx \equiv Px)]$$

The above proposition is in generalised from. We can apply UI on it either by supplying individual variable or by individual constant. If we apply UI on it by individual variable we get a propositional function from this proposition and if we apply UI on it by individual constant we get another proposition from the given one. Copi sums up the uniformity of UI by saying that in substituiting one variable instead of another to obtain a propositional function from a proposition, the same variable must be instantiated for every bound occurrence of the same variable for which it is substituted and in substituting constants for variable to obtain a proposition from another porposition, the same constant must be substituted for every bound occurrence of the same variable for which the constant is substituted. So the proposition under consideration can legitimately be instantiated (unifromly) by the following ways :

(i) By individual variable:

$(x)(Mx \,.\, Px) \supset (Mx \equiv Px)$

$(My \,.\, Py) \supset (My \equiv Py)$ UI

(ii) By individual constant:

$(x)(Mx \,.\, Px) \supset (Mx \equiv Px)$

$(Ma \,.\, Pa) \supset (Ma \equiv Pa)$ UI

It is clear from the above that whatever is substituted for 'x' :(an individual variable or an individual constant), it must also be substitued for all occurrences of 'x'. If an individual variable, say 'y' is substituted for 'x', it has to be substituted for all occurrences of 'x'. Likewise if an individual constant say 'b' is substituted for 'x', it has to be substituted for all occurrences of 'x'. But if we ignore the said rule of UI then the principle *that in order to substitute constant for variable or variable for variable to obtain either a proposition from a proposition or a propositional function from a proposition, the same constant or the same variable* *must be substituted for every bound occurrence of the same variable for which the constant or variable is substituted* has been ruled out. We never get a propositional function, say [(Ma . Pa) ⊃ (Mx ≡ Px)] from the given proposition, viz; (x)[(Mx . Px) ⊃ (Mx ≡ Px)] if all of the occurrences of the propositional variable are substituted uniformly by an individual constant. This can happen if we allow partial application of UI.

But what goes wrong with the pratial instantiation of UI ? It is claimed that if the given proposition is true, it must remain true in every unifrom substitution or intantiation of it. This also confirms that uniform instantiation of universal proposition preserves the same logical status of the given proposition. The propositional "All men are mortal" in symbol '(x)(Mx ⊃ Nx)' (where Mx : x is a man ; Nx: x is mortal) is genuinely a true proposition. This proposition can be characterized in terms of the propositional functions like this: 'For all values of x, if x is a man then x is mortal'. It remains true in every substitution since the universal quantification of a propositional function is true if and only if its substitution instances are true. But the truth of this posposition gets under way if we do not follow the rule of UI. If we allow to instantiate the given proposition ***partially*** by an individual constant, say 'a' (where 'a' stands for Alma), we get the propositional function 'Ma ⊃ Nx'. (If Alma is a man then x is mortal) which does not enjoy the same logical status of the given proposition.

(ii) In UI 'ν' must occur free in 'φ ν'at all places where 'μ' occurs free in 'φμ'

To explain the logical force of this rule, let us consider the following inference.

$$\frac{(x)[(\exists y)(Fx \equiv \neg Fy)]}{\therefore (\exists y)(Fy \equiv \neg Fy)} \quad UI$$

The above inference is erroneous since in this inference 'ν' ('y') does not occur free in 'φν'['(∃y)(*Fy* ≡ ¬*Fy*)'] at all places that 'μ'('x') occurs free in 'φμ'['(∃y)(*Fx* ≡ ¬*Fy*)']. Copi rules out the legitimacy of this inference in saying that '"(∃y)(Fy ≡ ¬*Fy*)' is not a legitimate 'φν' for use in applying UI where (μ) (φμ) is '(x)[(∃y)(Fx ≡ ¬*Fy*)]'"[3]. The above inference is invalid for the fact that the given premise is true whereas the conclusion is definitely self-contradictory. Our quip is: How do we get a false conclusion from a true premise by applying the valid rule of UI? We cannot have that. It can then be said that the application of UI in the above inference is not legitimate.

Let us now provide a test for showing that the said inference is invalid. Let us suppose that the world consists of just two individuals, namely, a,b. Let us further assume that a is F but b is not F. This means that Fa is true but Fb is false. On the basis of the presupposition the inference can then be paraphrased as logically equivlent to the following truth-functional argument.

