THE PROBLEM OF EXISTENCE IN QUANTIFICATION LOGIC

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I

The problem of existence not only vitiates the traditional system, it vitiates the quantification logic as well. In quantification logic one can legitimately infer 'Fy' from '(x) Fx' by applying UI on "(x) Fx'; and '(∃x) Fx' from 'Fy' by applying EG on 'Fy'. Since logic deals with the 'form', but not with the 'content' of proposition, prima facie, one should not be concerned about what the variable 'y' stands for. Of course, logicians have interpreted 'y' as an arbitrarily selected individual. But the term arbitrarily selected individual' is very much unperspicuous. Following Nicholas Rescher, Mackie says:"' ... that any talk of" arbitrarily selected individual "is inept and misleading, that if S is said to be an" arbitrary element" it is not an "element" or individual at all, but a notational device, a surrogate for individuals."[1] Now, if the variable 'y' being an arbitrarily selected individual, is supposed to be vague, unperspicuous, a problem may arise in its interpretation. The problem will come into existence when one will try to interpret the rules of UI and EG in terms of empty noun expressions. In this paper, I shall try to explain, as clearly as I can, where the problem of existence actually lies submerged in quantification logic. What will be the impact if quantification logic is to be understood with regard to the notion of existence? Or what will be the impact if the said logic is to be understood irrespective of the notion of existence?

II

The problem which I am going to envisage, has centred round in

Quine's many writings. In his celebrated article: "On What There Is", Quine remarks : " A curious thing about the ontological problem is its simplicity. It can be put into three Anglo - Saxon monosyllables: 'What is there?' It can be answered moreover in a word - 'Everything' - and everyone will accept this answer as true."[2] The same remak, of course, in different tone of voice, is found in his *Mathematical Logic*. There Quine says: "To say that *Something* does not exist, or that there is something which *is not*, is clearly a contradiction in terms, hence'. (x) (x exists)' must be true"[3] But in the subsequent lines of the same book, Quine draws our attention to an obvious predicament. He opines that on the basis of the true assertion '(x) (x exists)', one can infer true, false or even meaningless conclusion from it. He says, ' ... from the truth of '(x) (x exists)' one can logically derive not only to the true conclusion 'Europe exists' but also to the controversial conclusion 'God exists' and the false conclusion "Pegasus exists if we accept 'Europe' "God' and 'Pegasus' as primitive names in our language."[4] This makes sense to say when we infer 'Fy' from '(x) Fx' we may have different sorts of conclusions, subject to the interpretation of 'y'. If we make sense of 'F' as exists' and substitute 'Europe' for 'y', then the inference holds good; if we make sense of 'F' as exists ' 'and substitute 'Pegasus' for 'y', the inference is to be false; since 'Pegasus' lacks existence; and again if we make sense of 'F' as 'exists' and substitute 'God' for 'y' then the inference is to be controversial as the existence of 'God' is itself controversial. So for Quine whether the inference (1) (x) Fx, so Fy, and the inference (2) Fy, so (∃x)Fx are to be valid or not depends entirely on what the arbitrarily selected individual, viz, 'y' stands for. It seems clear from the above that if the variable 'y', being an arbitrarily selected individual, stands for non-empty noun-expressions, or in other words, if the variable 'y' is substituted only by non-empty noun expressions, then of course the above inferences hold good, but if 'y' stands for or is substituted by empty noun expressions, then the said inferences are thought to be illegitimate. According to Quine, the following two inferences are objectionable to our intuitions These are:

Inference : a

1. (x) Fx
   / ∴ Fy }   Corresponding to : (x) (x exists)
   So, Pegasus exists.
2. Fy 1, UI

Inference : b

1. Fy
   /∴ (∃x) Fx}   Corresponding to : Pegasus does not exist
   So, (∃x) (x does not exist)
2. (∃x) Fx 1, EG

Quine seems to have believed that the above two inferences are objectionable simply for the fact that in each inference we have a false conclusion from a true premise. But this is not the case which actually happens in all inferences. Let us consider the following two inferences which are not objectionable on the same ground.

