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Multi-objective non-linear programming problem based on Neutrosophic Optimization Technique and its application in Riser Design Problem

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Abstract- Neutrosophic set is a part of neutrosophy which studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra. Neutrosophic set is a powerful general formal framework that has been recently proposed. The paper aims to give computational algorithm to solve a multi-objective non-linear programming problem (MONLPP) using neutrosophic optimization method. The proposed method is for solving MONLPP with single valued neutrosophic data. We made a comparative study of optimal solution between intuitionistic fuzzy and neutrosophic optimization technique. The developed algorithm has been illustrated by a numerical example. Finally, optimal riser design problem is presented as an application of such technique.

Keywords: Neutrosophic set, single valued neutrosophic set, neutrosophic optimization method, Riser design problem.

1 Introduction

The concept of fuzzy sets was introduced by Zadeh in 1965 [1]. Since the fuzzy sets and fuzzy logic have been applied in many real applications to handle uncertainty. The traditional fuzzy sets uses one real value $\mu_A(x) \in [0, 1]$ to represents the truth membership function of fuzzy set A defined on universe X. Sometimes $\mu_A(x)$ itself is uncertain and hard to be defined by a crisp value. So the concept of interval valued fuzzy sets was proposed [2] to capture the uncertainty of truth membership. In some applications we should consider not only the truth membership supported by the evident but also the falsity membership against by the evident. That is beyond the scope of fuzzy sets and interval valued fuzzy sets. In 1986, Atanassov introduced the intuitionistic fuzzy sets [3], [5] which is a generalisation of fuzzy sets. The intuitionistic fuzzy sets consider both truth membership and falsity membership. Intuitionistic fuzzy sets can only handle incomplete information not the indeterminate information and inconsistent information. In neutrosophic sets indeterminacy is quantified explicitly and truth membership, indeterminacy membership and falsity membership are independent. Neutrosophy was introduced by Smarandache in 1995 [4]. The motivation of the present study is to give computational algorithm for solving multi-objective non-linear programming problem by single valued neutrosophic optimization approach. We also aim to study the impact of truth membership, indeterminacy membership and falsity membership functions in such optimization process and thus have made comparative study in intuitionistic fuzzy and neutrosophic optimization technique. Also as an application of such optimization technique optimal riser design problem is presented.

2 Some preliminaries

2.1 Definition -1 (Fuzzy set) [1]

Let X be a fixed set. A fuzzy set A of X is an object having the form $\tilde{A}$ = {(x,μ_A (x)), x ∈ X} where the function $\mu_A(x)$: X → [0, 1] define the truth membership of the element x ∈ X to the set A.

2.2 Definition-2 (Intuitionistic fuzzy set) [3]

Let a set X be fixed. An intuitionistic fuzzy set or IFS $\tilde{A}^i$ in X is an object of the form $\tilde{A}^i = \{< X, \mu_A(x), \nu_A(x) > / x \in X\}$ where $\mu_A(x)$: X→ [0, 1] and $\nu_A(x)$: X→ [0, 1] define the Truth-membership and Falsity-membership respectively , for every element of x∈ X , $0 \le \mu_A(x) + \nu_A(x) \le 1$.

2.3 Definition-3 (Neutrosophic set) [4]

Let X be a space of points (objects) and $x \in X$. A neutrosophic set $\tilde{A}^n$ in X is defined by a Truth-membership function $\mu_A(x)$, an indeterminacy-membership function $\sigma_A(x)$ and a falsity-membership function $\nu_A(x)$ and having the form $\tilde{A}^n = \{< X, \mu_A(x), \sigma_A(x), \nu_A(x) > / x \in X\}$.
$\mu_A(x)$, $\sigma_A(x)$ and $\nu_A(x)$ are real standard or non-standard subsets of]0^-, 1^+[. that is

$$\mu_A(x) : X \to]0^-, 1^+[$$
$$\sigma_A(x) : X \to]0^-, 1^+[$$
$$\nu_A(x) : X \to]0^-, 1^+[$$