Premise : $(\exists x)[(\exists y)(Fx \equiv \neg Fy)]$

$[(Fa \equiv \neg Fa)\ V\ (Fa \equiv \neg Fb)] . [(Fb \equiv \neg Fa)\ V\ (Fb \equiv \neg Fb)]$

T F F T (T) T T T F (T) F T F T (T) F F T F

Conclusion: $(\exists y)(Fx \equiv \neg Fy)$

$(Fa \equiv \neg Fa)\ V\ (Fb \equiv \neg Fb)$

T F F T (F) F F T F

It should be clear that the above truth-functional inference has been shown or proved to be invalid by assigning the truth value T to Fa and F to Fb. Copi says, "It should be obvious that the inference is invalid because it fails for a model or possible universe containing some things that are F and some things that are not F, which would make the premise true, whereas the conclusion, being self-contradictory is false for any model or possible universe."[4]

(iii) In UI there may be more free occurrence of 'ν' in 'φν' than the free occurrence of 'μ' in 'φμ'.

This rule is not so important as the previous one. Let us consider the following inference :

$$\frac{(x)[Fx \supset (Gy . Hx)]}{\therefore Fy \supset (Gy . Hy)} \quad UI$$

There are three free occurrence of 'ν'('y') in 'φν' ['Fy ⊃ (Gy . Hy)']; but there are just two free occurrence of ('x') in 'φμ' ['Fx ⊃ (Gy . Hx)']. This is possible only when in '(μ)(φμ)' some free occurrences are included. That could be had without violating the rule II of UI stated above. This means that we are permitted to apply. UI on any arbitrary individual variable which has already occured as free in a previous step.

(iv) In UI the quantifier that one is dropping in '(μ)(φμ)' has to cover the complete line of the proof.

In any use of UI, the range of the quantifier '(μ)' in '(μ)(φμ)' governs the whole of 'φμ'. For example, we cannot obtain '(Fx v Gx) ⊃ (z) Hz' from '(x)(Fx v Gx) ⊃ (z) Hz' by applying UI simply for the fact that in '(x)(Fx v Gx) ⊃ (z) Hz' the scope of the quantifier '(x)' is limited before the implication sign. Actually we do apply UI only on a universal proposition. But the proposition under consideration is not a universal proposition but an implicative proposition of which the antecedent is a qualified universal proposition and the consequent is an unqualified universal proposition. Being a conditional the proposition under consideration is a compound proposition. But Copi rules out UI of a compound proposition. He holds that in order to apply UI on any proposition, the proposition must have to be a noncompound proposition. So the given proposition is not in the from of '(μ)(φμ)' but actually in the from of '(μ)(φμ)⊃ (μ)(φμ)'. In a proposition such as this, UI cannot be applied legitimately on any part of the compound proposition.

EXISTENTIAL INSTANTIATION : (EI) : $\dfrac{(\exists\mu)(\phi\mu)}{\therefore \phi\nu}$

The procedure of EI is more tricky than other procedures, and it plays an important role in quantification logic. In Symbolic Logic we find different techniques through which this procedure has been introduced. There we find two versions of EI of which one is called the preliminary version, and the other is called an extended version of EI. In the preliminary version of EI, 'ν' in 'φν' means only individual constant, but in the extended version of EI 'ν' in 'φν' means only individual variable. In the extended version EI has been introduced as an assumption (hypothesis). But why should this shift be required? Copi, however, does not specify any reson for his preferring the shift from individual constant to individual variable in this procedure. It seems that in single general proposition he considers 'ν' in 'φν' as an individual constant, but as soon as he enters into multiply general propositions he uses indivudual variables instead of induvidual constants. Perhaps one reason he may have had in mind is that in EI the individual variables are in strict sense ambigous names since they are used similar to the way that we use individual constants. So the chief reason for not using individual constants in the extended version of EI is that individual constants are used only for individual but any given individual may not be an appropriate instance of the expression that we are instantiating. That is why Copi prefers using individual variables instead of individual constants in the extended version of EI Thus in the extended version of EI "ν" in 'φν' means only individual variables. This confirms that in the extended version of EI, one gets only a propositional function from a proposition. This again makes it clear that in this version of EI both "ν" and 'μ' are same as both of them consist of individal variables only. For example in the following inference:

$$\frac{(\exists y)\ (Fy \supset Gy)}{\therefore Fy \supset Gy}$$

Both "ν"('y') in "φν" '(*Fy* ⊃ *Gy*)' and 'μ' '(y)' in'φμ' '(*Fy* ⊃ *Gy*)' are identical.