Inference a'

1. (x) Fx
   / ∴ Fy }   Corresponding to : (x) (x exists)
   So, Socrates exists
2. Fy 1, UI

Inference b

1. Fy
   /∴ (∃x) Fx   Corresponding to : Socrates, does not exist
   So, (∃x) (x does not exist)
2. (∃x) Fx 1, EG

Unlike (a) and (b) we have a true conclusion from a true premise both in (a') and (b'). This makes sense to say that the rules of UI as well as EG do not hold good for each and every interpretation of 'F' (where F stands for predicate expression) and each and every substitution for the free variable.

Uptil now, we have examined, following Quine, in what situation the applications of UI as well as EG give rise to a false conclusion from a true premise; and in what set-up the applications of UI as well as EG give rise to a true conclusion from a true premise. It can be said that an inference can preserve its validity only if the applications of UI and EG are confined to the non-empty noun expressions for which the variable 'y' stands for. This view is taken into account simply for considering empty noun-expressions as meningless. If it has been presupposed that all noun-expressions, irrespective of empty or non empty, are meaningful, then no problem will arise in interpreting the above inferences. And at the same time the rules of UI as well as EG can be applied without imposing any reservation on them. So it is a matter of *decision* whether empty noun-expressions are to be regarded as meaningful or not. Philosophers, however, have disagreed on this vital issue. A few influential philosophers hold that a noun-expression can be meaningless if it lacks existence, i.e. if it fails to designate or denote anything. Quine, however, does not agree with the idea that empty noun-expressions are meaningless simply for the fact that they fail to designate anything. Unlike many other philosophers, he favours the use of such words as 'Pegasus', 'Cerberus' 'Centaur' etc., under certain reservations. Instead of considering such words as meaningless, he rather sets out different logical laws for contemplating such terms. He opines that there are certain unequivocal rules which can be proved as true for any noun-expressions, empty or non-empty, and also there are certain unequivocal rules which can be used only for non-empty noun-expressions. Since Quine solicits or aspires to different sets of rules for contemplating empty or non-empty noun-expressions, one has to make sure beforehand whether the noun-expression, he may like to employ, is empty or not. This, of course, is an empirical decision. Quine further claims that all the restrictions that we have, should be observed not only in the case of empty noun-expressions; but also in the case of those expressions which are yet to be identified specifically either as empty or non-empty.

III

Czeslaw Lejewski, however, does not agree with the conviction that some of our rules of inference are based on empirical enquiry. For him the so-called empirical enquiry is exotic to the character of logical enquiry. Instead of quizing empirical information, says Lejewski, one has to look into a thorough re-examination of the inference. He then goes on to examine on what ground the assertion '(x) (x exists)' is taken as true; while the assertion '(∃x) (x does not exist) is taken as false by Quine. Besides this, he also endeavours the logical method by which the rules of UI and EG can be applied legitimately to reasoning with empty noun-expressions. Finding out a solution on these quips, he, however, turns into the interpretation of the quantifiers.

According to Lejewski, as far as existence of the noun-expressions is concerned, we find two types of interpretations of the quantifiers, viz., the restricted as well as the unrestricted. Those who stand by the restricted interpretation have opted for substituting individual variable with regard to non-empty noun-expressions. On the contrary, those who comply with the unrestricted interpretation have gone for substituting individual variable with regard to both empty and non-empty noun-expressions. So far as the scope of the applications of UI and EG are concerned, the restricted interpretation is limited while the unrestricted interpretations is unlimited. Let us first explain the restricted interpretation of the qunatifiers.