There is no restriction on the sum of $\mu_A(x)$, $\sigma_A(x)$ and $\nu_A(x)$, so

$$^-0 \le \sup \mu_A(x) + \sup \sigma_A(x) + \sup \nu_A(x) \le 3^+$$

2.4 Definition-3 (Single valued Neutrosophic sets) [6]

Let X be a universe of discourse. A single valued neutrosophic set $\tilde{A}^n$ over X is an object having the form $\tilde{A}^n = \{< X, \mu_A(x), \sigma_A(x), \nu_A(x) > / x \in X\}$ where $\mu_A(x)$: X→ [0, 1], $\sigma_A(x)$: X→[0, 1] and $\nu_A(x)$: X→ [0, 1] with $0 \le \mu_A(x) + \sigma_A(x) + \nu_A(x) \le 3$ for all x ∈ X.

Example Assume that X = [x_1, x_2, x_3]. X_1 is capability, x_2 is trustworthiness and x_3 is price. The values of x_1, x_2 and X_3 are in [0, 1]. They are obtained from the questionnaire of some domain experts, their option could be a degree of “good service”, a degree of indeterminacy and a degree of “poor service”. A is a single valued neutrosophic set of X defined by

A = ⟨0.3,0.4,0.5⟩/x_1 + ⟨0.5,0.2,0.3⟩/x_2 + ⟨0.7,0.2,0.2⟩/x_3

2.5 Definition- 4(Complement): [6] The complement of a single valued neutrosophic set A is denoted by c(A) and is defined by

$$\mu_{c(A)}(x) = \nu_A(x)$$

$$\sigma_{c(A)}(x) = 1 - \sigma_A(x)$$

$$\nu_{c(A)}(x) = \mu_A(x)$$

for all x in X.

Example 2: let A be a single valued

neutrosophic set defined in example 1. Then, c(A) = ⟨0.5,0.6,0.3⟩/x_1 + ⟨0.3,0.8,0.5⟩/x_2 +⟨0.2,0.8,0.7⟩/x_3 .

2.6 Definition 5(Union):[6] The union of two single valued neutrosophic sets A and B is a single valued neutrosophic set C, written as C = A ∪ B, whose truth-membership, indeterminacy-membership and falsity-membership functions are are given by

$\mu_{c(A)}(x)$ = max $(\mu_A(x), \mu_B(x))$
$\sigma_{c(A)}(x)$=max($\sigma_A(x), \sigma_B(x))$
$\nu_{c(A)}(x) =$ min$(\nu_A(x), \nu_B(x))$ for all x in X

Example 3: Let A and B be two single valued neutrosophic sets defined in example -1. Then, A ∪ B = ⟨0.6,0.4,0.2⟩/x_1 + ⟨0.5,0.2,0.3⟩/x_2 + ⟨0.7,0.2,0.2⟩/x_3.

2.7 Definition 6(Intersection):[6] The Intersection of two single valued neutrosophic sets A and B is a single valued neutrosophic set C, written as C = A ∩ B, whose truth-membership, indeterminacy-membership and falsity-membership functions are are given by

$\mu_{c(A)}(x)$ = min $(\mu_A(x), \mu_B(x))$
$\sigma_{c(A)}(x)$ = min $(\sigma_A(x), \sigma_B(x))$

$\nu_{c(A)}(x)$ = max $(\nu_A(x), \nu_B(x))$
for all x in X

Example 4: Let A and B be two single valued neutrosophic sets defined in example -1. Then, A ∩ B = ⟨0.3,0.1,0.5⟩/x_1 + ⟨0.3,0.2,0.6⟩/x_2 + ⟨0.4,0.1,0.5⟩/x_3.

Here, we notice that by the definition of complement, union and intersection of single valued neutrosophic sets, single valued neutrosophic sets satisfy the most properties of classic set, fuzzy set and intuitionistic fuzzy set. Same as fuzzy set and intuitionistic fuzzy set, it does not satisfy the principle of middle exclude.