Another important freature of EI is that unlike UI in EI "ν" in 'φν' under no circumstance can be greater than 'μ' in'φμ'. This makes sense to say that there must be one-to-one correspondence between 'ν' in 'φν' and 'μ' in'φμ'. By applying UI we can get 'My ⊃ Wy' ('φν') from '(x) (Mx ⊃ Wy)' ['(μ)(φμ)'] in which 'ν'('y') in 'φν' is greater than 'μ' ('x') in'φμ'. But in EI we can never get 'My . Wy' from '(∃*x*)(*Mx. Wy*)'and to establish that 'ν' in 'φν'is greater than 'μ' in'φμ'. It is said that no existential instatiantion can be done legitimately by an individual variable/constant which has already occurred in the previous step. I shall return to this principle later. What we can at best say at this juncture is that in EI 'ν' in 'φν' can neither be smaller nor even be greater than 'μ' in'φμ'; but there has to be one-to-one correspondence between 'ν' and 'μ'. We have, for example, 'Fx' ('φν') from ['(∃μ)(φμ)'] or 'Fx . Wx' from '(∃x)(Fx . Wx)' in which both 'ν' in 'φν' and 'μ' in'φμ' are same.

Some Stipulations of EI

(i) Like UI, EI must be applied uniformly.

This stipulation is required mainly because without maintaining this principle the logical status of a proposition cannot be retained. I have explained in some detail in what sense in UI the violation of this principle does hamper the logical force of the given proposition. From the proposition 'Some men are mortal' in symbol, '(∃x)(Mx . Nx)' (where Mx: x is a man ; Nx: x is mortal) we get at least one propositional function for which this instantiation would be true. We can rightly instantiate the proposition '(∃x)(Mx . Nx)' for having either the propositional function 'Mx . Nx' just deleting the quantifier '(∃x)' or we get the prosoitional function 'My . Ny' or 'Mz . Nz' etc.; claiming that this propositional function is true in at least one instantiation of the proposition '(∃x)(Mx . Nx)'. But we cannot have the propositional function 'Mx . Nx' from the given proposition since in that case EI is not applied uniformly. The uniformity of EI is to be retained if and only if the same variable must be instantiated for every bound occurrence of the same variable for which it is instantiated.

(ii) Like UI, in EI the quantifier that one is dropping in '(∃μ)(φμ)'has to cover the complete line of the proof.

The logical force of this principle is that we can not apply EI on any compound proposition. For example, we get 'Wx . Nx' from '(∃x)(Mx . Nx)'just by deleting the existential quantifier '(∃x)'. Here the application of EI is valid since in this proposition the existential quantifier ranges over the whole proposition and the proposition is, of course, a noncompound proposition. But can we apply EI on '(∃x) Mx. (∃x) Wx'? If we are entitled to do so, we get a possible propositional function, such as, 'Mx . (∃x)Wx'. But this is not legitimate because here EI is applied on a compound proposition partially. We can apply EI on an existential proposition; but being a conjunctive proposition it is considered as a compound proposition of which two conjuncts are unqualified existential propositions. The above principle of EI can only be fulfilled if the scope/range of an existential proposition is extended till the end of the proposition.

(iii) In EI, 'ν' must occur free in 'φν' at all places that 'μ' occurs free in 'φμ'

This is exactly the same stipulation that we have previously considered in UI. For example, from (∃x)(Mx . Wx), we can get 'My . Wy' by applying EI legitimately. Here 'ν'('y') occurs free at two places in 'φν' ['My . Wy'] like 'μ'('x') occurs free at two places in 'φμ' [Mx . Wx]. But unlike UI, in EI 'ν' in 'φν' cannot ne greater than 'μ' in 'φμ'. This is mainly for this reason that UI can be legitimately done by an individual variable which has already occurred free in the previous step. But no EI can be legitimately applied by an indivudual variable which has already occurred in the previous step. This should lead us to the next stipulation of EI.

(iv) In EI, 'v' cannot occur free in any prior line of the proof.

This stipulation is considered as the basic rule of EI and it should be examined carefully. Let us first consider the consequence if someone violates the rule.