The Restricted interpretation of the quantifiers :

In outlining the restricted interpretation of the quantifiers, Lejewski, however, refers to Quine's interpretation of the quantifiers which has been approved by most of the modern logicians. In his *Methods of Logic*, Quine remarks: "If we think of the universe as limited to a finite set of objects a,b, ..., h , we can expand existential quantifications into alternations and universal quantification into conjunctions, '(∃x) Fx' and (x) Fx become respectively:

Fa V Fb ... V Fh, Fa . Fb ...b . Fh"[5]

Ensuing Quninian interpretation of the quantifiers, Lejewski, however, circumscribes the universe fictitiously. He says, let us suppose that the world consists of just two objects, such as, 'a' and 'b'. Accordingly, the corresponding expansions of '(x) Fx and '(∃x) Fx' become : Fa . Fb and Fa ∨ Fb', respectively. Having presupposed the universe in such a manner, Lejewski; however, does not rule out the appearance of any new noun-expressions in the universe. Precluding noun-expressions as a linguistic matter, he then goes on to say that the language we do practice and employ may leave room for new noun-expressions other than the singular names, such as, 'a' and 'b' cited above. Suppose, by taking the advantage of language, one may familiarise a new noun-expression, say c, which fails to designate either 'a' or 'b'. Since 'a' and 'b' are the only real names in the universe and 'c' fails to designate either of these; 'c' must be an empty noun-expression. Again one may also presuppose a noun-expression, viz, 'd', which designates either 'a' or 'b'. Since an appearance of a new noun-expression is a linguistic concern, it does not vitiate the universe comprising 'a' and 'b' only; nor does it produce irrelevant consequence in interpreting the universe. Again, as 'c' is an ampty noun-expression, the proposition regarding 'c' namely, 'c does not exit' turns out to be true. And moreover, by converting the noun-expression 'd' into a predicate-expression 'D', one may assert a true proposition '(x) Dx' by virtue of the fact that 'd' designates either 'a' or 'b' and hence the proposition (x) Dx' can be instantiated either in Da' or in 'Db'. Under this interpretation an entity belongs to the universe only if it exists. Or in other words, to say that something exists is tantamount to saying that it belongs to the universe. In this sense the proposition 'a exists' and proposition 'b' exists' turn out to be true and their negations such as' a does not exist' and 'b' does not exist' also turn out to be false. Again if the propositions 'a exists' and 'b exists' are taken to be true, we have a true proposition, say (∃x) Fx' from either anyone of these. And further if, (∃x) Fx' is taken to be true, then, of course, the proposition namely, '(∃x) (x does not exist)' becomes false. Again if '(∃x) (x does not exist)' is taken to be false, then its negation, namely, the proposition '(x) (x exists)', happens to be true. This is authenticated by the expansion of universal proposition. The expansion of '(x) Fx', as we saw earlier, is 'Fa. Fb'. Both 'Fa' and 'Fb' are held to be true in isolation, so their conjunction is also held to be true. But what can we think about 'c' or 'd' ? As they do not belong to the universe, the rules of UI and EG do not hold good if applied to 'c' or 'd'. Lejewski says, "Within the fictitious universe the rule of universal instantiation and the rule of existential generalisation fail if applied to 'c' or 'd'. They are valid rules of inference if their application is restricted to reasoning with 'a' and 'b'.

Having delineated the restricted interpretation of the quantifiers in the above manner, let us pass on to appraise, after Lejewski, the following two points put up by Quine. These are :

(i) In what sence Quine asserts that the proposition '(x) (x exists)' and the proposition (∃x) (x does not exist)' have turned out to be true if we adhere to the restricted interpretation of the quantifiers.

(ii) In what sense the inferences (1) and (2), cited earlier, do not hold good for every interpretation of F (where F stands for a predicate expression) and for all substitution for the free variable 'y'.

Let us try to answer the above two questions in the light of the restricted interpretation. Since the restricted interpretation of the quantifiers deals with the notion of existence, the assertions:

(a) (x) (x exists)

(b) (∃x) (x does not exist)

can be logically paraphrased as :

(a') (x) (x exists ⊃ x exists)

(b') (∃x) (x exists and x does not exist)

Now, on the basis of (a') and (b'), it is obvious why (a) is açcepted as true, and (b) is accepted as false. As far as the restricted interpretation is concerned both the pairs (a) and (a'); and (b) and (b') are equivalent in the sense that in each pair one logically entails and is entailed by the other. It is clear that (a') is a mere tautology and what it entails, namely (a), must be true. Again (b') is a mere self-contradiction, so what it entails must be false.