3 Neutrosophic Optimization Technique to solve minimization type multi-objective non-linear programming problem.

A non-linear multi-objective optimization problem of the form

Minimize $\{f_1(x), f_2(x), \dots \dots \dots f_p(x)\}$ (1)

$g_j(x) \quad \le b_j$ j=1, ………..,q

Now the decision set $\widetilde{D}^n$, a conjunction of Neutrosophic objectives and constraints is defined as

$\widetilde{D}^n = (\bigcap_{k=1}^{p} \tilde{G}_k{}^n) \cap (\bigcap_{j=1}^{q} \tilde{C}_j{}^n) = \{(x, \mu_{\widetilde{D}^n}(x), \sigma_{\widetilde{D}^n}(x), \nu_{\widetilde{D}^n}(x))\}$

Here $\mu_{\widetilde{D}^n}(x)$=min $(\mu_{\tilde{G}_1}{}^n(x), \mu_{\tilde{G}_2}{}^n(x), \dots \dots \dots . \mu_{\tilde{G}_p}{}^n(x);$ $\mu_{\tilde{C}_1}{}^n(x), \mu_{\tilde{C}_2}{}^n(x), \dots \dots . \mu_{\tilde{C}_q}{}^n(x)$

for $all\ x \in X$.

$\sigma_{\widetilde{D}^n}(x)$=min $(\sigma_{\tilde{G}_1}{}^n(x), \sigma_{\tilde{G}_2}{}^n(x), \dots \dots \dots . \sigma_{\tilde{G}_p}{}^n(x);$ $\sigma_{\tilde{C}_1}{}^n(x), \sigma_{\tilde{C}_2}{}^n(x), \dots \dots . \sigma_{\tilde{C}_q}{}^n(x))$

for all $x \in X$

$\nu_{\widetilde{D}^n}(x)$=max $(\nu_{\tilde{G}_1}{}^n(x), \nu_{\tilde{G}_2}{}^n(x), \dots \dots \dots . \nu_{\tilde{G}_p}{}^n(x);$

$\nu_{\tilde{C}_1}{}^n(x), \nu_{\tilde{C}_2}{}^n(x), \dots \dots . \nu_{\tilde{C}_q}{}^n(x))$

for all $x \in X$.

Where $\mu_{\widetilde{D}^n}(x)$, $\sigma_{\widetilde{D}^n}(x)$, $\nu_{\widetilde{D}^n}(x)$ are Truth membership function, Indeterminacy membership function, falsity membership function of Neutrosophic decision set respectively. Now using the neutrosophic optimization the problem (1) is transformed to the non-linear programming problem as

Max α………………(2)

Max γ

Min β

such that $\mu_{\tilde{G}_k}{}^n(x) \ge$ α,

$\mu_{\tilde{C}_j}{}^n(x\ge$α

$\sigma_{\tilde{G}_k}{}^n(x) \ \ge$ γ

$\sigma_{\tilde{C}_j}{}^n(x) \ge$ γ

$\nu_{\tilde{G}_k}{}^n(x) \le$ β

$\nu_{\tilde{C}_j}{}^n(x) \le$ β

α + β +γ ≤ 3

α ≥ β

α ≥ γ

α ,β, γ ∈ [0, 1]

Now this non-linear programming problem (2) can be easily solved by an appropriate mathematical programming algorithm to give solution of multi-objective non-linear programming problem (1) by neutrosophic optimization approach.

4 Computational algorithm

Step-1: Solve the MONLP problem (1) as a single objective non-linear problem p times for each problem by taking one of the objectives at a time and ignoring the others. These solution are known as ideal solutions. Let x^k be the respective optimal solution for the k^{th} different objective and evaluate each objective values for all these k^{th} optimal solution.