It has been said that if one violates the rule of EI, then one can logicaly derive a false conclusion from true premises. Let us consider the following inference.

Something is round
Something is square
So, something round is square.

The above argument can be symbolized in the following way:

(∃x) Rx
(∃x) Sx /∴ (∃x) (Rx . Sx)

where Rx stands for : x is round and Sx stands for : x is square.

Logical derivation

1. (∃x) Rx
2. (∃x) Sx /∴ (∃x) (Rx . Sx)
3. Rx 1, EI
4. Sx . 2, EI (Wrong)
5. Rx . Sx 3, 4, Conj.
6. (∃x) (Rx . Sx) 5, EG.

It is important to observe that in the above inference the conclusion is logically deduced from the given permises if we ignore the rules of EI mentioned above. The given premises are true since there is nothing wrong in admitting that something is round and something is square in the world. But the conclusion is self-contradictory as anything round cannot be square as well. Our quip is : How can we derive a false conclusion from the true premises if the rules that we have been given are validly applied by us ? The fundamental tenet of a valid natural deduction is that if the given premises are true then by applying valid rules we must have a conclusion which can never be false. But in the above case we have a conclusion which is false and it is obtained from true premises by applying valid inferential rules. Certainly, there must have undergone some mistake in the procedure of EI. Of course, step-4 in the above derivation is erroneous since in this step EI has been applied illegitimately. We get step-4 from step-2 just by deleting '(∃x)' by applying EI. But this step is erroneous since 'x' has already occured as free in step-3 and hence 'x' cannot further be instantiated in the subsequent line of the proof. If we do so, then it is possible for us to derive a false conclusion from a given set of true premises.

Why does this mistake occur ? What kind of reason lies behind it ? In predicate logic, the ordinary word 'some' is understood as 'at least one'. This makes the sense that when we say that 'something is round', we actually intend to say 'at least one thing is round'. Now, if the first premise of the above argument is instantiated by 'x' meaning that 'x is round' (where x stands for at least one), then how can we use the same symbol 'x' again to denote that it ('x') is square. A particular thing named as round can not be named as a square because same thing can not posses two contradictory attributes. To avoid an error of this sort, says Copi, we must obey the indicative restriction of EI mentioned above.

Let us turn to Existential Generazation.

EXISTENTIAL GENERAZATION : (EG) : $\dfrac{\phi\nu}{\therefore (\exists\mu)(\phi\mu)}$

In the above symbolic shema, 'ν' in 'φν' means either individual variables or indivdual constants, but 'μ' in 'φμ' means individual variables only. This means that in EG we may get a proposition either a proposition (if 'ν' in 'φν' stands for individual constant/s) or from a propositional function (if 'ν' in 'φν' stands for induvidual variable/s). For example, we get (∃x) Fx ['(∃μ)(φμ)'] from 'Fa' 'φν'. Likewise, we get the proposition '(∃x) Fx' ['(∃μ)(φμ)'] from the propositional function 'Fx'(φν). It is also important to note that we may get a propositional function from another propositional function by applying EG. This is one important peculiarity of EG which I propose to discuss later.

Some stipulatations of EG

(i) EG may or may not be applied uniformly

We already noted that EI and UI must be applied uniformly. We shall see later that UG must also be applied uniformly. For example, we get the proposition '(∃x)(*Mx* . *Wx*)' either from the proposition 'Ma . Wa' or from the porpositional function 'Mx . Wx' by applying EG uniformly. But quite interesting, we can also obtain the porpositional function (∃y) (My . Wx) from the propositional function 'Mx . Wx' or even we get the proposition (∃x) (*Mx* . *Wa*) from the proposition 'Ma . Wa' by applying EG. But the application of EG in the latter case is somehow different from the application of EG in the former case. In the former case EG has been applied uniformly; but in the latter two cases EG has not been so applied.

We have already justified the legitimacy of uniform instantiation. But how can we prove the legitimacy of pratical application of EG. Let us consider the following derivation in which the conclusion is obtained from the given premise by applying partial EG.