Now, it is clear in what sense Quine holds that the proposition (a) is true and the proposition (b) is false.

In answering to the second point, let us consider the following expressions in the light of the restricted interpretation :

1. $(x)\, Fx \supset Fy$

2. $Fy \supset (\exists x)\, Fx$

As in this interpretation, we are allowed to use the notion of existence, the above two expressions (i.e., 1 and 2) can be logically paraphrased as :

(1a) (x) (x exists $\supset$ Fa) $\supset$ Fy

(2a) Fy $\supset$ $(\exists x)$ (x exist . Fx)

Paraphrasing (1) and (2) into (1a) and (2a), let us presuppose that 'F' stands for 'exists' and 'y' stands for Pegasus'. On the basis of this presupposition it seems clear that the antecedent of (1a) becomes a tautology; but at the same time its consequent turns out to be false. Accordingly, the inference is proved to be illegitimate for having a false conclusion from a true premise. Again, if it is presupposed that 'F' stands for 'does not exist' and 'y' stands for 'Pegasus' then the antecedent of (2a) turns out to be true and its consequent becomes false, the inference is to be regarded as invalid again for having a false conclusion from a true premise.

Again, consider the expressions (1) and (2) cited above with regard to the following assertions.

Suppose 'F' stands for 'exists' and 'y' stands for 'Socrates' then from (1), we have the inference.

(1b) (x) (x exists $\supset$ Fx) $\supset$ Socrates exists.

which is supposed to be valid as here we have a true conclusion from a true premise. Again, if it is presupposed that 'F' stands for 'does not exist' and 'y' stands for 'Socrates'. then from (2), we have the following inference :

(2b) Socrates does not exist $\supset$ $(\exists x)$ (x exists . x does not exist) which again is supposed to be consistent as here we have a false conclusion from a false premise. The above consideration makes it clear why the above inferences do not hold good for every substitution for the free variable 'y'. If 'y' is substituted by an empty noun-expression, the inference under consideration does not hold good but if 'y' is substituted by a non-empty noun-expression, then the inferences hold good.

Uptil now we have considered the restricted interpretation of the quantifiers. As far as the existential import is concerned, we find a specific reservation on the uses of UI and EG. Logicians of course, do impose some restrictions on the uses of UI and EG, but the reservation that we have in the restricted interpretation is not similar to them. In quantification logic, when we have ('x) Fx' we can instantiate it by 'Fy' without knowing what 'y' stands for. An individual variable, 'y' may stand for any noun-expressions, empty or non-empty. Quantification logic should not be restricted or even perhaps cannot be apprehended in connection with the restricted interpretation of the quantifiers, rather it should be udnerstood in a pervasive point of view so that it includes all noun-expressions, empty or non-empty, as meaningful. In this regard Lejewski advocates the unrestricted interpretation of the quantifiers. Let us pass on to explain the unrestricted interpretation of the qunatifiers following Lejewski.

IV

The Unrestricted Interpretation of the Quantifiers :

In formulating the unrestricted interpretation of the quantifiers. Lejewski, however, goes along with Lesinewski who adopted a similar interpretation. Like the restricted interpretation, Lejewski, admits the universe consisting of just two objects, such as, 'a' and 'b'; but unlike the restricted interpretation empty noun-expressions do not function as the components of the expansions of the quantifiers; but in the case of unrestricted interpretation both empty as well as non-empty noun-expressions do function as the components of teh expansions of the quantifiers. Accordingly, under the present interpretation '(∃x) Fx' becomes 'Fa V Fb V Fc V Fd', and '(x) Fx' becomes : 'Fa . Fb . Fc . Fd', where 'a' and 'b' stand for non-empty noun-expressions; 'c' stands for an empty noun-expression and 'd' stands for a general non-empty noun expression. Here 'd' is a general non-empty noun-expression in the sense that it denotes either 'a' or 'b'.