Step-2: From the result of step-1, determine the corresponding values for every objective for each derived solution. With the values of all objectives at each ideal solution, pay-off matrix can be formulated as follows.

$$\begin{bmatrix} f_1{}^*(x^1) & f_2(x^1) \dots \dots \dots \dots f_p(x^1) \\ f_1(x^2) & f_2{}^*(x^2) \dots \dots \dots \dots f_p(x^2) \\ \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots .. \\ f_1(x^p) & f_2(x^p) \dots \dots \dots \dots f_p{}^*(x^p) \end{bmatrix}$$

Step-3. For each objective $f_k(x)$, find lower bound $L_k{}^{\mu}$ and the upper bound $U_k{}^{\mu}$.

$U_k{}^{\mu}$ = max $\{f_k(x^{r*})\}$ and $L_k{}^{\mu}$ = min $\{f_k(x^{r*})\}$ where r =1, 2,…,k.

For truth membership of objectives.

Step-4. We represents upper and lower bounds for indeterminacy and falsity membership of objectives as follows:

$U_k^{\nu} = U_k^{\mu}$ and $L_k^{\nu} = L_k^{\mu}$ + t (U_k^{μ} - L_k^{μ})

$L_k^{\sigma} = L_k^{\mu}$ and $U_k^{\sigma} = L_k^{\mu}$ + s ($U_k^{\mu} - L_k^{\mu}$)

Here t and s are to predetermined real number in (0, 1).

Step-5. Define Truth membership, Indeterminacy membership, Falsity membership functions as follows:

$$\mu_k(f_k(x)) = \begin{cases} 1 & if\ f_k(x) \leq L_k^{\mu} \\ \frac{U_k^{\mu} - f_k(x)}{U_k^{\mu} - L_k^{\mu}} & if\ L_k^{\mu} \leq f_k(x) \leq U_k^{\mu} \\ 0 & if\ f_k(x) \geq U_k^{\mu} \end{cases}$$

$$\sigma_k(f_k(x)) = \begin{cases} 1 & if\ f_k(x) \leq L_k^{\sigma} \\ \frac{U_k^{\sigma} - f_k(x)}{U_k^{\sigma} - L_k^{\sigma}} & if\ L_k^{\sigma} \leq f_k(x) \leq U_k^{\sigma} \\ 0 & if\ f_k(x) \geq U_k^{\sigma} \end{cases}$$

$$\nu_k(f_k(x)) = \begin{cases} 0 & if\ f_k(x) \leq L_k^{\nu} \\ \frac{f_k(x) - L_k^{\nu}}{U_k^{\nu} - L_k^{\nu}} & if\ L_k^{\nu} \leq f_k(x) \leq U_k^{\nu} \\ 1 & if\ f_k(x) \geq U_k^{\nu} \end{cases}$$

Step-6. Now neutrosophic optimization method for MONLP problem gives a equivalent non-linear programming problem as:

Max α - β + γ ……………… (3)

Such that

$\mu_k(f_k(x)) \geq \alpha$

$\sigma_k(f_k(x)) \geq \gamma$

$\nu_k(f_k(x)) \leq \beta$

α + β +γ ≤ 3

α ≥ β

α ≥ γ

α ,β, γ ∈ [0, 1]

$g_j(x) \leq b_j$, x ≥ 0,

K=1,2,…….p;

j=1,2,……..q

Which is reduced to equivalent non-linear-programming problem as:

Max α - β + γ …………… (4)

Such that

$f_k(x) + (U_k^{\mu} - L_k^{\mu})\ .\alpha \leq U_k^{\mu}$

$f_k(x) + (U_k^{\sigma} - L_k^{\sigma})\ .\gamma \leq U_k^{\sigma}$

$f_k(x)$ - $(U_k^{\nu} - L_k^{\nu})$.β ≤ L_k^{ν}

for k = 1, 2, …….,p

α + β +γ ≤ 3

α ≥ β

α ≥ γ

α ,β, γ ∈ [0, 1]

$g_j(x) \leq b_j$ for j=1,2,……..q.