1. Mr . Br	/ ∴ (∃x)(*Mx* . *Br*)
2. Mr	1, Simpl.
3. Br	1, Simpl.
4.(∃x) Mx	2, EG.
5.(∃x) Mx . Br	4,3, Conj.
6.(∃x) (Mx . Br)	5 by role of passage: "[(∃x) Fx. P] ≡ (∃x) (Fx . P)"

(ii) EG can be legitimetely applied on a singular proposition

This rule of EG permits that we get an existential proposition from a singular proposition - a proposition having at least one attribute predicate and an individual constant. For example, from the singular proposition 'Ram is beautiful', in symbol 'Br' (where B stands for attribute predicate and 'r' stands for Ram) we can deduce the proposition 'someone is beautiful', in symbol, '(∃x) Bx'. This can by shown by the following derivation :

1. Br / ∴ (∃x) Bx
2. (∃x) Bx 1, EG.

But can we derive the proposition '(∃x) Bx' from the propositional function 'Bx' (x is beautiful)? The answer may be both 'yes' or 'no' subject to clarification. If 'Bx' as a propositional function is given, then the answer should be negative. But if 'Bx' is not given, but is an intermediatory step of a derivation and is obtained either from the pervious proposition by applying valid UI or EI or is obtained from previous steps by applying valid rules, the answer should be positive. If 'Bx' is given, then as a propositional function it can neither be true nor be false. But as a proposition '(∃x) Bx' must be either true or false. So we can never derive something as either true or false from something as neither true nor false. But this is not the case if 'Bx' is obtained as an intermediatory step of a valid derivation.

(iii) In any use of EG the scope of the quantifier must include the complete line of the proof.

We have already shown that we can deduce the proposition '(∃y) (Wy . My)' or the propositional function '(∃x) (Wy . Hx)' from the propositional function 'Wx . Hx'. Or, we can validly deduce the proposition '(∃x) (Mx. Bx)' or the proposition '(∃x) (Mx . Br)' from the posposition 'Mr . Br'. But can we deduce '(∃x) Wy . Hx from 'Wx . Mx' ? We cannot have it because like the previous cases, here the scope of the quantifier does not include the complete line of the proof. It is true that when we apply EG on a compound proposition or propositional function we must get an existential proposition which must be treated as uncompound. But '(∃y) Wy . Hx' being conjunctive must be treated as compound of which one component is an existential proposition and the other component is a propositional function.

(iv) In EG single quantifier can not be used to bind more then one variable.

In EG single quantifier should be used to bind single varable only. For example, from the given propositional function 'Bxy' , we cannot have the proposition '(∃x) Bxx'. In 'Bxy' both 'x' and 'y' are two free variables. We have the proposition '(∃x) Bxx' by applying EG on 'y' in Bxy. But in doing so the single quantifier '(∃x) ' has been used to bind both free variables.

The violation of this principle leads us to derive a false conclusion from true premise/s. Let us consider the following argument.

Everyone talks to someone. Therefore, someone talks to himself.

The above argument can be symbolically represented as under:

1. (x) (∃y) Txy / ∴ (∃x) Txx
2. (∃y) Txy 1, UI
→ 3. Txy
4. (∃x) Txx 3, EG(wrong)
5. (∃x) Txx 2,3-4 CP.

In the above derivation the conclusion logically follows from the premise if we ignore the principle of EG. But truly speaking the argument is invaild. We can not assert 'Someone talks to himself' from 'Everyone talks to someone'. The invalidity of the argument can thus be shown in the following way.

Let us assume that the world consists of just two individuals such as a,b. Accordingly, the above argument is logically equivalent to the following truth-functional argument.

premise:

(∃x) (∃y) Txy

≡ (∃y) Tay . (∃y) Tby

≡ (Taa v Tab) . (Tba v Tbb)

F T T T T T F

Conclusion :

(x) Txx

≡ Taa v Tbb

F F F

It is clear from above that by assigning the truth value T to 'Tab' and 'Tba' and F to 'Taa' and 'Tbb' the given argument is proved invalid. But in this invalid argument the conclusion can be logically deduced from the premise if we violate the above mentioned rule of EG.

Now, let us pass on to Universal Generalization:

UNIVERSAL GENERALIZATION : (UG) : $\frac{\phi\nu}{(\mu)(\phi\mu)}$

In the above symbolic schema of UG 'ν' in 'φν' denotes only individual variables. So in UG both 'ν' and 'μ' are same in the sense that both of then are made of of individual variables only. If in 'ν' in 'φν' consists of only individual variables then in UG we get a proposition only from a propositional function. The principle of UG admits that from any arbitrary selected instance of a universally qualified statement, we can derive the universal quantification of that statement subject to needed restrictions. The concept of arbitary selected instance is very much dubious, and has come under fire in recent times. However, I do not propose to enter into the debate.