Let us examine in what sense the restricted interpretation differs from the unrestricted one. It is said that the restricted interpretation of quantifiers be at odds with the unrestricted on two important counts : (i) unlike the restricted interpretation here all noun-expressions, irrespective of empty or non-empty, are regarded as meaningful; and (ii) all empty noun-expressions do function as the components of the expansions of the quantifiers. Following the two important differences, let us find out ,more differences between the two interpretations. Unlike the restricted interpretation the proposition, (∃x) (x does not exist) happens to be true since in this interprectation the proposition 'c' does not exist' is true. Now, if it is asserted that the proposition '(∃x) (x does not exist)' is true, then on the same ground the proposition '(x) Fx' turns out to be false. But the proposition, and (x) Fx' is proved to be true in the restricted interpretation. This is made clear in another way. If it is said that 'c does not exist' (i.e., in symbol ~ Fc) is true; then of course 'c exists' (in symbol Fc) becomes false. Now, if 'Fc' turns out to be false, then the expansion of and '(x) Fx' i.e., 'Fa. Fb. Fc. Fd' also turns out to be false so '(x) Fx' is false. But in the restricted interpretation of the quantifiers, we have seen that both the propositions' (∃x) (x does not exist) and '(x) Fx' became false and true respectively. The outcome of this interpretation is that it does not set up any restriction regarding the application of the rules of UI and EG. Both the rules can be applied without knowing whether a noun-expression is empty or non-empty. Under this interpretation the variable 'y' can not only be substituted by non-empty noun-expression, but it can also be substituted by empty as well as non-empty general noun-expressions. Unlike the restricted interpretation here the general noun-expression 'd' need not be converted into the predicate-expression 'D'.

Again, we make a comparison between the restricted and the unrestricted interpretation of the quantifiers. We have seen that in the case of the restricted interpretation, the inferences (1) and (2) become true if the variable 'y' is substituted by a non-empty noun-expression, and the said inferences become false if the variable 'y' is substituted by an empty noun-expression. But in the case of unrestricted interpretation both the said inferences turn out to be consistent without specifying what the individual variable 'y' stands for. Unlike the restricted interpretation the propositions, namely, 'Pegasus exists' and 'Pegasus does not exist' are meaningful in the unrestricted interpretation. Here every noun-expression, empty or non-empty, must be regarded as meaningful and accordingly may be treated as a component of quantificational expressions. Now, the proposition 'Pegasus exists' is false as there is no Pegasus existing in the universe. If the proposition 'Pegasus exists, is false then the corresponding unviersal proposition '(x) (x exists)' must also be false. Since by the expansions of '(x) (x exists)' we have a conjunctive proposition of many components of which at least one component, namely, Pegasus exists, turns out to be false. Accordingly, the conjunctive proposition turns out to be false and hence '(x) (x exists)' becomes false. As a matter of fact the inference (1) turns out to be consistent as here we have a false conclusions from a false premise. But the same inference turns out to be false in the case of the restricted interpretation. Again, if it is asserted that the proposition 'Pegasus exists' is false, it is *ipso-facto* asserted that the proposition 'Pegasus does not exist' is true. If the proposition ' Pegasus does not exist' is asserted as true then the proposition '(∃x) (x does not exist)' also turns out to be true. Hence the inference (2) becomes consistent for having a true conclusion from a true premise. But the same inference became false in the restricted interpretation as there we had a false conclusion from a true premises. So Lejewski goes on to say that unlike the restricted interpretation; the two inferences thus considered "cannot be used as counter examples to disprove the validity of the rules of universal instantiation and existential generalisation in application to reasoning with empty noun-expressions"[7] in the unrestricted interpretation.