x ≥ 0,

5 Illustrated example

Min $f_1(x_1, x_2) = x_1^{-1} x_2^{-2}$

Min $f_2(x_1, x_2)$ =2 $x_1^{-2} x_2^{-3}$

Such that $x_1 + x_2 \leq 1$

Here pay-off matrix is $\begin{bmatrix} 6.75 & 60.78 \\ 6.94 & 57.87 \end{bmatrix}$

Here L_1^{μ} =6.75, $U_1^{\nu} = U_1^{\mu}$ = 6.94 and L_1^{ν} = 6.75 + 0.19 t

$L_1^{\sigma} = L_1^{\mu}$ = 6.75 and U_1^{σ} = 6.75 + 0.19 s

L_2^{μ} =57.87, $U_2^{\nu} = U_2^{\mu}$ = 60.78 and L_2^{ν} = 57.87 + 2.91 t

$L_2^{\sigma} = L_2^{\mu}$ = 57.87 and U_2^{σ} = 57.87 + 2.91 s

We take t = 0.3 and s = 0.4

Table-1: Comparison of optimal solutions by IFO and NSO technique.

Optimization	Optimal	Optimal	Aspiration	Sum of

techniques	Decision Variables x_1^*, x_2^*	Objective Functions f_1^* , f_2^*	levels of truth, falsity and indeterminacy membership functions	optimal objective values
Intuitionistic fuzzy optimization(IFO)	0.3659009, 0.6356811	6.797078 58.79110	α^* = 0.719696 β* = 0.022953	65.588178
Proposed neutrosophic optimization(NSO)	0.3635224, 0.6364776	6.790513 58.69732	α^*= 0.7156984 β*= 0.01653271 γ*= 0.2892461	65.487833

Table-1. Shows that Neutrosophic optimization technique gives better result than Intuitionistic fuzzy non-linear programming technique.

6 Application of Neutrosophic Optimization in Riser Design Problem

The function of a riser is to supply additional molten metal to a casting to ensure a shrinkage porosity free casting. Shrinkage porosity occurs because of the increase in density from the liquid to solid state of metals. To be effective a riser must solidify after casting and contain sufficient metal to feed the casting. Casting solidification time is predicted from Chvorinov's rule. Chvorinov's rule provides guidance on why risers are typically cylindrical. The longest solidification time for a given volume is the one where the shape of the part has the minimum surface area. From a practical standpoint cylinder has least surface area for its volume and is easiest to make. Since the riser should solidify after the casting, we want it's solidification time to be longer than the casting. Our problem is to minimize the volume and solidification time of the riser under Chvorinov's rule.

A cylindrical side riser which consists of a cylinder of height H and diameter D. The theoretical basis for riser design is Chvorinov's rule, which is $t = k\,(V/SA)^2$.

Where t = solidification time (minutes/seconds)

K = solidification constant for molding material (minutes/in^2 or seconds/cm^2)

V = riser volume (in^3 or cm^3)

SA = cooling surface area of the riser.

The objective is to design the smallest riser such that $t_R \geq t_C$

Where t_R = solidification time of the riser.

t_C = solidification time of the casting.

$$K_R\,(V_R/SA_R)^2 \geq K_C\,(V_C/SA_C)^2$$

The riser and the casting are assumed to be molded in the same material, so that K_R and K_C are equal. So $(V_R/SA_R) \geq (V_C/SA_C)$.

The casting has a specified volume and surface area, so V_C/SA_C = Y = constant, which is called the casting modulus.