Some stipulations of UG

(i) UG must be applied uniformly.

The unifromly of UG can be stated like this : For any use of UG, if a given variable such as 'y' is generalized on any line of a proof, then all occurences of that varable on that line have to be generalized. Accordingly, we get '(x)(Fx ⊃ Gx)' from 'Fy ⊃ Gy' since every occurrence of 'y' is generalized by '(x)'. But we do not have (x)(Fx ⊃ Gy) from 'Fy ⊃ Gy' since in this generalization every occurrence of 'y' is not generalized by '(x)'.

(ii) In UG 'v' in 'ϕv' is not a free variable on a line derived in the proof by the use of EI.

It is said that no UG can be applied legitimately on an individual variable which is obtained from EI Let us consider the following argument :

Everyone talks to someone
So, someone talks to everyone.

Symbolically we can represent the above argument as below :

1. (x) (∃y) Txy /∴ (∃y) (x) Tyx [where Txy : x talks to y].
2. (∃y) Tzy 1, UI
→ 3. Tzw
4. (x) Tzx 3, UG (wrong)
5. (∃y) (x) Tyx 4, EG
6. (∃y) (x) Tyx 2, 3-5, CP

In the above derivation, step-4 is erroneous since in this step UG is not applied legitimately. Here UG is applied on an individual variable, namely w, which is obtained from EI and remains free in the assumption of Tzw. In this case Tzw is not a legimate 'ϕv' for the given 'ϕμ'from which '(x) Tzx' can be derived as '(μ)(ϕμ)' by applying UG. Nevertheless the conclusion logically follows from the premise at the cost of the rule of UG mentioned above. The invalidity of the argument can be shown thus :

Let us suppose that the world consists of two individuals, namely a, b. On the basis of this presupposition the argument can be represented as logically equivalent to the following truth - functional argument.

premise : (x)(∃y) Txy

≡ (Taa v Tab) . (Tba v Tbb)

F T T (T) T T F

Conclusion : (∃y)(x) Tyx

≡ (Taa . Tab) v (Tba . Tbb)

F F T (F) T F F

It is clear that by assigning the truth-value F to 'Taa' and 'Tbb' and T to 'Tab' and 'Tba', the given argument can be proved invalid. But this argument can be validly deduced if we ignore the said rule of UG.

(iii) No UG can be legitimately applied on any individual constant occurring as a singular proposition.

The force of this rule is clear enough since if we violate this rule then we have to assert something more in the conclusion than the premise or premises. How can we derive a universal proposition from a singular proposition ? It is said that in a valid deduction the conclusion cannot overlap the premise/s. Accordingly, we can not deduce 'Everyone is beautiful' from the singular proposition 'Ram is beautiful'. The following inference must be regarded as invalid.

1. Br /∴(x)(Bx)
2. (x) Bx 1, UG (Wrong)

(iv) In UG single quantifier cannot be used to bind two free variables.

Let us consider the following argument:

Something is greater than everything
So, everything is greater than itself

In Symbols:

1. (∃y)(x)Gxy /∴(x) Gxx [Gxy : x is greater than y]
→ 2. (y) Gxy
3. Gxy 2, UI
4. (x)Gxx 3, UG (wrong)
5. (x)Gxx 1,2-4, CP

In the above derivation step-4 is erroneous since in this step single quantifier, namely, '(x)' has been used to bind two free variables, such as, 'x' and 'y'. It may be the case that 'Something is greater than everything' but from this it does not follow that 'Everything is greater than itself'. The conclusion is obviously false as nothing can be greater than itself. Here we deduced a false conclusion from a true premise by taking the advantage of an illegitimate application of UG. The invalidity of the above argument can be shown in the following manner:

Let us suppose that the world consists of two individuals, such as, a,b. On the basis of this presupposition the given argument can be shown to be equivalent to the following truth-functional argument:

Premise: (∃x)(y)Gxy

≡ (Gaa . Gab) v (Gba . Gbb)

F F T (T) T T T

Conclusion : (x)Gxx

Gaa . Gbb

F (F) T

It is clear from the above that by assigning the truth-value T to 'Gbb', 'Gab', 'Gba' and F to 'Gaa' the given argument is proved invalid. But this invalid argument can be validly deduced if we ignore the rule of UG mentioned above.