So far we have examined, following Lejewski, the restricted as well as the unrestricted interpretation of the quantifiers with regard to a finite universe. In both these itnerpretations, it is presupposed that the universe under consideration consists of just two singular names, viz, 'a' and 'b' and on the basis of this assumption the corresponding expansions of the quantifiers have been examined. It is said that the unviersal porposition '(x) Fx' is *equivalent* to its expansion, such as 'Fa. Fb; and the particular proposition '(∃x) Fx' is *equivalent* to its expansion, such as, {'Fa V Fb}. Under this presupposition the logical propositions (x) Fx ≡ {Fa. Fb} and (∃x) Fx ≡ {Fa V Fb} hold good but the principle of eauivalance between quantifier and its expansion is taken good enough as long as the unvierse is confined within a limited number of noun-expressions. Can we adopt the same relation between the quantifier and its expansion if the universe under consideration is supposed to be infinite? The answer, says Lejewski, is no. If the universe is supposed to be infinite having unknown number of objects, then we perhaps may not establish an equivalent tie between the quantifier and its expansion, because in such a case we cannot complete the expansion of our quantification. It would be imprudent for us to claim that : (x) Fx = {Fa . Fb . Fc ... Fx}; but it would logically be sound to say that (x) Fx ⊃ {Fa. Fb . Fc ... Fx}. Again , we should not say that: (∃x) Fx = {Fa ∨ Fb ∨ Fc ∨ ... Fn}; but it is rather logically justifiable to say that : (∃x) Fx ⊃ {Fa ⊃ Fb ⊃ Fc ... Fn}. That is why Lejewski has rightly pointed out by saying that in the case of infinite universe, one should dissolute equivalence in favour of implication. So instead of saying that a universal quantification is *equivalent* to any component of its infinite expansion, it is logically innocuous to say that a universal quantification *implies* any component of its infinite expansion. Similarly, instead of saying that an existential quantification is *equivalent* to any component of its infinite expansion, it is logically impregnable to say that an existential quantification is *implied* by any component of its infinite expansion. So whether the universe is to be presupposed as finite or infinite it is a matter of decision. It is a decision on the part of the interpreter. But whatever the universe may be, finite or infinite, it does not, masquerade the interpretation of the quantifiers. What is important to be observed here is that under the restricted interpretation every noun-expression must denote only one object that belongs to the universe. But in the unrestricted interpretation whether a noun-expression designates or not, is not important to be observed. Here a noun-expression may designate only one object; it may designate more the restricted and the unrestricted interpretation is that in the case of the restricted interpretation, whether a noun-expression is meaningful or not, is decided simply by the fact whether it designates or fails to designate any object in the universe. But in the case of the unrestricted interpretation every noun-expression is to be meaningful even if it fails to designate anything.

A few Advantages of the Unrestricted Interpretation of the Quantifiers :

Lejewski favours the unrestricted interpretation rather than the restricted one. For him, besides so many other advantages, it ensures a uniform application of the rules of UI and EG with regard to any noun-expressions, empty or non-empty. It enables us to overcome so many difficulties quantification logic suffers from. Let us focus on a few problems of quantification logic which can be solved by the unrestricted interpretation of the quantifiers.

We have already noticed that quantification logic suffers from a serious setback. If it is understood with regard to the restricted interpretation the inferences '$(\exists x)\ Fx \supset Fy$' and '$Fy \supset (\exists x)\ Fx$' are not universally valid. Their validity, we observe, depends on the very meaning of the idividual variable 'y'. But in the quantification logic we are not concerned about what 'y' stands for. We can infer 'Fy' from (x) Fx' by applying the rule of UI. We do, of course, make it sure that 'y' stands for any noun-expression, but we are at the same time at inconvenience to make it sure whether 'y' stands for either 'Pegasus', or 'Socrates', or 'Unicorn'.So when we have 'Fy' from '(x) Fx' without knowing for what noun expression 'y' stands for we are really acknowledging the unrestricted interpretation of the quantifiers. This is made possible mainly for considering every noun-expression as meaningful. If it is presupposed that each proposition stands for any noun-expression and thereby regarded as a component of the quantificational expansions, then it is *ipso-facto* claimed that every noun-expression is implied by the corresponding unviersal proposition and it implies the corresponding existential proposition. Suppose 'Fa' is a proposition in which 'a' stands for a noun-expression, empty or non-empty, then 'Fa' must be regareded as a component of the quantificational expansions. If this is so, then it implies '(∃x) Fx' and is implied by '(x) Fx'. In this regard both the inferences namely, (1) and (2), become true. But this would not be the case if the quantificational theory is to be understood in connection with the restricted interpretation.