$(V_R/SA_R) \geq Y$, $V_R = \pi D^2H/4$, $SA_R = \pi DH + 2\pi D^2/4$

$(\pi D^2H/4)/(\pi DH + 2\pi D^2/4) = (DH)/(4H+2D) \geq Y$

We take V_c = 2.8.6=96 cubic inch. and SA_C = 2.(2.8+2.6+6.8)= 152 square inch.

then, $\frac{48}{19}D^{-1} + \frac{24}{19}H^{-1} \leq 1$

Multi-objective cylindrical riser design problem can be stated as :

Minimize V_R (D, H) = π $D^2H/4$

Minimize t_R (D, H) = (DH)/(4H+2D)

Subject to $\frac{48}{19}D^{-1} + \frac{24}{19}H^{-1} \leq 1$

Here pay-off matrix is $\begin{bmatrix} 42.75642 & 0.631579 \\ 12.510209 & 0.6315786 \end{bmatrix}$

Table-2. Values of Optimal Decision variables and Objective Functions by Neutrosophic Optimization Technique.

Optimal Decision Variables	Optimal Objective Functions	Aspiration levels of truth, falsity and indeterminacy membership functions
D^* = 3.152158 H^* = 3.152158	V^*_R (D^*, H^*)= 24.60870, t^*_R (D^*, H^*) = 0.6315787.	α^* = 0.1428574 β^* = 0.1428574 γ^* = 0.00001

Conclusion: In view of comparing the Neutrosophic optimization with Intuitionistic fuzzy optimization method, we also obtained the solution of the numerical problem by Intuitionistic fuzzy optimization method [14] and took the best result obtained for comparison with present study. The objective of the present study is to give the effective algorithm for Neutrosophic optimization method for getting optimal solutions to a multi-objective non-linear programming problem. The comparisons of results obtained for the undertaken problem clearly show the superiority of Neutrosophic optimization over Intuitionistic fuzzy optimization. Finally as an application of Neutrosophic optimization multi-objective Riser Design Problem is presented and using Neutrosophic optimization algorithm an optimal solution is obtained.

References

[1] L. Zadeh, “Fuzzy sets” Inform and control vol-8 (1965), pp 338-353.

[2] I. Turksen, “Interval valued fuzzy sets based on normal forms”, Fuzzy set and systems, vol-20 (1986), pp-191-210.

[3] K. Atanassov, “Intuitionistic fuzzy sets”, Fuzzy sets and system, vol-20 (1986), pp 87-96.

[4] F. Smarandache, Neutrosophy, Neutrosophic probability, set and logic, Amer. Res. Press, Rehoboth, USA, 105 p, 1998.

[5] K. Atanassov “Interval valued intuitionistic fuzzy sets.” Fuzzy set and systems, vol-31 (1989), pp 343-349.

[6] H. Wang, F. Smarandache, Y.Zhang, and R. Sunderraman, single valued neutrosophic sets, multispace and multistructure, vol-4 (2010), pp 410-413.

[7] F.Smarandache, A generalization of the intuitionistic fuzzy set, International journal of pure and applied mathematics, vol-24 (2005), pp 287-297.

[8] S.K. Bharatiand and S.R. Singh, solving multi-objective linear programming problems using intuitionistic fuzzy optimization: a comparative study, International journal of modelling and optimization, vol 4 (2014), no. 1, pp 10-15.

[9] B. Jana and T.K. Roy, “Multi-objective intuitionistic fuzzy linear programming and its application in transportation model,”NIFS, vol. 13 (2007), no. 1, pp 1-18.

[10] G.S. Mahapatra, M. Mitra, and T.K. Roy, “Intuitionistic fuzzy multi-objective mathematical programming on reliability optimization model,” International journal of fuzzy systems, vol. 12 (2009), no. 3, pp 259-266.

[11] Chvorinov, N. "Control of the solidification of castings by calculation," Foundry Trade Journal, vol. 70 (1939), p. 95-98.

[12] Creese, R.C. "Optimal riser design by geometric programming," AFS cast metals research journal. Vol. 7 (1971), no.2, p.118-121.

[13] Creese,R.C. "Dimensioning of riser for long freezing range alloys by geometric programming," AFS cast metals research journal. Vol. 7 (1971), no. 4, pp-182-185.

[14] P.P Angelov, "Optimization in an Intuitionistic fuzzy environment" , Fuzzy set and system, vol 86, pp 299-306, 1997.

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