(v) No UG can be legitimately applied on 'v' when a) 'v' is derived from the assumed premise and b) 'v' remains free within the scope of that premise.

Let us consider the following derivation.

D-1

→ 1. (x)(∃y) (Gx . Hy)	/ ∴ (x) Gx . (∃y)Hy
2. (∃y) (Gx . Hy)	1, UI
3. Gx . Hy	
4. Gx	3, Simpl.
5. Hy	3, Simpl.
6. (x)Gx	4, UG (Wrong)
7. (∃y)Hy	5, EG.
8. (x)Gx . (∃y)Hy	6, 7 Conj.
9. (x)Gx . (∃y)Hy	2, 3-8, CP.

Step-6 in D-1 is wrong since UG is applied on 'v'('x') which is obtained from the assumed premise and remains free within the scope of that premise.

'v'('x') is free in step-2. We have step-3 from step-2 by an assumption of EI. In step-3 we get 'v'('x') as free and hence here it can be regarded as neither true nor false, and it remains neither true nor false within the scope of the assumption of EI. So within the assumption of EI, UG is not allowed on it. By applying UG on it in step-6, we get a proposition from a propositional function. But we are not allowed to have a proposition from a propositional function if the variable of the propositional function is obtained not from a proposition but from an assumption of EI. But UG can be legitimately applied on and within the scope of an assumption which is taken as bound. In such a case the assumption step is regarded as a proposition. We obtain a proposition from another proposition and hence no problem will arise. The validity of the above argument can be legitimately proved in the following way:

D-2

1. (x)(∃y) (Gx . Hy) / ∴ (x) Gx . (∃y)Hy
2. (∃y) (Gx . Hy) 1, UI
→ 3. Gx . Hy
4. Gx 3, Simpl.
5. Hy 3, Simpl.
6. (∃y)Hy 5, EG.
7. (x)Gx . (∃y)Hy 6,5, Conj.
8. (x)Gx . (∃y)Hy 2, 3-7, CP.
9. Gx 8, Simpl.
10. (x) Gx 9, UG,
11. (∃y)Hy 8, Simpl.
12. (x)Gx .(∃y)Hy 10,11, Conj.

Let us compare D-1 with D-2. Like D-1, UG is also applied in 'v'('x') in D-2, but unlike D-1 UG is applied not within the scope of the assumption of D-2, but outside the scope of the assumption of D-2. In D-2 the scope of the assumption of EI ranges over, step 3 to 7. Once the assumption is closed the steps within the scope of the assumption are not considered at all and hence cannot be further used in the derivation. It appears from the above that after having closed the assumption, the derivation is continued up to step-12. But no step between 8-12 is obtained within the scope of the assumption. We get step-11 from step-9 which is not included within the scope of the assumption.

Thus after having examined the procedure if UI, EI, EG and UG in quantification logic in a detailed manner, let us now sum up all of the rules in a simplified from:

(i) UI, EI, UG must be applied uniformly, but EG may or may not be applied uniformly.

(ii) Both in UI and EI, the quantifier that one is dropping has to cover the complete line of the proof.

(iii) Both in UI and EI, 'v' must occur free in 'ϕv' at all places that 'μ' occurs free in '$\phi\mu$'.

(iv) In UI 'v' can occur as free in any prior line of the proof; but in EI 'v' cannot occur as free in any prior line of the proof.

(v) EG can be applied on a singular proposition; but UG cannot be so applied.

(vi) Both in UG and EG the scope of the quantifer must include the com plete line of the proof.

(vii) Both in UG and EG single quantifier cannot be used to bind two variables.

(viii) In UG 'v' in 'ϕv'is not a free variable on a line derived in the proof by the use of EI.

(ix) No UG can legitimately applied on 'v' when (a) 'v' is derived from the assumed premise; and (b) 'v' remains free within the scope of the premise.

NOTES

1 Mackie, J. L. : The Symbolising of Natural Deduction; *Analysis*, December, pp. 25-37

2 Copi. I. M. : *Symbolic Logic*; Macmillan Publishing Co; Inc; 1997, pp. 91-92

3 *Ibid*; p.94

4 *Ibid*; p. 94