The unrestricted interpretation also gets rid of another important difficulty from quantification theory. Many philosophers have felt that the schemata, such as, (i), (∃ x) (Fx V- Fx)'and (ii) '(x) Fx ⊃ (∃x) Fx' can be valid only if the universe is conceived to be non-empty. They are not valid in the empty universe as their truth depends on their being something. This makes sense to say that whatever is true of the object named by a given singular term is true of something; and clearly the inferences under consideration lose their justification when the singular term in question does not happen to name. Quine remarks: "It has frequently been claimed that though the schemata: (i), (∃ x) (Fx V~ Fx)'and (ii) '(x) Fx ⊃ (∃x) Fx' are demonstrable in quantification theory, the statements of the forms which these schemata depict are not logically true. For, it is argued, such statements depend for their truth upon their being someting in the unvierse; and that there is something is, though true, not logically true."[8] Quine, then, rules out the empty universe for contemplating such schemata. He holds that in quantification logic the study of such schemata is entirely pointless if the universe is assumed to be empty. He, therefore, puts forward to dismantle the empty universe as relatively pointless and thereby holds that the study of quantification theory is to be worthwhile if the universe in known or confidently believed to be non-empty. This again confirms that the above schemata fail for the empty universe if we understand the quantifiers with regard to the restricted interpretation. This becomes evident from (1a) (∃x) (x exists.(Fx ∨ ~ Fx)' and (2a) '(x) (x exists ⊃ Fx) ⊃ (∃ x) (x exists.Fx), which are the corresponding translations of (1) and (2) respectively. It seems clear that both (1a) and (2a) turn out to be false if there exists nothing. If the universe is supposed to be empty, then (1a) turns out to be false simply for the fact that 'x exists' is false in (1a). (2a) also turns out to be false, because although its antecedent is true, its consequent becomes false and hence the inference becomes false for having a false conclusion from a true premise. But this would not be the case if we comply with the unrestricted interpretation. In this interpretation both the inferences turn out to be true irrespective of whether the universe is assumed to be empty or non-empty. Since this interpretation admits every noun-expression, empty or non-empty, as meaningful and thereby considers it as a component of the quantifier expansion, it follows logically that ' (∃x) (Fx V ~ Fx)' is implied by any component of the type 'Fa V ~ Fa'. Even if, it is asserted that ∃x) (Fx V ~ Fx) is implied by the quantifier expression (Fa V~Fa) where 'a' stands for an empty noun-expression, the expression '(∃x) (Fx V ~ Fx)' still turns out to be true. This is made possible, says Lejewski, mainly for the fact that the component 'Fa V ~ Fa' is not only true for a particular choice of universe; but also true for any choice of universe and hence true for (∃x) (Fx V ~ Fx). Again, if it is said that the antecedent of '(x) Fx ⊃ (∃x) Fx' turns out to be true, then by virtue of this assetion, it is *ipso-facto* asseted a proposition of type 'Fa' which is implied by '(x) Fx'. This is made possible in harmony with the unrestricted interpretation of the universal quantifier. Again, if we get 'Fa' as true from '(x) Fx', then we have' (∃x) Fx from 'Fa' too, since under this interpreatation any component of the quantificational expansions implies in turn the corresponding proposition of type '(∃x) Fx'. '. Accordingly, '(x) Fx ⊃ (∃ x) Fx' turns out to be true in the unrestricted interpretation. Lejewski says, ... in the establishing of the truth value of (∃ x) (Fx V-Fx) and (∃ x) Fx ⊃ (∃ x) Fx the problem of whether the universe is empty or non-empty is alogether irrelevant on condition, of course, that we adopt the unrestricted interpretation of the quantifier."[9]

Now if it is asserted that (∃ x) Fx' is logically implied by '(x) Fx' by virtue of the fact that (x) Fx' implies 'Fa' and 'Fa' implies (∃ x)Fx, then it is claimed that under this interpretation the existential quantifier lacks existential import. If 'a' in Fa stands for an empty noun-expression and 'Fa' logically implies '(∃ x) Fx' on account of the fact that any component of the quantificational expressions implies '(∃ x) Fx' then, of course. (∃ x) Fx' has no existential import. This makes sense to say that under this interpretation existential quantification has no existential import. So according to Lejewski under this interpretation the reading of '(∃x)' as there exists an x such that Fx' is wrong. He says. "... it would be misleading to read '((∃) Fx' as 'there exists an x such that Fx'. The non-committal' for some x, Fx' seems to be more appropriate."[10] Now, if the logical phrase 'there exists an x such that Fx' is replaced by for some x, Fx', then of course, the existential quantification should be replaced by 'particular quantification' as well as the existential quantifier should also be replaced by 'particular quantifier.'

Lejewski, then, points out another important feature for which he favours the unrestricted interpretation rather than the restricted one. For him unlike the restricted interpretation, the unrestricted interpretation has a close proximity to ordinary usage. Ordinarily, we do not believe that 'everything exists' and also we do not find any contradiction by saying that 'something does not exist.' But the logician's ingenuinity or innovativeness apprehends these statements rather differently. If we abide by the restricted interpretation, it seems clear to us that the proposition 'Something does not exist" turns into a contradiction and accordingly, the proposition 'Everything exists' becomes true. But this line of logical thinking is contrary to the ordinary line of thinking. By way of expounding the unrestricted interpretation of the quantifiers, Lejewski observes, logicians not only overcome the boom of the restricted interpretation, but also establish 'a nearer approximation to ordinary usage.' But this does not mean that by way of formulating the unrestricted interpretation, logicians deviate from quantificational theory. Rather it is claimed that the unrestricted interpretation of the qualifiers has a close approximation to the formal quantification theroy.

Claiming the inviolability or sanctity of the unrestricted interpretation, Lejewski, however, throws a challenge to find out a formula which is demonstrable in the formal quantification theory, but which is not applicable to the unrestricted interpretation of the quantifiers. He says, "I do not know of any formulae which are demonstrable in the formal quantification theory and which, under the unrestricted interpretation, are not applicable to reasoning with empty noun-expressions or do not hold for universe of some specific size."[11] Lejewski opines that it is a great slip-up on the logicians part not to comply with the restricted interpretation; but to work out this interpretation with a strict adherence to the notion of existence. Quantification logic, should not or even perhaps cannot be interpreted with the notion of existence. If it does, then it would be the logic of 'content' rather than the 'form' of proposition. Following Von Wright we can say that the sole and whole business of quantificational logic is to detect the 'form' but not the 'content' of the proposition. The restricted interpretation of the quantifiers should not be accepted mainly for considering the idea of quantification with the notion of existence. Lejewski says, "In' my opinion the most serious disadvantage of the theory of the restricted quantification is that by merging the idea of quantification with the notion of existence it has put logicians and philosophers on a wrong track in their endeavours to elucidate the problem of existence in logic."[12] If Lejewski is right in his own standpoint, we think actually he is, then our all important conclusion is that the commendation of the unrestricted interpretation is an advancement in solving the problem of existence in quantification theory.

NOTES

1. Mackie, J.L. : "The Rules of Natural Deduction", *Analysis*: 19.2, December 1958; p.27
2. Quine, W.V. :"On What There Is, "in *From* a *Logical Point of view*; Harper & Row Publishers, New York, 1953, p.1
3. Quine, W.V. : *Mathematical Logic*, Harvard University Press, 1951, p. 150
4. *Ibid.*, p.150
5. Quine, W.V. : *Methods of Logic*, Routledge & Kegan Paul Ltd., 1952, P. 117.
6. Lejewski, C. : "Logic and Existence" in *Logic and Philosophy*, Edited by J Buchler and S. P. Lamprechat, The Century Philosophy Series, P. 167.
7. *Ibid.*, P.175

8. Quine, W.V. : *From* a *Logical Point of View, op. cit.*, P.160

9. Lejewski, C. *: op. cit., P. 177*

10. *Ibid.*, P. 177

11. *Ibid.*, p. 177

12. *Ibid.*, p. 177